SHIMMY PROBLEMS OF LANDING GEARS CAUSED BY ELASTIC DEFORMATION OF TIRES

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E. Sperling
Dr.-Ing., Oberingenieur
Lehrstuhl für Leichtbau, Technische Universität München
München, Bundesrepublik Deutschland

Abstract

Airplanes taxying on runways are - like every land vehicle with wheels - endangered by self-exited vibrations which may lead to severe accidents.
Since the first papers concerned with this problem more than one dozen of theories has been proposed. As every author thinks his theory to be the best, for the design engineer of today there exists the problem to select one of the theories under the viewpoints (a) physical clearness, (b) computational costs, (c) description of the essential phenomena. To support the selection in this paper the substances of the most important theories are systematically compared and classified starting with the model rigid wheel on rigid runway.
Finally the theories are applied to simple examples and the computational results compared.
The results of the theoretical and the computational comparison are clearly arranged in synoptical tables and diagrams. Future theoretical investigations and crucial experiments are proposed.

1. Introduction

Especially nose landing gears but also main landing gears are endangered by self-exited vibrations which sometimes are called "shimmy", "flutter" or "kinetic or dynamic instability". This phenomenon may be described in the following way: When an airplane is accelerated on the runway at a "critical velocity" v_{crit} the landing gear starts to vibrate violently with increasing amplitude perpendicular to the rolling direction. These vibrations may lead to a rupture of the landing gear followed by severe accidents.

The principles of this dangerous phenomenon can be described in a qualitative way by the very simple model "rigid wheel on rigid runway"(1)- see also chapter 2. But to describe it quantitatively one has to consider the (elastic) deformation of the tire, chapter 3. These additional degrees of freedom enormously increase the difficulties of the mathematical formulation. Therefore it is not surprising that since the first papers concerned with this problem more than one dozen of theories has been proposed to answer the relative simple question whether a landing gear runs stable or not. This question may be called "simple" because due to an important result of the theory of stability it is only necessary to treat the linearized differential equations(2). That is why this reduction is used throughout this paper.

The following theories have been considered in the author's dissertation(1), where also the voluminous and systematic derivations are given:

1. Slip Theory
2. Kalker's Theory
3. B. de Carbon's Elementary Theory considering or neglecting the elastic deformations of the tire
4. B. de Carbon's Complete Theory
5. Curvature Theory considering or neglecting the elastic deformations of the tire
6. Moreland's Theory
7. Modified Moreland's Theory
8. Improved Modified Moreland's Theory
9. Theory of v.Schlippe and Dietrich/Smiley/Pacejka (Approximation I) - String Model
10. Pacejka's Theory (Approximation II) - String Model
11. Generalisation of the Theory of v.Schlippe and Dietrich/Smiley/Pacejka - String Model
12. Transmission of the Generalisation of the Theory of v.Schlippe et al. to the beam model
13. B. de Carbon's Elementary Theory applied to a wheel set

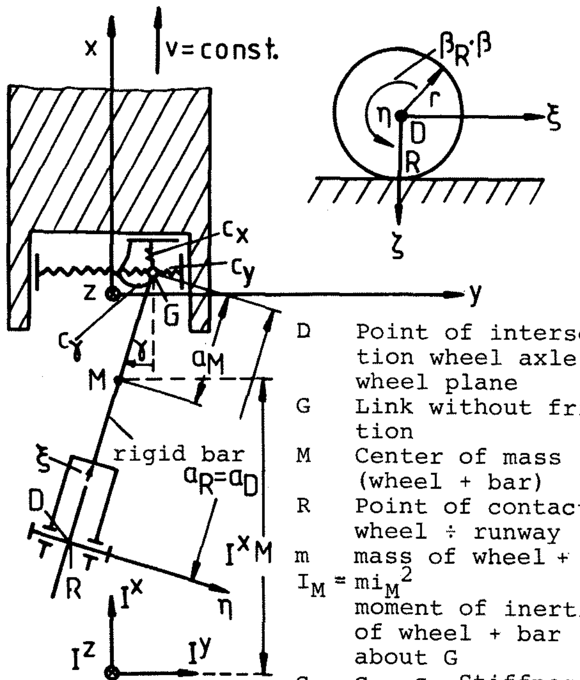
New theories are underlined.

For the design engineer of today there exists the problem to select one of the theories under the viewpoints
- Physical clearness
- Computational costs
- Description of the essential phenomena.

To support the selection in this paper the substances of the listed theories are systematically compared and classified starting with the model "rigid wheel on rigid runway".

2. Model "Rigid Wheel on Rigid Runway"

This model, shown in Figure 2.1, is the simplest one to describe the kinetic instability(1). It is very useful to work out the mechanism of self-excitation as the cause of instability and to find essential parameters. The landing gear is represented by a wheel fixed to a rigid bar which is suspended in the airplane by springs.



D Point of intersection wheel axle ÷ wheel plane
G Link without friction
M Center of mass (wheel + bar)
R Point of contact wheel ÷ runway
m mass of wheel + bar
$I_M = mi_M^2$ moment of inertia of wheel + bar about G
c_x, c_y, c_γ Stiffness of springs
a_R caster

I^x, I^y, I^z inertial coordinate system
ξ, η, ζ coordinate system fixed to D, the ξ-η-plane is always parallel to the x-y-plane
x, y, z coordinate system fixed to the airplane, running with constant velocity v with respect to the inertial system

Degrees of freedom: x, y, γ

Figure 2.1. Model "Rigid Wheel on Rigid Runway", coordinate systems and parameters.

