PERFORMANCE EVALUATION OF A LINEAR RECURSIVE TECHNIQUE FOR AIRCRAFT ALTITUDE PREDICTION IN AIRBORN COLLISION AVOIDANCE SYSTEMS

A. Badach and P. Form
Technical University of Braunschweig, West Germany

Abstract

This paper analyses the performance of a linear recursive technique for the aircraft altitude prediction. This prediction technique has been employed in the collision avoidance system (TCAS II). It is based on altitude reports which are derived from barometric altimeters and are quantized in 100-foot increment. This prediction technique employs the observed level occupancy time, i.e. the time within the aircraft crosses one increment of 100 feet. It is shown that the estimate of the aircraft is biased. The bias value is evaluated in dependence on the aircraft velocity, the update time and the velocity estimation time. Formulas for the probability density function of the velocity estimator is given. The use of this probability distribution for the aircraft altitude prediction is presented.

I. Introduction

In this paper we study the following problem. Consider an aircraft as a moving object. Its altitude at time t is $z(t)$ and its vertical velocity is constant. We will determine this velocity.

The estimate of the vertical aircraft velocity is based upon altitude reports derived from encoding barometric altimeters and are quantized in q-foot (q=100 or q=25) altitude increments.[1,2] When simple linear recursive tracking techniques (for example alpha-beta smoothing filter) are applied to such quantized altitude reports, certain errors in estimation of vertical velocity can be directly attributed to the altitude quantization. These errors can be reduced by use of the estimation technique which explicitly recognizes the quantized nature of the altitude measurements. This velocity estimate technique has been employed in the collision avoidance system TCAS II. It is based on the observed level occupancy time,[1] i.e. the time within the aircraft crosses verticaly one increment of q feet. The measured value of the level occupancy time is the time difference between observed altitude transitions of two succesive quatization levels. Errors in this measurement are attributable to the sampling (update) interval. In TCAS (Threat Alert and Collision Avoidance System) this interval is 1 second. If altitude tracking is based on ground-based sensor data, the sampling interval may be 4 seconds or greater.

II. Characteristics of the level occupancy time technique

Let the altitude of an aircraft be $z(t)$ at the time t and its velocity at time t be

$$\dot{z}(t) = \frac{d}{dt} z(t) \qquad (2.1)$$

For simplicity we assume that $\dot{z}(t)=v_o$ = const. and $v_o > 0$. The considerations retain their generality since the case of $v_o < 0$ is obtained by reflecting the altitude $z(t)$ around the origin of the coordinate system.

Let t_o be the level occupancy time, i.e. the time required for $z(t)$ to change by an amount q. The value q we call here altitude quantization level. Therefore, the level occupancy time is the amount of time which is required for the aircraft to cross a single quantization level.

The altitude $z(t)$ is observed at the times $\tau_o, \tau_1, \tau_2, \ldots$ with the sampling interval , i.e.

$$\tau_i = \tau_o + i \cdot \tau \quad , i=1,2, \ldots \qquad (2.2)$$

To describe the altitude measurement error we define the following function

$$\left.\begin{aligned} INT_q(x) &= x - R_q(x) = n \cdot q \\ 0 &\leq R_q(x) < q \end{aligned}\right\} \qquad (2.3)$$

where n is a positive integer number. The function $R_q(x)$ denotes the fractional remainder of q.

The measured altitude position z_k^* at time τ_k^* is given by

$$z_k^* = INT_q(z(\tau_k^*)) \qquad (2.4)$$

The altitude at time τ_k^* is measured with the error

$$e_k = R_q(z(\tau_k^*)) \qquad (2.5)$$

For our investigations we distinquish two cases: $t_o > \tau$ and $t_o \leq \tau$.

The case $t_o > \tau$

In this case the level occupancy time is greater than the sampling interval and the k-th estimation t_k^* of the level occupancy time t_o may be calculated as follows

$$t_k^* = l_k^* \cdot \tau \qquad (2.6)$$

where l_k^* is a positive integer number.
An illustration of this situation is shown in Figure 1.

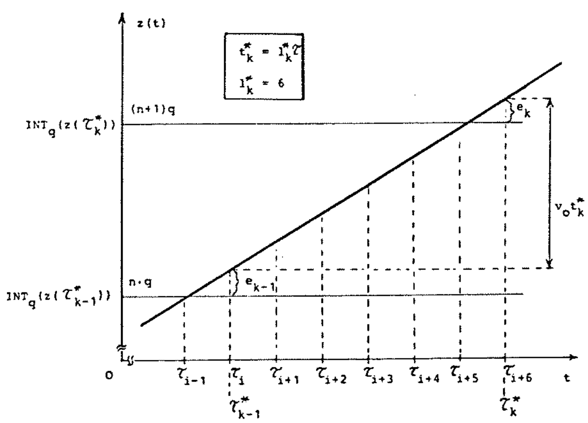

Figure 1. Illustration of the situation when the level occupancy time t_o is greater than the sampling interval

This number l_k^* must satisfy the following condition:

$$INT_q(z(\tau_k^* + l_k^* \tau)) - INT_q(z(\tau_k^*)) = q \qquad (2.7)$$

After some algebraic manipulations we get the following formula for the k-th estimation of the level occupancy time t_o:

$$t_k^* = t_o + (e_k - e_{k-1})/v_o \qquad (2.8)$$

where e_{k-1}, e_k denote the altitude observation errors.[1,3]

If we introduce the relative error

$$\mathcal{E}_k = e_k/q \qquad (2.9)$$

we get from Eq. (2.8):

$$t_k^* = t_o + t_o(\mathcal{E}_k - \mathcal{E}_{k-1}) \qquad (2.10)$$

The error in t_k^* is proportional to the difference of the initial and final values of the observation errors.

