SENSITIVITY ANALYSIS AND MULTIDISCIPLINARY OPTIMIZATION FOR AIRCRAFT DESIGN: RECENT ADVANCES AND RESULTS.

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Abstract

Optimization by decomposition, complex system sensitivity analysis, and a rapid growth of disciplinary sensitivity analysis are some of the recent developments that hold promise of a quantum jump in the support engineers receive from computers in the quantitative aspects of design. Review of the salient points of these techniques is given and illustrated by examples from aircraft design viewed as a process that combines the best of human intellect and computer power to manipulate data.

Introduction

Development trends in advanced civil and military aircraft are toward longer time separating major projects and toward greater capability increment of each next project over its predecessor. Together with rapidly increasing vehicle complexity, this trend works toward reducing usefulness of the statistical information and design by precedent, and toward increasing importance of computer-based, multidisciplinary analysis and formal synthesis (optimization) among the designer's tools.

Progress in multidisciplinary analysis and synthesis was discussed in the context of aircraft design in [1] presented to the 14th ICAS in September 1984. The major points made in that reference were: 1. the currently prevailing sequential manner of conducting design process is likely to produce suboptimal results; 2. the systems (vehicles) decomposable into top-down hierarchy of engineering disciplines and subsystems may be optimized by multilevel procedure made up of sub-optimizations performed concurrently at each level of the hierarchy and linked by optimum sensitivity derivative information.

The purpose of this paper is to update the review given in [1] by referring to new information on a test of multilevel, multidisciplinary optimization based on a hierarchal, top-down decomposition of the type postulated in [1], and by bringing into focus two new advances: the emergence of the engineering sensitivity analysis in individual disciplines and in analysis of complex, coupled systems, and the adaptation of a formal system approach to aircraft design. Sensitivity analysis offers a practical tool to answer the "what if" questions that so often arise in design, and the system approach enables one to exploit interdisciplinary synergism while dividing the large design task into smaller, concurrently executable tasks, without being limited to formalism of the top-down, hierarchal decomposition reported in [1].

Hierarchal Decomposition in Design

To provide background for discussion of the recent developments, the description of hierachal decomposition given in [1] is restated in abridged manner using generic terms. It is followed by a discussion of application experience available to date, including an application example.

Optimization Procedure Formulation

An example of a hierarchal system is depicted in Fig.1. Each box in the figure stands for a mathematical model representing an aspect of the system behavior (an engineering discipline, e.g., aerodynamics), or a physical subsystem, e.g., propulsion plant, or both. Mathematically speaking, a typical box in the midst of the hierarchal system is a converter transforming an input received from its "parent" black box, or from outside of the system, to an output transmitted to its "daughter" black box or to outside of the system. Consistent with the black box concept our attention here is on the input, output, and their transmission paths, but not on the details of the converter.

The system levels are numbered from the top, and the system is regarded as hierarchal if j-th black box at i-th level is linked to only one parent in a level above and is linked to no other black box at the same hierarchy level, although it may be connected to several black boxes below forming a cluster such as those indicated by dashed envelopes in Fig.1. In other words, there are no lateral transmission links in the system, and the output O_{ij} from a black box is a function of an input from its parent and its own design variables

$$O_{ij} = f(X_{ij}, I_{ij}) \quad (1)$$

where the input

$$I_{ij} = O_{pr} \quad (2)$$

and

$$r > i \quad (3)$$

The output defined in eq.1 may include also the design variables X_{ij}, if they are needed as inputs elsewhere in the cluster below.

As shown in [1], optimization of such a hierarchal system may be carried out by a procedure that begins with an initialization of all the constants and X's, fol-

lowed by a top-down sweep of individual analyses in which each black box output is generated and transmitted to the appropriate boxes below. Boxes at the same hierarchy level may be analysed concurrently since they are isolated from each other.

When the analysis sweep is completed, a sweep of individual optimizations performed in each black box proceeds from the bottom-up. An optimization problem solved in each black box except the top one is:

$$\min_{X_{ij}} C_{ij}(X_{ij}, I_{ij}) \tag{4a}$$

subject to constraints (STOC):

$$h(X_{ij}, I_{ij}) = 0 \tag{4b}$$

$$L_{ij} \le X_{ij} \le U_{ij} \tag{4c}$$

The equality constraints in eq.4b restrict the changes to X's so as to conform to the inputs received from the parent black box. The upper and lower limits in eq.4c include the physical bounds and the move limits temporarily imposed on the design variables.

The objective function of the problem, eq.4a, is a cumulative constraint representing inequality constraints g in box ij and the cumulative constraints C_{uv} from its daughters. A typical inequality constraint function is formulated by comparing an output quantity with its allowable value in a dimensionless expression constructed so that a positive constraint is a violated constraint, negative constraint is a satisfied constraint, and a zero constraint is critical (active)

$$g^k_{ij}(X_{ij}, O_{ij}) = \frac{O_{ij}}{A_{ij}} - 1 \le 0 \tag{5}$$

The allowable quantity in the above expression may also be a function of design variables

$$A_{ij} = f(X_{ij}) \tag{6}$$

The cumulative constraint used in [1] is a differentiable envelope of the constraint functions g taken in a form of the Kreisselmeier-Steinhauser function (KS function) from [2]

$$C_{ij} = KS(g_{ij}, C_{uv}) = \frac{1}{r} ln \left(\sum_k e^{r g^k_{ij}} + \sum_{u=i+1,v} e^{r C_{uv}} \right) \tag{7}$$

which has a property of

$$MAX(g_{ij}, C_{uv}) \le KS \le MAX(g_{ij}, C_{uv}) + \frac{ln(m)}{r} \tag{8}$$

where r is a user-controlled factor whose increase draws the KS closer to the maximum constraint, and m is a total number of constraints g_{ij} and C_{uv}.

