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WIND TUNNEL BLOCKAGE CORRECTIONS FOR BLUFF BODIES WITH LIFT.

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Abstract:

The effect of Solid boundaries on the forces and moments on an arbitrary lifting body has been analysed using a flow model in the inviscid domain. The flow model consists of the free streamlines representing the viscous separated shear layer and stagnant fluid of constant width, the width being determine by equating the momentum loss of the fluid in the far wake to the upstream momentum of the stagnant fluid. This momentum surface model has yielded results which are comparable to those of some other models in the sense that blockage is seen to decrease the drag co-efficient at the same back pressure. Some other models like the displacement surface model suggest increased drag co-efficient with blockage at the same back pressure. These results seem to suggest that inviscid modelling does not necessarily yield proper trends, and it is suggested that additional constraints, yet to be identified, like the Kutta Condition in airfoil theory, may have to be invoked.

I.Introduction

Developments in the methods for blockage corrections in wind tunnels have a long history going back to about 1929[1]. The earlier developments are covered in the books by Pankhurst and Holder[2] and Ray and Pope[3], and in the monographs by Garner et al[4] Agard CP 335[5] and Mokry et al[6]. The most important of the limitations of the early methods are that the flow must be attached to the body, and blockage, (which may be quantified as the ratio of maximum cross-sectional area of the body perpendicular to the flow to the cross sectional area of the wind tunnel test section), must be small, say less than 2%. With the use of low aspect ratio wings with favourable flow separation, and flight at large angles of attack, it became necessary, particularly after 1950, to develop methods which do not have the limitations of the old methods. Initially the development of such methods followed several parallel courses. A semi-empirical approach was developed by Maskell[7] in 1963 for blockage corrections with flow separation and high blockages. His method has been extended to several environments, [8,9,10,11] and some of its limitations have been clarified [12,13]. It is now generally believed that Maskell's original formulation over-predicts the correction[13].

The second approach to the prediction of blockage effect, has been through the use of free-streamline theory, particularly in two-dimensional flows[14,15,16,17,18]. More recently [19,20] there has been a revival of the singularity methods, some what similar to the original approach of Lock[1] but more extended in scope, namely to cover the cases of flow separation and large blockages also. The availability of powerful computers has stimulated development of CFD methods, panel methods[21], boundary measurement methods and adaptive wall methods[22]. Of these, the application of free streamline theories seem to have been confined almost to water tunnels to take into account cavitation but its possibly use in wind tunnels do not seem to have been well-explored. Possibly the main limitation of free-streamline theory at present is that it is not convenient to use in three-dimensional flow. However, recent developments in free-streamline theories have seen the emergence of a number of inviscid flow models[23] which yield flow patterns which are reasonably similar to those in real viscous fluids. With this observation in mind, we present here the results of two-dimensional blockage study with flow separation using one of the recent models and later present results with different models to highlight the importance of the choice of proper model.

In the separated flow of a real fluid, one observes that the separation streamlines meet each other approximately one chord length away from the body, at a point called the, "Reattachment Point". The flow at and around the reattachment point is turbulent and fluid is swept downstream from the reattachment point was a wake. The wake thus contains the fluid which has lost its energy due to the effects of viscosity. In the domain of inviscid fluids, the separation bubble is modelled by free-streamlines and the far-wake is modelled by a cavity. The near-wake bubble and the far-wake cavity may be modelled as continuous as in the original model of Kirchhoff or its recent variant as 'Choked flow' of Ai[15]. It is probably more appropriate to model the near wake bubble and the far-wake cavity as two distinct entities, as proposed by Roshko[24] and further developed by Tulin[25], Oba [16], Wu[26] and Sato[17]. In this approach, there is a near-wake bubble of constant pressure and a far-wake cavity of different pressure or pressures. The near and far-wake has to be postulated to account for pressure discontinuities. Three of the well known models are the nuities. Three of the well known models are the parallel wake model[4], Displacement surface model[17] and momentum surface model[16]. In the parallel wake model, the bubble streamlines of the near wake are assumed to become parallel to the free stream smoothly at the singular surface separating the near and far wakes and proceed to infinity as parallel surfaces. The displacement surface model does not stipulate any such tendency but in fact, the far-wake boundaries of the cavity need not be parallel to the free stream at down-stream infinity. while the near wake bubble is modelled in the momentum surface model as in the other models, the far-wake is modelled as a cavity with walls parallel to the from stream smoothly at the singular surface separating the near and far wakes and proceed to infinity as parallel surfaces. The displacement surface model does not stipulate any such tendency but in fact, the far-wake boundaries of the cavity need not be parallel to the free stream except at down-stream infinity. While the near wake bubble is modelled in the momentum surface model as in the other models, the far-wake is modelled as a cavity with walls parallel to the free stream such that the separation gap of the walls is a measure of momentum loss in the free stream due to drag and there is a discontinuity in slope at the junction of the near-wake bubble boundaries and the far-wake walls. In the study presented below, this momentum surface model is employed to study blockage in lifting two-dimensional separated flow, and the results are compared with others predictions.

II. Mathematical Formulation

Consider an arbitrary shaped body, placed between two parallel solid walls of width h, Fig:1. The Stagnation

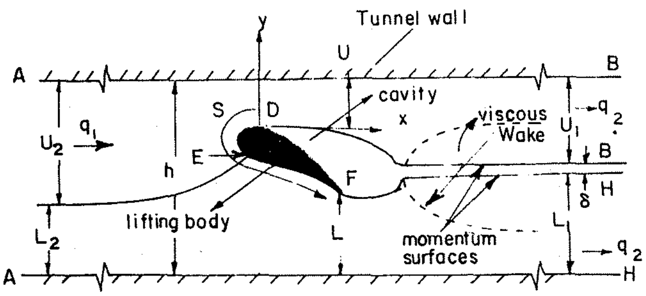

FIG.1 FLOW CONFIGURATION IN THE PHYSICAL PLANE

streamline separates on the body surface at D and F, which are fixed but arbitrary points. The arc-length between the separation points D and F is assumed to be S along the attached flow portion. The free stream velocity q_1 reaches a value q_c along the free-streamlines and finally q_2 far-downstream.