Assumed
- the airplane is running with constant speed v
- the bar is rigid
- the only degrees of freedom are x,y,γ
- the link G is frictionless
- the wheel is represented by its center plane

the following system of linear differential equations with constant coefficients can be derived with help of the Lagrangean equations with nonholonomic constraints in Pfaffian form(1), see also Figure 2.2,

Newton's second law for y-direction

$$m\ddot{y}-ma_M\ddot{\gamma}+c_y y = \lambda_1 \qquad (2.1)$$

Euler's equation about G

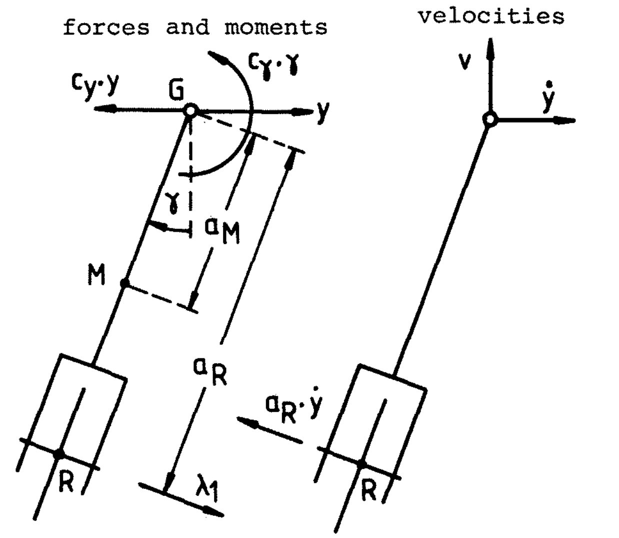
$$m(a_M^2+i_M^2)\ddot{\gamma}-ma_M\ddot{y}+c_\gamma\gamma = -\lambda_1 a_R \, , \qquad (2.2)$$

Rolling (nonholonomic) constraint perpendicular to wheel plane

$$\dot{y}-a_R\dot{\gamma}-v\gamma = 0 \, . \qquad (2.3)$$

Newton's second law for the x-direction is not included here because its differential equation is decoupled from equations (2.1) to (2.3) and is of no importance in the stability analysis(1).

The rolling constraint of equation (2.3) can be explained twice:
- The wheel runs without sliding
  or
- The velocity of wheel contact point R perpendicular to the wheel center plane vanishes (is zero), see Figure 2.2b.



a) b)

Figure 2.2. Model "Rigid Wheel on Rigid Runway", forces and moments and velocities.

3. Model "Elastic Tire on Rigid Runway"

The model of chapter 2 has to be completed by considering the following facts of an elastic tire, Figure 3.1:
- The contact point R becomes a contact area; in most theories the wheel is replaced by its center plane - so we have a contact line.
- The rigid wheel center plane becomes flexible, in other words, an infinite number of degrees of freedom is added.
- The tire is able to transmit a moment M_ζ about the vertical ζ-axis.

Due to these additional effects the system of differential equations (2.1) to (2.3) must be altered and enlarged:
- The equation due to Newton's second law remains unchanged

$$m\ddot{y}-ma_M\ddot{\gamma}+c_y y = \lambda_1 \, . \qquad (3.1)$$

- In Euler's equation about G appears the additional moment M_ζ

$$m(a_M^2+i_M^2)\ddot{\gamma}-ma_M\ddot{y}+c_\gamma\gamma = -\lambda_1 a_R+M_\zeta \, . \qquad (3.2)$$

The mathematical description of the soil reactions is discussed in sub-chapter 3.1.
- Due to the additional degrees of freedom of the wheel center plane the rolling constraint has to be completed by the associated velocities which are symbolised in equation (3.3) by a circle

$$\dot{y}-a_R\dot{\gamma}-v\gamma+\bigcirc = 0 \, . \qquad (3.3)$$

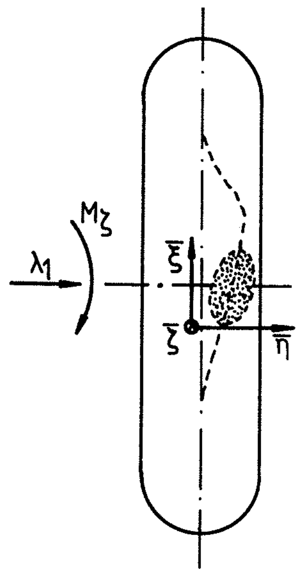


Figure 3.1. Elastic tire on rigid runway; deformations and soil reactions λ_1 and M_ζ.

The "contents" of the circle is discussed in sub-chapter 3.2.

- Also due to the additional degrees of freedom which are - in other words - additional unknowns an associated number of additional equations has to be created, symbolised in equ. (3.4) by a dotted line

$$\cdots\cdots\cdots\cdots\cdots = 0 \; . \qquad (3.4)$$

Details are given in sub-chapter 3.3.

In the following theoretical comparison of the theories (chapter 3) listed in chapter 1 two groups can be distinguished (see also Figures 3.3, 3.5, 3.6 in which the separation is symbolised by a double line):

1. group (Theories 2, 3, 4, 5, 6, 7, 8, 13)
Here no concrete model is used:
The elasticity of the tire is taken into account by considering its kinematical behaviour under special conditions; an overview is given in Figure 3.7.

2. group (Theories 9, 10, 11, 12)
Here a concrete model is used:
Prestressed string on elastic foundation, called "string model", Figure 3.2. Only in theory 12 a "beam model" is considered.

In the theoretical comparison it is not necessary to distinguish in the theories nrs 3 and 4 wheather the elastic deformations are considered or neglected, but in the computational comparison of chapter 4 this distinction will be of importance.

Theory nr. 12 is not treated here in detail because its system of differential equations is congruent to that in theory nr. 11; only the coefficients have a different physical meaning[1] - see also Figure 3.10.

Kalker's theory has been developed in connection with contact problems at railway vehicles[3] and is considered here because of its close relationship to B. de Carbon's Elementary Theory. Kalker used in his derivation the model of a paraboloid rolling on a half space but the result justifies to classify his theory to the first group - in the context of this paper.

