COMPUTER-AIDED STRUCTURAL OPTIMISATION OF AIRCRAFT STRUCTURES

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Abstract

This paper describes the principal methods used within the computer program STARS for the computer-aided design of optimum structures subject to a variety of constraints.

Based on the foundations set forth regarding both structural and optimisation aspects, a description is given of the NEWTON Method as applied in STARS. Likewise, the shape optimisation developed in the form of a hierarchical approach is described. Test problems connected with this are presented.

Practical examples are given that show how research originated at RAE has been continued and applied at MBB. This includes various components which are typical in aircraft construction and also a description of the manner in which flutter optimisation is being accomplished with STARS at MBB in combination with the in-house aeroelastic program.

1 INTRODUCTION

The design of structural components is an iterative process in which the aim is to achieve a structure which is adequate in strength and stiffness, favourable to manufacture and inexpensive: that is, in some sense, an optimum design. The design procedure can take a very long time if approached conventionally, and it is unlikely that components will in fact be optimised in detail against all important criteria.

The intensive use of computer methods, involving finite element codes together with CAD systems and FE pre- and post-processors, has provided an important step towards shortening the design process, and structural optimisation provides a further valuable aid in this context.

Time and cost benefits have been found from using structural optimisation for:

- weight assessment of designs using various constructions and materials at a pre-dimensioning stage, since it is then that essential decisions are taken with regard to shaping of components or assemblies;
- modification of structures, often at short notice, in the case of changes of specification, load changes and the occurence of resonance or flutter;
- further weight reduction during productionisation.

The references [1, 2, 3, 4] contain general information on the state-of-the-art in structural optimisation. This paper outlines the principal methods used in the program STARS and shows how the research originated at RAE has been continued and applied at MBB.

2 PRINCIPAL METHODS

2.1 Foundations

2.1.1 Structural Aspects. The presumption in STARS is that we are dealing with large structural models not amenable to exact solution. Thus the approach is based on the finite element method in which the equations of linear elasticity are reduced to a set of simultaneous algebraic equations

$$\mathbf{K}\, u = p, \tag{2.1.1}$$

where the unknown displacement freedoms u are related to prescribed load vectors p by the stiffness matrix **K**. The stiffness matrix is a symmetric, positive-definite, banded matrix and these equations may be readily solved by Choleski factorisation. The size of the system of equations is such, maybe involving 1000 - 10000 freedoms, that analyses must be called upon extremely sparingly.

The most straightforward structural optimisation problem would seek to minimise the weight

$$W = \sum_{k=1}^{N} \rho_k A_k l_k \tag{2.1.2}$$

with respect to the individual cross-sectional areas A of bars or, correspondingly, the thicknesses of plates, while satisfying behavioural constraints of the form

$$g_i = e_i^T u \leqslant c_i , \tag{2.1.3}$$

where ϱ and l are the density and length of bar elements and e is a vector of coefficients which, together with the bounds c, defines the constraint. The constraint g has an implicit dependence on A through **K** in the governing equation (2.1.1).

Even to reduce the size of the optimisation problem to the order of 50 - 1000 design variables requires the use of devices such as design variable linking. The use of design variable linking also has further advantages, such as providing the means to impose symmetry or fabrication requirements or to embody the designer's insight and prior experience. In addition, apart from problems involving simple bars or beams which can be modelled exactly, failure to employ design variable linking correctly will lead to false solutions in which the optimised structure is not correctly modelled by the analysis mesh. In such a case there is a danger that optimisation will merely serve to increase analysis errors.

Hence, in the following, it is assumed that elements are linked into groups, each controlled by a single design variable x. The areas A are then given by:

$$A_k = \sum_{j=1}^{n} a_k \mathbf{B}_{kj} x_j , \; k = 1 \ldots . N , \tag{2.1.4}$$

where **B** is a Boolean matrix and a is a reference area for each element. The structural weight reduces to

$$W = \sum_{j=1}^{n} w_j x_j = \sum_{j=1}^{n} w_j / z_j \tag{2.1.5}$$

where the coefficients w are component masses given by

$$w_j = \sum_{k=1}^{N} \rho_k a_k l_k \mathbf{B}_{kj} \tag{2.1.6}$$

and z are reciprocal variables, $z_j = 1/x_j$, whose use is discussed later.

To be efficient, an optimisation method requires knowledge of design sensitivities of the constraints with respect to these variables. STARS maintains a tight active-set strategy and therefore requires relatively few sensitivities to be calculated at any iteration. It uses fully analytic derivatives, to be contrasted with the semi-analytic approach employed within NASTRAN, and the calculation employs the adjoint, or pseudo load, method.

That is, analytic differentiation of the governing structural equation yields

$$\nabla u = -\mathbf{K}^{-1} \nabla \mathbf{K}\, u, \tag{2.1.7}$$

where ∇ is the gradient operator denoting vectors of partial derivatives with respect to z. As it stands this expression gives the sensitivity of the entire structural response to design change. To calculate the sensitivity of a particular active constraint $g(z) = e^T u$, the constant vector e is treated as an adjoint load and the equation

$$\mathbf{K}^T v = e \tag{2.1.8}$$

solved using previously calculated Choleski factors. The combined effect of the use of reciprocal variables and design variable linking is to give a form for the derivatives

$$\nabla_g = v^T \left[\sum_{k=1}^{N} \frac{a_k \mathbf{B}_{kj} \mathbf{K}_k x_j^2}{A_k} \right] u \tag{2.1.9}$$

involving a summation over all the elements linked to a given design variable.

Next we turn our attention to the standard equations of mathematical optimisation, since it is they that provide the context in which mathematical programming and engineering methods must be understood.

