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A PARALLEL ALGORITHM OF AF-2 SCHEME FOR PLANE STEADY TRANSONIC POTENTIAL FLOW WITH SMALL TRANSVERSE DISTURBANCE

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Abstract

In this paper, a parallel algorithm for implicit approximate-factorization scheme-2 (AF-2) has been developed to solutions of the plane steady transonic potential equation with transverse small disturbance for flows about NACA-0012 airfoil. Multi-pivot elements parallel elimination method for solving a system of equations with tridiagonal coefficients matrix is presented. The computation efficiencies of parallel and serial operation are compared by computation experiment.

The freestream Mach numbers considered are 0.8, 0.85 and 0.9. The angles of attack are 0°, 1°, 2° and 4°. The solutions are obtained on a variable spaced grid with 39*31 x, y points and 22 points along the chord. The x-mesh range is about twenty times the chord itself as is the y-mesh range. The iterations are started from uniform flow.

The conclusions indicate that parallel operation is about sixteen times faster than serial operation.

I. Introduction

Low computation efficiency and slow convergence is still one of the main problems in finite diference computation for transonic flow. The objective of this paper is to investigate a fast and simple computer method with satisfactory precision for solving plane steady transonic potential flow.

The implicit approximate-factorization scheme-2(AF-2) [1] has been proved to give the same stable and reliable convergence as SLOR [2] but faster than SLOR. Good examples are given by ref. [1,3] for plane small disturbance and full potential equation respectively. The precision of full-potential equation is high but its computation is complex. The calculation of small-disturbance equation is simple but its precision is low, especially near the stagnation point. Here, AF-2 scheme is used to solve the plane steady transonic potential equation with longitudinal large disturbances and transversal small disturbances [4] which is simple for computation and has satisfactory precision.

The advantage of parallel operation is that datums can be treated with high- efficiency. But a good parallel algorithm is very important to obtain high-efficiency parallel operation. In this article, the parallel algorithm of the AF-2 scheme will be investigated. All the computations are made on the computer Yinghe-1 (YH-1).

II. Mathematical model and AF-2 iteration scheme

A body-fixed Cartesian coordinates system with origin at the leading edge is used as illustrated in Fig. 1. Assume that the disturbance is large in the x-direction and small in the y-direction. The potential equation of steady transonic flow around the airfoil is

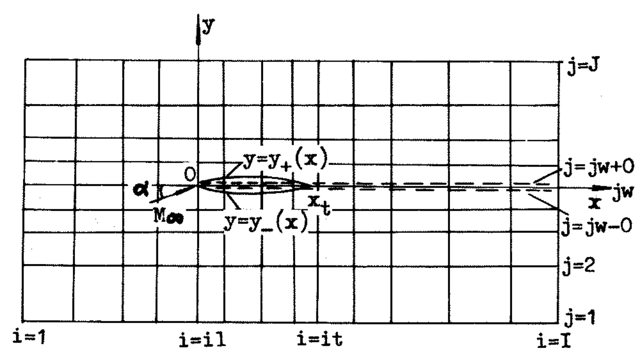


Fig.1 Illustration of coordinate and mesh spacing

$$(1-M^2)\varphi_{xx}+\varphi_{yy}=0 \tag{1}$$

$$1-M^2=\frac{1-M_\infty^2-\frac{\gamma+1}{q_\infty}M_\infty^2\varphi_x-\frac{\gamma+1}{2q_\infty^2}M_\infty^2\varphi_x^2}{1-\frac{\gamma-1}{q_\infty}M_\infty^2\varphi_x-\frac{\gamma-1}{2q_\infty^2}M_\infty^2\varphi_x^2} \tag{2}$$

where M is the local Mach number; q_∞ and M_∞ represent the uniform oncoming velocity and Mach number respectively, γ is the ratio of specific heats.