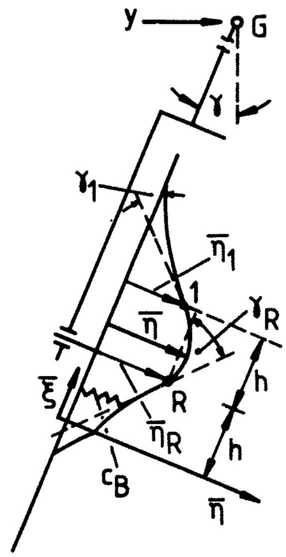


Figure 3.2. String model

Additional assumptions with respect to chapter 2:

- Wheel is represented by its center plane
- Curved string is replaced by a straight infinite one
- Mass of string is neglected
- Damping of tire is neglected

Additional degrees of freedom:

$\bar{\eta}(\bar{\xi},t), \bar{\eta}_1(t), \bar{\eta}_R(t), \gamma_1(t), \gamma_R(t)$

Additional parameters:

c_B Spring stiffness of elastic foundation
2h Contact length
S Tension force in string (not shown in Figure).

3.1 Comparison of the Mathematical Description of the Soil Reactions

Figure 3.3 gives an overview:

- Most theories use a linear spring law, relating the soil reactions λ_1 and M_ζ to the degrees of freedom η_R and γ_R respectively by the coefficients of stiffness $c_{\lambda 1}$, $c_{M\zeta}$. These coefficients can be either gained by static experiments or by analysis from the string model using the following general formulas (see Figures 3.2 and 3.4)

$$\lambda_1 = \underbrace{c_B L(\bar{\eta}(2h,t)+\bar{\eta}(0,t))}_{\text{transition curve}} + \underbrace{c_B \int_0^{2h} \bar{\eta}(\bar{\xi},t)\,d\bar{\xi}}_{\text{contact line}} \qquad (3.5)$$

$$M_\zeta = \underbrace{c_B L(h+L)(\bar{\eta}(2h,t)-\bar{\eta}(0,t))}_{\text{transition curve}} + \underbrace{c_B \int_0^{2h} \bar{\eta}(\bar{\xi},t)(\bar{\xi}-h)\,d\bar{\xi}}_{\text{contact line}} \qquad (3.6)$$

with $L = \sqrt{S/c_B}$ = relaxation length.

Rigid wheel with nonholonomic constraint		$M_\zeta = 0$
Slip Theory	$\lambda_1 = \infty \cdot 0 =$ finite value	$M_\zeta = 0$
Kalker's Theory		$M_\zeta = \infty \cdot 0 =$ finite value
B. de Carbon's Elementary Theory		
Curvature Theory		
B. de Carbon's Complete Theory		constants $c_{\lambda 1}$, $c_{M\zeta}$ from
Moreland's Theory	Linear Spring Laws $\lambda_1 = c_{\lambda 1} \cdot \eta_R$; $M_\zeta = c_{M\zeta} \cdot \gamma_R$	static experiments
Modified Moreland's Theory	"average displacement"	
Improved Modified Moreland's Theory	"average rotation"	
Theory of v.Schlippe, Dietrich/Smiley/Pacejka (Appr.I)		$c_{\lambda 1}$, $c_{M\zeta}$ calculable; contact line assumed
Theory of Pacejka (Appr.II)		as straight line - see Figure 3.4
Generalisation of the Theory of v.Schlippe et al.	No Spring Law; contact line with arbitrary shape, approximated by (e.g.) Fourier series - see Figure 3.4	

Figure 3.3. Comparison of the mathematical description of the soil reactions, overview.

- The assumptions made in the first theories of Figure 3.3 may be interpreted as a limit case of the linear spring law with infinite stiffness and vanishing degree of freedom.
- The Generalisation of the Theory of v.Schlippe et al. is the most advanced theory:
  -- No spring law is used for the soil reactions
  -- Shape of contact line may be arbitrary and approximated by any kind of series, e.g., Fourier series in any desired order of accuracy.

3.2 Comparison of the Rolling Constraint(s)

Figure 3.5 gives an overview:

- Proceeding from the top line to the bottom the stepwise generalisation of the simple *nonholonomic* constraint for the rigid wheel can be regarded.
- The most advanced rolling constraint is formulated in the Generalisation of the Theory of v.Schlippe et al.: It is a partial differential equation and has the physical meaning: No point of the contact line slides.

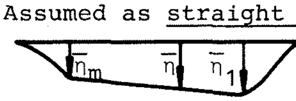
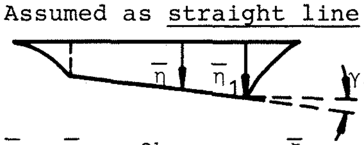
	Description of the contact line
Theory of v.Schlippe, Dietrich/ Smiley/ Pacejka (Appr.I)	Assumed as <u>straight line</u>  $\bar\eta = \bar\eta_m + \frac{\bar\eta_1 - \bar\eta_m}{2h} \bar\xi$
Theory of Pacejka (Appr.II)	Assumed as <u>straight line</u>  $\bar\eta = \bar\eta_1 - 2h\gamma_1 + \gamma_1 \bar\xi$
Generalisation of the Theory of v.Schlippe et al. (Transmissable to beam model)	<u>Arbitrary shape</u>  $\bar\eta(\bar\xi, t) = b_1(t) + b_2(t)\frac{\bar\xi}{2h} + \sum b_{\nu+2}(t) \cdot \sin\frac{\nu\pi}{2h}\bar\xi = \{\varphi\}^T\{b\}^*$

Figure 3.4. String model; description of the contact line.
* Any desired order of accuracy is possible by approximation, e.g. with collocation method.
$\{\varphi\}$ Vector of shape functions
$\{b\}$ Vector of constants.

