FATIGUE LIFE PREDICTIONS UNDER COMPLEX LOADING

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Abstract

Fatigue (information) life predictions under complex loading are discussed in this paper. The principle and application of local strain method is presented in detail and the fatigue information lives of an aluminum alloy specimen with a central hole under random loading and spectrum type loadings using the local strain method are also presented. The traditional nominal stress method is used directly. In the nominal stress method, we use different calculations and simplified methods to treat the random load (strain) time history in order to obtain different program spectra. Then we use the linear damage theory and S-N curve of the material to calculate the fatigue lives of the same aluminum alloy specimen under random loading and different program spectrum loadings.

The results of the fatigue life calculation are compared with experimental ones and it is thought that the local strain method is a better fatigue information life estimation method, but the traditional nominal stress method still has its use value in engineering.

I. Introduction

Fatigue life includes mainly both crack initiation life and crack propagation life. In engineering, the fatigue crack initiation life generally indicates that the crack has nucleated and has propagated to an detectable crack length. The methods for the prediction of fatigue crack initiation life will be discussed in this paper. As we know, there are many prediction methods. One of them is the so called nominal stress method which makes use of linear accumulative damage theory and has found wide application in engineering. But there are some inevitable shortcomings in the nominal stress method, such as that the effect of loading sequence and that the variation of local residual stresses cannot be considered. The local strain method (or local stress-strain method), which has been developed since the sixties, is another fatigue life prediction method based on studying the local stress-strain history of a small piece of material at critical position in specimen. This method will be the main one considered in this paper.

II. Some Major Problems in Local Stress-Strain Method

1. The Determination of Local Stress-Strain at Notch Root.

Fatigue failures always occur in stress concentration positions such as hole sides and so on. Under complex alternating loads, it is a difficult problem to determine local stress-strain response at stress concentration positions. Cyclic stress-strain curves and hysteresis curves of material must be used in calculating local stress and strain, and the "memory characterestic" of material must also be considered.

Metallic materials, under cyclic strain (or stress), exhibit the transient characterestic of cyclic hardening/softening and cyclic relaxation/creep. In general, the degree of hardening/softening of a material gradually tends to stabilize, as the number of cycles increases (aluminum alloys generally stabilize faster). In an engineering calculation, we can neglect these transient effects, and use the stable cyclic stress-strain curve of a material. Moreover, as an approximate estimate, the stable cyclic stress-strain curve can be used right at the start of prediction. For most engineering materials (except gray cast iron etc.), the stable hysteresis curve of a material is geometrically similar to the cyclic stress-strain curve of the same material but magnified by a scale factor of two, so they can be represented by equations(1) and (2) respectively as follows:

$$\Delta \varepsilon = \frac{\Delta \sigma}{E} + 2\left(\frac{\Delta \sigma}{2K'}\right)^{1/n'} \quad (1)$$

$$\varepsilon_a = \varepsilon_{ea} + \varepsilon_{ep} = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{1/n'} \quad (2)$$

where $\varepsilon_a = \frac{\Delta \varepsilon}{2}$, $\sigma_a = \frac{\Delta \sigma}{2}$;

ε_{ea} and ε_{ep} are elastic component and a plastic component of strain amplitude respectively;
$\Delta \varepsilon$ and $\Delta \sigma$ are strain range and stress range respectively;
E is the elastic modulus;
K' is the cyclic strength coefficient;
n' is the cyclic strain hardening exponent.

We can use experimental or calculation methods to determine the relationship between local stress and strain at notch root. Some methods which are often used in engineering will be introduced as follows:

(1) Neuber's rule

In 1961, H. Neuber took a prismatic body und shearing strain and showed that, with a certain plastic range, the theoretical stress concentration factor K_t is equal to the geometric mean of the local stress concentration factor K_σ and the local strain concentration factor K_ε, i.e, $K_t = \sqrt{K_\sigma K_\varepsilon}$. The application of this rule was later extended to other loading conditions. Under cyclic loading, when the nominal stress is within the elastic region but the local stress is within the plastic region (it is the most practical condition), the following expression can be derived

$$\Delta \sigma \Delta \varepsilon = \frac{(K_t \cdot \Delta S)^2}{E} \quad (3)$$

where ΔS is the nominal stress range.