The boundary conditions on the airfoil surfaces are

$$\varphi_y(x,\pm 0)=\left[q_\infty\cos\alpha+\varphi_x(x,\pm 0)\right]\frac{dy_\pm}{dx}-q_\infty\sin\alpha \tag{3}$$

where α is the angle of attack; y_+ and y_- represent the upper and lower airfoil surface y-coordinates respectively. At the blunt leading edge,

$$\varphi_x=-q_\infty\cos\alpha \tag{4}$$

will be satisfied. The boundary conditions on the trailing vortex surfaces are:

$$\varphi(x,+0)-\varphi(x,-0)=\varphi(x_t,+0)-\varphi(x_t,-0) \tag{5}$$

$$\varphi_y(x,+0)=\varphi_y(x,-0) \tag{6}$$

where x_t is the x-coordinate at the trailing edge of the airfoil. A far-field asymptotic solution [4] is used at the far-field.

The relaxation iteration equations of AF-2 scheme at the subsonic and supersonic points are respectively:

$$(\sigma\overleftarrow{\delta}_x-\delta_{yy})f_{i,j}^{(n)}=\sigma\Omega L\varphi_{i,j}^{(n-1)} \tag{7a}$$

$$(\sigma-A_{i,j}\overrightarrow{\delta}_x)C_{i,j}^{(n)}=f_{i,j}^{(n)} \tag{7b}$$

and

$$(\sigma\overrightarrow{\delta}_x-\delta_{yy})f_{i,j}^{(n)}=\sigma\Omega L\varphi_{i,j}^{(n-1)} \tag{8a}$$

$$(\sigma-A_{i-1,j}\overleftarrow{\delta}_x)C_{i,j}^{(n)}=f_{i,j}^{(n)} \tag{8b}$$

where

$$A_{i,j}=(1-M^2)_{i,j}$$

$$L=\begin{cases}A_{i,j}\delta_{xx}+\delta_{yy} & \text{(for subsonic)}\\ A_{i-1,j}\overleftarrow{\delta}_{xx}+\delta_{yy} & \text{(for supersonic)}\end{cases}$$

and $\overleftarrow{\delta}_x$, $\overrightarrow{\delta}_x$ are first-order, first forward, and backward difference operators. $\overleftarrow{\delta}_{xx}$, δ_{xx} and δ_{yy} are second-order forward and central difference operators respectively. $C_{i,j}^{(n)}=\varphi_{i,j}^{(n)}-\varphi_{i,j}^{(n-1)}$. σ and Ω are acceleration convergence parameter and relaxation parameter respectively. Von-Neumann analysis of the stability (See III) indicate that $0\leq\Omega\leq 2$ and fast convergence can be obtained by circulary using a σ-sequence

$$\sigma_k=\sigma_H\left(\frac{\sigma_L}{\sigma_H}\right)^{\frac{k-1}{K-1}},\quad k=1,2,\cdots,K$$

where the optimal values of σ_L, σ_H and K shoud be determined by numerical experiments. $f_{i,j}$ is an intermediate variable, its boundary conditions can be derived as follows:

On upper and lower airfoil surface,

$$f_y(x,\pm 0)=0 \tag{9}$$

On upper and lower surface of trailing vortex

$$f_y(x,+0)=f_y(x,-0) \tag{10}$$

$$f(x,+0)-f(x,-0)=f(x_t,+0)-f(x_t,-0) \tag{11}$$

and at the far-field,

$$f(x,y)=0 \tag{12}$$

Two algebraic systems of equations with tridiagonal coefficients matrix can be obtained by expanding Eq. (7) or Eq. (8) and imbeding boundary conditions.

III. Von-Neumann analysis of stability and convergence

Assume that the coefficient of φ_{xx}, $1-M^2$, is a constant, thus, set $A_{i,j}$=A=constant. Eq. (7) can be written as:

$$(\sigma\overleftarrow{\delta}_x-\delta_{yy})(\sigma-A\overrightarrow{\delta}_x)(\varphi_{i,j}^{(n)}-\varphi_{i,j}^{(n-1)})-\sigma\Omega L\varphi_{i,j}^{(n-1)}=0 \tag{13}$$

The solution of Eq. (13) may be expressed as φ(x, y, t) + ε(x, y, t), where φ(x, y, t) is the exact solution and ε(x, y, t) is the error. ε(x, y, t) also satisfies Eg. (13) and can be represented by

$$\varepsilon(x,y,t)=e^{\lambda t}e^{i_0 ax}e^{i_0 by} \tag{14}$$

where $i_0=\sqrt{-1}$, a and b are wave number. For the m-th wave component of the error, $a=m\pi/N\Delta x$, m=0, 1, 2, ..., N; $b=m\pi/P\Delta y$, m=0, 1, 2, ..., P. λ is the function of a and b, and may be complex number.