Rigid wheel with nonholonomic constraint	Velocity of point R perpendicular to wheel center plane is zero: $\dot{y}-a_R\dot{\gamma}-v\gamma = 0$
Slip Theory	Velocity of point R perpendicular to wheel center plane is not zero but has the finite slip velocity v_{SGL}
Kalker's Theory	which is proportional to the side force λ_1: $v_{SGL} \sim \lambda_1$
B. de Carbon's Elementary Theory	
Curvature Theory	
B. de Carbon's Complete Theory	Velocity of point R perpendicular to contact line in point R is zero:
Moreland's Theory	$\dot{y}-a_R\dot{\gamma}+\dot{\bar{\eta}}_R-v(\gamma+\gamma_R) = 0$
Modified Moreland's Theory	
Improved Modified Moreland's Theory	
Theory of v. Schlippe and Dietrich/Sniley/Pacejka (Appr. I)	a) Rolling constraint for point R and point 1 b) No point of contact line slides: Point 1 becomes R after rolling of h: $y_1=y_R({}_Ix+h)\approx y_R+h\dot{y}_R/v+h^2\ddot{y}_R/2v^2+\ldots$
Theory of Pacejka (Appr. II)	Rolling constraint for point 1
Generalisation of the Theory of v. Schlippe et al.	Rolling constraint for each point of contact line: $\dot{y}-(a_R-(\bar{\xi}-h))\dot{\gamma}+\dot{\bar{\eta}}(\bar{\xi},t)-v(\gamma+\bar{\eta}^{\nabla}(\bar{\xi},t)) = 0$

Figure 3.5. Comparison of the Rolling Constraint(s), overview.
$\bar{\eta}^{\nabla} = \partial\eta/\partial\bar{\xi}$ = Inclination of contact line with respect to wheel center plane.

3.3 Comparison of the Additional Equations

Here the greatest differences between the theories appear. In Figure 3.6 the two groups of theories already mentioned in chapter 3 can be distinguished:

1. group: No concrete model is used; the elasticity of the tire is taken into account by describing its kinematical behaviour under special conditions which are shown in Figure 3.7: The wheel is pressed to the soil by a vertical load (not drawn in Figure 3.7), then loaded by a side force λ_1 or by a moment about the vertical axis M_ζ and rolled slowly ahead. During this motion the wheel is forced to remain perpendicular to the surface. In the left two cases shown in Figure 3.7, also the tuning degree of freedom about the vertical axis is blocked, while it is free in the right two cases.

In all four cases the wheel tends to a stationary behaviour after a short transition phase.

This different behaviour of the wheel is formulated mathematically as shown and introduced as additional equations to the various theories cited at the bottom of Figure 3.7.

Most theories use the kinematical behaviour of the wheel with turning axis being blocked. Only the Curvature Theory assumes the wheel to be free to turn about the vertical axis. B. de Carbon's Complete Theory is a mixture between the left and the right two cases and is difficult to understand from the physical point of view. The two equations found in Figure 3.7, side slip formula

$$\gamma_R = -(1/\kappa_{s\eta})\cdot\lambda_1 \quad \text{and} \qquad (3.7)$$

curvature formula

$$\ddot{y}/v^2 = -(\kappa_{\lambda 1}\lambda_1+\kappa_{M\zeta}M_\zeta) \;, \qquad (3.8)$$

seem to be contradictory extremes. But the curvature formula can be reduced to the side slip formula as shown in Figure 3.8.

2. group: String model is used.

The additional equation is found by mathematically formulating the observation from experiments that the transition between contact line in point 1 (Figure 3.2) and the transition curve of the string is "continuous" and the inclination γ_1 being proportional to $\bar{\eta}_1$:

$$\gamma_1 = -(1/L)\bar{\eta}_1 \;. \qquad (3.9)$$

Additional remarks are given in sub-chapter 3.4.

Rigid wheel with nonholonomic constraint	
Slip Theory	
Kalker's Theory	
B. de Carbon's Elementary Theory	
Curvature Theory	1. group: No concrete model is used; the elasticity of the tire is taken into account by describing its
B. de Carbon's Complete Theory	kinematical behaviour under special conditions shown in Figure 3.7
Moreland's Theory	
Modified Moreland's Theory	
Improved Modified Moreland's Theory	
Theory of v.Schlippe, Dietrich/Smiley/Pacejka (Appr.I)	2. group: String model is used.
Theory of Pacejka (Appr.II)	The inclination γ_1 of the contact_line in point 1 is proportional to the displacement $\overline{\eta}_1$ at point 1:
Generalisation of the Theory of v.Schlippe et al.	$\gamma_1 = -(1/L)\overline{\eta}_1$. The constant of proportionality may be derived from theory or measured by experiments

Figure 3.6. Comparison of the additional equations, overview.