In a real fluid, the separated shear layers DC and FG form a time averaged steady state constant pressure near-wake bubble, and meet in the region of CG representing the reattachment region. From this turbulent mixing region, a viscous and rotational far-wake follows and has been observed to grow nearly parabolically[16]. In order to deal with separated flows within the limits of potential flow theory, the wake is replaced by momentum surfaces at a distance $\bar{\theta}$ apart. The upper momentum surface is at a distance U_1 from the upper tunnel wall and the lower momentum surface is at a distance L_1 from the lower tunnel wall. We define a parameter σ, which may be called as the wake under pressure coefficient,

$$\sigma = (q_c^2 - q_1^2) / q_1^2 \qquad \text{...(1)}$$

σ is the negative of the base pressure coefficient, a term perhaps more commonly used. It is called, "Cavitation Number" by hydrodynamicists.

Since the flow is assumed to be irrotational, in-compressible and two-dimensional, one can find a velocity potential ϕ and a stream function ψ in such a way that a complex potential function $f(z)=\phi+i\psi$, which is analytic in the flow region can be defined.

The flow region in the physical Z-plane transforms into a slitted infinite strip in the f-plane (Fig:2).

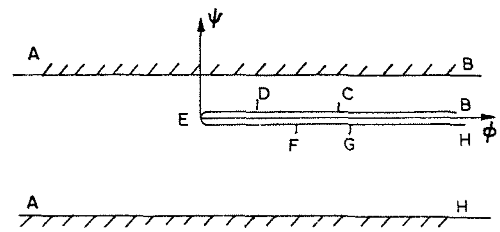

FIG.2 FLOW CONFIGURATION IN COMPLEX POTENTIAL PLANE : $f = \phi + i\psi$

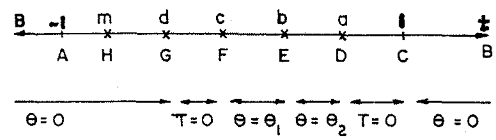

FIG.3 FLOW CONFIGURATION AUXILARY PLANE

The flow and boundary regions in the f-plane are further transformed onto the upper half and the entire real axis of an auxiliary g-plane by the Schwarz-Christoffel transformation,

$$f=(hq_1/\pi \,/\, (1+b))\ ((m-b)\ln(g-m)/cb-m)) +$$

$$(1+b)\ln(g+1) \,/\, (b+1))) \qquad \text{... (2)}$$

The flow problem in the physical plane can be solved in the g-plane (Fig:3) as a mixed-boundary value problem for the logarithmic hodograph function

$$W(g) = \theta + i\ \ln(q/q_c) \qquad \text{...(3)}$$

Since either the real or imaginary part of the logarithmic hodograph function is known along various line segments of the real axis in the g-plane, a mixed boundary value problem can be formed for the function W(g) as follows:

$$\theta = 0 \qquad -\infty<t<d \quad ; \quad 1<t<\infty$$

$$T = 0 \qquad d<t<c \quad ; \quad n<t<1$$

$$\theta = \theta_1(t) \qquad c<t<b$$

$$\theta = \theta_2(t) \qquad b<t<n \qquad \text{...(4)}$$

The mixed boundary value problem can be solved as a Riemann-Hilbert problem for the half-plane (Muskhelishvili[27]), by considering an auxiliary function in the running variable t of the g-plane.

$$\nu(t)=i \;/\; (t-d)(t-c)(t-n)(t-1) \qquad \ldots(5)$$

From Eqns. 4 and 5, we can construct the general solution to the boundary value problem as,

$$W(g)=(-1/\pi/\nu(g) \int_c^b [\theta 1 \;(t) \;+\; \int_b^n \theta_2 \;(t)]$$

$$\{ \frac{dt}{\sqrt{(t-g)(t-d)(t-c)(n-t)(1-t)}} \} + F(g) \qquad \ldots(6)$$

where $F(g)=i\;(A+Bg)\;\sqrt{\{(g-c)(g-n)/(g-1)(g-d)\}}$ ..(7)

is a singular solution.

The introduction of the singular solution F(g) requires explanation. Since the far-wake and the near-wake have been considered as separate entities with different pressures, one has to postulate a singular surface separating the two. This singular surface may be conveniently identified with the real fluid turbulent mixing reattachment region. The type of singularity that should exist here can be inferred from the requirements of the particular integral of Riemann-Hilbert problem which suggests a square root singularity. As Oba[16] has remarked, many authors who have considered the solution of the Riemann-Hilbert problem have also found the need for square root singularity.

III. UNKNOWN PARAMETERS

The solution (Eqn.7) is observed to contain nine unknown parameters, A,b,m,d,c,b,n, σ and q_2. Since the pressure in the near-wake region can, to some extent be altered by artifical means, the parameter σ can be left as a free parameter. To get the remaining eight unknown parameters one needs to have eight independent equations involving these parameters. The required constraints to obtain them are described below.