Using Egs. (13) and (14), we can obtain

$$e^{\lambda\Delta t}=\frac{b_1+(1-\Omega)a_1}{b_1+a_1} \tag{15}$$

where

$$b_1=\frac{\sigma}{\Delta x}(1-e^{-i_0 a\Delta x})+\frac{2A}{\sigma\Delta x\Delta y^2}(1-\cos b\Delta y)(1-e^{i_0 a\Delta x}) \tag{16}$$

$$a_1=\frac{2A}{\Delta x^2}(1-\cos a\Delta x)+\frac{2}{\Delta y^2}(1-\cos b\Delta y) \tag{17}$$

Stability requires that $\left|e^{\lambda\Delta t}\right|\leq 1$. Thus, the linear stability condition of AF-2 scheme at subsonic point can be derived from Eg. (15) as follows:

$$0\leq\Omega\leq 2+\frac{2Re(b_1)}{a_1} \tag{18}$$

Set Ω=2, optimal σ value which minimizes $\left|e^{\lambda\Delta t}\right|$ can be obtained from Eg. (15) as follows:

$$\sigma_L=(1+A)\left(\frac{2\pi}{L_{max}}\right)^2 \tag{19}$$

for the longest wave and

$$\sigma_H=2/\Delta x \tag{20}$$

for the shortest wave, where L_{max} is the longest wave length. Hence, the range of optimal σ values is O(1)~O(1/x) when Ω=2.

Making the similar analysis for Eq. (8), we can obtain for supersonic points:

$$e^{\lambda\Delta t}=\frac{c_1+(1-\Omega)c_2}{c_1+c_2} \tag{21}$$

where

$$c_1=\left[-\frac{\sigma}{\Delta x}+\frac{2A}{\sigma\Delta x\Delta y^2}(1-\cos b\Delta y)\right](1-e^{-i_0 a\Delta x}) \tag{22}$$

$$c_2=\frac{A}{\Delta x^2}(1-2e^{-i_0 a\Delta x}+e^{-i_0 2a\Delta x})-\frac{2}{\Delta y^2}(1-\cos b\Delta y) \tag{23}$$

For long-wave components, the stability condition can be obtained from Eq. (21) as follows:

$$0\leq\Omega\leq 2 \tag{24}$$

IV. The parallel algorithm of AF-2 scheme

Expanding Eqs. (7a, 8a) and (7b, 8b), we can obtain for f and c tridiagonal matrix equations respectively as follows:

$$-a_{i,j}f_{i,j-1}^{(n)}+b_{i,j}f_{i,j}^{(n)}-e_{i,j}f_{i,j+1}^{(n)}=h_{i,j}+g_{i,j}f_{i-1,j}^{(n)}$$
$$j=2,3,\cdots,J-1; \quad i=2,3,\cdots,I-1. \tag{25}$$

and

$$-\bar{a}_{i,j}c_{i-1,j}^{(n)}+\bar{b}_{i,j}c_{i,j}^{(n)}-\bar{e}_{i,j}c_{i+1,j}^{(n)}=\bar{d}_{i,j}$$
$$i=2,3,\cdots,I-1; \quad j=2,3,\cdots,J-1. \tag{26}$$

All of the coefficients in the Egs. (25) and (26) only depend on known values and are independent of each other, so parallel operation can be organized. Parallel operation can also be organized for solving each tridiagonal matrix equation. The parallel algorithm is as follows:

1. For $2\leq i\leq I-1$, step 1, calculate the following vectors in parallel:

$$(a_{i,2},a_{i,3},\cdots,a_{i,J-1})^T$$
$$(b_{i,2},b_{i,3},\cdots,b_{i,J-1})^T$$
$$(e_{i,2},e_{i,3},\ldots,e_{i,J-1})^T$$
$$(h_{i,2},h_{i,3},\cdots,h_{i,J-1})^T$$
$$(g_{i,2},g_{i,3},\cdots,g_{i,J-1})^T$$

2. For $2\leq i\leq I-1$, step 1, make the following cycles

   a). calculate the vector

$$(d_{i,2},d_{i,3},\cdots,d_{i,J-1})$$

   The caculation formula is

$$d_{i,j}=h_{i,j}+g_{i,j}*f_{i-1,j}^{(n)}$$

   b). Solving the tridiagonal matrix equations (25) in parallel, we can obtain the vector

$$(f_{i,2}^{(n)},f_{i,3}^{(n)},\cdots,f_{i,J-1}^{(n)})$$

3. For $2 \le i \le I-1$, step1, calculate the following vectors

$$(\bar{a}_{i,2}, \bar{a}_{i,3}, \cdots, \bar{a}_{i,J-1})^T$$

$$(\bar{b}_{i,2}, \bar{b}_{i,3}, \cdots, \bar{b}_{i,J-1})^T$$

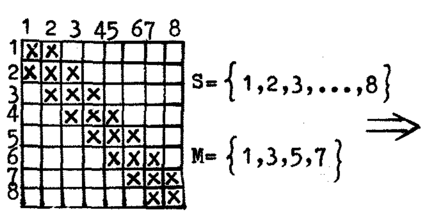
$$(\bar{e}_{i,2}, \bar{e}_{i,3}, \cdots, \bar{e}_{i,J-1})^T$$

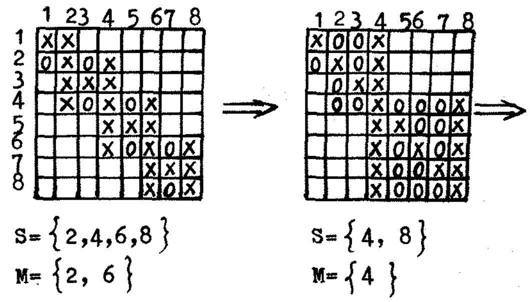
$$(\bar{d}_{i,2}, \bar{d}_{i,3}, \cdots, \bar{d}_{i,J-1})^T$$

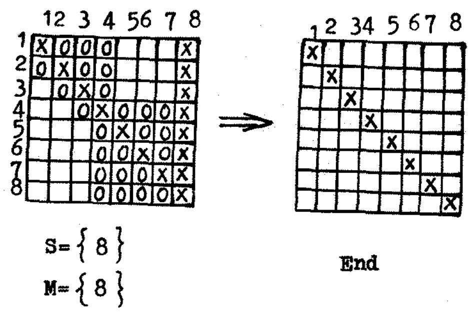
4. Simultaneously solving all the tridiagonal matrix equations corresponding to formula (26), we can obtain at a time all of the $C_{i,j}^{(n)}$.

In this algorithm, steps 1 and 3 are parallel operation for the coefficients of the Eqs. (25) and (26) respectively. The parallel operations are organized for the internal and boundary points respectively, because the caculation formula of the coefficients of internal and boundary points are different and computer YH-1 can make parallel opeation only for the same calculation formula. The vector length for internal points is (I-2)* (J-4). If the vector length is larger than the capability of the computer, we can organize parallel operations piecewise. For boundary points, the vector length is the number or mesh points on which the same computation formulae can be used.

