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ORTHOGONALISATION : A TOOL FOR IMPROVED TEST DATA

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ABSTRACT

A very important property of a set of modal parameters describing the dynamic behaviour of a structure is that the mode shapes form an orthogonal set. This assumption is basic to many analytical methods used to further process modal data for example, structural coupling, flutter analysis, etc. The test for orthogonality of a set of data is simply performed. If the data quality is poor the test engineer may decide to re-acquire some or all of the test data. However there comes a time when the limitations of the test equipment and analysis methods defy further improvement of the data by experimental means, especially where there are closely spaced vibration modes.

This paper discusses three different methods whereby orthogonality may be imposed on a set of measured data by analytical techniques. Three methods of orthogonalisation have been investigated and are compared. Any of the three methods result in an orthogonal mode set, the purpose of the paper is to ascertain whether the calculated modes are an improvement on the measured mode set, and are in fact representative of the structure itself.

Both theoretical and practical test cases are used to demonstrate the effect of orthogonalisation under various conditions.

It is concluded that the availability of an accurate mass matrix of the structure is essential to obtain good results. Bearing this in mind, the modified McGrew method is found to be the most effective of the three techniques of orthogonalisation. The method provides a useful tool for modal data improvement.

INTRODUCTION

Perspective of the Paper

This paper is concerned with the improvement of ground vibration test (GVT) data which has been obtained via normal mode testing techniques. In the field of aeroelasticity, ground vibration testing is an important preliminary step in the clearance of aircraft for flight. The combination of structural and aerodynamic data is a requirement for flutter analysis. The structural data is often acquired using normal mode testing techniques, and as with any measurement technique, there are inaccuracies inherent in the results. The data may be subject to several stages of theoretical analysis prior to flutter predictions and flight tests. With so much dependent on the results of the GVT it is essential that the degree of accuracy is as high as possible.

Ground Vibration Testing

It is not proposed to discuss methods in detail as these are adequately covered elsewhere[1]. However, it is necessary to discuss briefly the nature of the results obtained and the limitations of the equipment which may affect their accuracy.

The aim of the GVT is to determine the lower modes of vibration of the structure, in terms of frequency of vibration, generalised mass, damping coefficient and the mode shape for each of the first 10 to 20 modes. This data set forms a structural representation of the aircraft in terms of its lower modes of vibration.

The data is collected from the structure via a matrix of accelerometers, and from the energy input to the structure via a number of electromagnetic exciters. Problems in data accuracy arise from the inherent systematic errors of the system, and the inability of the operator to isolate closely spaced modes of vibration. If two modes are closely spaced in frequency it can be almost impossible to separate them, and each measured mode will contain elements of the other mode.

Orthogonality

In qualitative terms, orthogonality is a property of a modal set that each mode is independent of all other modes in the set, cf two orthogonal vectors, at 90 degrees to one another which are each independent of the other, having no component in the direction of the other. In this study, three methods of imposing orthogonality on a set of data were investigated. It is relatively easy to impose orthogonality on a set of data, but it is necessary that the solution is in fact more representative of the structure than the original measured data. To ascertain whether this was the case a comprehensive set of analytical tests was conducted. Some practical results are subjected to orthogonalisation and the results are discussed.

LITERATURE SURVEY

Surprisingly few papers on this subject are available. Gravitz[2] proposes a method of orthogonalisation which makes use of the property of an orthogonal mode set that the structural influence coefficient matrix is symmetric. The method involves the construction of this matrix from the GVT data, and averaging the off-diagonal terms of the matrix to make it symmetric. The Eigen value problem is then solved to produce a revised set of modes and frequencies. The appeal of the method is its simplicity. It claims to average out imperfect measurement techniques, instrumentation errors and the effects of structural non-linearities.

Targoff[3] avoids the necessity of performing an eigen solution which can be costly in terms of computing time for large matrices. He proposes a corruption matrix which gives a measure of the inaccuracy of the measured modes. Using this matrix and the mass matrix of the structure a revised mode shape matrix may be calculated. The method assumes that errors are primarily due to symmetric elements in the off-diagonal terms of the orthogonality matrix (these should be zero) and shows that the effect of asymmetric errors is small.