The case $t_o \leq \tau$

In this case the level occupancy time is not greater than the sampling interval and the k-th estimation t_k^* of the level occupancy time t_o is given by:

$$t_k^* = \tau/m_k^* \qquad (2.11)$$

where m_k^* is a positive integer number. This situation is shown in Figure 2.

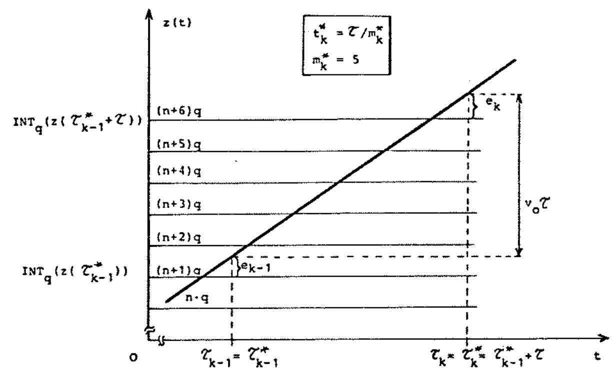

Figure 2. Illustration of the situation when the level occupancy time t_o is not greater than the sampling interval τ

It is obvious that

$$m_k^* = \frac{1}{q}(INT_q(z(\tau_k^* + \tau)) - INT_q(\tau_k^*)) \qquad (2.12)$$

It may be shown that the number m_k^* of quantization level transition observed within on the sampling interval τ can be written as follows:

$$m_k^* = \begin{cases} INT(\tau/t_o), & \text{if } e_k/q < 1-R(\tau/t_o) \\ INT(\tau/t_o)+1, & \text{if } e_k/q \geq 1-R(\tau/t_o) \end{cases} \qquad (2.13)$$

where $INT(\tau/t_o) = INT_1(\tau/t_o)$. The value of $R(\tau/t_o)$ is defined as the fractional remainder of the ratio τ/t_o, i.e. $R(x) = R_1(x)$ (see Eq. (2.3)).

After simple algebraical manipulations we get the following formula

$$t_k^* = t_o + \frac{t_o}{m_k^*}(\mathcal{E}_{k-1} - \mathcal{E}_k) \qquad (2.14)$$

In this case the error in t_k^* is reciprocally proportional to the number of observed quantization level transitions within the sampling interval τ . Therefore, the quality of the estimation t_k^* is proportional to the vertical aircraft velocity.

Let v_k^* be the k-th observation of the vertical aircraft velocity. Taking into account the relationship

$$v_k^* = q/t_k^* \qquad (2.15)$$

we get

$$v_k^* = \begin{cases} v_o/(1+ \varepsilon_k - \varepsilon_{k-1}), & \text{if } t_o > \tau \\ v_o/(1+(\varepsilon_{k-1} - \varepsilon_k)/m_k^*), & \text{if } t_o \leqslant \tau \end{cases} \qquad (2.16)$$

III. Probability distribution of the level occupancy time observations

Let

$$\delta t_k^* = (t_k^* - t_o)/t_o \qquad (3.1)$$

denote the relative error of k-th level occupancy time observation t_k^*. Thus, from (2.10) and (2.14) we get

$$t_k^* = \begin{cases} \varepsilon_k - \varepsilon_{k-1}, & \text{if } t_o > \tau \\ \varepsilon'_{k-1} - \varepsilon'_k, & \text{if } t_o \leqslant \tau \end{cases} \qquad (3.2)$$

where $\varepsilon'_k = \varepsilon_k/m_k^*$.

To determinate the probability distribution of the level occupancy time observations we introduce the following parameter

$$a = \begin{cases} \tau/t_o, & \text{if } t_o > \tau \\ 1/\mathrm{INT}(\tau/t_o), & \text{if } t_o \leqslant \tau \end{cases} \qquad (3.3)$$

It is assumed that errors ε_k, k=0,1, ... are independent of each other and their statistic is stationary. Therefore, the error sequence δt_k^*, k=0,1, ... has stationary statistics, as well.