Cumulative constraints C_{uv} in the formulation of the cumulative constraint C_{ij} are approximated as linear functions of X_{ij}

$$C_{uv} = \overline{C}_{uv} + \frac{\partial \overline{C}_{uv}}{\partial O_{ij}} \frac{\partial O_{ij}}{\partial X_{ij}} \Delta X_{ij} \tag{9}$$

In the above, the derivative $\partial \overline{C}_{uv}/\partial O_{ij}$ is an optimum sensitivity derivative in the sense of [3] or [4]. A vector of these derivatives is obtained by performing optimum sensitivity analyses using algorithms described in [3] and [4] of the optimum found by solving the problem stated by eq.4 for black box uv - a daughter of box ij (see Fig.1). The optimum sensitivity derivatives are obtained with respect to each element of the output O_{ij} transmitted from box ij to box uv. Thus, the optimizations in the black boxes are recursively related throughout hierarchy, and the reason that their executions have to begin at the bottom level is that the black boxes there have no daughters and, consequently, the C_{uv} constraints do not enter into the subotimizations at that level.

The derivative $\partial O_{ij}/\partial X_{ij}$ is a behavior sensitivity derivative obtained from behavior sensitivity analysis of box ij. It is assumed that such analysis is included in the top-down analysis sweep.

The physical meaning of optimization defined by eq.4 is that the black box design variables are manipulated so as to reduce as much as possible the constraint values (in other words, to achieve maximum feasibility) in the black box itself and in the entire cluster of the boxes related to it in the levels below, while conforming to constant inputs received from the parent black box. The system performance does not enter these optimizations. It is solely accounted for in the black box on the top of the hierarchy for which the optimum problem is defined as

$$minF(X_{11}), \quad STOC \tag{10a}$$

$$C_{uv} \le 0, \quad uv \in cluster \tag{10b}$$

$$g_{11} \le 0 \tag{10c}$$

$$L_{11} \le X_{11} \le U_{11} \tag{10d}$$

Inequality constraints C_{uv} in eq.10b are approximated as in eq.9 for the daughters that appear in level 2. Owing to the recursivity of the C_{uv} formulation in eq.4a, 7, and 9, these constraints have the effect of guarding against constraint violations in the entire hierarchy below the top level. Inequality constraints g_{11} in eq.10c represent the system level constraints, and eq.10d is analogous to 4c. The objective function in eq.10a is a measure of the system performance formulated so that the performance is maximized when this function is minimized.

Since the linearization errors occur due to the use of eq.9 in optimizations at all levels, the analytical data have to be updated after the optimization sweep is completed. Hence, the procedure alternates the analysis and optimization sweeps until convergence is attained.

The merit of optimization by decomposition lies in its breaking up of what would be a very large optimization problem, if all design variables were manipulated simultaneously, into a set of much smaller optimization problems many of which may be solved concurrently. It also subordinates all the lower level optimizations to the dominant goal of bettering the system performance while preserving the constraints of the entire system and those that are local to its parts. This automatically resolves the trade-offs occuring among the disciplines and physical subsystems represented by the black boxes in the hierarchy.

Application Experience

Since publication of [1], the above decomposition method was used in [5] to formulate optimization of structures by substructuring with unlimited number of levels. Numerical results presented in that reference demonstrated satisfactory accuracy and convergence characteristics of the algorithm.

As far as multidisciplinary applications are concerned, the decomposition method was reported in [6] as an effective tool for aerodynamics-structure-performance optimization of a glider, and [1] gave a status report limited to a problem formulation for a similar application to a transport aircraft wing. Now, a report on the experience obtained from that application became available in [7].

The object of the application was a transport aircraft depicted in Fig.2a. The objective function was block fuel consumption for an assumed typical commercial flight. A total of 1950 constraints were accounted for in the aircraft performance (e.g., the take-off field length, and the rate of climb), in the aerodynamics of the wing, and in the wing structure. The latter included detail stress and local buckling constraints in each of 316 individual wing cover panels. Optimization affected the wing shape and structure cross-sectional sizing only, while the remainder of the aircraft and its engines remained unchanged. Aerodynamic analysis of the wing pressure distribution was carried out by CFD panel code, and structural analysis was based on a finite element model depicted in Fig.2b.

A total of 1303 design variables were included ranging from those governing the airfoil shape to the detailed dimensions of the wing cover panel skins and reinforcing stringers. The optimization procedure outlined in [1] was organized in three levels shown in Fig.3 and defined in Table 1 which displays also the information transmitted between the levels. Optimization in each black box employed a nonlinear mathematical programing (NLP) optimizer.

Procedure Performance in Aircraft Application

The procedure performance reported in [7] was satisfactory as illustrated by the histograms for the fuel objective and structural weight in Fig.4. It demonstrated that the decomposition approach makes it possible to perform a NLP type of optimization for a complex system including diverse disciplines and using a large number of design variables that would be far beyond practical limitations of a conventional single level procedure. It has also shown for the first time that a rigorous, mathematical link may be established from a design detail all the way up to system performance.

Procedure Limitations

The hierarchal, top-down nature of the system depicted in Fig.1 makes it difficult to apply the above decomposition scheme to systems with lateral links, and to those whose analysis requires iterating between parent-daughter black boxes. Such systems are known as networks and some problems in aircraft design fall into that category. For example, consider a flexible wing with active controls described in [8] and, to keep the discussion simple, limit the active control in that case to just one function: load alleviation to reduce the root bending stress. Then, the information links among the black boxes of aerodynamics, structures, and active control form a system shown in Fig.5 in a graph-theoretic format. Indeed, the aerodynamic loads are input into structural analysis which outputs elastic deformations that affect the loads, a stress signal from the wing root is transmitted to the active control system whose actuators add hinge moments to the wing structure loads, the active control receives information (direct or indirect) about the wing pressure distribution it needs to decide how to move the control surfaces whose deflections are input into aerodynamic analysis.