CONSTRAINTS IMPOSED:

The first two conditions are given by the boundedness of the function W(g) as $|g| \to \infty$. This condition yields,

$$RAJAM = \sqrt{(t-d)(t-c)(n-t)(1-t)} \qquad \ldots(8)$$

$$\pi A=\pi(q_2/q_c) + \int_c^b \theta_1(t) + \int_b^m \partial_2 \;(t) \;]$$

$$[\;(t/RAJAM) \quad dt \;] \qquad \ldots(9)$$

and

$$\pi B \int_c^b [\theta_1 \;(t) + \int_b^n \theta_2 \;(t)]\;[(1/RAJAM)\;dt] \quad \ldots(10)$$

The third condition is given by the known arc-length of the wetted portion of the body to the value computed from the integrals representing the same arc-length in the auxiliary plane (See Figs:1 & 3).

$$S= \{ \int_b^n - \int_c^b \} \quad \frac{df}{dt} \quad \frac{dt}{q} \qquad \ldots(11)$$

The fourth condition is given by equating the vertical distance between the momentum surface at near-wake closing region to the vertical distance between the terminus points C and G of the shear layers.

$$Y|_c \;-\; Y|_g = \bar{\theta} \qquad \ldots(12)$$

where

$$Y|_c = \int_n^f \frac{df}{dt} \quad \frac{\sin\theta}{q}\,dt \qquad \ldots(13)$$

and

$$Y|_G = Y|_F - \int_d^c \frac{df}{dt} \quad \frac{\sin\theta}{q} \quad dt \qquad \ldots(14)$$

and $\bar{\theta}$ is given by conservation of mass condition,

$$\bar{\theta} = h(q_2 - q_1) \;\;/\;\; q_2 \qquad \ldots(15)$$

The fifth condition is given by the vertical distance consideration from the geometrical requirement of Fig:1.

$$U_1 \;-\; U+Y|_c \;=\; 0 \qquad \ldots(16)$$

where

$$Y|_c = \int_n^1 \frac{df}{dt} \quad \frac{\sin\theta}{q} \quad dt \qquad \ldots(17)$$

and

$$U_1 = q_1 h\;(1+m) \;\;/\;\; q_2/(1+b) \qquad \ldots(18)$$

The sixth condition is obtained by equating the value of the logarithmic hodograph function at far-upstream point in the physical Z-plane to the estimated value by theory in the auxiliary g-plane.

$$W(g=-1) \;=i \;\ln\,(q_1/q_c) \qquad \ldots(19)$$

The eighth and last condition is obtained from the consideration of pressure integral around the body surface, momentum considerations and drag force.

The drag coefficient of the body with flow separation can be obtained as the pressure integral

$$C_D = \text{Real part of } \{-i \int_{\text{Point-F}}^{\text{Point-D}} (q_c^2 - q^2) \ / \ q_1^2 \ \} \quad \ldots(21)$$

The existance of velocity defect in the far-wake may be viewed as equivalent to the superposition of an in-flow of uniform velocity from downstream infinity towards the body. The far-wake in a viscous fluid is then modelled as a cavity of width $\bar{\theta}$ parallel to the walls, right from the cavity closure to downstream infinity (See Tulin[25], Oba [18]). The velocity defect in the far-wake has been viewed as a source like contribution, whose strength is obviously θq_2.

The coefficient of drag due to the fictitious source contribution is

$$C_s = 2\bar{\theta} \ q_2^2 \ / \ q_1^2 \quad \ldots(22)$$

The drag coefficient exerted on the semi-infinite body consisting of the body, near wake-boundaries and momentum surfaces, can be calculated by applying momentum theorem to a rectangular control volume having breadth equal to the width of the tunnel and length extending to regions up and downstream of the body in such a way that the speed is uniform across the tunnel.

The drag coefficient exerted on the semi-infinite body is given by C_c and can be worked out as,

$$C_c = 2[hq_1^2 - (h-\bar{\theta} \ q_2^2 + h(q_2^2 - q_1^2)/2] \ q_1^2 \quad \ldots(23)$$

and obviously,

$$C_D = C_c + C_S \quad \ldots(24)$$

The pitching moment coefficient of the arbitrary body about its upper separation point is given by

$$C_M = \text{Real part of } \{ \int_{\text{Point-D}}^{\text{Point-F}} (q_c^2 - q^2) \ Z \ d\bar{Z} \ / \ q_1^2 \quad \ldots(25)$$

IV: APPLICATION OF THE GENERAL THEORY TO THE FLOW PAST A FLAT-PLATE AIRFOIL:

The above analyses from Eqn.(6) to Eqn.(25) Completes the search for the solution for an arbitrary lifting body. One observes that the solution in Eqn.(6) requires a knowledge of the distribution of the inclination θ of every wetted point on the body to the wind tunnel axis. It is very unlikely that a closed form evaluation of the integrals in Eqn.(6) can be found for an arbitrary distribution of θ as a function of t, where t is the running variable along the real axis in the g-plane. However, to illustrate the type of results that one is likely to obtain, calculations have been performed for a flat plate airfoil at incidence, where the quantities θ_1 and θ_2 are constant and the relavant integrals can be evaluated in closed form.