The magnitude of the relative error ε_k is limited to

$$\varepsilon_k \leqslant \min\{\tau, t_o\}/t_o \qquad (3.4)$$

Furthermore we assume that errors ε_k are uniformly distributed over the interval

$$(0, \min\{\tau, t_o\}/t_o)$$

Let ΔT^* represent the relative errors (3.1) of level occupancy time observations. Under the forementioned assumptions it follows, that in the case $t_o > \tau$ the random variable ΔT^* has the probability density function

$$p_{\Delta T^*}(x) = \begin{cases} \frac{1}{a}(1 + \frac{x}{a}), & -a < x \leqslant 0 \\ \frac{1}{a}(1 - \frac{x}{a}), & 0 \leqslant x < a \end{cases} \qquad (3.5)$$

To determinate the density function of ΔT^* in the case $t_o \leqslant \tau$, we assume that the numbers m_k^*, k=0,1, ... , are independent of each other and their statistics are stationary. Let M^* represent these numbers. It follows from (2.13) that the random variable M^* has the following probability distribution:

$$P(M^* = \mathrm{INT}(\tau/t_o)) = 1 - R(\tau/t_o)$$

$$P(M^* = \mathrm{INT}(\tau/t_o)) = R(\tau/t_o) \qquad (3.6)$$

Therefore, in the case $t_o \leqslant \tau$, the probability density function of ΔT^* may be expressed as follows:

$$p_{\Delta T^*}(x) = p_{\Delta T^*}(x | M^*=\mathrm{INT}(\tau/t_o))\,P(M^*=\mathrm{INT}(\tau/t_o)) + p_{\Delta T^*}(x | M^*=\mathrm{INT}(\tau/t_o)+1)\,P(M^*=\mathrm{INT}(\tau/t_o)+1) \qquad (3.7)$$

The conditional density functions have the form:(3)

$$p_{\Delta T^*}(x | M^*=\mathrm{INT}(\tau/t_o)) = \begin{cases} \frac{1}{a}(1+\frac{x}{a}), & -a < x \leqslant 0 \\ \frac{1}{a}(1-\frac{x}{a}), & 0 \leqslant x < a \end{cases} \qquad (3.8a)$$

$$p_{\Delta T^*}(x | M^*=\mathrm{INT}(\tau/t_o)+1) = \begin{cases} \frac{1}{a_1}(1+\frac{x}{a_1}), & -a_1 < x \leqslant 0 \\ \frac{1}{a_1}(1-\frac{x}{a_1}), & 0 \leqslant x < a_1 \end{cases} \qquad (3.8b)$$

where

$$a_1 = a/(1+a) \qquad (3.9)$$

Finally in the case $t_o \leqslant \tau$ we get the following formula for the probability den- density function of ΔT^*:

$$p_{\Delta T^*}(x) = \begin{cases} \frac{1-R}{a}\left(1+\frac{a}{x}\right), & -a < x \leq -a_1 \\ \frac{1+aR}{a} + \frac{1+(2a+a^2)R}{a^2}x, & -a_1 \leq x \leq 0 \\ \frac{1+aR}{a} - \frac{1+(2a+a^2)R}{a^2}x, & 0 \leq x \leq a_1 \\ \frac{1-R}{a}\left(1-\frac{x}{a}\right), & a_1 \leq x < a \end{cases}$$

where $R=R(\tau/t_o)$.

The probability density function of ΔT^* is shown in Figure 3.

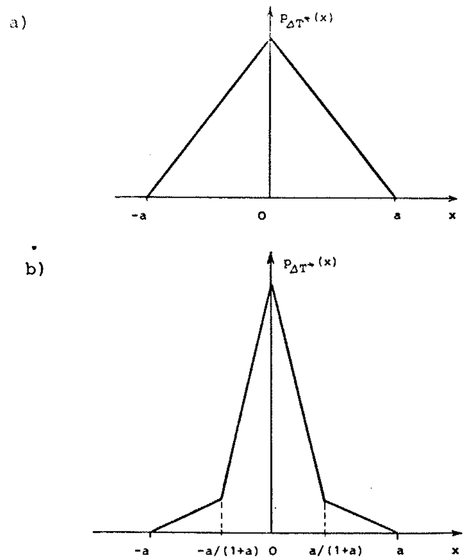

Figure 3. Probability density function of ΔT^*; a) the case $t_o > \tau$, b) the case $t_o \leq \tau$

The parameter a determines the maximum value of relative errors t_k^*, k=0,1, ... Figure 4 shows the parameter a vs the aircraft vertical velocity v_o.

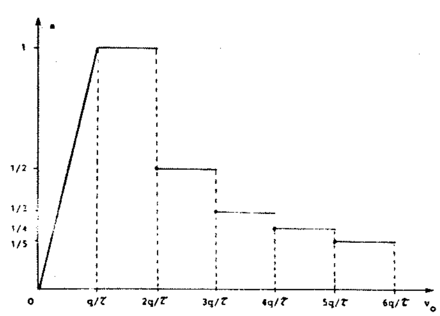

Figure 4. Parameter a vs the aircraft vertical velocity v_o

The above results allow the following practical and important conclusion: When the values of a are nearly egual to 1, then the quality of the level occupancy time technique is highly degraded.

IV. Probability distribution of vertical velocity observations

We will now calculate the probability distribution of velocity observations v_k^*, k=0,1, ..., (see Eq. (2.16)).