McGrew[4] proposes a method whereby a Gramm-Schmidt orthogonalisation procedure is used, modified to include mass weighting. The method put forward by McGrew is based on the assumption that the lowest measured mode is the most accurate, and that higher frequency modes become progressively less accurate, each being a combination of a true mode and a component of all preceding modes. In aircraft ground vibration testing this assumption is certainly not valid, since many factors are contributory to the accuracy of a mode, especially the presence of another mode at a very close frequency.

Baruch and Bar Itzhack[5] propose a method whereby an orthogonal mode set is calculated which is as close as possible to the original measured mode set. The method is basically to minimise a Euclidean norm of the errors, subject to the orthogonality requirement. The work goes on to show how a corrected stiffness matrix may be constructed from the mode shape data, and this is used to calculate new mode shapes and frequencies.

The work of Targoff is cited as being an optimal method for correcting modes in the same sense as that proposed in (5).

A further paper (6) by Baruch puts forward the idea of orthogonalising modes using a matrix of proportionality. This takes into account the credibility of the original modes as judged by the engineer.

CHOICE OF METHODS

Aims of the Method

The aim of this paper is not to find a new solution to an old problem, but rather to arrive at a tried and tested method which will solve the problem for a particular application. The emphasis is on simplicity for a number of reasons.

Requirements of the Method

The chosen method should be ideally suited for interactive use in a test environment. For example, once a number of modes have been isolated for a particular configuration under test, the engineer should perform an orthogonality check as a test of the accuracy of the measured data. If large errors are evident the orthogonalisation method may be used to estimate a better mode shape. The engineer can then repeat the test with the revised shape in mind. Once a "reasonable" mode set is achieved then the orthogonalisation procedure can be used to refine the data to produce a realistic orthogonal mode set. Thus for interactive use in a test situation a neat concise program is required which will not occupy large quantities of valuable storage space, and which will run quickly and efficiently. There are always measurement errors inherent in any measurement technique. While it is hoped that an orthogonalisation procedure will reduce these errors, absolute accuracy will never be achieved. The complexity of the method should be balanced against the anticipated percentage improvement.

The methods of Targoff and Gravitz were chosen for further investigation due to their simplicity. McGrew's method was also investigated, but the assumption that the lowest frequency was the most accurate has been discarded. Instead an initial orthogonality check is used to investigate the order of integrity of the modes. The Gram-Schmidt orthogonalisation with mass weighting then proceeds on the basis of this decision.

DEFINITION OF ORTHOGONALITY

In order to define clearly what is meant by the term orthogonality as applied to experimental modal data, it is necessary to consider the parameters of the structure as they are measured in test or derived by other means.

Mass Matrix. Must relate to the measurement points on the structure. It is essential that this matrix is accurate.

Frequency. The natural frequency of each mode of vibration, measured in Hz.

Generalised Mass. A quantity related to the energy input required to excite the mode, and forms a matrix for the complete mode set.

Damping Coefficient. May be defined in a number of ways, not relevant to this particular study.

Mode Shape. The individual displacements of each accelerometer. Usually these displacement are normalised such that the maximum displacement is 1 and all other displacements are factored accordingly.

For the subsequent analysis the following terms are defined:

$\phi_{(m*n)}$ the matrix of mode shapes.

$\phi^{T}_{(n*m)}$ the transpose of ϕ.

$M_{g(m*m)}$ the matrix of generalised masses.

$M_{(n*n)}$ the mass matrix.

where m is the number of modes

and n is the number of accelerometers.

For an orthogonal mode set, then,

$$\phi^{T} M \phi = M_{g}$$

where M_g is a diagonal matrix containing the calculated values of generalised mass for the mode set. If ϕ does not describe an orthogonal mode set M_g will have off-diagonal terms.

If ϕ is normalised such that each element of mode i is divided by the square root of M_{gi} then the following equation defines an orthogonal mode set:

$$\psi^{T} M \psi = I$$

where ψ is the mass-normalised mode shape matrix. This will be used for the definition of orthogonality.