Under a certain nominal stress, when material and notch shape of a specimen is determined, the right side of equation (3) is equal to a constant, called Neuber's Constant. Equation (3) represents a Neuber's hyperbola. The intersection of Neuber's hyperbola and a stable cyclic stress-strain curve (or a stable hysteresis curve) defines the desired local stress and local strain. In calculation, we can use Newton's iteration method to solve for the desired local stress and local strain, but there are two questions that should be paid attention to. The first one is that whether the stable $\sigma-\varepsilon$ curve or the hystoresis curve should be used. The second one is how to determine the starting point of a loading or an unloading course in order to consider the "memory characteristic" of a material. Regarding the first question, we think that the stable $\sigma-\varepsilon$ curve should be used as long as the applied load exceeds the maximum load that have been arrived at in previous loading history and stable hysteresis curve should be used under other conditions. According to the counting principle of rainflow method, the second question is equivalent to looking for maximum or minimum load during a section of random loading. The scheme of the procedure to solve for local stress and strain is shown in figure 1.

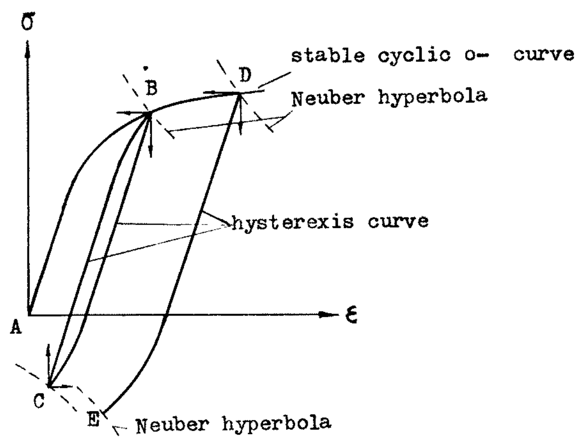


FIG. 1

(2) Modified Neuber's rule

Neuber's rule is often conservative in life predictions, so equation (3) should be modified, usually the fatigue notch factor K_f is used instead of K_t. K_f can be given by the following formulas:

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{\frac{A}{R}}} \tag{4}$$

or

$$K_f = 1 + \frac{K_t - 1}{1 + \frac{A}{R}} \tag{5}$$

where A is a material constant, R is the radius of notch root. The values of K_t and A of common used materials can be found in relevant references (as references (5), (6)).

There are other equations for calculating K_f, which in fact, varies with the amptitude of nominal stress. So which equation conforms to reality needs to be further explored.

(3) Modified Stowell's rule(7)(8)

This rule assumes that, when the material's deformation reaches the plastic stage, the stresses and strains satisfy the following equation:

$$\Delta\sigma = \frac{\Delta S \Delta\varepsilon}{\Delta\varepsilon - (K_t - 1)\frac{\Delta S}{E}} \tag{6}$$

This rule is similar to Neuber's rule. In modified Stowell's rule, equation (6) is used instead of Neuber's hyperbola (i.e equation (3))

(4) Linear Strain rule

This rule assumes that although the material's deformation reaches the plastic stage, the local strain and the nominal stress are still linear related, so we have

$$\Delta\varepsilon = \frac{K_t \Delta S}{E} \tag{7}$$

then the local stress can be obtained from equation (1) or (2) by using the value of $\Delta\varepsilon$ obtained from equation (7).

2. The Strain-Life Curve and Its Modification

The strain-life curve of a material is usually obtained by small specimens under full reversed constant amplitude strain fatigue test, as shown in Fig.2.

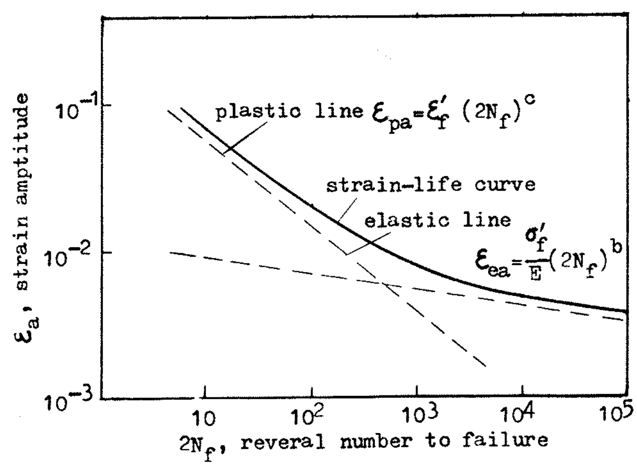


FIG. 2

The (total) strain amplitude ε_a can be separated into elastic component ε_{ea} and plastic component ε_{pa}. Most material test data show that there exists an approximitely linear relation between ε_{ea} or ε_{pa} and fatigue life $2N_f$ (i.e reversal number to failure) on double logarithm plot, as shown in Fig.2. The strain-life curve can be expressed by

$$\varepsilon_a = \varepsilon_{ea} + \varepsilon_{pa} = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c \tag{8}$$

where σ_f', b are the fatigue strength coefficient

and exponent respectively;
ε'_f, c are the fatigue ductility coefficient and exponent respectively.