The wing may be considered as a system whose output consists of the data on aerodynamic pressure, structural deformations, and active control hinge moments and deflection angles. This output is influenced by design variables of aerodynamic shape, structural sizing, and active control law coefficients (gains). Tha system is a non-hierarchal network in which there is no inherent mathematical reason (other then the historically evolved practice of considering aerodynamics first, structure next, and active control last) to place one black box above another. Even without active control the system would be a non-hierarchal one because

of the two-way link between aerodynamics and structures. Neglecting the elastic deformation feedback in that link made it possible to decompose the test case in [7] into a pure top-down hierarchy. It was justified for a long range transport that spends most of its life in a cruise mode in which that feedback may be compensated for by building the wing to a jig shape that offsets the elastic deformation effect on aerodynamics. However, this assumption does not carry over to a multimission aircraft such as a fighter.

The need to optimize non-hierarchal systems was addressed by developing a method for sensitivity analysis which may be coupled with judgmental and formal optimization. That method to be introduced next is meant to be an efficient substitute for the inaccurate and often cost-prohibitive, but currently prevailing, approach of computing sensitivity derivatives of complex, coupled systems by finite differencing that requires repetition of the entire analysis of the system for every design variable perturbation.

Non-Hierarchal Sensitivity Analysis and Optimization

A method for solving the system sensitivity problem was developed in [9] from the implicit function theorem, and initial experience with its use was reported in [10,11] and also in [8] and [12] in the program of this congress (ICAS-16).

Sensitivity Analysis

The generic sensitivity analysis method from [9] is introduced here using as an example an actively controlled wing shown as a network system in Fig.5. For the purposes of sensitivity analysis a network system is abstracted in a manner shown in Fig.6. The outer box represents the system made up of internal black boxes labeled A, S, and C for aerodynamics, structures and controls, respectively. As in the preceding section, each black box is regarded as a set of mathematical operations that convert input listed in the inner parentheses to output denoted Y subscripted with the label of the box, e.g., Y_A. Coupling of the inner black boxes illustrated in Fig.5 by arrows is reflected in Fig.6 by the outputs being fed to the inputs, also illustrated by arrows. The design variables are denoted by X and unlike in the definition used in the preceding section they are not included in the outputs Y. In the general case, the quantities Y and X are vectors. Usually, the data vectors X and Y are input selectively. For instance, the subset of Y_A entered into B may be different than the subset of Y_A entered into C, although the subsets may overlap. This selectivity is tacitly understood but not reflected in the notation in order to keep the nomenclature simple.

In more precise mathematical terms, A, B, and C are vectors of functions of the arguments shown in the corresponding parentheses, and setting them to zero forms the set of simultaneous equations that govern the system. The number of equations in each box is sufficient to solve for the unknown elements of its output vector Y. A set of vectors Y constitutes a solution of the system if the Y's substituted simultaneously in A, B, and C produce zeros on the right hand side. In many practical applications, such solution can only be found by iterating among the black boxes, as in the case of converging aerodynamic loads and elastic deformations of a wing using nonlinear aerodynamic analysis.

Sensitivity analysis from [9] enables one to calculate the sensitivity derivatives of the system solution with respect to a design variable X_k from a set of simultaneous equations, termed sensitivity equations

$$\begin{bmatrix} I & \frac{-\partial Y_A}{\partial Y_B} & \frac{-\partial Y_A}{\partial Y_C} \\ \frac{-\partial Y_B}{\partial Y_A} & I & \frac{-\partial Y_B}{\partial Y_C} \\ \frac{-\partial Y_C}{\partial Y_A} & \frac{-\partial Y_C}{\partial Y_B} & I \end{bmatrix} \begin{Bmatrix} \frac{dY_A}{dX_k} \\ \frac{dY_B}{dX_k} \\ \frac{dY_C}{dX_k} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial Y_A}{\partial X_k} \\ \frac{\partial Y_B}{\partial X_k} \\ \frac{\partial Y_C}{\partial X_k} \end{Bmatrix} \quad (11)$$

Regardless of the mathematical nature of the governing equations of the system and regardless whether they do or do not require iteration for solution, these sensitivity equations are always linear and algebraic.

The sensitivity derivatives appear in eq.11 as the vector of unknowns, the partial derivatives of outputs with respect to a particular design variable X_k form the right hand side vector, and the matrix of coefficients is independent of the design variables. The matrix is formed from identity submatrices along the diagonal and the matrices of partial derivatives of a black box output with respect to its input (the Jacobian matrices) positioned off the diagonal. These partial derivatives appear only where output from a particular black box is affected by input received from another black box, so that the matrix of coefficients reflects the system couplings. Consequently, this matrix needs only be formed and factored once for a given system, and then backsubstituted with as many right hand side vectors as there are design variables for which one wishes to obtain the sensitivity derivatives.

The partial sensitivity derivatives entered in the matrix of coefficients and the right hand side vectors are by definition computable from each black box treated in isolation from each other, and they may be computed concurrently. Thus, it may be said that the system is being decomposed for the purposes of sensitivity analysis and, yet, the solution vector of eq.11 produces the system sensitivity derivatives that fully account for coupling among the black boxes. In this vein,

it may be instructive to observe that it is the presence of the off-diagonal submatrices that makes the vector of unknowns in eq.11 different from the right hand side vector. In other words, in a system of uncoupled black boxes the system sensitivity is the same as that of each black box directly affected by a particular design variable, but it is not so when the black boxes are coupled.