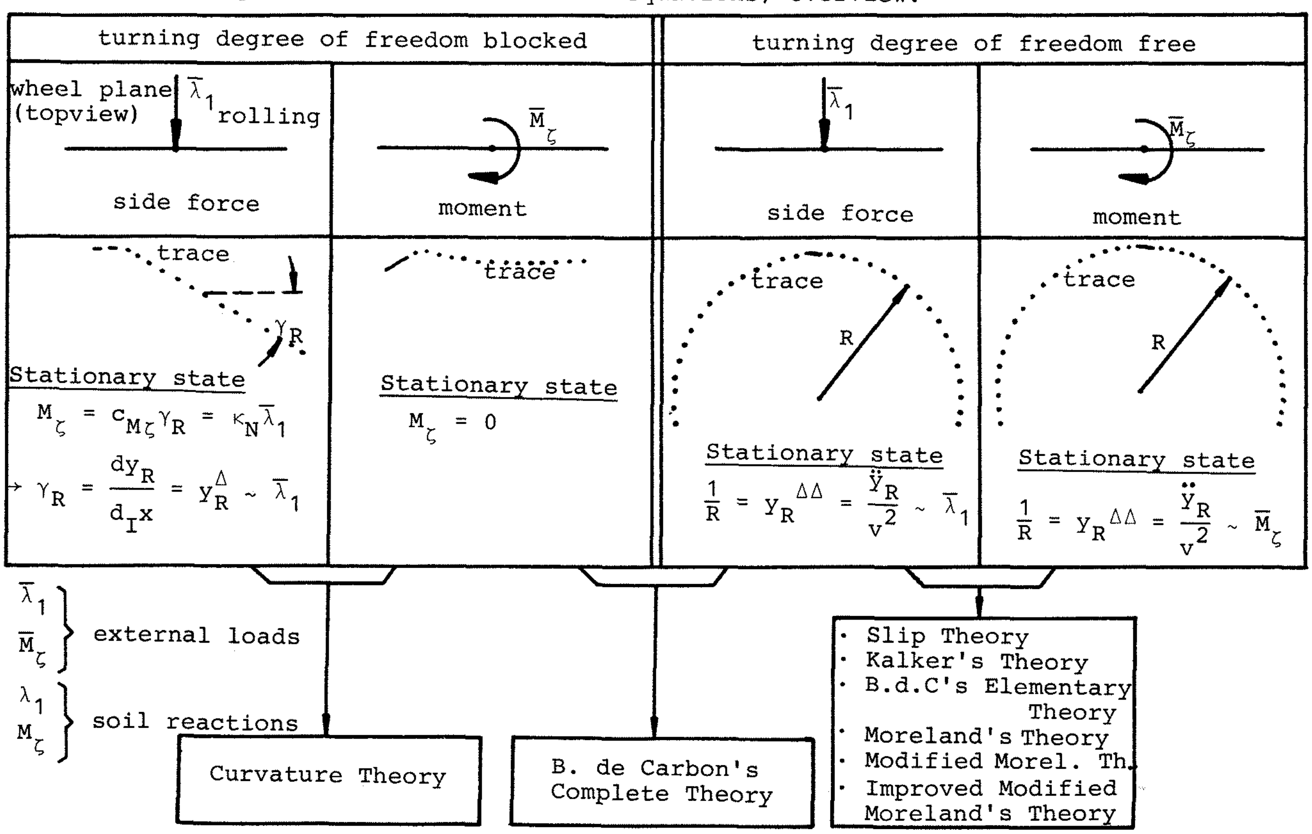


Figure 3.7. Comparison of the additional equations for the 1. group of theories. The kinematical behaviour shown in the right two rows has been proved by experiments(1) (4).

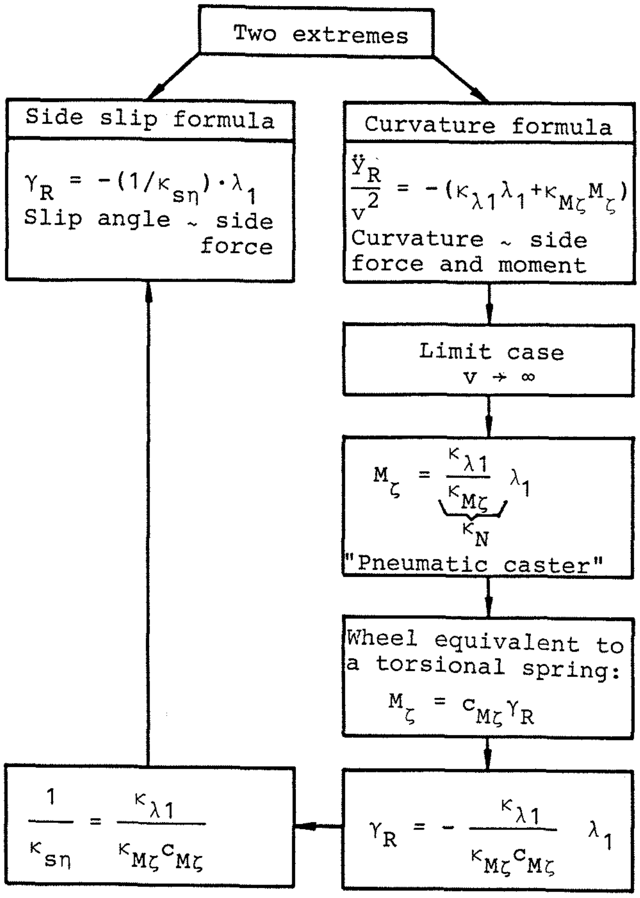


Figure 3.8. Connection between side slip formula and curvature formula.

3.4 Comparison of the Systems of Differential Equations

This voluminous comparison has been given in detail by the author[1]. Here only some remarkable facts are listed:

a) All systems of differential equations are formed by linear, homogenious ordinary differential equations with constant coefficients. There is only one exception: The Generalisation of the Theory of v.Schlippe et al. and its transmission to the beam model. These theories are described by an integro-differential equation; the rolling constraint is a partial differential equation.

b) Moreland's Theory using a "time-lag-term" is isolated completely from all other theories. The "distance-lag-term" used in the Modified Moreland's Theory makes it an integral part of the systematics of theories, see Figure 3.10.

c) Some additional equations of those compared in sub-chapter 3.3 can be shown - after some lengthy conversions - to be special cases of the Theory of v.Schlippe et al. An overview is given in Figure 3.9.

d) It is possible to derive Kalker's Theory by applying B. de Carbon's Elementary Theory to a wheel set with a finite width when the rolling constraint is formulated for a point lying at a distance κ_N ("pneumatic caster") before point R.