OUTLINE OF TARGOFF'S METHOD

Full details of Targoff's method may be found in the reference, (3), however it is relevant to give a brief outline at this stage. Assumptions are that the frequency is accurately measured and that the mass matrix is accurate.

For a suitably normalised mode set,

$$\psi^T M \psi = I$$

If the measured mode shape is called ψ_m where

$$\psi_m^T M \psi = I$$

a corruption matrix C may be introduced such that

$$\psi_m = \psi C$$

OR defines the results of the orthogonality check on the measure modes, where

$$OR = \psi_m^T M \psi_m$$

$$= C^T \psi^T M \psi C$$

$$= C^T C$$

or, since C is symmetric,

$$OR = C^T C = C^2$$

For small measurement or separation errors, C will be close to I, the identity matrix.

i.e. $C = I + \alpha$

where α is small.

Hence,

$$OR = C^2 = (I + \alpha)^2 = I + S$$

where S is easily calculated following the orthogonality check.

It follows that

$$S = 2\alpha + \alpha^2$$

$$\alpha = \tfrac{1}{2} S\left(I + \tfrac{1}{2}\alpha\right)^{-1}$$

To solve this equation a simple iteration scheme is used where

$$\alpha_o = 0$$

$$\alpha_1 = \tfrac{1}{2} S \quad \text{etc.}$$

Having calculated α, and hence C the revised mode shapes may be obtained:

$$\psi_m = \psi C$$

$$\psi = \psi_m C^{-1}$$

Only the mode shapes are altered by this method, the other modal parameters do not enter into the equations.

OUTLINE OF McGREW'S METHOD

Once again the assumption that an accurate mass matrix of the structure is available is made and that the frequency is measured accurately. The third assumption made by McGrew is that the modal amplitude and phasing errors increase with increasing frequency, i.e. the lowest measured mode is the most accurate, and subsequent modes contain components of all lower modes. For the present application this assumption is not considered valid. For example, two low frequency modes may be less than 1 Hz apart, both of very high energy. To isolate these two modes might prove a very difficult task for the engineer. However, a higher mode of vibration, well separated from other modes of vibration, would result in a much more accurate representation of a normal mode of the structure. It is proposed to modify McGrew's method by introducing a matrix of confidence whereby the engineer decides on the order of reliability of the measured modes.

The final assumption is related to the third one and proposes that the structural damping effects are small, but tend to cause high modes to excite lower modes. It is considered that to establish an order of confidence in the modes will provide such improved accuracy as to swamp the effects of damping in exciting lower modes.

Let us assume that the mode shapes and other modal matrices have been ordered from the most reliable mode to the least. This ordered measured modal matrix is called ϕ_{om}. If each measured mode is considered separately taking the most accurate first, then the ith mode may be expressed as a function of the i-1 more accurate modes:

$$\left\{h_m^i\right\} = \left\{h^i\right\} + \left\{h_R | h^1 | \ldots h^{i-1}\right\}\left\{L^i\right\}$$

h_R represents the rigid body modes which may be included in the solution. L is the modal coupling matrix, each term of which describes the degree of coupling between a pair of modes.

For an orthogonal mode set the off-diagonal terms in the orthogonality matrix should be equal to zero. Hence for the ith mode

$$0 = \left\{h_R | h^1 | \ldots h^{i-1}\right\}^T [M] \left\{L^i\right\}$$

where [M] is the mass matrix for the structure.

Substituting for h_i gives

$$\left\{h_R | h^1 | \ldots h^{i-1}\right\}^T [M] \left\{h_m^i\right\}$$

$$= \left\{h_R | h^1 | \ldots h^{i-1}\right\}^T [M] \left\{h_R | h^1 | \ldots L^{i-1}\right\} \left\{L^i\right\}$$

The RHS contains the expression for the generalised mass matrix, up to the i-1th mode.