To illustrate the physical meaning of partial derivatives in eq.11, consider the submatrix in the upper right hand corner of the matrix of coefficients, assuming that the aerodynamic analysis outputs N_a pressure coefficients at discrete locations at the wing, and that the active control system influences the wing by N_c control surfaces and receives pressure data from N_p sensors on the wing. A column in that submatrix contains N_a partial derivatives of the pressure coefficients with respect to a particular control surface deflection angle, so that there are Nc columns in the submatrix.

The opposite submatrix in the lower left corner contains N_p columns, each containing N_c partial derivatives of the control surface deflections with respect to the reading of a particular pressure sensor. The two submatrices are not mutually symmetric - a point to bear in mind when choosing an off-the-shelf program to solve the sensitivity equations.

Let us assume that the design variables of interest are the sweep angle and a composite laminate orientation angle in the wing cover. The sweep angle directly affects the aerodynamics and structures, but not the active control. Hence, its right hand side vector will have a null partition corresponding to the active control as seen in the first term of eq.12:

$$\begin{Bmatrix} \partial Y_A/\partial X_k \\ \hline \partial Y_S/\partial X_k \\ \hline 0 \end{Bmatrix}, \begin{Bmatrix} 0 \\ \hline \partial Y_S/\partial X_k \\ \hline 0 \end{Bmatrix} \qquad (12)$$

The laminate orientation angle affects directly the structual deformations only, so that its right hand side vector will have null partitions corresponding to the aerodynamics and active control as in the second term of eq.12. However, the system sensitivity derivatives obtained from eq.11 for two such design variables will, in general, be nonzero for all Y's, that is, for the aerodynamic pressure coefficients, structural deformations, control surface deflections, and for the hinge moments.

The above introduction to system sensitivity analysis is based for simplicity on an example of only three black boxes. However, it establishes a pattern that, as [9] shows, extends readily to an unlimited number of black boxes. The pattern is also a recursive one because a black box may be regarded as a system within itself. Additional examples of the use of the system sensitivity analysis are given in [8,9,10,11] and [12].

Availability of Partial Sensitivity Derivatives

The partial derivatives needed to build the matrix of coefficients and the right hand sides of eq.11 may be generated, in order of preference, analytically, semi-analytically, and by finite differencing. Regarding the latter, one should emphasize that this finite differencing is performed on an isolated black box, therefore, it is not nearly as computationally costly as the finite difference performed on the entire system analysis and it should also be more accurate (see [9] for the cost and accuracy discussion).

The fact that the partial derivatives are generated, by definition, within each black box separately is an important advantage because it enables one to use specialized sensitivity analysis methods whose development has recently been rapidly progressing in many engineering disciplines as evidenced by large number of papers collected in [13], and a survey in [14]. In structures, sensitivity analysis has become available in a major, production-level program, [15]. In computational aerodynamics, at least one production level program is now available, [16], and a generic sensitivity analysis method based on the implicit function theorem and proposed in [17] has been built upon in [18] and [19]. On the other hand, the rapidly increasing speed of computers makes it also viable to perform aerodynamic sensitivity analysis by finite differencing as demonstrated in [8] and [11]. Thus, the two disciplines known for their relatively greatest demand put on the computer resources are moving toward making sensitivity analysis routinely available. It appears that other disciplines will follow that lead.

Another important advantage of the black box formulation underlying eq.11 is that it accepts the partial sensitivity derivatives obtained experimentally, or from any other external source, e.g., statistics or judgment. That was not possible in a conventional finite differencing performed on the entire system.

System Sensitivity Derivatives in Design Process

Optimization: Formal and Judgmental

The sensitivity analysis is shown incorporated in a procedure for quantitative support to design process in a flowchart in Fig.7. The first task of the procedure is to obtain a converged (but not necessarily feasible) solution of a trial design of the system by any suitable method. The next set of separate tasks is to calculate partial sensitivity derivatives for each black box in the

system - these tasks may be executed concurrently - and collecting these derivatives in equations such as eq.11. Once the system sensitivity derivatives are obtained, they may be used to guide the design toward improvement, either by judgmental modifications, or by execution of formal optimization, or both.

Used judgmentally, the system sensitivity derivatives indicate by their relative magnitude which design variables are the most influential ones and whether their influence is positivie or negative. This is very useful information for deciding how to modify the design. Without sensitivity analysis, that information tends to be obscured in complex systems by conflicting trends and trade-offs.

In conjuction with formal optimization, the system sensitivity derivatives are used to establish an approximate model of the system. A linear extrapolation equations based on these derivatives are the simplest example of such a model that may be attached to an optimization algorithm which will, then, modify the design toward improving its measure of performance (objective function) within constraints, and within move limits which should be judiciously set by the user to avoid excessive extrapolation errors. Example of a suitable optimization algorithm is the usable-feasible directions method from [20]. When the optimization algorithm attains its termination criteria, the system design variables are updated and its analysis and sensitivity analysis have to be repeated to refresh the approximate model data before the next optimization may begin.

Derivative-Based Approximate Model of System Behavior

Effectiveness and efficiency of the above process is strongly influenced by the move limits which in turn depend on the nonlinearity of the problem at hand. High nonlinearity forces narrow move limits and frequent updates of the system analysis. Therefore, results reported in [21] are of interest here because they show that at least some of the physical phenomena encountered in aircraft design are only mildly nonlinear so that the derivative-based linear extrapolations are good approximation of the true behavior within fairly wide move limits. For example, Fig.8 shows an elastic wing trimmed angle of attack as a function of the wing sweep angle for a converged aerodynamic loads and structural displacements (it also shows how much the loads-displacements coupling affect that function: its slope is actually reversed comparing to that of a rigid wing). The function slope is a derivative of the aerodynamic-structure coupled system that could have been calculated from sensitivity equations such as eq.11. The figure shows that the extrapolation based on such derivative would be quite accurate over the sweep angle interval of about +- 25% of the reference sweep angle. Examination of similar functions in aircraft design literature shows this to be a fairly typical situation, although exceptions do occur, therefore, it is important to keep human intelligence in the process.