2.1.2 Optimisation aspects. In structural optimisation the primary goal is to satisfy a set of inequality constraints

$$g_i(z) \leqslant c_i, \quad i = 1 \ldots m, \tag{2.1.10}$$

which establish the behavioural response considered acceptable for the structure. At the same time there is also the secondary goal of minimising some objective functions, for aerospace application this is usually the weight W(z), with respect to design variables, here denoted by z.

Some mathematical programming techniques based on hill-climbing approaches address this problem directly. However, for the present purposes, it is better to employ an equivalent formulation based on use of the Lagrangian function $L(z, \lambda)$, where

$$L(z, \lambda) = W(z) + \sum_{i=1}^{m} \lambda_i (g_i - c_i). \tag{2.1.11}$$

The Lagrangian function depends on two sets of variables: the primal variables z and the dual variables λ (otherwise known as Lagrange undetermined multipliers). A necessary condition for the minimisation of the original constrained optimisation problem is that the Lagrangian function should be stationary with respect to both primal and dual variables. Differentiating yields the well-known Kuhn-Tucker conditions:

$$\frac{\partial W(z)}{\partial z_j} + \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial z_j} = 0 \qquad j = 1 \ldots, n \tag{2.1.12 a}$$

$$\left.\begin{aligned} \lambda_i (g_i - c_i) &= 0 \quad &(2.1.12\,b) \\ \lambda_i &\geqslant 0 \quad &(2.1.12\,c) \end{aligned}\right\} i = 1 \ldots, m$$

The location of a stationary point thus requires the solution of a system of (n+m) simultaneous non-linear equations, as difficult a task as the original minimisation problem!

Nonetheless, this Lagrangian form offers a variety of insights, both for the original, primal problem and in providing the basis for deriving the dual problem, a maximisation problem [5].

Whichever method of optimisation is adopted, the purpose remains the same, the satisfaction of the Kuhn-Tucker necessary conditions. The goal is to achieve this effectively and economically, particularly for the large structural system optimisation problems where a vast number of design freedoms may exist. The very success of non-mathematical programming techniques such as the stress-ratio method and optimality criterion methods show that it should be possible to make good progress.

2.2 Newton-Based Methods

As discussed elsewhere [6], methods derived from considerations specific to the optimisation of engineering systems, and owing little to classical mathematical programming techniques, led to fully-stressing design and to optimality criterion methods. Each sets up formulae which may be applied iteratively to solve equations representing a sub-set of the Kuhn-Tucker conditions. In each instance limitations to the applicability of the methods arise from the fact that the whole of the set of equations is not addressed simultaneously.

The goal of the STARS Newton method was to evolve a technique, based as rigorously as possible on mathematical concepts, but without losing the features which made the engineering-intuitive techniques work so well on large problems.

The emphasis was on complementing the optimality criterion method by providing estimates for the dual variables. It is assumed that the critical constraints have been identified, and so the Kuhn-Tucker equations are formulated for a set of active equality constraints.

As a first step towards solving the set of nonlinear simultaneous equations representing the Kuhn-Tucker conditions, a linear approximation is formed about the current point, giving equations

$$(\nabla W + \sum_{i=1}^{m} \lambda_i \nabla g_i) + \left[\nabla^2 W + \sum_{i=1}^{m} \lambda_i \nabla^2 g_i \right] \delta z + \sum_{i=1}^{m} \nabla g_i \, \delta\lambda_i = 0 \qquad (2.2.1\,a)$$

$$(g_i - c_i) + \nabla g_i \, \delta z = 0\,, \qquad (2.2.1\,b)$$

which determine the iteration step $\delta z, \delta \boldsymbol{\lambda}$. For brevity, gradient and second partial derivative matrices have been denoted by ∇W and $\nabla^2 W$ and the summation over j representing the inner-product with δz is not shown explicitly. Like any application of Newton's method, the repeated solution of this set of linear equations does not necessarily converge; but provided the starting point lies within the domain of convergence, then that convergence will be quadratic. Unfortunately the requirement that second derivatives should be provided for all constraints in the active set requires excessive computation.

Thus, rather than employing an exact Newton step, the equations are further approximated by neglecting second derivatives of the constraints. Such approximations are already implicit in both the stress-ratio and optimality criterion methods, and are known to be exact for statically determinate structures, optimised with respect to reciprocal variables z.

$$\left[\frac{2 w_j}{z_j^3} \right] \delta z + \sum_{i=1}^{m} \nabla g_i \, \delta\lambda_i = \left\{ \frac{w_j}{z_j^2} \right\} - \sum_{i=1}^{m} \lambda_i \, \nabla g_i \qquad (2.2.2a)$$

$$\nabla g_i \, \delta z = -(g_i - c_i) \qquad (2.2.2b)$$

For a more general class of problems, this need to depart from the strict Newton form will lose the quadratic convergence properties, indeed it is quite possible that the iteration may diverge from any solution. In practice, however, many structural problems appear to be exceptionally well-behaved, giving good convergence to minimum weight designs.

Omitting the second derivatives of constraints, in fact, gives a very simple form for the linear equations. The weight as objective function is convex and separable, giving a diagonal Hessian matrix with positive coefficients. Thus, each of the sets of equations (2.2.2 a) may be used to eliminate one of the primal variables from (2.2.2 b) explicitly, giving a reduced system of equations in which the dual variables are the unknowns. When these are found they may then be substituted into the linearised optimality equations (2.2.2 a).

The step taken by the Newton algorithm within STARS can be shown to be equivalent to an optimality criterion step combined with a weighted least-squares restoration step which moves into the tangent space of constraints [6]. The effect of constraint curvature is also discussed in some detail in the same reference.