B. de Carbon's Elementary Theory	0	$= -\frac{c_{\lambda 1}}{\kappa_{s\eta}}\eta_R - \gamma_R$ side force ~ slip angle
Modified Moreland's Theory	$\underbrace{\frac{d\gamma_R}{d_I x} = \frac{1}{v}\frac{d\gamma_R}{dt} = \frac{1}{v}\dot{\gamma}_R}$ distance-lag-term	$= -\frac{c_{M0}c_{\lambda 1}}{\bar{c}_{M01}}\eta_R - \frac{1}{\bar{c}_{M01}}\gamma_R$
Improved Modified Moreland's Theory	$\frac{1}{v}(\dot{\gamma}+\dot{\gamma}_R)$ turning of wheel plane about vertical axis allowed	$= -\frac{c_{M0}c_{\lambda 1}}{\bar{c}_{M01}}\eta_R - \frac{1}{\bar{c}_{M01}}\gamma_R$
Curvature Theory	$\frac{\ddot{y}_R}{v^2} = \frac{1}{v}(\dot{\gamma}+\dot{\gamma}_R)$ curvature	$= -\kappa_{\lambda 1}c_{\lambda 1}\eta_R - \kappa_{M\zeta}c_{M\zeta}\gamma_R$ side force and moment
Theory of v.Schlippe and Dietrich/Smiley/Pacejka	$\frac{\ddot{y}_R}{v^2} + \chi_1\frac{\dddot{y}_R}{v^3} + \ldots = \frac{1}{v}(\dot{\gamma}+\dot{\gamma}_R) + \frac{\chi_1}{v^2}(\ddot{\gamma}+\ddot{\gamma}_R) + \ldots$ derivatives of curvature	$= -\chi_2\bar{\eta}_R - \chi_3\gamma_R$

Figure 3.9. Additional equations as special cases of the Theory of v.Schlippe and Dietrich/Smiley/Pacejka.
Additional terms with respect to the preceding line are underlined by ---.
c_{M0}, $\bar{c}_{M01}$ are constants.

e) Some additional remarks to the Generalisation of the Theory of v.Schlippe et al.:
This theory is described by the equations (3.1), (3.2), (3.5), (3.6) and the rolling constraint given in the last line of Figure 3.5. These equations are 5 partial integro-differential equations for the 5 unknowns y, γ, γ_1, M_ζ, $\bar{\eta}$. That means no additional equation is necessary! Trials to gain an approximate solution by the collocation method fail because the resulting system of algebraic equations becomes singular. This is due to the fact that the rolling constraint, being represented by a partial differential equation, is integrable!(1). Its solution is

$$\bar{\eta}(\bar{\xi},t) = -y(t)+(a_R-(\bar{\xi}-h)\gamma(t)-\underline{\Phi_1(vt-\bar{\xi})} . \qquad (3.10)$$

This means that the degrees of freedom are linearly dependent.
The last underlined term in equation (3.10) is determined by the initial conditions. The voluminous discussion of the initial conditions leads to the following results:

- Only when the contact line is straight and lies on the $_I$x-axis (see Figure 3.2) stable rolling of the system is possible: The wheel runs like in a groove and behaves like a conservative oscillator.
- Otherwise unstable rolling occurs because the wheel leaves the neutral position without return.
- The cause for this behaviour of the model: The assumption "No point of the contact line slides" (see sub-chapter 3.2) is too rigorous. It can be avoided by allowing the contact line to slide at least in the foremost portion of the contact line. The most simple form to allow this is to use the "additional equation" given in the last line of Figure 3.6 in a somewhat different form:

$$\bar{\eta}^{\nabla}(2h,t) = -\frac{1}{L}\,\bar{\eta}(2h,t) . \qquad (3.11)$$

- With respect to mathematical classification equation (3.11) is no "additional equation" but a boundary condition.
- Thus it is to solve a system of partial integro-differential equations completed by the boundary condition (3.11). This can be done by any approximation method, e.g. collocation method, which is close to the classical solution by v.Schlippe et al.(1). Better numerical result may be expected by Galerkin's method or others.
- These considerations are transmissable analogously to the beam model(1) with some additional considerations.

3.5 Summary of the Theoretical Comparisons

The most important differences and similarities are summarized in the flow chart of Figure 3.10. The symbol →○← means that the theories are described by a system of differential equations of the same mathematical form. Only the physical significance of the constants is different. In Figure 3.10 some branches are marked by dotted lines. They have been added in the author's dissertation(1) and allow to construct important correlations between the separate branches developed in the literature until that time.

4. Computational Results of Special Cases

Computational results have been gained for two special cases
- Kinematical Shimmy(5)
- Wheel without caster(6),(7).

4.1 Kinematical Shimmy

Here the wheel rolls so slowly that mass forces may be neglected compared to spring forces and soil reactions. By this assumption the order of the characteristic equation is decreased; for all theories an analytic solution may be given.

The results in Figure 4.1 show larger discrepancies in the region of small caster; here crucial experiments will be necessary and helpful.

If in B. de Carbon's Elemenatary Theory the finite width of the wheel is considered an additional region of instability appears (fourth line in Figure 4.1). In the author's opinion a similar phenomenon will occur in the other theories and the finite width must be included in future computations.

4.2 Wheel without Caster

In this special case the wheel has no caster ($a_R = a_M = 0$). Contrary to the Kinematical Shimmy the mass forces are considered here. In Figure 4.2 only some typical results are summarized. More details are given in references (6) and (7).

The results show significant differences in the region near the frequency ratio $\Omega = (\omega_\gamma/\omega_y)^2 = 1$ and $V = 0$ where $\omega_\gamma^2 = c_\gamma/(mi_M^2)$, $\omega_y^2 = c_y/m$ and V is the dimensionless velocity of the airplane.

Two results should be mentioned especially.
- The difference between the second and the fourth line in Figure 4.2 is due to the connection between these theories shown in Figure 3.8: For high velocity the curvature formula reduces to the side slip formula.
- The last two lines show the great importance of the finite width of the tire which has already been mentioned in subchapter 4.1.