System Sensitivity Derivatives for Interdiciplinary Communication

Recognizing the central role of human intellect in design, one can think of a design process organization in which the system sensitivity derivatives would be used as principal means of communication regarding the quantitative side of design among the disciplinary specialists, as shown in Fig.9. The scheme depicted in the figure is an adaptation of the flowchart from Fig.7 to the workings of an engineering design organization. It calls upon the specialists to generate information in their disciplines, and to augment it with the partial sensitivity derivatives of their outputs with respect to inputs and to the design variables. After the partial sensitivity derivatives are used in the system sensitivity equations to calculate the system sensitivity derivatives, the specialists are being called upon again to decide on the design modifications using system sensitivity derivatives with the aid of formal optimization and, or, judgmentally including due consideration to the non-quantitative aspects of design.

Application to Entire Aircraft Configuration Treated as a System

Although experience with the use of the system sensitivity derivatives available to date and referenced above is limited to aircraft subsystems such as a wing, the concept is readily extendable to include entire aircraft considered as an engineering system.

This may be shown by examining a typical textbook aircraft design procedure, for example the one from [22] illustrated in Fig.10. Consistent with the prevailing practice, this procedure is a sequential one, so that as implied by the module labeled "CHANGE WEIGHT, WING & ENGINE SIZE" in the upper right hand corner, it would be repeated for each design variable perturbation in order to assess the influence of that variable on design.

Casting the set of modules in Fig.10 as a set of coupled black boxes results in a system shown in a graph-theoretic format in Fig.11 - analogous to Fig.5. Information transmitted between the boxes is represented by vectors Y_i - the subscript identifies the vector source box. The figure shows also the design variables X input into the black boxes. Examples of the content of the Y and X vectors are given for each black box in Table 2. Consistent with Fig.6, Fig.11 does

not show that transmissions of the data from one box to another and the X inputs are selective.

The sensitivity equations for the system from Fig.11 take on this form

$$\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 \\ -Y_{21} & I & -Y_{23} & -Y_{24} & -Y_{25} & 0 & -Y_{27} \\ -Y_{31} & 0 & I & 0 & 0 & 0 & 0 \\ -Y_{41} & -Y_{42} & -Y_{43} & I & -Y_{45} & 0 & -Y_{47} \\ -Y_{51} & 0 & 0 & 0 & I & 0 & 0 \\ 0 & -Y_{62} & 0 & 0 & -Y_{65} & I & 0 \\ -Y_{71} & -Y_{72} & -Y_{73} & -Y_{74} & 0 & 0 & I \end{bmatrix}$$

$$\begin{Bmatrix} \frac{dY_1}{dX_k} \\ \frac{dY_2}{dX_k} \\ \frac{dY_3}{dX_k} \\ \frac{dY_4}{dX_k} \\ \frac{dY_5}{dX_k} \\ \frac{dY_6}{dX_k} \\ \frac{dY_7}{dX_k} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial Y_1}{\partial X_k} \\ \vdots \\ \frac{\partial Y_i}{\partial X_k} \\ \vdots \end{Bmatrix} \cdots \begin{Bmatrix} \frac{\partial Y_1}{\partial X_L} \\ \vdots \\ \frac{\partial Y_i}{\partial X_L} \\ \vdots \end{Bmatrix} \cdots \quad (13)$$

in which, for compactness, Y_{ij} denotes the partial derivative of Y_i with respect to Y_j. In these equations the matrix of coefficients is block-sparse because the system in Fig.11 is not fully coupled. Solution of these equations yields a measure of influence of the design variables X_k on design through the sensitivity derivatives of all the Y vectors with respect to these variables, without finite differencing or performing parametric studies implied in Fig.10.

Sensitivity Analysis vs. Parametric Study

Since the sensitivity study approach is a prevailing industry practice to achieve quantitative improvement of design, it should be useful to compare the information generated by such study with that produced by system sensitivity analysis coupled with optimization. As pictured by an example in Fig.12 (from [22]), a parametric study determines a function character over the entire range of interest and tells whether extrema exist and where are they located, but it does that for one variable at a time at the price of solving the system at discrete points within that range. A minimum of three data points along one axis are needed to establish the simplest nonlinear function in this manner, hence, one needs 3^n points to do that for n design variables. Skillful engineers use intuition and judgment about relative importance of the design variables and combine variables into groups to keep the number of data points required within practical bounds, nevertheless, in advanced projects where little guidance from the past experience is available the pressure on that number to escalate beyond acceptable limits is relentless.

In contrast, sensitivity analysis provides the function slope information at a single point but may do it for all n design variables at hand while solving the system only once. Furthermore, that multivariable slope information may be translated by a single inexpensive execution of an optimization algorithm such as the one from [20] into a pointer in the design space of n variables showing how to modify the design in order to realize an improvement of its objective function within constraints - an advantage which the human mind can hardly match for n greater than 2 or 3. Its drawback is that a piecewise linear path has to be traced toward optimum - a process during which it is easy to lose the physical insight and understanding of the reasons that drive the design changes.

One may assert that tools are now available to evolve a compromise practice that will exploit the best characteristics of each approach. That is to use the sensitivity analysis coupled with optimization to navigate the design space toward improvement, and to rely on the parametric studies in conjunction with computer graphics to visualize the system performance, and the critical and near critical constraints, as functions of a limited number of dominant variables (selected on the basis of their sensitivity derivatives) in the vicinity of the improved design. That combined approach should be efficient computationally and would still provide the physical insight necessary for an engineer to develop confidence in the design he is evolving.

