WIND INFLUENCE ON THE RANGE OF JET OR PROPELLER AIRCRAFT

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Abstract

It has been found that the maximum range of aircraft subjet to wind effect is obtained when it flies at a lift coefficient which at every instant satisfies the following relations, respectively:

$$C_{L_a} = C_{L_o}\left(1 + \frac{w}{V + w/2}\right)^{\frac{1}{2}} \quad \text{for prop.}$$

$$C_{L_a} = C_{L_o}\left(\frac{1 + 2w/V}{1 + 2w/3V}\right)^{\frac{1}{2}} \quad \text{for jet}$$

where C_{L_o} is the lift coefficient giving maximum range in still air in the corresponding case, its value being

$$C_{L_o} = (C_{D_o}\pi\lambda)^{\frac{1}{2}} \quad \text{and} \quad C_{L_o} = (C_{D_o}\pi\lambda/3)^{\frac{1}{2}}.$$

respectively; they were derived from the general equation by putting w=0; C_{D_o} is minimum drag coefficient of the aircraft, λ is the wing effective aspect ratio and w is the wind velocity component along the route, positive in the case of a tail wind.

Analysis of functions for small values of w/V show that the airspeed for maximum range can be approximated by the following expressions:

$$V_{m_a} = V_{m_o} - w/4 \quad \text{for prop.}$$

$$V_{m_a} = V_{m_o} - w/3 \quad \text{for jet}$$

where V_{m_o} is the instantaneous speed for maximum range in still air in the corresponding case.

Numerical integration for jet aircraft showed: a) Favorable though negligible differences in range compared with the C_L constant method, b) A decrease in the elapsed time on route in the presence of a head wind, compared against the C_L constant case.

Constant specific fuel consumption has been assumed and no compressibility effects has been considered.

I. Introduction

Range calculations for both jet and propeller aircraft are widely known provided the flight is conducted in still air, in which case the maximum range is obtained at some particular constant value of the lift coefficient.

They were predicted for instance by the classical Breguet formula for propeller aircraft or by a similar expression for jet aircraft.

Little is found in the literature on the case when the aircraft flies subjected to a component of the wind along the route, in which case the lift coefficient for best range is no longer constant.

In this paper it is intended to determine the conditions that the lift coefficient must fulfil in order to maximize the range.

II. Propeller Aircraft

Fuel consumption by weight equals the change in total weight in an elapsed time dt:

$$c(P/\eta)dt = -dG \tag{1}$$

where P is the engine shaft power , c the specific fuel consumption (kg/kg.m), η the propeller efficiency and G the instantaneous weight of the aircraft.

An element of the range is given by

$$da = (V+W)dt = -\frac{\eta}{c}\frac{V+W}{P}dt \tag{2}$$

wher w is the wind direction, positive for a tail wind; $P=(C_D/C_L)G.V$; then:

$$a = -\frac{\eta}{c}\int_{G_o}^{G_f}\frac{V+w}{V}\frac{C_L}{C_D}\frac{dG}{G} \tag{3}$$

where η and c were considered as constants.

Since $C_L = f_1(G,V)$ and $C_D = f_2(C_L)$, the integrand function for a given aircraft at constant altitude and wind magnitude depends only on G and V:

$$F(G,V)= \frac{V+w}{G.V}\frac{C_L}{C_D} \tag{4}$$

Therefore we have to find a function F(G,V) which should maximize the integral in (3), a problem of Calculus of Variations.

The first necessary condition to extremize (maximize or minimize) the integral (3) is given by the Euler-Lagrange equation (Ref.1) which adapted to the present nomenclature can be written as:

$$\frac{\partial F}{\partial V} + \frac{d}{dG}\left(\frac{\partial F}{\partial V'}\right) = 0 \qquad (5)$$

where $V'=dV/dG$. Considering the physical nature of the problem we interpret this condition as maximizing the integral.

Since $\partial F/\partial V' = 0$ the condition (5) reduces itself to:

$$\frac{\partial F}{\partial V} = 0 \qquad (6)$$

Now let us replace in (4) the explicit values of:

$$C_L = 2G/\rho S V^2 \quad \text{and} \quad C_D = C_{D_o} + \frac{C_L^2}{\pi\lambda} \qquad (7)$$

we obtain:

$$F(G,V) = \frac{2(V+w)}{\rho S V^3}\left[C_{D_o} + \frac{1}{\pi\lambda}\left(\frac{2G}{\rho S V^2}\right)^2\right]^{-1} \qquad (8)$$

By performing a partial derivative of (8) with respect to V, eqating to zero and arranging, we obtain the condition

$$C_{D_o}(4V+6w) - \left(\frac{2G}{\rho S}\right)^2 \frac{4V+2w}{\pi\lambda V^4} = 0 \qquad (9)$$

from where:

$$G = \frac{1}{2}\rho V^2 S (C_{D_o}\pi\lambda)^{\frac{1}{2}}\left(1+\frac{w}{V+w/2}\right)^{\frac{1}{2}} \qquad (10)$$