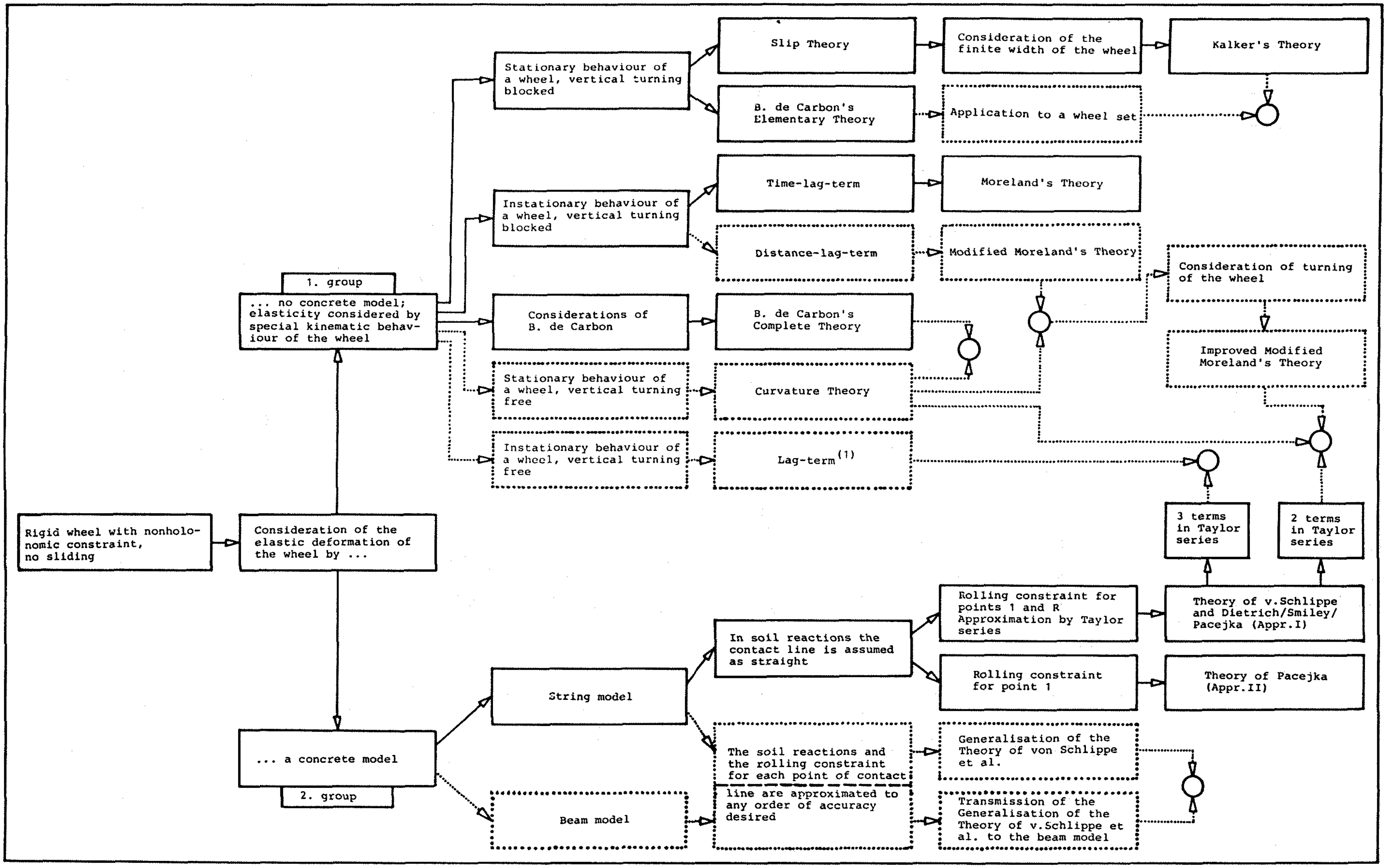


Figure 3.10. Flow chart summarizing the differencies and similarities of the theories