Conclusions

Several new tools have become available to designers of complex enginering systems of which aircraft is a prime example. The common problem addressed in developing these tools is the control of interactions that occur among disciplines and physical subsystems in order to improve the entire system performance.

One such tool is optimization by decomposition illustrated by an example of a transport aircraft wing optimization for improved fuel economy. The method demonstrated the ability to handle in excess of a thousand design variables and to link the design detail with system performance, provided that the system may be decomposed into a strictly top-down hierarchy.

That limitation may be removed by new method based on sensitivity analysis of a complex, coupled system which yields derivatives of the system behavior with respect to design variables fully accounting for the interactions among the parts of the systems and among the disciplines that govern its design. The sensitivity derivatives of the system are computed from the partial sensitivity derivatives of its parts. These partial derivatives may be generated by specialized discipli-

nary sensitivity analysis methods currently undergoing vigorous development, and they may also be obtained experimentally. This new system sensitivity analysis enables one to bypass the heretofore prevailing approach of finite differencing performed on the entire system analysis.

The system sensitivity data may be used to determine how to improve the design, either by quantitatively supported judgment, or by formal optimization, or both. They provide a numerically precise and comprehensive answer to the "what if" questions frequent in design process, and may be regarded as a communication device informing each specialist supporting that process how his decisions will affect the other specialists' domains and the system as a whole.

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TABLE 1. COUPLING DATA FOR SYSTEM IN FIG. 3

Box 1	**Aircraft performance and aerodynamic analysis. Design variables: airfoil relative thickness. Objective funtion: fuel consumption.**
Box 2	**Structural finite element analysis. Design variables: wing cover equivalent skin thicknesses.**
Box 3	**Strength, buckling, and local buckling of the wing cover panel skin and reinforcing stringers for each of 316 panels. Design variables: detailed dimensions of of the cross-sections for each panel.**
Arrow 1	**Aerodynamic loads, flight parameters, load factors, configuration data.**
Arrow 2	**Edge forces, equivalent skin thicknesses including stringer material.**
Arrow 3	**Minimized cumulative constraint and its optimum sensitivity derivatives. Cumulative constraint represents strength and buckling constraints of the wing cover panel.**
Arrow 4	**Minimized cumulative constraint and its optimum sensitivity derivatives. Cumulative constraints represents the wing box constraints and the cumulative constraints of the individual panels.**

TABLE 2. EXAMPLES OF COUPLING DATA FOR SYSTEM IN FIG. 11.

Vector Y	Content Examples
1	**See the box labeled INPUT.**
2	**Wing area, aspect ratio, taper, sweep angle, airfoil geometry data. Engine thrust.**
3	**Fuel tank locations and assumed volumes.**
4	**Wing structural weight and internal volume.**
5	**Take-off Gross Weight.**
6	**See box 6.**
7	**Landing gear weight and location, in stowed and extended position. Take-off field length.**

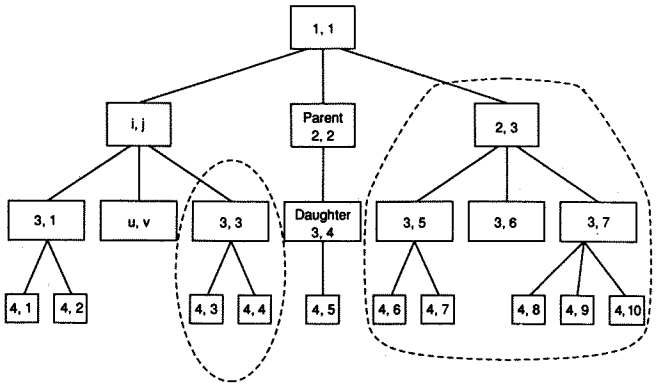

FIGURE 1. Example of a hierarchal system.

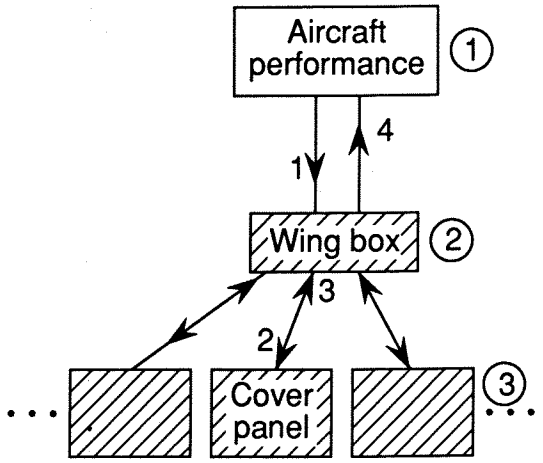

FIGURE 3. Hierarchal, three-level decomposition for aircraft wing optimization.