In practical loading spectra, it is necessary to modify the $\varepsilon_a - 2N_f$ curve in order to consider the effects of mean stress and mean strain when the $\varepsilon_a - 2N_f$ curve is obtained in full reversed constant amplitude strain control test.

There are many modification methods for considering the effects of mean stress and mean strain. The relatively simple one amony them assumes that it is feasible only to consider the effect of mean stress. Then equation (8) should be rewritten as follows:

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E}(2N_f)^b + {}_f(2N_f)^c \qquad (9)$$

The effect of mean strain ε_m can also be considered. But, in general, $\varepsilon_m \ll \varepsilon'_f$ and mean strain ε_m tends to disappear gradually in cyclic loading. So the modification of the effect of mean strain is not necessary.

3. The Statistical Treatment of Random Load-Time History

In statistical treatment of random load-time history, rainflow method is a more logical counting method, which is used in this paper. Considering that the program load spectra are still used in fatigue test recently, we use such simplifying principles as the variable mean value, the constant mean value and the equivalent damage simplifing principles to get different program load spectra. The main peak value counting method is also used. (In this paper, the counting condition of main peak is that the range from a peak (or a valley) to a valley (or a peak) is greater than or equal to half of the peak (or valley) value).

From the viewpoint of mechanics, it is more reasonable to calculate damage according to the whole cycles filtered out by rainflow method than to calculate damage according to the ranges from peaks to neighbouring valleys. We use both methods to calculate lives and compare the results obtained.

III. The Results of Fatigue Life Prediction and Comparison With Experimental Results

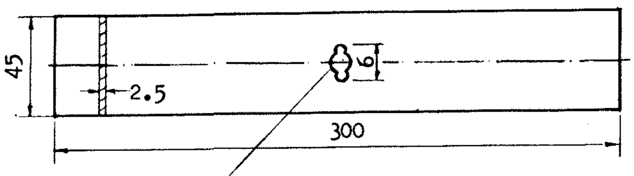
When we calculate fatigue life using local stress-strain method we use rainflow method to treat strain time history in order to obtain a series of strain ranges in the form of whole cycles or half cycles. Newton's iteration method is used according to equation (9) to calculate the damage. The damage of half a cycle is assumed as half a damage of the whole cycle in calculation.

Linear cumulative damage theory is used for the calculation of total damage D_T. As the random load spectrum used in this paper corresponds to 50 flight hours, the life T is

$$T = \frac{50}{D_T} \qquad (10)$$

S-N curve is used in the nominal stress method. To estimate fatigue life, we use a S-N curve of LY12-CZ with K_t=4, S_m=7 kg/mm^2 (from reference (12)). In calculation, we use Goodman linear equation to convert various alternating stresses into equivalant alternating stresses Saeq corresponding to S_m=7kg/mm^2. Then, on S-N curve, we obtain the equivalent N value from Saeq interpolation.

The fatigue life predictions under spectrum loads of the aluminum alloy specimen with center hole (K_t=4), as shown in Fig.3



(ϕ4.5 centre hole and two ϕ3.0 end holes)

FIG.3

are given in table 1. The experimental results of fatigue life under 50 flight hours random load spectrum as well as the various program load spectra obtained by different counting methods and simplifying principles from the random load spectrum are given in table 1.

The random load spectrum in calculation is taken from (12), and it is also used in experimentation. The material characteristics of LY12- CZ are taken from (14) , and the similar American material 2024-T4 data are taken from (15), as shown in table 3 in detial.

A brief discussion of calculation and experimental results in table 1 and table 2 is given as follows:

(1) Comparing table 1 with table 2 shows that the fatigue lives obtained from Neuber's rule (in table 1) are lower than the experimental lives obtained under the random load spectrum (in table 2), and the fatigue lives obtained from modified Neuber's rule are closer to the experimental values. Meanwhile, we can see the calculated results of fatigue lives are affected greatly by K_f. Moreover, the various calculated lives by using LY12- CZ material data are lower than by using 2024- T4 ones. The possible reason is that the material fatigue characteristics of 2024- T4 may be better than of LY12- CZ.