The equation becomes:

$$\left\{L^i\right\}\left[M_g^{i-1}\right] = \left[h_R|h^1|\ldots h^{i-1}\right] + [M]\left\{h_m^i\right\}$$

Substitution gives:

$$h^i = \left[(I) - \left[h_R|h^1|\ldots h^{i-1}\right]\left[M_g^{i-1}\right]^{-1}\left\{h_R|h^1|\ldots h^{i-1}\right\}^T[M]\right]\left\{h_m^i\right\}$$

or $$\left\{h^i\right\} = \left[MC^i\right]\left\{h_m\right\}^i$$

where $[MC^i]$ is the modal correction matrix for the ith mode. Each successive "correct" mode may then be calculated, starting with the most accurate measured mode.

OUTLINE OF GRAVITZ'S METHOD

Once again the assumption is made that an accurate mass matrix of the structure is available. The method incorporates an eigen solution, such that new frequencies and mode shapes are calculated.

The method is described in reference (3), and is based on the fact that the structural flexibility influence coefficient matrix is symmetric for an orthogonal mode set. This fact may be easily derived.

For an orthogonal mode set:

$$\phi^T M \phi = M_g$$

where M_g is diagonal.

The generalised stiffness influence coefficient matrix may be expressed:

$$K_g = M_g \omega^2$$

ω^2 is the diagonal matrix of model frequencies, which implies K_g is diagonal.

C_s is now defined as the structural flexibility influence coefficient matrix, which is the inverse of the structural stiffness coefficient matrix.

The structural stiffness matrix may be written in terms of the generalised stiffness matrix

$$K_g = \phi^T K_s \phi$$

$$K_s = \phi^{T-1} K_g \phi^{-1}$$

since ϕ is orthogonal.

$$K_s^{-1} = C_s = \phi^T(\phi^T)^{-1} K_g^{-1}(\phi)^{-1}$$

$$C_s = \phi K_g^{-1} \phi^T$$

If K is diagonal, its inverse is diagonal, so C_g is symmetric for ϕ orthogonal.

The matrix C_s is calculated using the above equations but with the non-orthogonal measured modes. The matrix is then averaged to impose symmetry upon it.

$$C_{sa} = \tfrac{1}{2}(C_s + C_s^T)$$

where C_{sa} is the averaged structural flexibility matrix.

It only remains to find the set of modal parameters which correspond to the new structural flexibility matrix. This is achieved through the following standard eigen value equation:

$$(C_{sa}M - \frac{1}{\omega^2} I)\phi = 0$$

One would expect frequency changes resulting from this process to be relatively small.

TESTING THE VALIDITY OF THE METHODS

The above methods have been applied by the various authors to experimental data sets. However, by nature of experimental data an accurate assessment of the methods cannot be made since the data is only approximate. It is necessary to start with a "perfect" data set, i.e. an orthogonal set, which is then corrupted in a manner representative of the errors introduced by experiment. The orthogonalisation procedures should then be able to return the corrupted data to its uncorrupted state.

An ideal base set is a finite element model which necessarily produces orthogonal mode sets. The details of the model are unimportant, and a small model of a truss, with 36 degrees of freedom was chosen, and the first ten vibration modes were taken into consideration. Mode shapes were mass-normalised to give a value of 1 on the leading diagonal of the orthogonality matrix, i.e.

$$\psi^T M \psi = I$$

The procedure of the test will be to corrupt the data in a number of ways and to apply the three methods of orthogonalisation to the corrupted data. The resulting "orthogonalised" modes will then be compared with the original data to see if the corrupted data is in fact improved by each method.

MEASURE OF IMPROVEMENT

In order to compare a complete matrix of data with another similar matrix it is desirable to use one or more simple parameters which will summarise the overall data quality. In fact there are two properties of the orthogonality matrix which are of value in assessing the results. The first is the overall orthogonality of the results, and the second is the correspondence of the "corrected" results to the "correct" or original mode shapes.

Comparison of Matrices.

As a means of comparison between two matrices A and B a standard method is the Euclidean norm.

$$E = \frac{\|A-B\|}{\|A\|}$$

$$E = \frac{\sum_{ij} (a_{ij} - b_{ij})^2}{\sum_{ij} a_{ij}^2}$$

where a_{ij} are the elements of the reference matrix A, i.e. the correct orthogonality matrix, and b_{ij} are the elements of the corrected orthogonality matrix B

For the correct matrix A,

$$a_{ij} = 0 \quad \text{for} \quad i \neq j$$

and

$$a_{ij} = 1 \quad \text{for} \quad i = j$$

Hence the equation reduces to

$$C = \frac{\sum_{i}(1-b_{ii}) + \sum_{i \neq j} b_{ij}^2}{n}$$

where n is the order of the matrix.