The Newton method is the primary technique used in the industrial applications which follow in section 4. Before moving to such applications it is shown how equations similar to (2.2.2) are also obtained by considering the dependence of a size-optimised structure on an embedded parameter, representing geometric or material variation.

3 SHAPE OPTIMISATION

In order to enable the large system optimisation problems to be solved efficiently, considerable simplification of the design problem has been assumed: neither change of geometry nor material has been considered.

As a first step to broaden the basis of the optimisation, a one-parameter search over geometry has been considered. Rather than simply expanding the dimension of the design space, a hierachical approach [7]has been adopted in which the parameter is used to move through a sequence of size-optimised designs. This both capitalises upon the achievement of efficient size optimisation and enables the shape optimisation to be terminated at any point with an efficient size-optimised design.

Firstly, straightforward size optimisation is performed for the extreme aspect ratios. In each case selection of an optimum thickness variation of the reinforcement leads to a considerable reduction in the concentration of the von-Mises equivalent stress, which was used as a strength criterion for the sheet material.

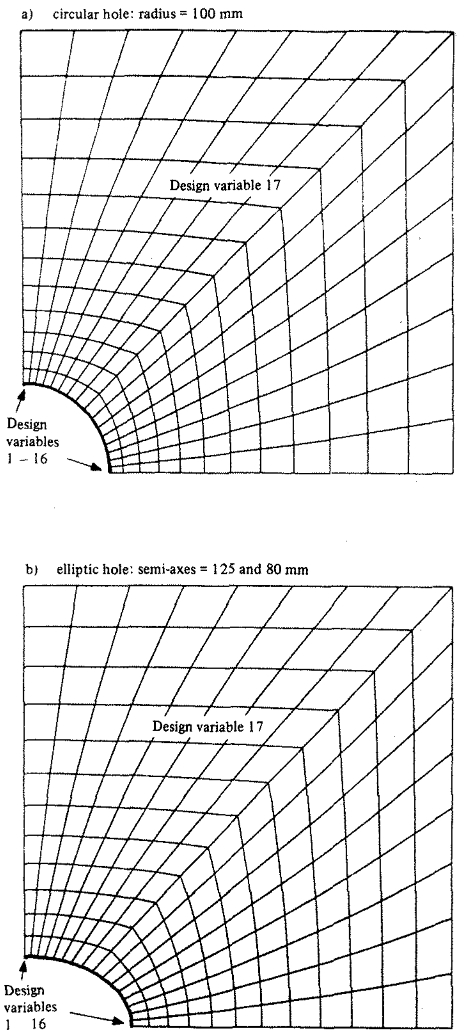


Fig. 1 Meshes used for shape optimisation

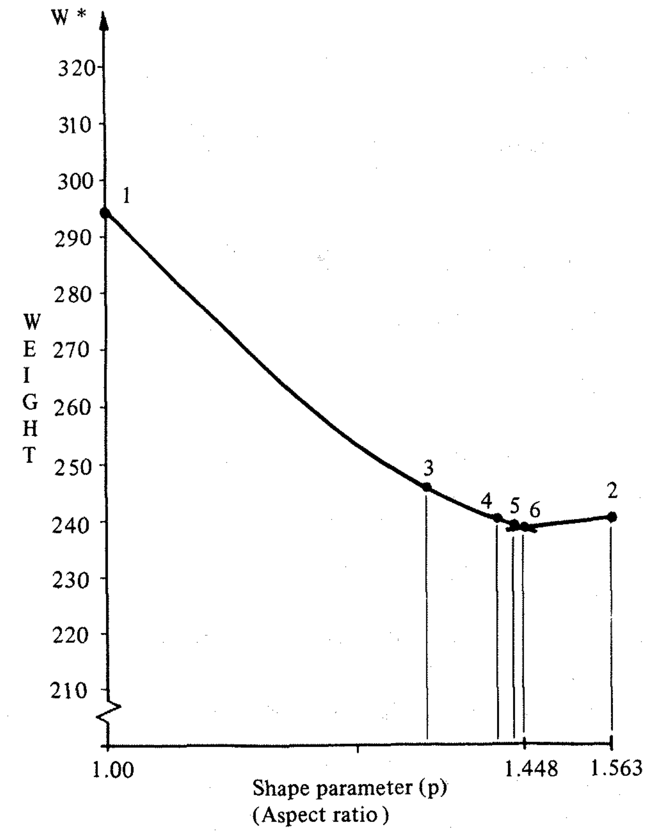


Fig. 2 Location of minimum weight for shape optimisation

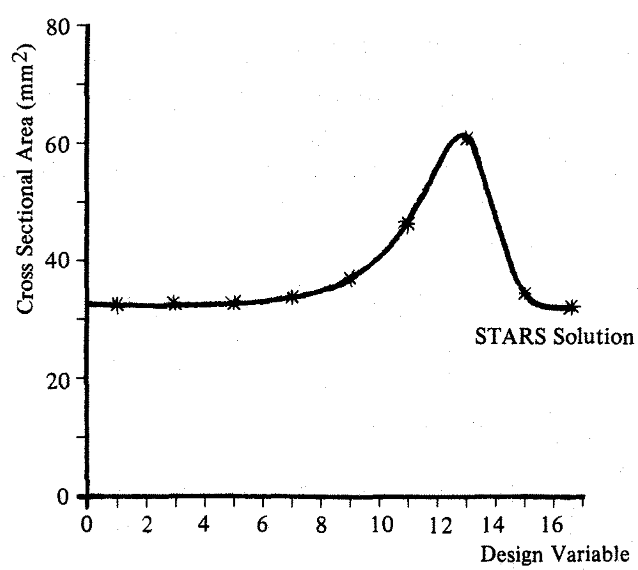


Fig. 3 Areas of edge reinforcement for normal mesh

Using the sensitivity calculation as the basis of linear or cubic interpolation of weight, the one-parameter shape optimisation algorithm converged to the optimum in six steps, giving an aspect ratio of 1.45 : 1, as shown in Fig. 2. The variation in thickness of reinforcement around the cut-out, corresponding to this aspect ratio, is shown in Fig. 3.