Let

$$v_{r,k}^* = v_k^*/v_o \qquad (4.1)$$

be the relative vertical velocity. It is assumed that velocity observations $v_{r,k}^*$, k=0,1, ... , have stationary statistics. Let V_r^* be a random variable that represents these velocity observations. It may be shown that the probability density function of V_r^* has the following form:[3]

- The case $t_o > \tau$

$$p_{V_r^*}(x) = \begin{cases} \frac{1}{a^2}\,\frac{(1+a)x-1}{x^3}, & 1/(1+a) < x \leq 1 \\ \frac{1}{a^2}\,\frac{1-(1-a)x}{x^3}, & 1 \leq x < A \end{cases} \qquad (4.2a)$$

- The case $t_o \leq \tau$

$$p_{V_r^*}(x) = \begin{cases} \frac{1-R}{a^2}\,\frac{(1+a)x-1}{x^3}, & 1/(1+a) < x \leq (1+a)/(1+2a) \\ \frac{1}{a^2}\,\frac{(1+a+2(a+a^2)R)x-(1+(2a+a^2)R)}{x^3}, & (1+a)/(1+2a) \leq x \leq 1 \\ \frac{1}{a^2}\,\frac{(1+a(2+a)R)-(1-a+2aR)x}{x^3}, & 1 \leq x \leq 1+a \\ \frac{1-R}{a^2}\,\frac{1-(1-a)x}{x^3}, & 1+a \leq x < A \end{cases} \qquad (4.2b)$$

where $R=R(\tau/t_o)$ and

$$A = \begin{cases} 1/(1-a), & a < 1 \\ \lim_{a \to 1^-} 1/(1-a) = \infty, & a = 1 \end{cases}$$

Figure 5 shows how the values of the parameter a influence the probability distribution of the aircraft velocity observations (4.1). If the level occupancy time t_o and the sampling interval τ are nearly equal, i.e. $a \approx 1$, then we have a great deviation between the observations v_k^*, $k=0,1,\ldots$, of the vertical velocity and its real value v_o.

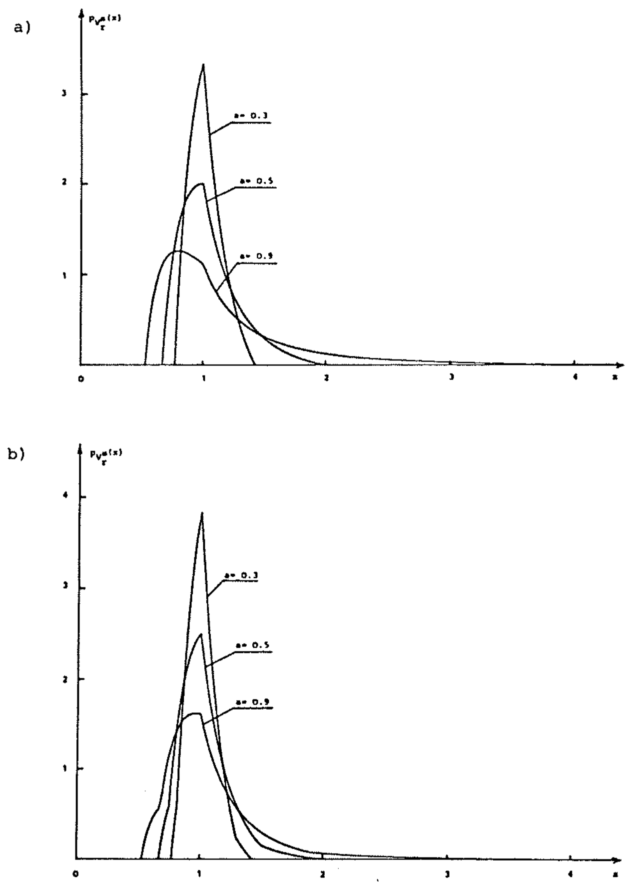

Figure 5. Probability density function $p_{V_r^*}(x)$ vs the parameter a; a) the case $t_o > \tau$ or $t_o \leq \tau$ and $R(\tau/t_o)=0$, b) the case $R(\tau/t_o)=0.5$

If the mean value of V_r^* is equal to 1, i.e.

$$\bar{v}_r^* = E(\, V_r^* \,) = 1$$

then the observation v_k^* of the vertical velocity is unbiased. It may be shown that the mean value $\bar{v}_r^*$ has the form:[3]

- The case $t_o > \tau$

$$\bar{v}_r^* = \frac{1+a}{a^2}\ln(1+a) + \frac{1-a}{a^2}\ln(1-a) \quad (4.3a)$$

- The case $t_o \leq \tau$

$$\bar{v}_r^* = \frac{1}{a^2}\Big[(1-2R)(1+a)\ln(1+a) + (1-R)(1-a)\ln(1-a) - R(1+a)(1+2a)\ln\frac{1+a}{1+2a}\Big] \quad (4.3b)$$

It is advantageous to introduce an important parameter

$$B_r^* = E(\, V_r^* \,) - 1 \quad (4.4)$$

which denotes a bias measure of velocity observations. Figure 6 shows the bias of velocity observations vs the ratio τ/t_o.