In the case of the flat plate, one can easily observe that θ_1 and θ_2 reduce to the simple forms $-\alpha$ and $(\pi-\alpha)$ respectively, where α is the inclination of the plate to the wind tunnel axis (Fig:1). We assume that the flow separates at the leading and trailing edges, and for a plate of unit length, trailing edge co-ordinates then become (Cos α, -Sin α). The integrals in Eqns.(9) and (10) can then be explicitly evaluated and the results are as follows.

$$\int_c^b (t\theta_1 \ (t) \ / \ \text{RAJAM}) \ dt = 2\alpha[dF(B1, \ CK) + (c-d) \ \pi(B1, \ (n-c) \ / \ (n-d), \ CK)] \ / \ \text{RAMA} \quad \ldots(26)$$

$$\int_b^n (t\theta_2(t) \ / \ \text{RAJAM}) \ dt = 2(\pi-\alpha)[F(B^2, \ CK) - (1-n) \ \pi(B2, \ (n-c)/(1-c), \ CK)] \ / \ \text{RAMA} \quad \ldots(27)$$

$$\int_c^b (\theta_1 \ (t) \ / \ \text{RAJAM}) \ dt = 2\alpha F(B1, \ CK)/\text{RAMA} \quad \ldots(28)$$

and

$$\int_b^n (\theta_2(t)/\text{RAJAM}) \ dt = 2(\pi-\alpha)F(B2,CK)/\text{RAMA} \quad \ldots(29)$$

The complete solution to the problem, along the various line segments on the real axis of g-plane, is given in Appendix-B.

V. RESULTS AND DISCUSSIONS

Numerical calculations were made for two wind tunnel blockages of 2.5% and 20% with the wake under pressure coefficient σ as a given parameter. Because of the implicit nature of the problem, trial values of various parameters were fed into the Dec-1090 digital computer of the Indian Institute of Science, Bangalore and a library algorithm based on the steepest gradient method was used to arrive at the converged values of the parameters which satisfied the imposed constraints. Final numbers which typically varied within .00001 in 1.0 could be obtained. But in the final phase, a C.P.U. time of about 2.5 secs was required for each data point. The results are given in Figs:4 to 12. The available results of other studies, which can be compared with the present values, are also shown.

The author is aware of only two other theoretical studies on inclined flat plate with finite near-wake region viz, those by Ota[28] and Cohen et al[29]. Ota's[28] calculations were made by assuming that the stagnation streamline lies at the centre of the channel far-upstream of the plate. This impli-es that the flat plate at incidence is not likely to be located symmetrically with respect to the mid line of the tunnel.

Cohen et al[29] have made the drastic assumption that the stagnation streamline always meets the flat plate at its leading edge and the leading edge of the flat plate is placed on the mid line of the channel. Because of the obviously incorrect modelling of the flow by Ota[28] and Cohen et al[29], their results are not quoted here.

Using the parallel wake model, Wu[30] has given results for unbounded flow past an inclined flat plate and his results have been plotted as "Open delta points" in Figs: 4 to 9.

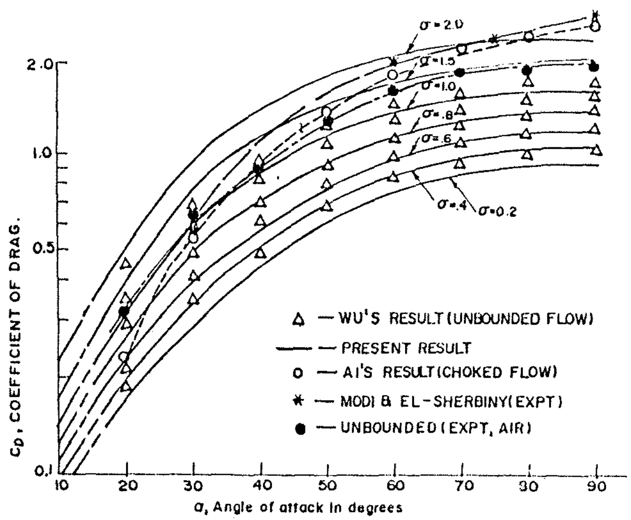

FIG. 4 VARIATION OF DRAG COEFFICIENT WITH BACK PRESSURE AND ANGLE OF ATTACK. (20 % Blockage)

Another related result is that of Ai[15] who made computation for what is called, "Choked flow". The term, "Choked flow" is used to denote the flow model where the separation streamlines ultimately become parallel to the free stream with no singular surface between the near-wake and far-wake region. Such a concept gives a closure hypothesis and it is not necessary to choose the wake under pressure coefficient as a given parameter, for a given blockage and angle of attack. For zero blockage, the parameter σ becomes a unique value of zero for all angles of attack, but, for a non-zero blockage, σ is stri-ctly not equal to zero and is a finite quantity.

So far as experimental results are concerned, the authors have been able to procure only two publications involving two-dimensional flat plates. Hoerner[31] has given values of lift and drag coefficients in his book for unbounded flow in air and these results are given in Figs: 4 and 6.

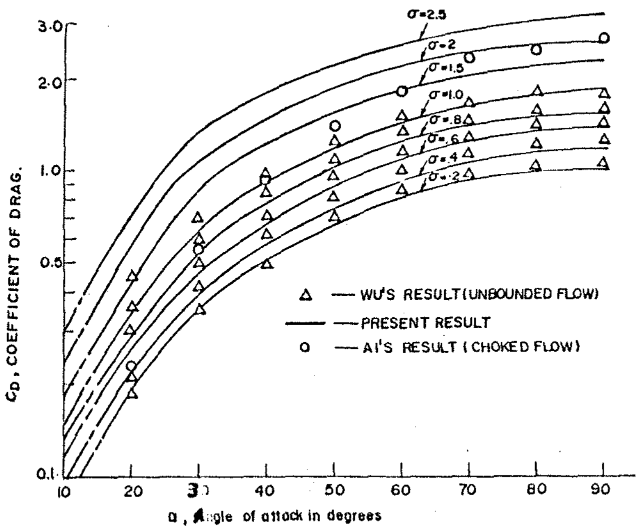

FIG. 5 VARIATION OF DRAG COEFFICIENT WITH BACK PRESSURE AND ANGLE OF ATTACK (2.5 % Blockage)

The measured values in wind tunnel for angles of attack α=30° to 90° from Modi and El-Sherbiny[8] for a blockage of 20.5% are also given in the Figs: 4 and 6.