3.1 Theory

The method is based on variation of Kuhn-Tucker necessary conditions which define the size optimum. Just as for the Newton step itself, a linear approximation is formed about the current point, giving equations

$$\left(\frac{\partial W}{\partial p} + \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial p}\right) + \left[\nabla^2 W + \sum_{i=1}^{m} \lambda_i \nabla^2 g_i\right] \frac{dz}{dp} + \sum_{i=1}^{m} \nabla g_i \frac{d\lambda_i}{dp} = 0 \qquad (3.1.1\,a)$$

$$\frac{\partial g_i}{\partial p} + \nabla g_i \frac{dz}{dp} = 0, \qquad (3.1.1b)$$

which determine the sensitivity of the solution, dz/dp, $d\boldsymbol{\lambda}/dp$, with respect to the parameter p.

Again, rather than employing an exact Newton step, the equations are further approximated by neglecting second derivatives of the constraints.

$$\left[\frac{2w_j}{z_j^3}\right] \frac{dz}{dp} + \sum_{i=1}^{m} \nabla g \frac{d\lambda_i}{dp} = -\left\{\frac{\partial W}{\partial p} + \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial p}\right\} \qquad (3.1.2\,a)$$

$$\nabla g_i \frac{dz}{dp} = -\frac{\partial g_i}{\partial p} \qquad (3.1.2b)$$

This approximation introduces some errors into the estimate of sensitivity of the size-optimising values but, although this makes the shape optimisation more difficult, the errors have not been sufficient to prevent convergence of the shape optimisation.

Omitting the second derivatives of constraints again gives a very simple form for the linear equation. A further point of note is that slope discontinuities in dW/dp arise whenever a change of active set occurs in the underlying size optimum. The difficulties this causes, together with the cost of semi-analytic approach to calculating the partial derivatives with respect to p, make it unlikely that the shape facility within STARS will be extended beyond the one-parameter capability in the short term.
Nonetheless, the one-parameter capability opens the way to automating a series of parametric studies as shown in the next section.

3.2 Test problem

The test problem comprises a large sheet of uniform thickness, under a 2 : 1 bi-axial load, and containing a cut-out of specified area. The cut-out is taken to be an ellipse of unknown aspect ratio, and the sheet is reinforced by a variable-thickness bead, capable of carrying end-load only, around the perimeter of the cut-out.

A one-parameter family of meshes is created by interpolating nodal co-ordinates between a mesh on a sheet with a circular cut-out and a similar mesh corresponding to an elliptic cut-out of aspect ratio 25 : 16, see Fig. 1.

At the optimum design, although the stress components vary from point to point in the vicinity of the cut-out, there is no concentration in the von-Mises stress. A finer mesh was also tried to eliminate the possibility that the coarser mesh had missed a slight stress concentration, which would result in considerably heavier solution if the sheet thickness had to be increased to compensate. The finer mesh, however, serves to substantiate the earlier run.

An alternative application of the one-parameter variation of size-optimised structure, currently under investigation, is achieved by linking the parameter to material properties, in particular the orientation of composite material.

For example, this should enable composite materials to be tailored to couple wing-bending loads and torsional deformation, in order to achieve static aeroelastic objectives in a forward swept wing.

Having considered small test problems, used to validate the methods of structural optimisation, we now proceed to industrial problems where the major challenges lie both in the size of the problems addressed and in bridging the gap between theoretical and coding developments and practical engineering requirements.

4 INDUSTRIAL USAGE OF STARS

At MBB, STARS is used for static, dynamic and combined static-aeroelastic optimisations. [8]

STARS' modular structure makes it quite easy to incorporate additional modules (user-written software, FE-programs or pre- and postprocessors). This possibility is successfully used at MBB. As far as the static side is concerned, MSC-NASTRAN, for example, is used in this manner both for analysis und for determination of the sensitivities required for optimisation. This is of great practical importance, because it enables the stress engineers to use one and the same analysis program in every step of structural design work (from projecting stage to strength analysis, including structural optimisation).

For the combined static-aeroelastic optimisation, STARS and the MBB in-house aeroelastic program AEROOPT [9] are modularly coupled. Fig. 4 illustrates the systematics of this procedure for the flutter optimisation process. The wing of an airliner has been taken as an example of a component that is to be optimised.

The basic idea is that the FE-model of the entire aircraft is used for the aeroelastic analysis and computation of flutter derivatives whereas only the FE-model of the wing is used for the static analysis and determination of static derivatives.

Redesign in the course of flutter optimisation is achieved with the Newton Method (so-called Pseudo Newton Method – PNM) described in chapter 2.2.
First flutter optimisations with this process, based on a submodel of the civil aircraft wing described in chapter 5.2, have proved to be successful.