For the purpose of comparison between A and B, B is normalised with respect to the generalised masses associated with A, so that the diagonal terms of B are not equal to 1, but the deviation from 1 reflects the error in the generalised masses associated with B.

Mass Error Criterion

To quantify the error in generalised mass (and hence indirectly in mode shape) only diagonal terms need be considered.

$$C_m = \frac{\sum_{i}(1-b_{ii})^2}{n}$$

where C_m is defined as the mass error criterion.

Orthogonality Error Criterion

To assess the error in terms of the orthogonality of the corrected mode set the off-diagonal terms are considered.

$$C_o = \frac{\sum_{i \neq j} b_{ij}^2}{n}$$

where C_o is defined as the orthogonality error criterion.

It is noted that the overall comparison between the matrices A and B in terms of the Euclidean norm may be expressed:

$$C = C_m + C_o$$

Assessment of Improvement

For the perfect data set C_m and C_o should be equal to zero. For a corrupted data set, C_m and C_o will have some positive value which should be reduced after the application of the orthogonalisation procedure.

CORRUPTION OF DATA

Random Corruption

A typical set of measured data contains a certain degree of random error in the measured values. Typically up to 10% error may exist in the mode shape displacement. Frequency is normally considered to be measured accurately and was not corrupted in the test for random corruption.

The perfect data set was corrupted in its mode shape values by applying standard Fortran number generation. Errors of up to 5% and of up to 10% were applied to the mode shape data, which was then re-normalised to a maximum deflection of 1. This is though to represent fairly the typical measurement errors experienced in a test situation. Obviously this causes calculated generalised mass to be in error.

Coupling of Modes

A frequent occurrence in modal isolation is that two modes are closely spaced in frequency and cannot be separated adequately. This was simulated in test by adding a small factor of one mode to another and re-normalising. This method was used to couple only two modes and again to couple several modes.

If a mode is inadequately isolated then the frequency of the mode will also be affected.

Consider an impure mode ϕ_x composed of a proportion of ϕ_1 and a proportion of ϕ_2.

$$\phi_x = \alpha_1\phi_1 + \alpha_2\phi_2$$

Since ϕ_1 and ϕ_2 are pure and hence orthogonal modes then

$$\phi_1^T K \phi_2 = \phi_1^T M \phi_2 = 0$$

and

$$\phi_1^T K \phi_1 = \omega_1^2$$

where ω is the matrix of frequencies associated with ϕ.

From standard vibration theory it can be said that:

$$\omega^2 = \frac{\phi^T K \phi}{\phi^T M \phi}$$

$$\omega^2 = \frac{\alpha_1^2\phi_1^T K \phi_1 + \alpha_2^2\phi_2^T K \phi_2}{\alpha_1^2\phi_1^T M \phi_1 + \alpha_2^2\phi_2^T M \phi_2}$$

and if

$$\phi_2^T M \phi_2 = 1$$

then

$$\omega_2^2 = \phi_2^T K \phi_2$$

Hence we obtain

$$\omega^2 = \frac{\alpha_1^2\omega_1^2 + \alpha_2^2\omega_2^2}{\alpha_1^2 + \alpha_2^2}$$

Thus the modified frequency resulting from the coupling of two modes may be calculated, so that a realistic corruption of the data may be performed.

Corruption of Mass Matrix

One of the basic assumptions of all the methods investigated in this study is that an accurate mass matrix of the structure is available. On a complex structure such as an aircraft, the mass matrix will be in error to some degree. The effect of random errors in the mass matrix was investigated to see how the end results were affected.

TESTING OF THE THREE METHODS

A series of tests was applied to all three methods according to the corruptions described above.

The results are summarised in Table 1.

Test 1 indicated that the "perfect" data was in fact very close to being orthogonal, and differences being probably due to numerical errors. In the cases where random errors were applied in nearly all cases a small improvement was noted (the exception is Targoff's method on the 10% corruption where a very slight increase in C_m is noted).