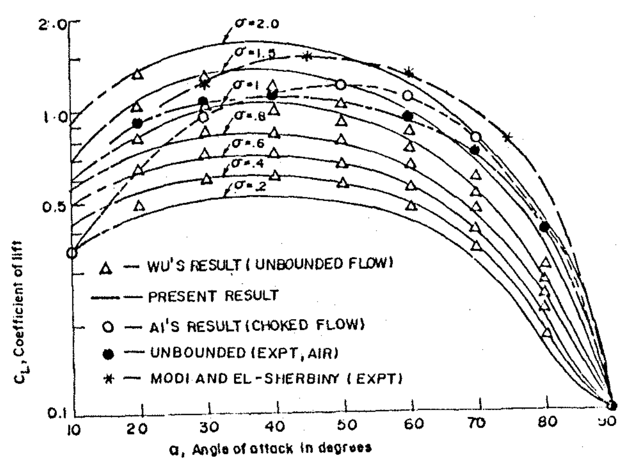

FIG. 6 VARIATION OF LIFT COEFFICIENT WITH BACK PRESSURE AND ANGLE OF ATTACK (20 % BLOCKAGE)

An experimental investigation by Parkin[32] has not been available to the authors in spite of serious efforts to procure them. However, Wu[26] states in his paper that Parkin's[32] experimental results with blockage are matching with his unbounded fluid flow results.

A comparison of C_D at the same value of wake under pressure coefficient at a blockage of 2.5% and 20% indicates the surprising result that there is only about 10% difference between them. At first sight, this result was intriguing since it is well known in solid walled, wind tunnels that blockage increases the value of C_D quite significantly.

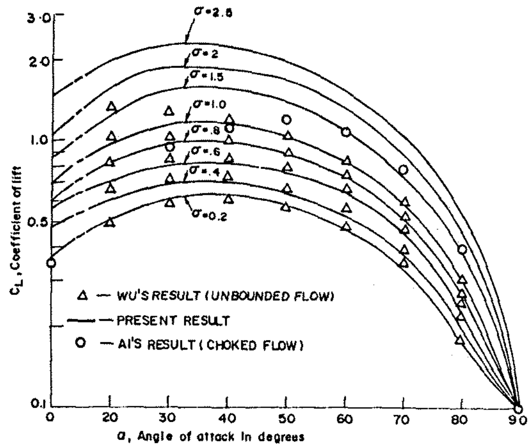

FIG.7 VARIATION OF LIFT COFFICIENT WITH 2.5% BLOCKAGE

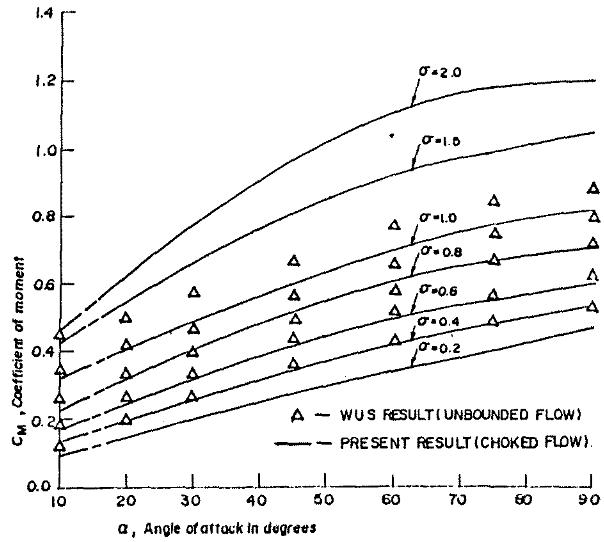

FIG.8 VARIATION OF PITCHING MOMENT COEFFICIENT WITH 20 % BLOCKAGE

Further consideration however, showed that the reason for only a small change in C_D due to blockage was the assumption of constant back pressure in both blockages. In real fluids, increase of blockage increases negatively the back pressure increasing drag, if no artificial means are used to increase the back pressure in the positive direction. This means that the back pressure at the higher blockage of 20% will be considerably more negative than at the value of the blockage of 2.5% if natural flow separation takes place. If one assumes the same back pressure at a blockage of 20% as at a blockage of 2.5%, one is implying that the near-wake pressure increased in the positive sense at the higher blockage. The positive pressure in the near-wake will act like a thrust on the plate and decreases the drag. It is therefore not surprising that increased blockage at the same value of σ does not show much change in drag coefficient, since the increase in negative pressure is neutralised by the imposition of a positive pressure. Surprisingly, in this momentum surface model, the drag coefficient at the higher blockage of 20% is observed to be less than the drag coefficient at a blockage of 2.5%.

The effect of blockage with force coefficient for different models has been analysed in Fig :10. The choked flow model of Ai[15] , parallelwake model of Wu[14] and momentum surface model by the authors show decrease in force coefficients with increase of blcokage. But, the displacement surface model study by Dheenadhayalan and Rao[18] shows incre -ase in force coefficients with increase of blockage. Some of the earlier theoretical results like those of Birkhoff and Zarantonello suggest an increase of force coefficient with increased blockage even at the same value of σ.

Unfortunately, it is not possible at this stage to pass judgement on the comparison of calculated results with experiments, since none of the authors who carried out their experiments have given the base pressure coefficient for the corr sponding force coefficients, except, apparently, Parkin [31].