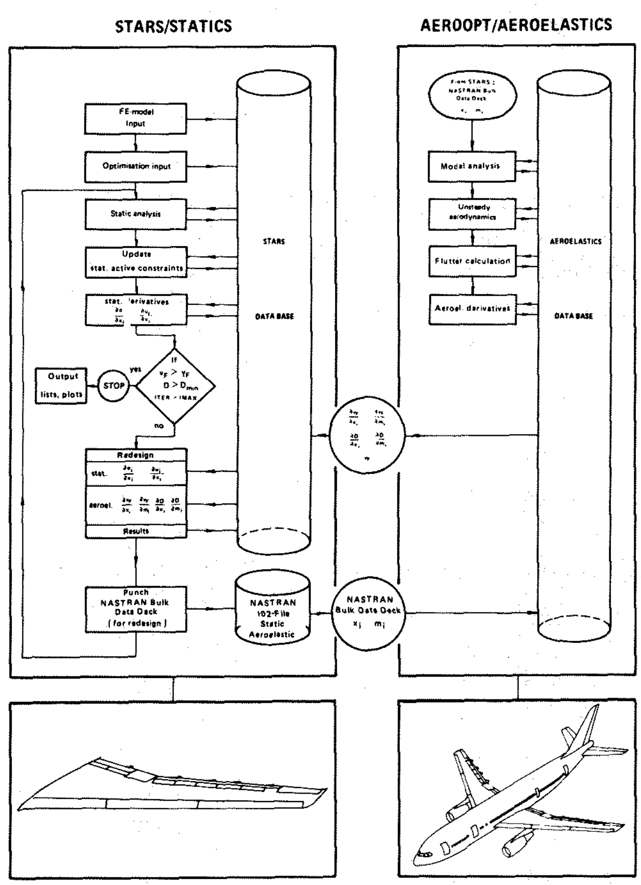


Fig. 4 Flutter optimisation with STARS in a "multi-model" process

5 INDUSTRIAL APPLICATIONS

This chapter describes some characteristic examples of the practical application of STARS at MBB. These examples relate to stress and stiffness optimisation of metal and composite components.

In industrial application it showed that time and cost advantages can be achieved particularly for the following fields if structural optimisation is used:

- pre-dimensioning:
  - early weight-optimal designs for various constructions and materials
- weight-optimal modification of structures at short notice in the case of
  - specification changes
  - load changes
  - occurrence of resonances, fluttering, ...
- productionisation / value analysis
  - optimal weight reduction of structures.

It is very important to commence structural optimisation already during the pre-dimensioning phase since essential decisions with regard to the shaping of components or assemblies are taken during this phase.

Consequently, the detailing of FE-models used for optimisation will differ, depending on the phase of application. The characteristic values of the FE-models and optimisation models are shown in the different illustrations of the application examples.

5.1 Military Aircraft Frame

Fig. 5 shows the FE-model of the frame for a modern fighter aircraft. This aluminium frame is subjected to the wing attachment forces.

From the optimisation results obtained, Fig. 6 plots the weight curve versus the iterations. The weight curve shows clearly that the optimum weight is achieved within 10 iterations.

In this case, the Stress Ratio Method (SRM) was used as the optimisation procedure for pure stress optimisation.
The minimum gauges for all finite elements to be optimised were specified as initial design.