Two modes were coupled in the next case, and the orthogonality matrix for this case is shown in Table 2. Modes 1 and 2 are 28% coupled. Dramatic improvements were achieved by orthogonalisation, particularly in the case of the McGrew method where the modes were ordered in terms of integrity. The two modes were successfully decoupled, with values of zero for both C_o and C_m.

The coupling of several modes is shown in Table 3. The results of the orthogonalisation demonstrate the importance of ordering the modes in the McGrew method. If the modes are not ordered the orthogonalisation produces worse results than the original corruption. However, when the modes are ordered significant improvements are achieved.

In the cases where random errors were introduced as well as coupling of modes, only the McGrew method with ordered modes showed any improvement over the corrupted data. The other methods generally corrupted the data still further.

The last case was the corruption of the mass matrix. The modal data was perfect, but the mass matrix was corrupted with random errors up to about 10%, so that it no longer corresponded to the structural modes. Each of the orthogonalisation methods succeeded in corrupting the data using this matrix. This emphasises the need for an accurate mass representation of the structure.

Practical Test Case

The test rig used to demonstrate the orthogonalisation of real modal data is shown in Figure 1. A total of ten natural modes of vibration were excited using normal mode techniques. It was difficult to produce an accurate mass matrix of the structure since electromagnetic exciters were used to excite the modes which were not placed in the same positions for each mode. The mass of the exciter moving parts totalled about 100 g which has a significant effect on the results of the orthogonality check. A best estimate was used which would adequately compensate for this additional mass except where more than one bar was moving significantly in the mode.

The results of the orthogonality check are shown in Table 4, and some degree of coupling (more than 10% is evident between modes 4 and 5, and to a lesser extent between modes 8 and 9). Referring to Figures 2 and 3 these are the measured modes for mode 4 and mode 5. Mode 4 could be described as S3 horizontal bending, with S1 out of phase, and mode 5 is S1 horizontal bending with S3 in phase. Figures 4 and 5 show how these modes were modified by orthogonalisation. Mode 4 was altered very little but the modified mode 5 shows a much greater degree of movement on S3, in phase with S1. The frequencies of modes 4 and 5 differ by less than 1 Hz, and it is thought that the strong mode 4 movement of S3 at 14,7 Hz inhibited the in-phase movement of S3 at 15,31 Hz. Once the influence of mode 4 had been removed S3 was able to move freely in its natural mode.

The apparent coupling between modes 8 and 9 which are much more widely spaced in frequency can be attributed to inaccuracy in the representative mass matrix due to the effect of the exciter moving parts.

CONCLUSIONS

Three methods of orthogonalisation have been implemented and thoroughly tested. They have been applied to theoretical and measured data sets and the following conclusions have been drawn:

1. In the case where random errors are present in a set of modal data, each method of orthogonalisation generally improves the data in terms of mode shape accuracy.

2. Where two modes are coupled, each method produces a dramatic improvement in mode shape data.

3. Where several modes are coupled in a manner which is typical of a measured data set both Targoff and Gravitz improve the data. McGrew does not improve the data unless the modes are ordered with respect to confidence in the mode. Then it produces the best results of all the methods.

4. When random error is imposed on data where several modes have been coupled only the McGrew method with ordered modes achieves a significant improvement on the data.

5. If the mass matrix is inaccurate orthogonalisation will probably corrupt the data still further.

6. In the case of test results where data quality is reasonable but isolation is difficult due to close spacing of modes, orthogonalisation is a useful and practical tool. An accurate matrix of the structure is however essential. The McGrew method with ordered modes is recommended.

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Table 1 : Results of orthogonalisation in terms of mass error and orthogonality error criteria for each of the methods considered.