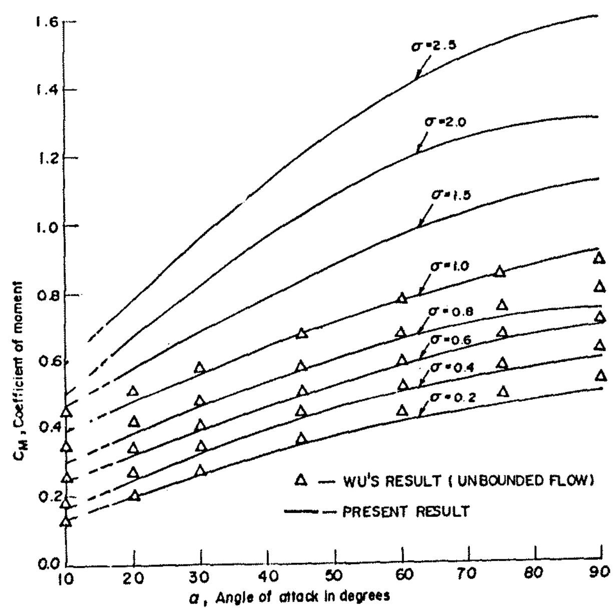

FIG.9 VARIATION OF PITCHING MOMENT ABOUT L.E WITH 2.5 % BLOCKAGE

However, it is known that the value of the base pressure coefficient even with blockage lies within the range of values covered in the computations and the fact that the experimental points straddle the theoretical curves may be taken as a fair indication that the theoretical results are useful.

To confirm the correct trend of force coefficients with blockage in the lifting case of

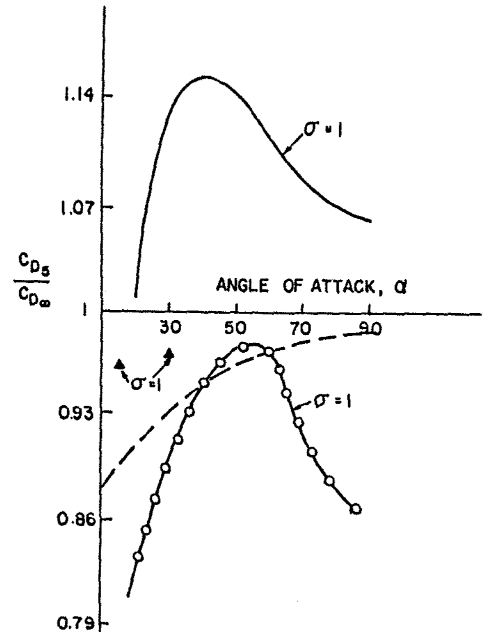

FIG.10 PREDICTION OF RELATIVE VARIATION OF DRAG COEFFICIENT BY DIFFERENT MODELS

▲ WU (1969) THEORY
—o— MOMENTUM SURFACE MODEL BY DHEENADHAYALAN AND RAO
----- AI (1966), CHOKED FLOW THEORY
—— PRESENT DISPLACEMENT SURFACE MODEL

separated flows, one needs a arge number of experi -mental data or theoretical investigation taking the viscous effects also into account.

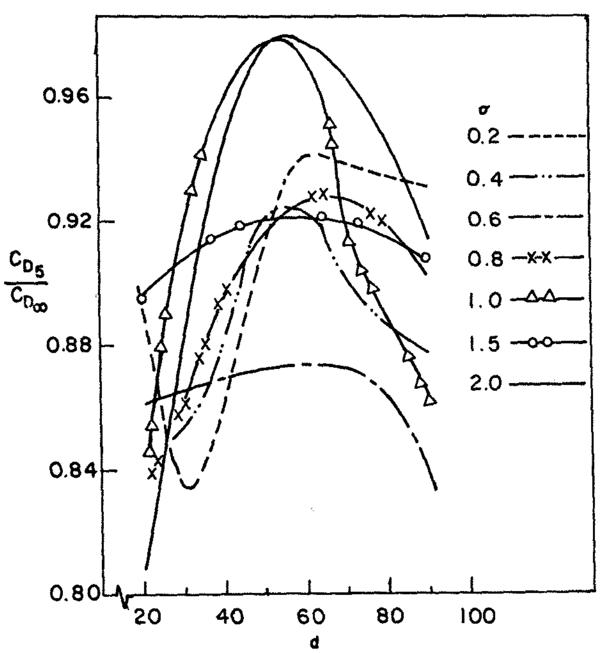

FIG. 11 EFFECT OF BLOCKAGE ON DRAG COEFFICIENT

A final question that will still have to be answered is how σ varies with blockage, other flow parameters remaining constant. This question cannot be answered unless one imposes an additional constraint as Ai[15] has done. Other constraints are possible such as those requiring equality of the distance of reattachment points from the two separation points, parallellism ~f the reattachment line to the chord of the plate etc,[18]. The problem here seems to be similar to the need to impose a condition like the Kutta condition in airfoil theory. It must be admitted that Ai's modelshows a flow pattern which is further removed from the real flow patterns (in requiring the wake pressure to be constant throughout the wake region) but its results seem to match the experimental data of Modi and El-Sherbiny or of Horner quite closely.