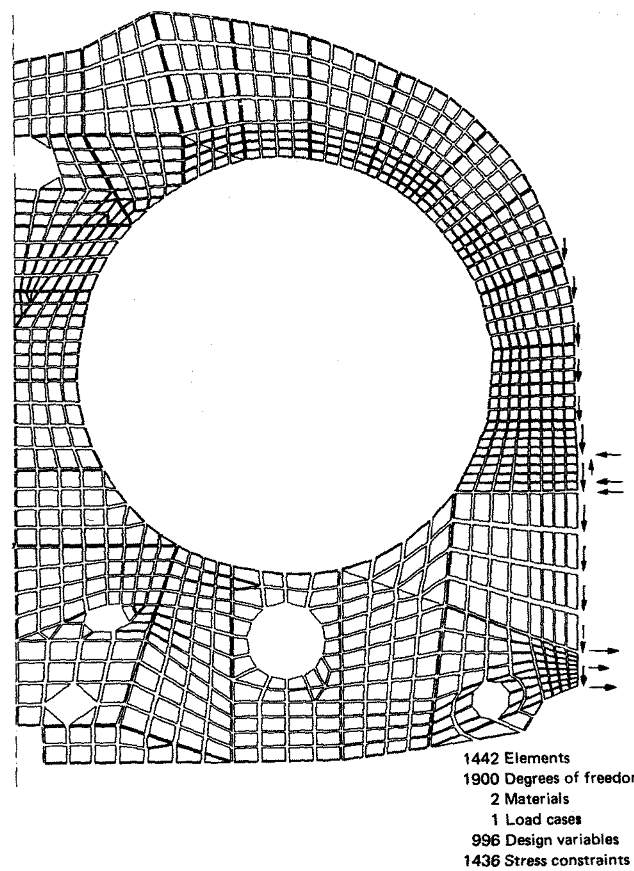


Fig. 5 FE-model of frame

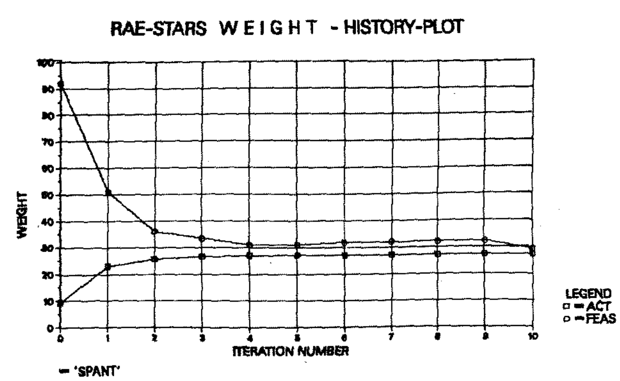


Fig. 6 Optimisation results for frame

5.2 Civil Aircraft Wing

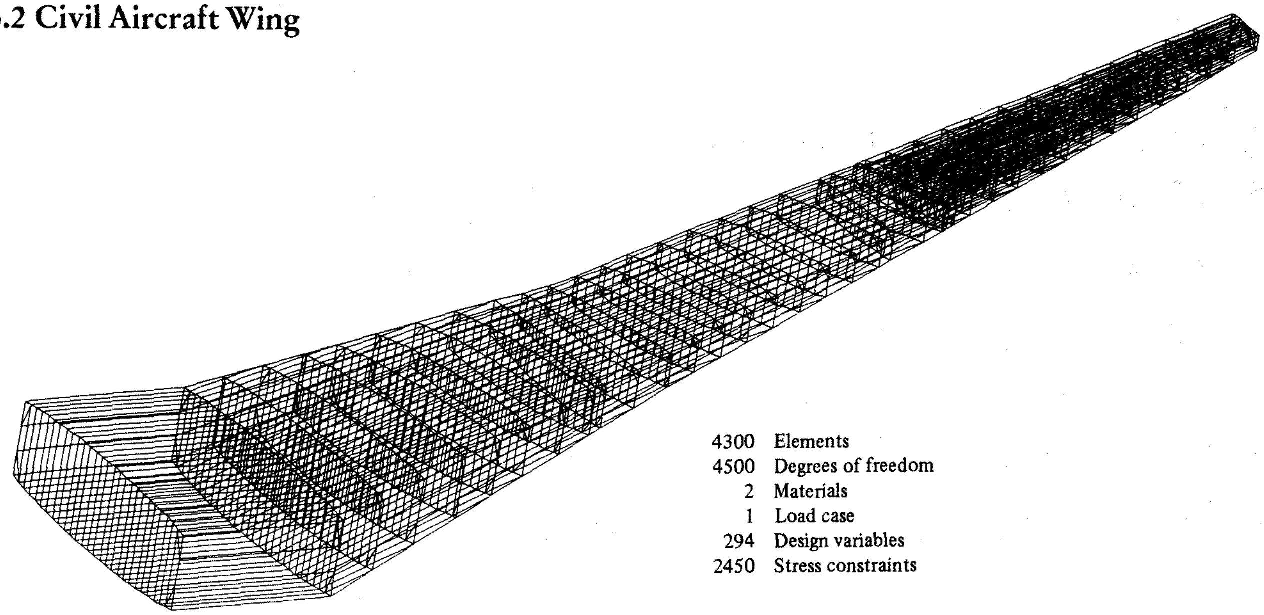


Fig. 7 FE-model of wing box

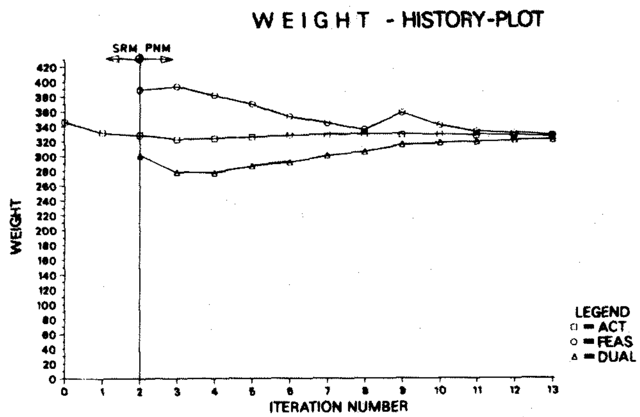


Fig. 8 Optimisation result for wing box

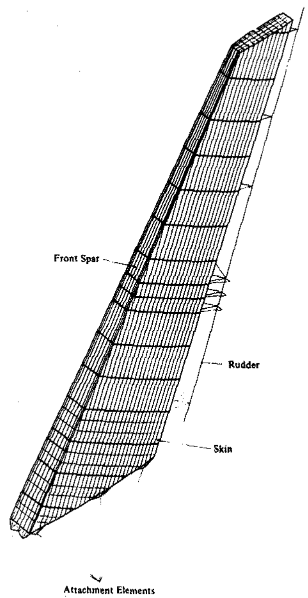
Fig. 7 gives a further practical example of the FE-model of the wing box of a modern airliner. This FE-model (metal inner wing/composite outer wing) served as the basis for stress optimisations at the inner and outer wing within the scope of a study. In these optimisation runs the number of DV's was systematically increased in successive runs.
Fig. 8 gives the example of a weight history plot for an optimisation run of the outer wing with 294 DV's.

Initially, the inexpensive SRM was used for two iteration steps in order to obtain an 'almost optimum' structure relatively quickly. This was then followed by the more comprehensive PNM until the optimum structure was reached.
The curves show that the SRM quickly converges and reaches the 'almost optimum' structure. The subsequent PNM introduces some corrections of the design, thus resulting in the optimum.

In practical industrial application SRM and PNM have proved to be very successful for redesign purposes. Moreover, experience has shown that a combination of both procedures can be very advantageous for many practical problems [8].

5.3 Airbus Tail Fin

Fig. 9 shows the FE-model of the composite fin box. The basic structure of the FE-model is shown in Fig. 10 [10].