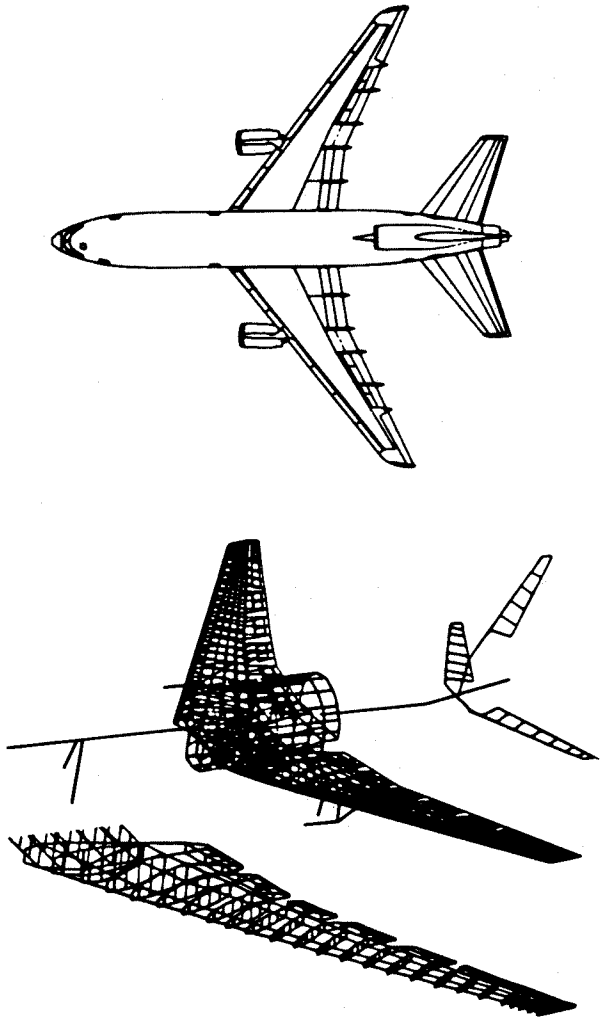

FIGURE 2. A transport aircraft and its finite element model.

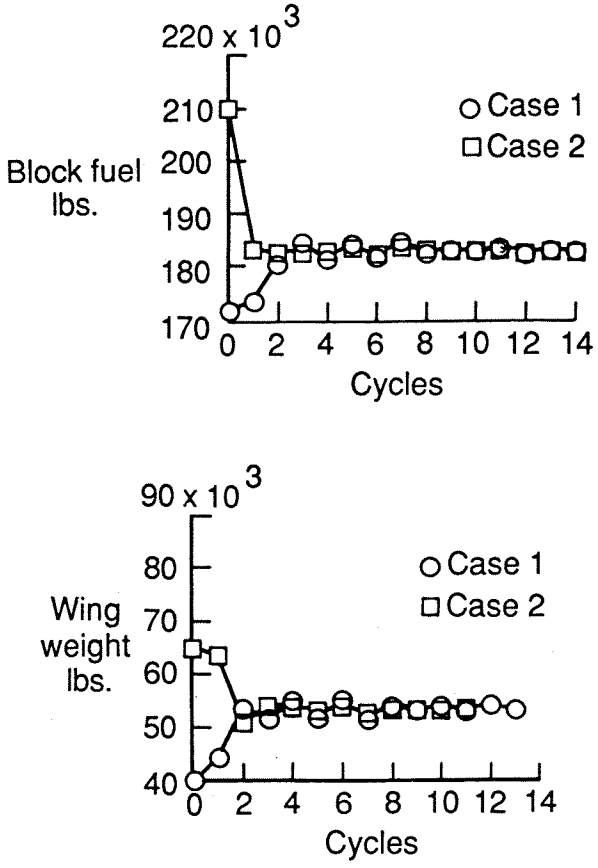

FIGURE 4. Histograms of three-level optimization of a transport aircraft; cases 1 and 2: initial design infeasible and feasible, respectively.

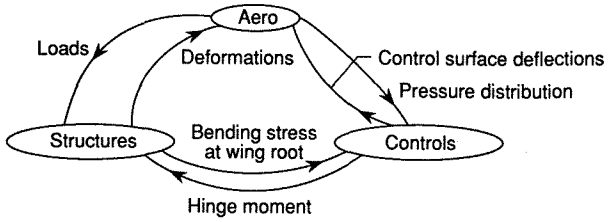

FIGURE 5. Graph representation of actively controlled, flexible wing as an example of a coupled system.

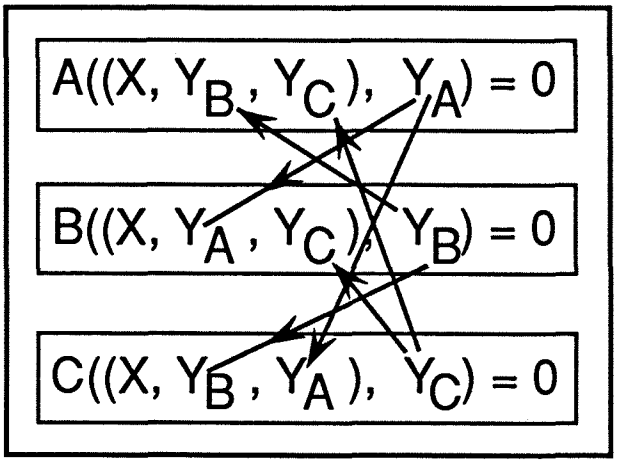

FIGURE 6. Governing equations of the actively controlled, flexible wing as an example of a coupled system.

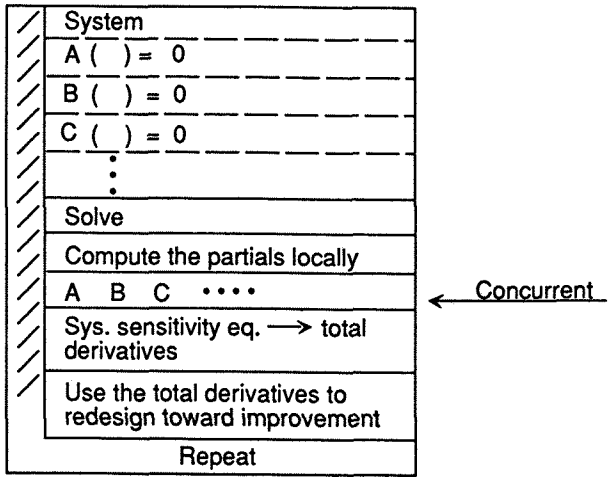

FIGURE 7. Flowchart of a procedure for quantitative support of design process incorporating system sensitivity analysis.