Using in (10) the expression of C_L as given in (7) and solving for C_{L_a}:

$$C_{L_a} = (C_{D_o}\pi\lambda)^{\frac{1}{2}}\left(1+\frac{w}{V+w/2}\right)^{\frac{1}{2}} \qquad (11)$$

Putting w=0 in (11) we obtain the lift coefficient for maximum range in still air $C_{L_o} = (C_{D_o}\pi\lambda)^{\frac{1}{2}}$, and replacing in (11) we have:

$$\frac{C_{L_a}}{C_{L_o}} \cdot = \left(1+\frac{w}{V+w/2}\right)^{\frac{1}{2}} = 0 \qquad (12)$$

Since $C_{L_a}/C_{L_o} = (V/V_o)^2$, replacing in (12) we have the alternative form:

$$u = \frac{2(v-v^5)}{3v^4-1} \qquad (13)$$

where $u=w/V_o$ and $v=V/V_o$.

Note that V_o is the airspeed that gives the maximum range in still air and corresponds to the constant C_{L_o} defined in the foregoing paragraph. Therefore V_o is variable during the flight according to the variation in G, the instantaneous weight of the aircraft.

This result for the propeller aircraft, jointly with that of the next section for jet aircraft, will be used in Section IV for an approximate calculation of the speed for maximum range for both types of aircraft.

III. Jet Aircraft

Following a method similar to that outlined for the propeller aircraft in the preceding section, we can express the range for the jet aircraft as:

$$a = -\frac{1}{c}\int_{G_o}^{G_f} \frac{V+w}{C_D q S}\, dG \qquad (14)$$

Where c is the specific fuel consumption per unit of thrust per unit of time, (kg/kg.sec); q is the dynamic pressure $\rho V^2/2$.

Performing steps similar to those we carried out in the case of propeller aircraft we can obtain the form for the function under the integral above, as:

$$F(G,V) = (V+w)\quad C_{D_o}\frac{\rho V^2 S}{2} + \frac{1}{2\pi\lambda}\frac{4G^2}{SV^2} \qquad (15)$$

Taking the derivative $\partial F/\partial V = 0$ and solving for G, we obtain:

$$G = \left(\frac{C_{D_o}\pi\lambda}{3}\right)^{\frac{1}{2}}\left(\frac{1+2w/V}{1+2w/3V}\right)^{\frac{1}{2}} \frac{\rho S V^2}{2} \qquad (16)$$

Setting $(C_{D_o}\pi\lambda/3)^{\frac{1}{2}} = C_{L_o}$; $C_{L_a} = G/qS$

we have:

$$\frac{C_{L_a}}{C_{L_o}} = \left(\frac{1+2w/V}{1+2w/3V}\right)^{\frac{1}{2}} \qquad (17)$$

replacing $C_{L_a}/C_{L_o} = (V/V_o)^2$ and rearranging:

$$u = 3\,\frac{v-v^5}{6v^4-2} \qquad (18)$$

Where $u=w/V_o$ $v=V/V_o$, V_o being the airspeed that gives the maximum range in still air and C_{L_o} the corresponding lift coefficient.

IV. Approximate Speed for Maximum range

To calculate the speed which maximize the range, it is required to solve fifth-degree equations (13) and (18).

The foregoing deductions suggest that this airspeed, V_a, is placed somewhere about V_o, the airspeed for maximum range in still air.

Therefore we can obtain an aproximation to its values by expanding functions (13) and (18) in Taylor series assuming w/V to be small enough in order to represent the function with only the first two terms of the expansion:

$$V_a = V_o + \left(\frac{dV}{dw}\right)w \qquad (19)$$

where dV/dw = dv/du must be evaluated at $V=V_o$ or, which is the same, at v=1. From (13) and (18) we can obtain:

$$\left(\frac{du}{dv}\right)_{v=1} = -4 \quad \text{for prop.} \qquad (20a)$$

$$\left(\frac{du}{dv}\right)_{v=1} = -3 \quad \text{for jet} \qquad (20b)$$

which allow us to write:

$$V_a = V_o - w/4 \quad \text{for prop.} \qquad (21a)$$

$$V_a = V_o - w/3 \quad \text{for jet} \qquad (21b)$$

where V_o is the instantaneous airspeed, variable along the flight path, calculated for zero wind, and w is the wind component along that path, positive if from tail.

V. Numerical Integration for Jet Aircraft

After many attempts to integrate equations (3) and (14) in a simple analytical way without success, it was decided to attain the solution, for the time being, through a numerical method which was applied to the jet aircraft case.