9001	Elements
8398	Degrees of freedom
1	Material
2	Load cases
130	Design variables
5689	Stress constraints
6	Displacement constraints

Fig. 9 FE-model for fin box

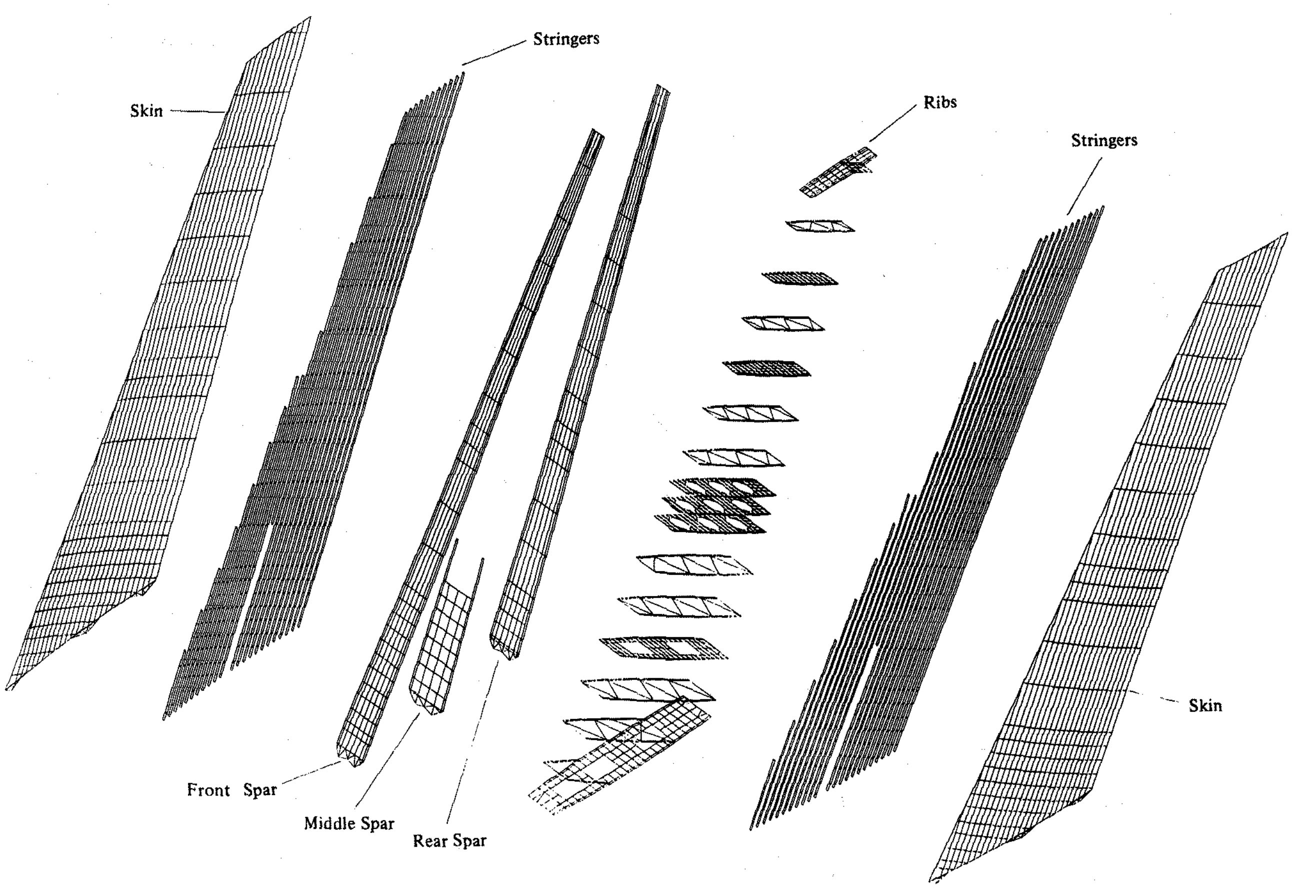


Fig. 10 Basic structure of the FE-model

In the composite design, layers with identical fibre direction were combined and idealised by a membrane element in the FE-model (see Figs. 11 and 12).

Weight optimisation of the fin box was carried out for the two dimensioning load cases, lateral gusts and maneuvers.

Allocation of the design variables to the element thicknesses and element cross-sections of the individual component areas is shown in Fig. 13 for the skin, taking into consideration the following aspects:

- Since the 0°/90° and ± 45° layers of the webs used are always a part of one and the same web and can thus be changed only as a whole, the thicknesses of the respective elements were combined to one design variable each (0° and 90°; +45° and −45°).
- Owing to the symmetrical structure of the vertical tail, the respective LH and RH elements of the skin and of the stringers could be combined to one design variable each.

The following elements were fixed, i. e. they were not optimised:

- connections of the fin box to the fuselage

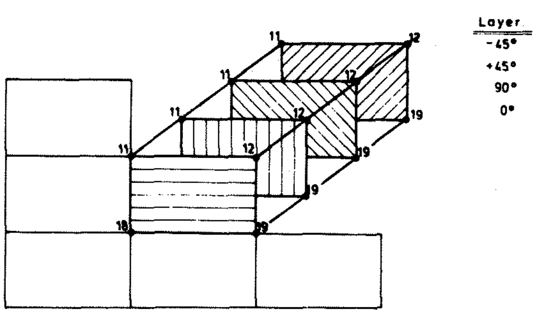


Fig. 11 FE-model structure for composite optimisation

- connection areas of skin and spars to the fuselage
- spar edge reinforcements
- stringers in the connection areas mentioned above
- all ribs.

Minimum limits of the element cross-sectional values to be optimised were governed by the specified structural and manufacturing requirements.

Displacement constraints at characteristic points of the front and rear spars and strain constraints for all elements to be optimised had been specified as constraints.