TEST	CORRUPTED DATA		TARGOFF		GRAVITZ		McGREW UNORDERED		McGREW ORDERED	
	C_o	C_m	C_o	C_m	C_o	C_m	C_o	C_m	C_o	C_m
Uncorrupted data	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000		
5% random corruption	.0249	.0614	.0010	.0523	.0000	.0529	.0000	.0534		
10% random corruption	.0343	.0971	.0031	.0981	.0000	.0888	.0000	.0943		
Two modes coupled	.1313	.0139	.0014	.0017	.0000	.0016			.0000	.0000
Several modes coupled	.0119	.0223	.0053	.0195	.0000	.0209	.0000	.0355	.0000	.0161
5% random and two modes coupled	.0118	.0522	.0083	.0547	.0000	.0550	.0000	.0619	.0000	.0468
10% random and two modes coupled	.1187	.1022	.0129	.1021	.0000	.1017	.0000	.1106	.0000	.0928
10% corruption in the mass matrix	.0303	.0175	.0004	.0274	.0000	.0284	.0000	.0404		

Table 2 : Orthogonality check for two modes coupled.

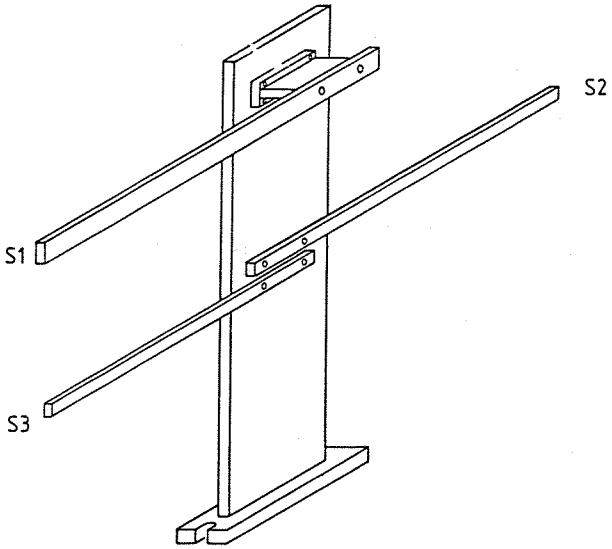
MODE	1	2	3	4	5	6	7	8	9	10
FREQ	8.95	10.18	33.21	35.32	39.57	44.87	68.96	70.84	98.88	116.10
	1.000									
	.287	1.000								
	.000	.000	1.000							
	.000	.000	.000	1.000						
	.000	.000	.000	.000	1.000					
	.000	.000	.000	.000	.000	1.000				
	.000	.000	.000	.000	.000	.000	1.000			
	.000	.000	.000	.000	.000	.000	.000	1.000		
	.000	.000	.000	.000	.000	.000	.000	.000	1.000	
	.000	.000	.000	.000	.000	.000	.000	.000	.000	1.000

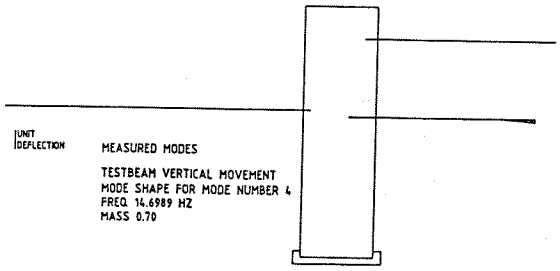
Table 3 : Orthogonality check for several modes coupled.

MODE	1	2	3	4	5	6	7	8	9	10
FREQ	8.89	10.65	33.22	35.37	39.57	45.17	68.96	70.87	98.88	116.10
	1.000									
	.195	1.000								
	.000	.099	1.000							
	.000	.000	.050	1.000						
	.000	.000	.000	.099	1.000					
	.000	.000	.000	.000	.000	1.000				
	.000	.000	.000	.000	.000	.099	1.000			
	.000	.000	.000	.000	.000	.000	.000	1.000		
	.000	.000	.000	.000	.000	.000	.000	.030	1.000	
	.000	.000	.000	.000	.000	.000	.000	.000	.000	1.000

Table 4 : Orthogonality check for measured data.