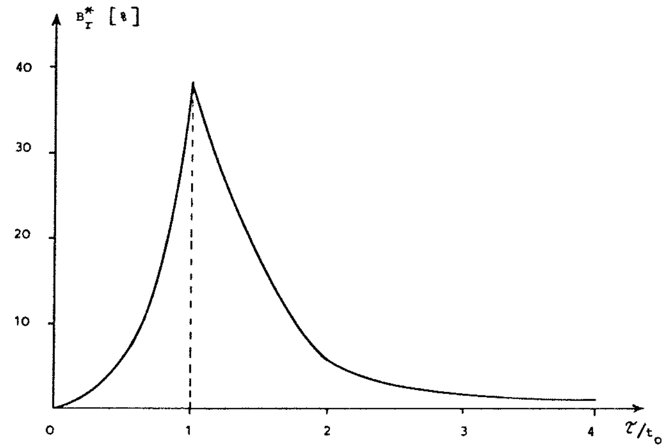

Figure 6. Bias of velocity observations (2.15) vs the ratio τ/t_o, τ - sampling interval, t_o-level occupancy time

The achieved results allow the conclusion that the bias of one velocity observation (measurement) can be nearly 40% of the real velocity value v_o.

V. Estimation of the level occupancy time

The estimate of the level occupancy time over K observations can be expressed as follows:

$$\hat{t} = \begin{cases} \dfrac{1}{K}\displaystyle\sum_{k=1}^{K} t_k^* , & t_o > \tau \\ K \Big/ \displaystyle\sum_{k=1}^{K} m_k^* , & t_o \leq \tau \end{cases} \quad (5.1)$$

Taking into account the definition

$$\hat{t}_r = \hat{t}/t_o = 1 + \delta\hat{t} \qquad (5.2)$$

we get from Eq. (5.1) the following formula for the error $\delta\hat{t}$ of level occupancy time estimation $\hat{t}$:

$$\delta\hat{t} = \frac{t - t_o}{t_o} = \begin{cases} (\varepsilon_K - \varepsilon_O)/K, & t_o > \tau \\ (\varepsilon_O - \varepsilon_K)\Big/\sum_{k=1}^{K} m_k^*, & t_o \leq \tau \end{cases} \qquad (5.3)$$

Let $\Delta\hat{T}$ be a random variable that represents the relative errors (5.3). Under the forementioned assumptions (Section III) the variable $\Delta\hat{T}$ has the probability density function:[3]

- The case $t_o > \tau$

$$p_{\Delta\hat{T}}(x) = \begin{cases} \frac{1}{d}\left(1 + \frac{x}{d}\right), & -d < x \leq 0 \\ \frac{1}{d}\left(1 - \frac{x}{d}\right), & 0 \leq x < d \end{cases} \qquad (5.4)$$

where

$$d = a/K = \tau/(K \cdot t_o) \qquad (5.5)$$

- The case $t_o \leq \tau$

$$p_{\Delta\hat{T}}(x) = \begin{cases} B_k + A_k x, & -d_k < x \leq d_{k+1} \\ & k=0,1,\ldots,K-1 \\ B_K + A_K x, & -d_K \leq x \leq 0 \\ B_K - A_K x, & 0 \leq x \leq d_K \\ B_k - A_k x, & d_{k+1} \leq x < d_k \\ & k=K-1, K-2, \ldots, 0 \end{cases} \qquad (5.6)$$

where

$$d_i = 1/(Km^* + i) \qquad (5.7)$$

and

$$m^* = \mathrm{INT}(\tau/t_o) \qquad (5.8)$$

For the parameters A_k and B_k we have:

$$A_k = \sum_{i=0}^{k} \binom{K}{i} (Km^* + i)^2 R^i (1-R)^{K-i}$$

$$B_k = \sum_{i=0}^{k} \binom{K}{i} (Km^* + i) R^i (1-R)^{K-i} \qquad (5.9)$$

for $k=0,1,\ldots,K-1,K$.

Figure 7 shows the distribution of the relative error $\delta\hat{t}$ in the level occupancy time estimate $\hat{t}$.