To this effect the following expression, derived from (14) and combined with (15) to (18), was used, after some steps and rearrangement:

$$a = \frac{1}{2}\frac{C_{L_o}}{cC_{D_o}}\int_{V_o}^{V_f}\left(\frac{3w}{Z^{\frac{1}{2}}} - \frac{Z^{\frac{1}{2}}}{V}\right)dV \qquad (22)$$

where Z = (3V+6w)(3V+2w); V_o and V_f the initial and final speeds.

For a comparison the range was also calculated for the case $C_L = C_{L_o}$ = const. superimposed with the displacement due to the wind, wt, where t is the duration of the flight, the expression being:

$$a' = \frac{4559}{c}\left(\frac{\pi\lambda}{C_{D_o}^3}\right)^{\frac{1}{4}}\left(\frac{G_o}{\sigma S}\right)^{\frac{1}{2}}\left[1-\left(\frac{G_f}{G_o}\right)^{\frac{1}{2}}\right]\times$$

$$\cdot\left[1 + \frac{1}{2}\frac{w}{V_o}\frac{\ln(G_o/G_f)}{1-(G_o/G_f)^{\frac{1}{2}}}\right] \qquad (23)$$

The number "4559" is the result of grouping all the numerical mathematical and physical constants (π, ρ_o, etc.)

The limits of integration of equation (22), V_o and V_f, initial and final airspeed, were obtained by solving the fifth-degree equation (18).

In both formulas the units used correspond to the Technical System (kg-force, meter, second). Accordingly fuel consumption is in kg/kg.sec (sec^{-1}) for jets and kg/kg.m (m^{-1}) for propeller aircraft.

The numerical integration has been applied, with various wind conditions, to two different aircraft representative of their respective types: one four-jet transport and one twin-jet executive, the features of which are shown in Table 1.

The principal results of these calculation are resumed in Table 2.

TABLE 1 Assumed Characteristics for two Representative Airplanes

		Four-Jet Transp.	Twin-Jet Exectv.
Máx.T.O.Weight, G_o	kg	148325	11433
Final Weight, G_f	kg	76086	7625
Wing Area S	m^2	279	41
Eff.Aspect R. λ	-	5,64	5.18
Max.Wing Load.	kg/m^2	530	279
Min.Drag Coeff. C_{D_o}	-	0.0243	0.0246
Density ratio σ at flight altitude z	m	0.3098 10660	0.4484 7610
Spec.Fuel Cons. c 10^4 kg/kg.sec	s^{-1}	1,41	1,81

VI. Comments on the Numerical Results

Examination of the values in Table 2, suggests the following coments.

1. When flying with a "variable C_L" according to the method of this paper, formula (18), the range is slightly greater than that predicted on the standard assumption of "constant C_L" (formula 23), regardless of whether there is a tail or head wind; but its value is practically negligible.

2. Flight time in maximum range at variable C_L shows an increase over that of the constante C_L case, when flying in a tail wind; and a decrease in a head wind.

We must point out that in the C_L constant case it is assumed that the flight time corresponds to the endurance at that lift coefficient, and, therefore, the value of flight time is also constant, independent of the wind condition. This value is indicated in Table 2 for reference and is the same in both methods when w=0.

TABLE 2. Principal Numerical Results

w	a	a'	t	V_r	V'_r	Δa	Δt
m/s	km	km	h	km/h	km/h	km	h
Four-Jet Transport, G_o = 148325 kg.							
+40	14220	14166	15.3	929	970	54	0.7
+20	13123	13112	15.0	875	898	11	0.4
0	12057	12057	(14.6)	826	826	0	0
-20	11110	11096	14.1	788	760	14	-0.5
-40	10022	9947	13.6	737	681	75	-1.0
Twin-Jet Executive G_o = 12100 kg.							
+40	4757	4721	7.4	643	684	36	0.5
+20	4277	4268	7.2	594	618	9	0.3
0	3724	3724	(6.9)	540	540	0	0
-20	3283	3271	6.6	497	474	12	-0.3
-40	2829	2773	6.0	471	402	56	-0.9

Notes: h=hour; number within parenthesis denote flight time for w=0; V_r and V'_r are the average cruising speed.Ground speed.

3. From the contents of the latest paragraph we infer that flying according to the"variable C_L"method is somewhat advantageous when dealing with a head wind, not because of the better range, but because of the lesser time aloft.

4. In the case of tail wind, the small increase obtained hardly justifies the longer time on route.

VII. Additional Comment

The assumption of the constant specific fuel consumption is a rather restrictive one.

It was adopted at the beginning of the work in the hope of obtaining a simpler formula for educational purposes.

As this intended simplicity was not been achieved it was necessary to resort to numerical methods, which now suggest the possibility of an additional assumption on the variation of the specific fuel consumption, a proposal which can be dealt with in a further study on this subjet.

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References

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