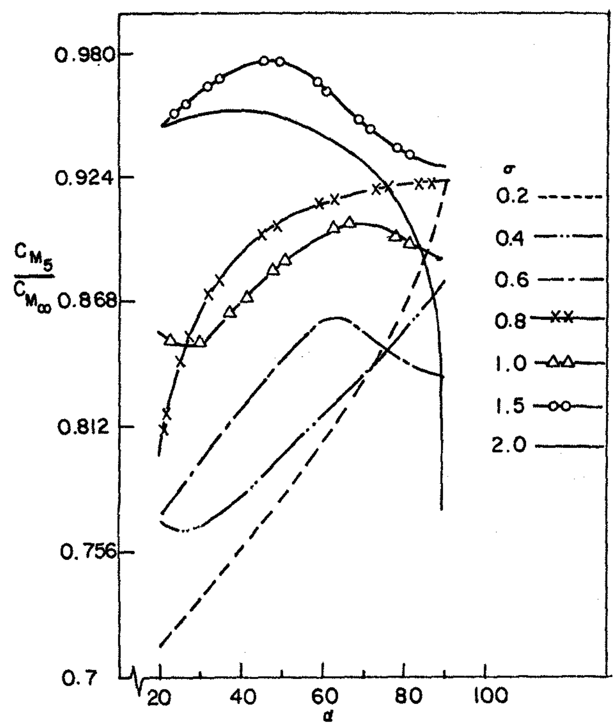

FIG. 12 EFFECT OF BLOCKAGE ON PITHING MOMENT

Other approaches to evaluate blockage-free conditions are possible, which may be termed semi-expirical. Three such proposals are described in Appendix-A. One can expect that a judicious use of the theoretical results presented here, together with the methods given in the Appendix-B will yield satis_actory blockage free results in separated flows.

APPENDIX - A

Hsu-Smith Method:

Recently, Hsu[33] has deduced a relation from the linearized theory of Smith [34] that

$$\sigma_\infty = \frac{\sigma_h \, [\cosh(\pi L/h) - 1]^{1/4}}{\sqrt{(\pi L/2/h)}\,[\cosh(\pi L/h)+1]^{1/4}}$$

where

$\sigma_h = 2(q_c^2 - q_1^2) / q_1^2$ = Cavitation number measured in a tunnel of width -h. $= -C_{p_b}$

q_c = Speed along the near-wake bubble boundary.

q_1 = Upstream Speed

h = Width of the tunnel

L = Length of the body and near-wake system

σ_∞ = Cavitation number estimated for the unbounded stream using the experimentally measured or theoretically calculated values in a tunnel of height-h.

Although Smith's Theory was developed for very low Reynolds numbers, HSu seems to suggest the use of the above relation for all Reynolds numbers.

Difference Method: In this method, σ_∞ has to be known such as by HSu-Smith Methods or as a measured quantity.

Then, let $C_{D_h}\Big|^{\sigma_h}_{EXP}$ = Measured Drag Coefficient of a body at an angle of attack α and cavitation number σ_h in a tunnel of height - h, in an experiment and,

$C_{D_h}\Big|^{\sigma_h}_{THEORY}$, $C_{D_\infty}\Big|^{\sigma_h}_{THEORY}$

the theoretically calculated values of drag coefficient at the same cavitation number σ_h, but in a tunnel of height h and h = ∞ respectively.

Then calculate,

$$C_{D_\infty}\Big|^{\sigma_h}_{Estimation} = \frac{\left\{ C_{D_h}\Big|^{\sigma_h}_{EXP} \right\}}{\dfrac{C_{D_h}\Big|^{\sigma_h}}{C_{D_\infty}\Big|}_{THEORY}}$$

= Estimated drag coefficient for the cavitation number σ_h and tunnel height h, using the above defined quantities.

Also, calculated $\Delta C_D = C_{D_\infty}\Big|^{\sigma_\infty}_{THEORY} - C_{D_\infty}\Big|^{\sigma_h}_{THEORY}$

where, $C_{D_\infty}\Big|^{\sigma_\infty}_{THEORY}$ and $C_{D_\infty}\Big|^{\sigma_h}_{THEORY}$ are theoretically calculated values of drag coefficient for the tunnel height h = ∞, for the cavitation numbers σ_∞ and σ_h respectively.

The final, required Drag Coefficient in unbounded flow at the same angle of attack α, with the estimated cavitation number σ_∞ is

$$C_{D_\infty}\Big|^{\sigma_\infty}_{EST} = C_{D_\infty}\Big|^{\sigma_h}_{EST} + \Delta C_D$$

Ratio Method: The ratio method is an alternate to the difference method and is perhaps more logical. After first estimating σ_∞ by experience or by using HSu-Smith method, one can compute

$$C_{D_\infty}\Big|^{\sigma_\infty}_{EST} = \frac{C_{D_\infty}\Big|^{\sigma_\infty}_{THEORY}}{C_{D_h}\Big|^{\sigma_h}_{THEORY}} \; C_{D_h}\Big|^{\sigma_h}_{EXP}$$

Terminologies given here are as in the difference method.

APPENDIX - B

The Symbols used in Equations are explained below:

Rama $= \sqrt{(n-d)\,(1-c)}$

T $= \ln(q/q_c)$

$T_1 = \ln(q_1/q_c)$

$T_2 = \ln(q_2/q_c)$

$A1(t) = \sqrt{(c-t)(n-t)/(1-t)/(t-d)}$

$A_2(t) = \sqrt{(c-t)(t-d)/(n-t)/(1-t)}$

$A_3(t) = \sqrt{(c-t)(t-d)(n-t)/(1-t)}$

$A_4(t) = \sqrt{(n-c)^2(b-d)(1-t)(t-d)/(1-b)(c-d)(n-d)(n-t)(c-t)}$

$A5(t) = \sqrt{(t-c)(n-t)/(1-t)/(t-d)}$

$A6(t) = \sqrt{(n-t)(1-t)(t-c)/(t-d)}$

$A7(t) = \sqrt{(t-d)(t-c)/(1-t)/(n-t)}$

$A8(t) = \sqrt{(t-d)(t-c)(n-t)/(1-t)}$

$A9(t) = \sqrt{(t-n)(1-t)(t-c)/(t-d)}$

$A10(t) = \sqrt{(n-c^2)(1-b)(1-t)(t-d)/(t-n)(1-c)(t-c)(1-n)(b-d))}$

$B1 = \sin^{-1}\sqrt{(n-d)(b-c)/(n-c)/(b-d)}$

$$B2 = \sin^{-1}\sqrt{(1-c)(n-b)/(n-c)/(1-b)}$$

$$B3 = \sin^{-1}\sqrt{(t-c)(n-d)/(t-d)/(n-c)}$$

$$B4 = \sin^{-1}\sqrt{(n-t)(1-c)/(1-t)/(n-c)}$$

$$C1(t) = (n-c)(1-t)/(1-c)/(n-t)$$

$$C2(t) = (1-d)(n-t)/(n-d)/(1-t)$$

$$C3(t) = (1-d)(t-c)/(1-c)/(t-d)$$

$$C4(t) = (t-d)(n-c)/(t-c)/(n-d)$$

$$CK = \sqrt{(1-d)(n-c)/(1-c)/(n-d)}$$

$$DK = \sqrt{1-CK^2}$$

$$VK = \sqrt{(1-n)(b-d)/(n-d)/(1-b)}$$

$$WK = \sqrt{(1-b)(c-d)/(1-c)/(b-d)}$$

In the interval of t between d and c, the speed is given by q_c and the flow inclination θ is given by,

$$\theta=-(A+B\ t)\ A1(t)+(2/\pi/\ Rama)]$$

$$[\theta_1[(1-n)A2(t)\ \Pi\ (B1,\ C1(t),\ CK] + A3(t)F(B1,\ CK)+$$

$$Rama\ \tan^{-1}(0.5\ A4(t)\ \sin(2\ B1))] + \theta_2\ A2(t)[(n-t)F(B2,CK]+$$

$$(1-n)\ \Pi\ (B2,\ C1\ (t),\ CK))]$$

In the interval of t between C and b, the flow inclination is given by θ_1 and the speed q is given by,

$$\ln (q/q_c) = (A+B\ t)\ A5\ (t)+ (2\ \theta_1/\ \pi)$$

$$[A6(t)\ F\ (B1,\ CK] /\ Rama-F\ (B1,\ CK)$$

$$[E(B3,\ CK)-E(CK)-E(CK)F(B3,\ CK)/\ K(CK)]-$$

$$\sum_{P=1}^{\infty}\ EXP\ (-P\ \pi\ K(DK)\ /\ K\ (CK)]$$

$$\sin\ (P\ \pi\ F\ (B3,\ CK)\ /\ K(CK)]\ \sin(P\ \pi\ F(B1,CK)/K(CK)]/$$

$$(P\ \sinh\ (P\ \pi\ K(DK)/\ K(CK))]-0.5\ \ln(\sin(0.5\ \pi(F(B1,CK)-$$

$$F(B3,CK)]/K(CK)]/\ \sin(0.5\ \pi\ (F(B1,\ CK)\ +F(B3,CK)]/$$

$$K(CK)))] + (\ \theta_2\ /\ \pi\ )\ [\ (-2\ \ A7\ (t)\ /\ Rama)$$

$$[(n-t)F(B2,\ CK)+\ (1-n)(F(B2,CK)-\ \ \Pi\ (B2,C2(t),CK))] +$$

$$\ln\ [(VK-(1-n)A7(t)\ \tan\ (B2)/Rama)\ /\ VK+(1-n)A7(t)$$

$$\tan\ (B2)/Rama))]$$

In the interval of t between b and n, the flow inclination is given by θ_2 and the speed q is given by,

$$\ln\ (q/q_c) = (A+B\ t)A5(t) + (\theta_1\ /\ \pi\ )\ [2/Rama/A7(t))$$

$$[(t-d)F(B1,\ CK)\ -\ (c-d)\ \ \Pi\ (B1,\ C3(t),CK)] +$$

$$\ln([WK+(c-d)\ \tan\ (B1)/Rama/A7(t)/(WK-(c-d)\ \tan\ (B1)/$$

$$Rama/A7(t)))]+(2\ \ \theta_2/\ \pi)\ [-A8(t)F(B2,CK)/\ Rama\ +$$

$$F(B2,\ CK)\ [E(B4,\ CK)\ -\ E(CK)\ F(B4,\ CK)/K(CK)]\ -$$

$$\sum_{P=1}^{\infty}\ \ EXP(-P\ \ \pi\ \ K(DK)/K(CK))\ \sin(P\pi\ F(B4,CK)/K(CK)]$$

$$\sin(P\ \pi\ F(B2,CK)/K(CK))/[P\ \sinh(P\pi\ K(DK)/K(CK))]+$$

$$0.5\ln\ (\sin(0.5\ \pi\ [F(B2,CK)-F(B4,CK)]/K(CK))/$$

$$\sin(0.5\ \pi\ [F(B2,CK)+F(B4,\ CK)]/K(CK)))]$$

In the interval of t between n and 1, the speed is given by q_c and the flow inclination θ is given by,

$$\theta = (A+B\ t)\ A1\ (t)\ \ +\ \ (2/\ \pi\ /Rama)[\theta_1\ [(t-c)F(B1,CK)+$$

$$(c-d)\ \Pi\ (B1,\ C4(t),\ CK)]\ /\ A2(t)+\theta_2\ [A9(t)F(B2,CK)\ +$$

$$(c-d)\ \Pi\ (B2,\ C4(t),\ CK)/A2(t)]]+(2\ \theta_2\ /\ \pi)$$

$$\tan^{-1}\ (0.5\ \ A10\ (t)\ \sin\ (2B2)]$$

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