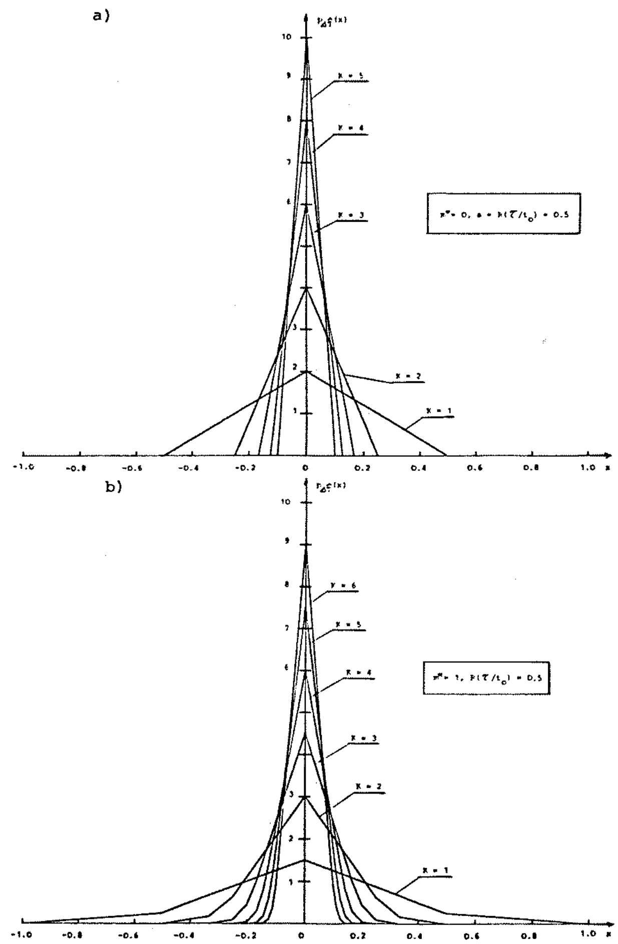

Figure 7. Probability density function of $\Delta\hat{T}$ vs K (i.e. number of observations); a) the case $m^*=0$ and $R(\tau/t_o)=0.5$; b) the case $m^*=1$ and $R(\tau/t_o)=0.5$

It may be shown that the estimate (5.1) of the level occupancy time is unbiased.

VI. Estimation of the vertical aircraft velocity

The estimate of the vertical aircraft velocity v_o over K observations is calculated according to

$$\hat{v} = q/\hat{t} \qquad (6.1)$$

where q denotes the altitude quantization level. We introduce the relative velocity

$$\hat{v}_r = \hat{v}/v_o \qquad (6.2)$$

Then from (5.1) and (6.1) we get

$$\hat{v}_r = 1/(1 + \delta\hat{t}) \qquad (6.3)$$

Let $\hat{V}_r$ be a random variable that represents the relative velocity $\hat{v}_r$. It may be shown that the probability density function of $\hat{V}_r$ has the form:[3]

- The case $t_o > \tau$

$$p_{\hat{V}_r}(x) = \begin{cases} \frac{1}{d^2}\frac{(1+d)x-1}{x^3}, & \frac{1}{1+d} < x \leq 1 \\ \frac{1}{d^2}\frac{1-(1-d)x}{x^3}, & 1 \leq x < \frac{1}{1-d} \end{cases} \qquad (6.4a)$$

- The case $t_o \leq \tau$

$$p_{\hat{V}_r}(x) = \begin{cases} \frac{(A_k + B_k)x - A_k}{x^3}, & \frac{1}{1+d_k} < x \leq \frac{1}{1+d_{k+1}} \\ & k=0,1,\ \ldots\ ,K-1 \\ \frac{(A_K + B_K)x - A_K}{x^3}, & \frac{1}{1+d_K} \leq x \leq 1 \\ \frac{A_K - (A_K-B_K)x}{x^3}, & 1 \leq x \leq \frac{1}{1-d_K} \\ \frac{A_k - (A_k-B_k)x}{x^3}, & \frac{1}{1-d_{k+1}} \leq x < \frac{1}{1-d_k} \\ & k=K-1,\ K-2,\ \ldots\ ,1,\ 0 \end{cases} \qquad (6.4b)$$

The parameters d, d_k, A_k and B_k are calculated according to Eqs. (5.5), (5.7) and (5.9).

The probability density function $p_{\hat{V}_r}(x)$ vs K (i.e. number of observations) is shown in Figure 8.

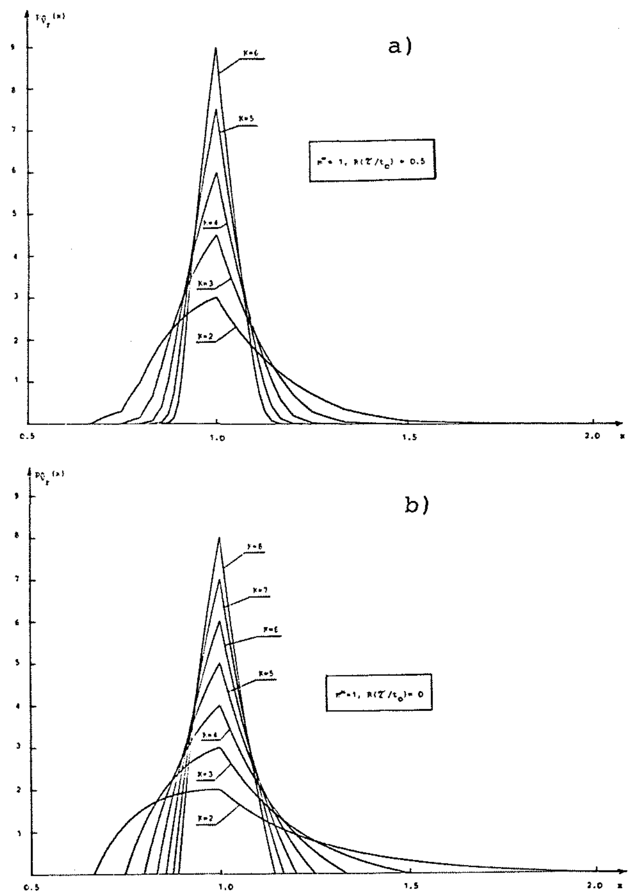

Figure 8. Probability density function of $\hat{V}_r$ vs K; a) the case m*=1 and $R(\tau/t_o)$=0.5, b) the case m*=1 and $R(\tau/t_o)$=0.5