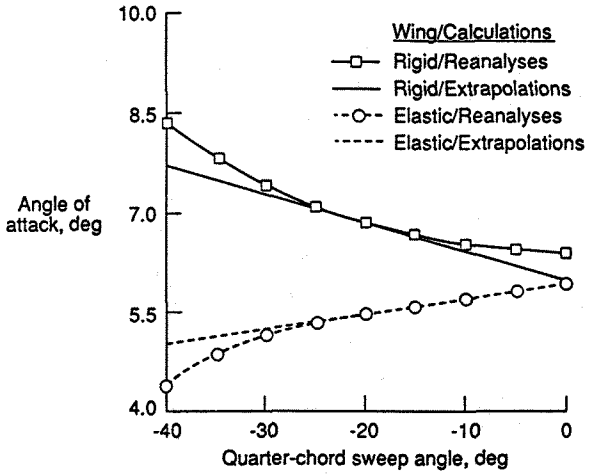

FIGURE 8. Trimmed angle of attack as a function of the sweep angle for rigid and elastic wing.

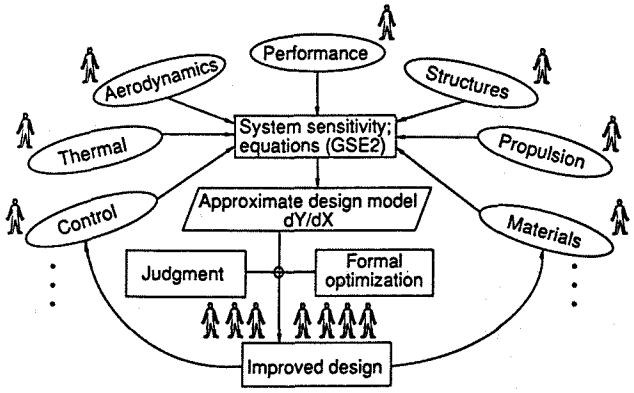

FIGURE 9. System sensitivity analysis as means of interdisciplinary communication in a design organization.

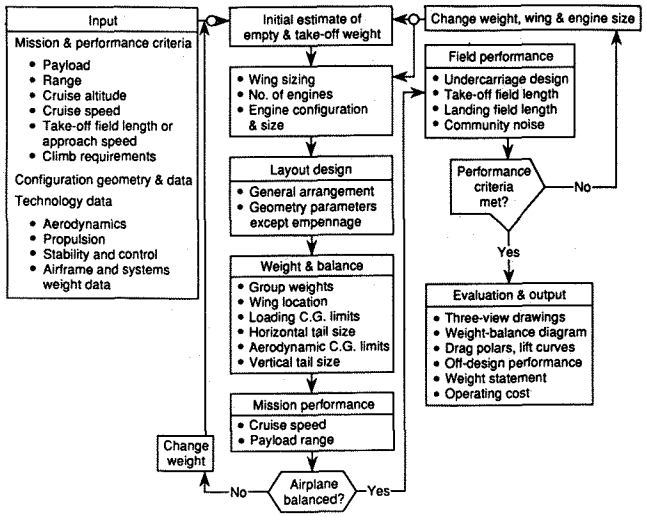

FIGURE 10. Aircraft design process arranged sequentially.

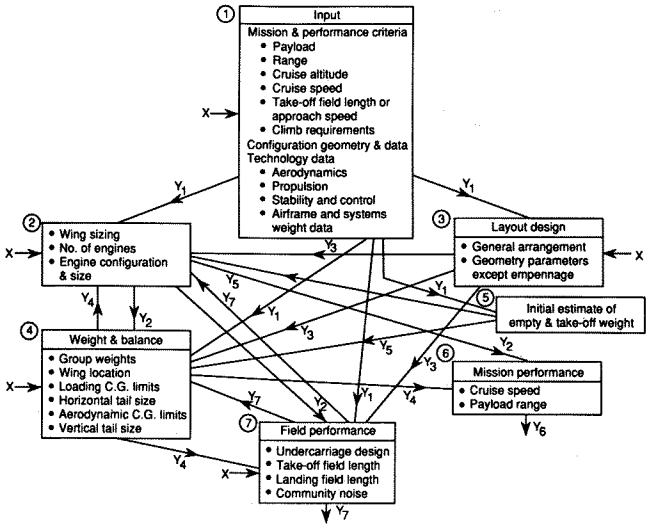

FIGURE 11. Aircraft design process rearranged from a sequential procedure into a non-hierarchal decomposition shown as a graph amenable to sensitivity analysis.

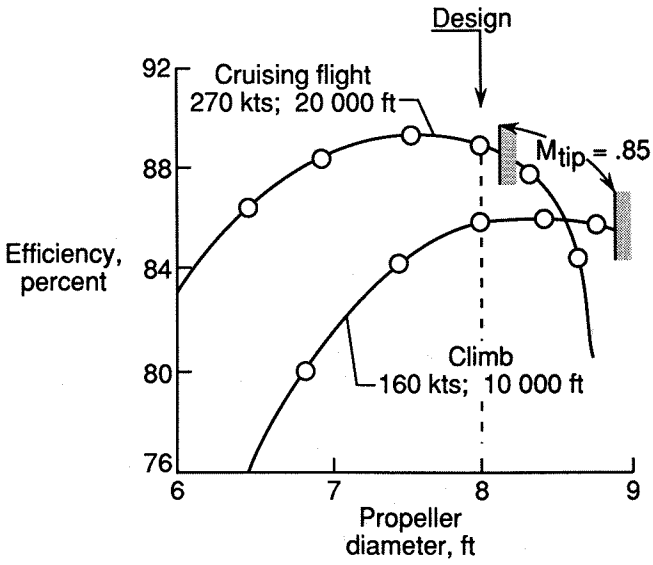

FIGURE 12. Typical parametric study results.