MODE	1	2	3	4	5	6	7	8	9	10
FREQ	7.22	9.66	12.55	14.70	15.31	26.57	28.69	35.58	66.18	72.10
	1.000									
	.016	1.000								
	.166	.010	1.000							
	.008	.030	.005	1.000						
	-.019	-.022	-.018	-.173	1.000					
	.002	.036	.002	-.004	-.019	1.000				
	.007	.015	.000	.061	-.010	-.011	1.000			
	.110	-.032	.029	.013	-.006	.010	-.003	1.000		
	.072	.070	-.016	-.010	-.087	-.021	.014	.113	1.000	
	.007	.041	-.008	.017	-.002	-.003	.026	.034	-.073	1.000





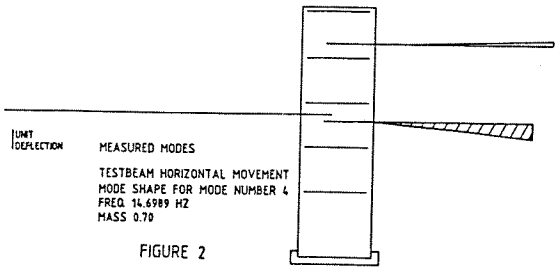
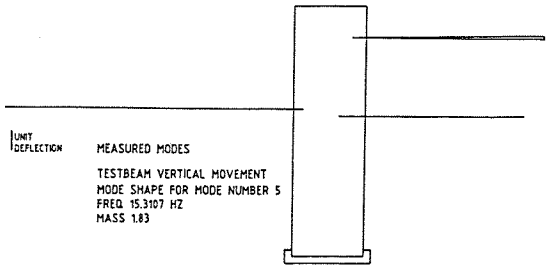


FIGURE 2



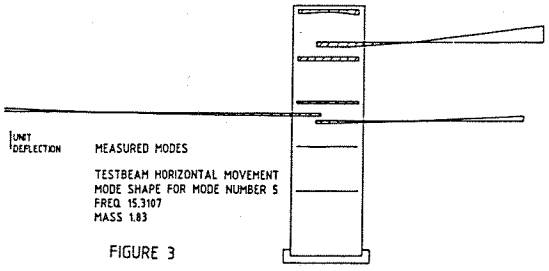
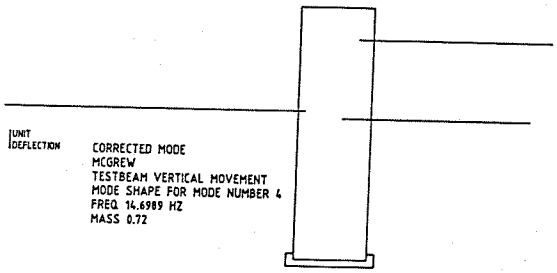


FIGURE 3



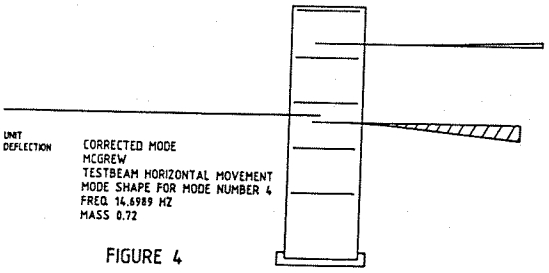
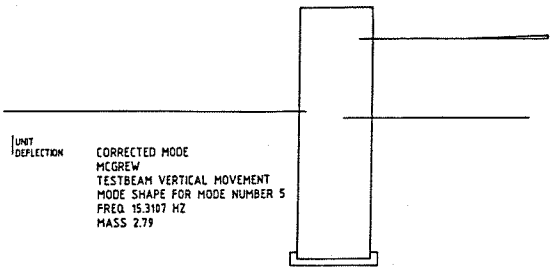


FIGURE 4



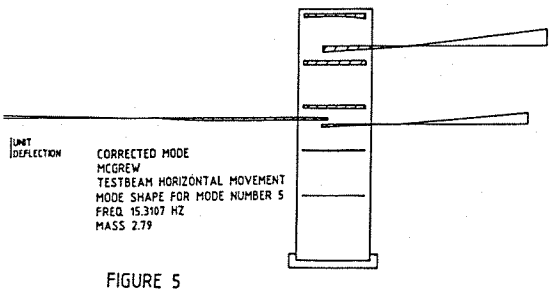


FIGURE 5