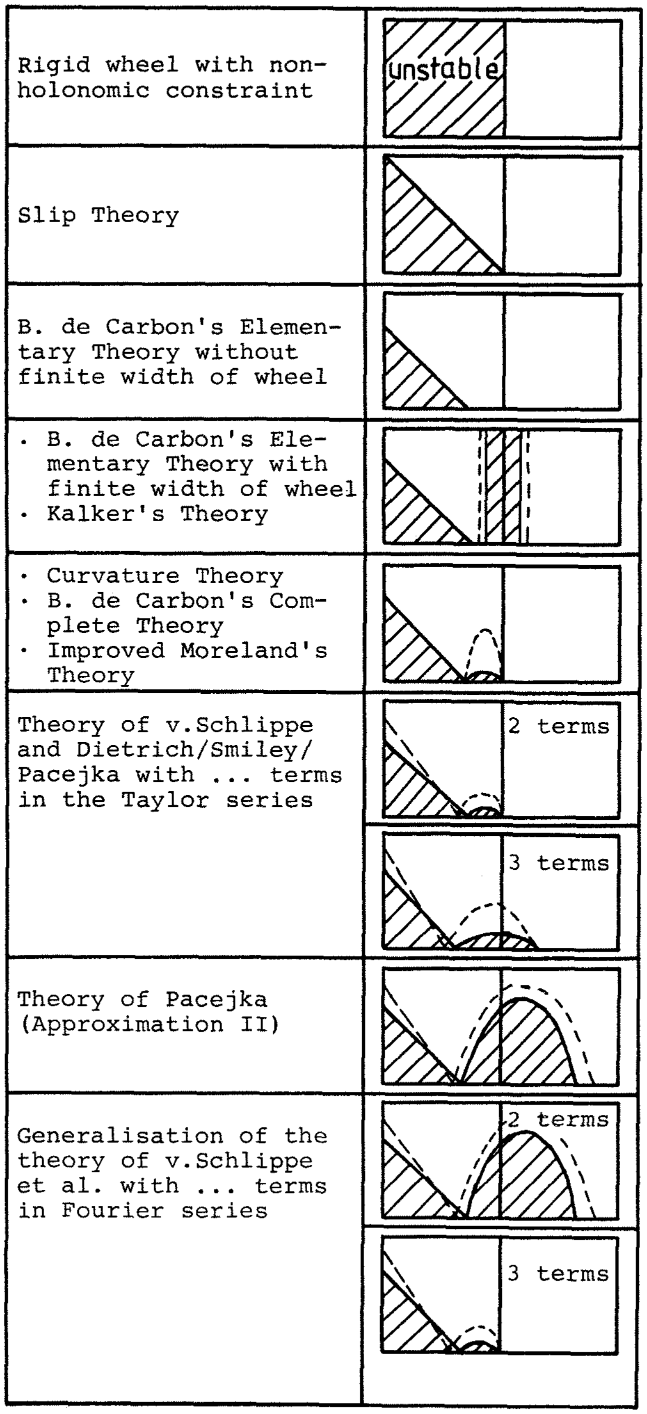


Figure 4.1. Computational results for the Kinematical Shimmy.
Abscissa: Dimensionless caster a_R
Ordinate: Dimensionless stiffness c_γ
If the magnitude of an instability region is determined by an additional parameter this fact is symbolized by dotted lines.

5. Conclusions and Outlooks to Future Research

In chapter 1 the problem was mentioned to select one of the theories under three viewpoints. To summarize the results gained until now the author proposes an order of theories being - in his opinion - the "best" ones:

- Physical clearness
  - -- Theories without concrete model
    - Curvature Theory
    - Improved Modified Moreland's Theory
  - -- Theories with concrete model
    - Generalisation of the Theory of v.Schlippe et al. and its transmission to the beam model
- Computational costs
  - -- Slip Theory
  - -- Curvature Theory
  - -- Improved Modified Moreland's Theory
- Description of the essential phenomena
  Here no answer can be given by the author because of lacking test facilities and crucial experiments respectively.

Future research should be done with respect to three fields:

- <u>Theory</u>
  - Examination of a "jumping phenomenon" in the Theory of v.Schlippe et al., when the number of terms in the Taylor series is increased(7). This phenomenon was also mentioned by Collins(8).
  - Stability analysis of the wheel <u>with</u> caster.
  - Analysis of the Generalisation of the Theory of v.Schlippe et al. by more efficient numerical approximations (e.g. Galerkin's method).
  - Considering the influence of the finite width of tire in all theories.
  - Considering damping and mass forces of the tire.
  - Relation of the results given in this paper to theoretical and experimental results found by considering a wheel excited harmonically(9).
- <u>Experiments</u>
  - Gathering of further experiments given in the literature.
  - Examination of the theoretical stability regions shown in Figures 4.1 and 4.2 and the relations between the parameters of different theories given in references (1) and (7) by systematic crucial experiments.
- <u>Practice</u>
  - Reduce the costs for design, development, manufacturing and service of Shimmy dampers.

6. References

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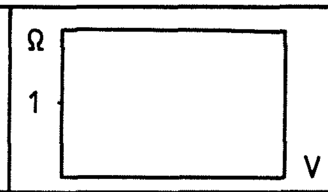
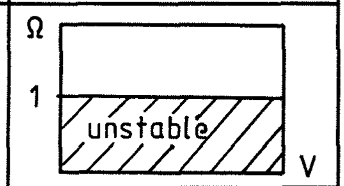
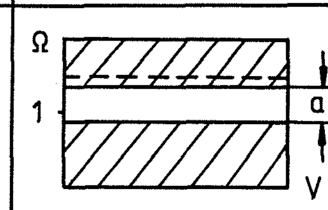
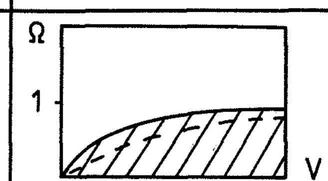
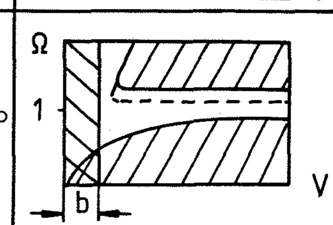
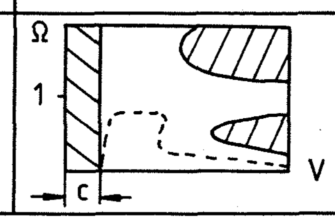
• Rigid wheel with nonholonomic constraint • Slip Theory	
• B. de Carbon's Elementary Theory neglecting the elastic deformations of the tire	
• B. de Carbon's Elementary Theory considering the elastic deformations of the tire • Modified Moreland's Theory • Theory of Pacejka (Approximation II) • Generalisation of the Theory of v.Schlippe et al. (GOTSA) with 3 terms in Fourier series - then a = 0 (Instability everywhere in the plane!)	
• Curvature Theory neglecting the elastic deformations of the tire • Kalker's Theory • B. de Carbon's Elementary Theory applied to a wheel set	
• Curvature Theory considering the elastic deformations of the tire • B. de Carbon's Complete Theory • Theory of v.Schlippe et al. (2 terms in Taylor series; zero width of tire) - then b = 0 • Theory of v.Schlippe et al. (3 and 4 terms in Taylor series; zero width of tire) - then b ≠ 0 • GOTSA (4 terms in Taylor series) - then b = 0	
• Theory of v.Schlippe et al. (2 terms in Taylor series; zero width of tire) - then c = 0 • Theory of v.Schlippe et al. (3 and 4 terms in Taylor series, finite width of tire) - then c ≠ 0	

Figure 4.2. Computational results for wheel without caster
Abscissa: Dimensionless velocity V
Ordinate: Frequency ratio $(\omega_\gamma/\omega_y)^2 = \Omega$

If the magnitude of an instability region is determined by an additional parameter this fact is symbolized by dotted lines.

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