Step 2.b is a parallel operation for the single tridiagonal matrix equation. Multi-pivot elements parallel elimination method for solving tridiagonal matrix equations is used here. It's basic idea is to select in the matrix at a time all odd or even rows as pivot rows from the rows in which pivot elements were not selected , then, eliminate in parallel the elements which are not in the main-diagonal in the other rows until the coefficient matrix of the equations becomes a single diagonal. For example, for an (8*8) trigiagonal matrix, the schematic explanation of the history of multi-pivot elements parallel elimination are as follows:





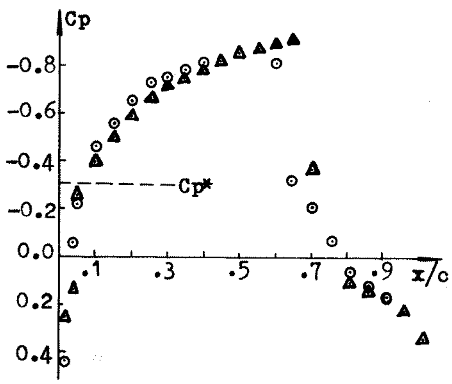


In above diagrams, the bland (or "O") and "x" represent zero and nonzero elements respectively, S represent the set of order number of the rows in which pivot elements have not been selected yet and M represent the set of order number of the rows in which pivot elements will be selected respectively. In this example, only 4 steps are needed for solution, and all calculations can be made in parallel for the each step.

Step 4 is to solve in parallel at a time all tridiagonal matrix equations corresponding to Eq. (26). Here, the method of solving each system of equations is the same as that of serial algorithm. This algorithm for step 4 can reduce the time of parallel operation with no increase in the total amount of computation and is about 23% faster than the algorithm used in step 2.b.

V. Computed Results

The computations are made for an NACA0012 airfoil. The freestream Mach numbers considered are 0.8, 0.85 and 0.9. The angles of attack are 0°, 1°, 2° and 4°. The solutions are obtained on a variable spaced grid with 39*31 x, y points and 22 points along the chord. The x-mesh and y-mesh extends about ten times the chord ahead and behind, above and below the chord respectively. The iterations are started from uniform flow (φ=0).



Δ Present computations

⊙ Experiments [5]

Fig.2. Surface pressure coefficient on NACA0012 airfoil at $M_\infty = 0.85$, $\alpha = 0°$.

The converged solutions for $M_\infty = 0.85$, $\alpha = 0°$, in terms of surface pressure coefficients, are compared with known wind tunnel test results [5] as shown in Fig.4. The computed results agree well with the test results. The comparisons of the computational efficiency of the parallel and serial operations for AF-2 scheme are given in table 1. The results indicate that the parallel operation is about sixteen times faster than serial operation.

VI. Concluding Remarks

The present study indicates that the computational efficiency can be greatly increased by using parallel algorithm presented in this paper. The solutions with satisfactory precision for transonic plane steady flow can be quickly obtained by solving Eq. (1), using the AF-2 scheme of difference computation and the parallel algorithm.

Present investigations is only for two-dimensional cases. We believe that the present method is sufficient motivation for fast solving three-dimensional problems.

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M_∞	α°	Parameters		n	$\left\lvert\frac{\Delta\phi^{(n)}}{\Delta\phi^{(1)}}\right\rvert_{max} * 10^4$	CPU TIME (Seconds) Serial	Parallel
0.80	0		$\Omega=1$ $K=5$	87	.9321	25.3	1.57
0.80	1			87	.9836	25.3	1.57
0.80	2			66	.9652	19	1.20
0.80	4		$\Omega=0.8$ $K=8$	74	.6180	21.5	1.34
0.85	0	$\sigma_L=0.4$ $\sigma_H=80$	$\Omega=1$ $K=8$	97	.4846	28.1	1.74
0.85	1			89	.5635	25.8	1.60
0.85	2		$\Omega=0.8$ $K=8$	90	.9328	26.1	1.62
0.85	4			113	.5716	32.7	2.02
0.90	0		$\Omega=0.8$ $K=10$	101	.8965	29.2	1.81
0.90	1		$\Omega=0.7$ $K=8$	131	.9473	37.8	2.34
0.90	2			145	.6938	41.9	2.59
0.90	4			122	.9625	35.2	2.18

Table 1. The comparisons of the computational efficiency of serial and parallel operations for AF-2 scheme.