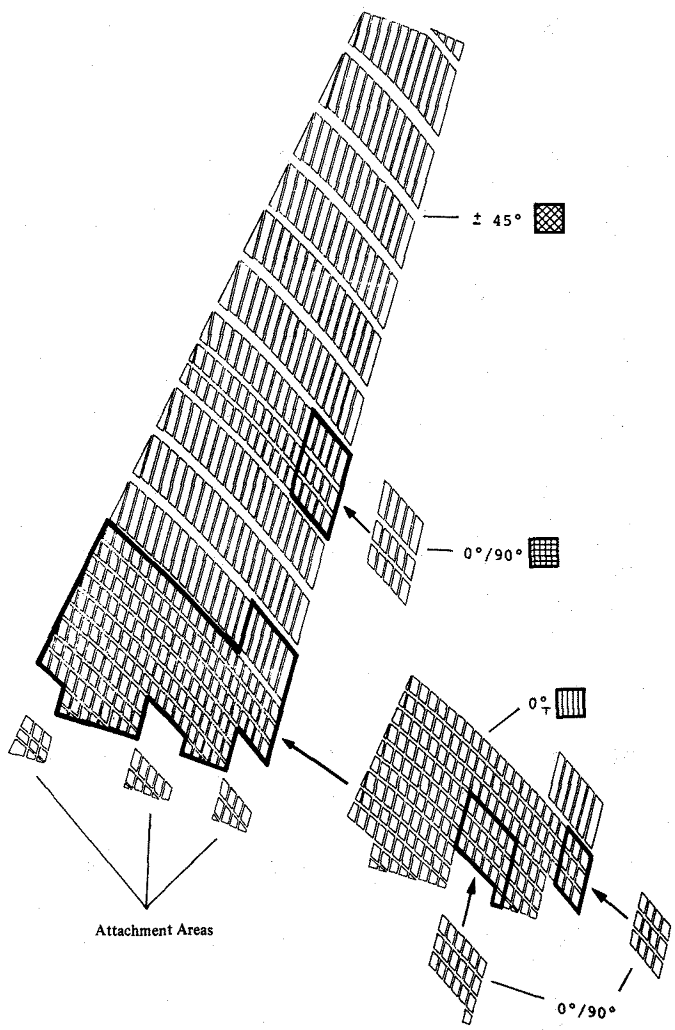


Fig. 12 Construction of skin FE-model

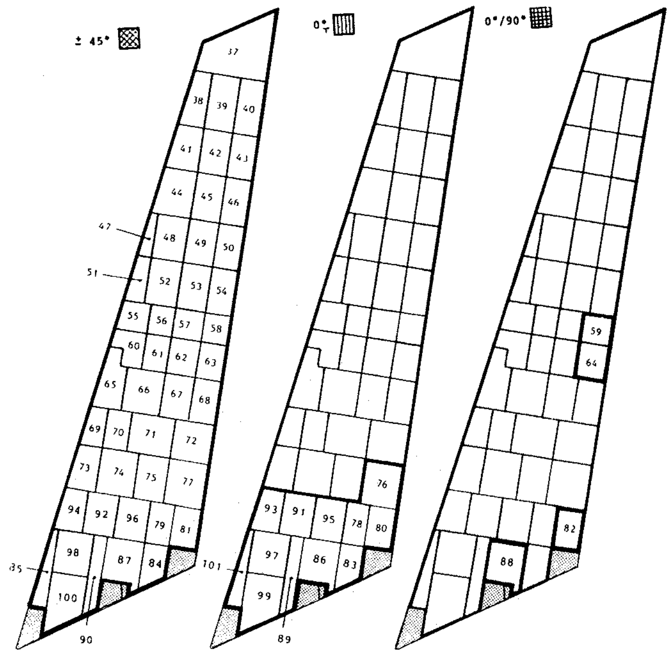


Fig. 13 **Design variables of the skin**

No detailed stability analysis was carried out within the scope of optimisation. A subsequent simplified analysis of local stability was performed for the optimised structure based on the existing strains. However, only strains parallel to the stringers were taken into account.

The Pseudo Newton Method was selected as optimisation procedure since, in the present case, stress and displacement constraints are specified.

Fig. 14 shows the standardised weight development versus the number of iterations as optimisation results.

It can be seen that

- a significant weight reduction as compared to the original component was achieved
- an optimum weight was achieved after the 7th iteration step and virtually none of the constraints was any longer violated.

The CPU time for 7 iterations amounted to approx. 1.8 hours.

A subsequently performed, simplified analysis of local stability based on strains resulted in the fact that the skin of the optimised structure had to be thickened in parts.

The optimisation described above applied to the idealised component structure. In this case, the results and the resulting element cross-sections and layer thicknesses were defined solely numerically. Therefore, adaptation of these values to the production requirements will again result in some increase of the optimised model weight.

This example of practical application for the CFRP Airbus fin box shows that STARS can also be used for weight optimisation of large structures under realistic conditions.

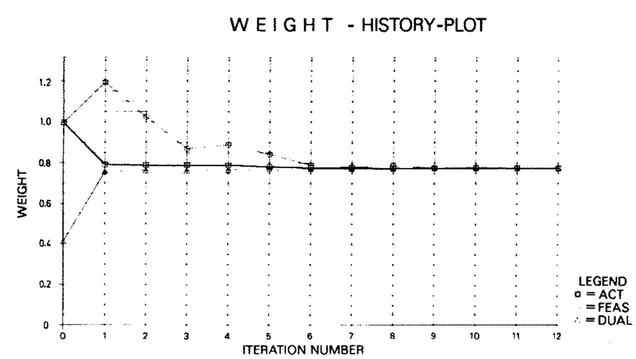


Fig. 14 **Optimisation results for fin box**

6 CONCLUSIONS

Industrial application of structural optimisation at MBB has convincingly demonstrated many advantages. Large and complex structural components made of metal and composite materials have been weight-optimised using the method described in this paper and considerable weight savings over conventional component design are possible.

Structural optimisation has become an efficient "design tool" at both preliminary and main design phases, making it possible to develop components at minimum weight and low cost within a relatively short time and thus improves competitiveness.

ACKNOWLEDGEMENTS

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