The estimate $\hat{v}$ (see Eqs. (5.1) and (6.1)) of the vertical aircraft velocity v_o is unbiased, when the mean value of $\hat{V}_r$ is equal to 1, i.e.

$$\hat{\bar{v}}_r = E(\hat{V}_r) = 1$$

It may be shown that $\hat{\bar{v}}_r \geq 1$ and the mean value $\hat{\bar{v}}_r$ can be calculated according to:

- The case $t_o > \tau$

$$\hat{\bar{v}}_r = \frac{1+d}{d^2}\ln(1+d) + \frac{1-d}{d^2}\ln(1-d) \qquad (6.5a)$$

- The case $t_o \leq \tau$

$$\hat{v}_r = \sum_{k=0}^{K-1} \left[A_k \ln \frac{1-d_k^2}{1-d_{k+1}^2} + B_k \ln \frac{(1+d_k)(1-d_{k-1})}{(1+d_{k+1})(1-d_k)} \right]$$

$$+ A_K \ln(1-d_K^2) + B_K \ln \frac{1+d_K}{1-d_K}, \quad K \geq 2, m^* > 1 \qquad (6.5b)$$

The bias measure

$$\hat{B}_r = \hat{v}_r - 1 \qquad (6.6)$$

of the velocity estimation $\hat{v}$ versus the ratio τ/t_o is shown in Figure 9.

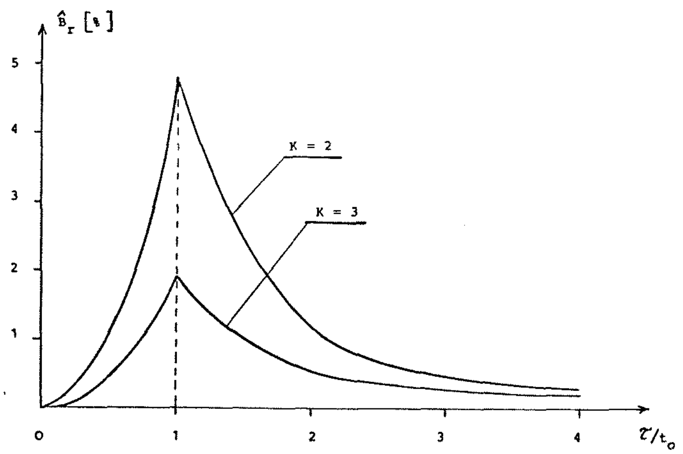

Figure 9. Bias of the velocity estimation (6.1) vs the ratio τ/t_o ; τ- sampling interval, t_o- level occupancy time

In Figures 6 and 9 we can observe that the parameter K has a great influence on the bias of velocity estimation (6.1).

VII. Distribution of the aircraft altitude prediction

In collision avoidance applications, it is of interest to know the aircraft position at some future time. Altitude prediction is obtained by a simple linear prediction of the aircraft motion according to

$$\hat{z}(t) = z^*(t_o') + v\,(t-t_o') \qquad (7.1)$$

where $z^*(t_o')$ is the measured altitude at time t_o' (see Eq. (2.4)); $\hat{v}$ is the estimated vertical velocity (see Eqs. (5.1) and (6.1)).

If $z^*(t_o') = n \cdot q$, then the true altitude at time t_o' (i.e. $z(t_o')$) belongs to an interval $[n \cdot q, (n+1) \cdot q)$ and is unknown.

The outlined probability distribution (6.4) of the vertical velocity allows to obtain the probability distribution of the predicted aircraft altitude (7.1). If we consider only the case $t_o > \tau$, it can be shown that the probability density function of $\hat{z}(t)$ has the following form:

$$p(\hat{z}) = \begin{cases} 0, & \hat{z} \leq nq + \frac{\hat{v}t}{1+d} \\ \frac{(1+d)^2}{2qd^2} + \frac{(\hat{v}t)^2}{2qd^2(\hat{z}-nq)^2} - \frac{(1+d)\hat{v}t}{qd^2(\hat{z}-nq)}, & nq + \frac{\hat{v}t}{1+d} < \hat{z} \leq nq + \hat{v}t \\ \frac{d^2+2d-1}{2qd^2} + \frac{(1-d)\hat{v}t}{qd^2(\hat{z}-nq)} - \frac{(\hat{v}t)^2}{2qd^2(\hat{z}-nq)^2}, & nq+\hat{v}t < \hat{z} \leq nq + \frac{\hat{v}t}{1-d} \\ \frac{1}{q}, & nq + \frac{\hat{v}t}{1-d} < \hat{z} \leq (n+1)q + \frac{\hat{v}t}{1+d} \\ \frac{(1+d)\hat{v}t}{qd^2(\hat{z}-(n+1)q)} - \frac{1+2d-d^2}{2qd^2} - \frac{(\hat{v}t)^2}{2qd^2(\hat{z}-(n+1)q)^2}, & (n+1)q + \frac{\hat{v}t}{1+d} < \hat{z} \leq (n+1)q + vt \\ \frac{(1-d)^2}{2qd^2} - \frac{(1-d)\hat{v}t}{qd^2(\hat{z}-(n+1)q)} + \frac{(\hat{v}t)^2}{2qd^2(\hat{z}-(n+1)q)^2}, & (n+1)q + \hat{v}t < \hat{z} < (n+1)q + \frac{\hat{v}t}{1-d} \end{cases}$$

The parameter d is calculated according to (5.5).

Figure 10 shows the distribution of the predicted aircraft altitude. It can be seen that the longer the prediction time (i.e. $t-t_o'$), the greater the significance of the estimation $\hat{v}$ in comparison with the altitude quantization error.

If the level occupancy time t_o and the sampling (update) interval τ are nearly equal and only one velocity observation v_k^* (2.16) is assumed as the velocity estimation $\hat{v}$ (i.e. $d \approx 1$ and $K=1$) then the altitude prediction error may be longer than the altitude quantization level q.

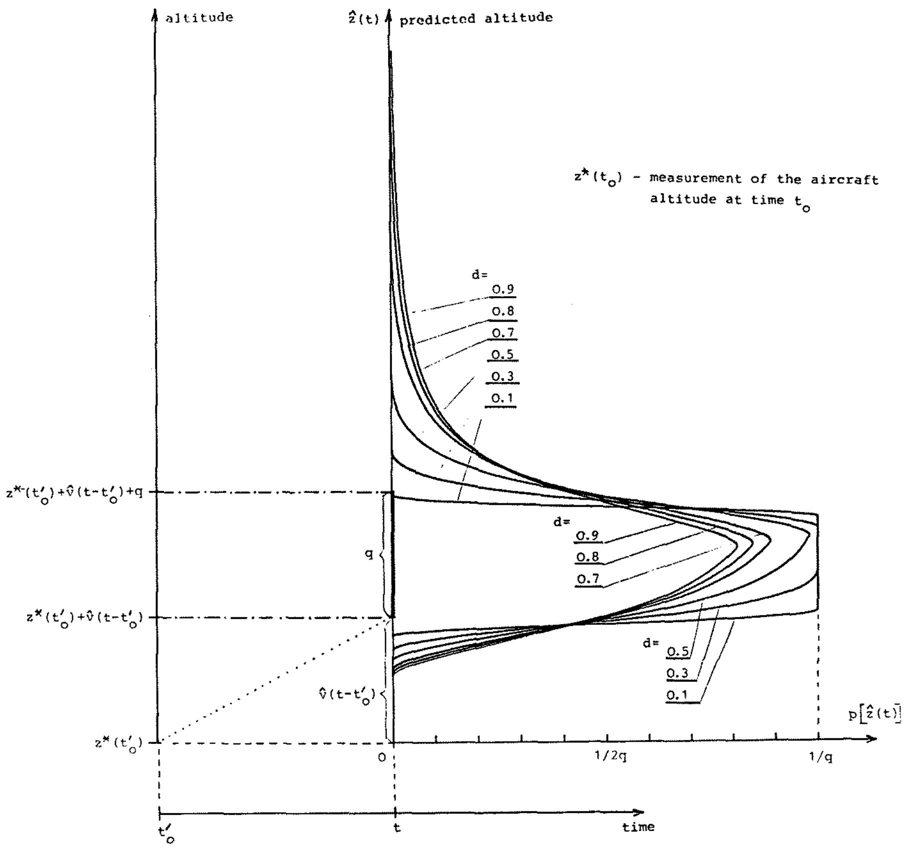

Figure 10. Probability density function $p[\hat{z}(t)]$ of the linearly predicted aircraft altitude

VIII. Conclusions

In this paper we have presented a performance evaluation of a linear technique for aircraft altitude prediction in airborn collision avoidance systems. This technique is based on observation of the level occupancy time.[1]

We have outlined the probability distribution functions for the estimations of level occupancy time and aircraft vertical velocity. It is shown that the vertical velocity estimation which is based on the evaluation of level occupancy time is highly biased. If one velocity observation (measurement) is assumed as the estimated velocity and the level occupancy time and the sampling (update) interval are nearly equal, then the bias of this velocity estimation can be nearly 40% of the real velocity value.

It should be noted that the results presented here may be useful for the development of airborn collision avoidance systems. Particularly, they may be used to investigate a statistical test for detection of the aircraft velocity changes.

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