(2) When the effect of mean stress or mean strain is considered, only the elastic component in strain-life curve is modified in medified Neuber's rule, but the plastic component is not modified. Substituting $\varepsilon'_f - \varepsilon_m$ for ε'_f to consider the effect of mean strain we can find (from table 1) that the calculated results for $\varepsilon'_f - \varepsilon_m$ and for ε'_f differ by only 4.3% for material 2024-T4. So we don't need to modify mean strain in calculations from now on.

(3) To consider local stress-strain response of the notch root in complex rondom load-time history, it is necessary in calculating fatigue

	life prediction method	material	K_t or K_f	life (hours)	remarks
local stress-strain method	Neuber's rule	2024 - T4	K_t=4	1138	
	Modified Neuber's rule	2024 - T4	K_f=3.68*	1869	
	Modified Neuber's rule	2024 - T4	K_f=3.68*	1789	mean strain modified
	Modified Neuber's rule	2024 - T4	K_f=3.68*	5487	damage calculation according to simple range.
	Modified Stowell's rule	2024 - T4	K_t=4	1528	
	Linear strain rule	2024 - T4	K_t=4	2314	
	Neuber's rule	LY12 - CZ	K_t=4	644	
	Modified Neuber's rule	LY12 - CZ	K_f=3.68*	996	
	Modified Neuber's rule	LY12 - CZ	K_f=3,23	1803	$K_f = 1+ \frac{K_t - 1}{1+\sqrt{a/R}}$
	Modified Stowell's rule	LY12 - CZ	K_t=4	745	
	Linear strain rule	LY12 - CZ	K_t=4	858	
Nominal stress method	variable mean value random load spectrum	LY12 - CZ	K_t=4	2539	
	variable mean value program load spectrum	LY12 - CZ	K_t=4	2572	
	constant mean value program load spectrum	LY12 - CZ	K_t=4	3119	
	equavilant damage program load spectrum	LY12 - CZ	K_t=4	2596	
	main peak value program load spectrum	LY12 - CZ	K_t=4	2717	

$$* \; K_f = 1+ \frac{K_t - 1}{1+ a/R}$$

TABLE 1 CALCULATED RESULTS OBTAINED FROM VARIOUS FATIGUE LIFE PREDICTION METHODS

types of loading spectrum	number of specimens	failure life (hours)	crack initiation life (hours)	
			*a=0.5mm	*a =0.25
random load spectrum	6	2787	2051	1848
variable mean value program load spectrum	6	2778	2041	1817
constant mean value program load spectrum	6	3737	2085	1854
equavilant damage program load spectrum	3	1712	1024	865

*a—side crack length

TABLE 2 EXPERIMENTAL FATIGUE LIVES BY USING DEFFERENT LOADING SPECTRA

material	σ_f' (kg/mm²)	ε_f'	b	c	K' (kg/mm²)	n'	E (kg/mm²)
2024-T4	64.50	0.420	-0.0575	-0.645	86.90	0.11	7170
LY12-CZ	68.34	0.665	-0.1027	-0.5114	67.75	0.067	7079

TABLE 3 THE MATERIAL CHARACTERISTICS OF 2024-T4 AND LY12-CZ ALUMINUM ALLOY

life to apply logical counting method (such as rainflow method). The damage calculations by using a series of whole cycles and half cycles obtained by rainflow method are closer to the experimental results than by using directly the ranges from peaks to neighbouring valleys, as shown in table 1.

(4) The medified stowell's rule is also conservative in life predictions, but better than Neuber's rule. The calculated result of Linear Strain rule generally gives longer life than experimental result. As a preliminary fatigue life prediction, it is appropriate for us to use the conservative Modified stowell's rule.

(5) The fatigue lives calculated by nominal stress Method under various load spectra (in table 1) are compared with the test fatigue failure lives in table 2 under the similar load spectra. In general, the calculated results from the nominal stress method, except for the equivalent damage load spectrum, are in fairly good agreement with the experimental results. Moreover, the calculated fatigue lives both from the main peak value program load spectrum and variable mean value spectrum are close to the experimental results, so we consider that the main peak value counting method is still an applicable counting method for fighter-type spectra.

To sum up, we find that the local stress-strain method is a good fatigue (crack initiation) life prediction method. The results calculated by using modified Neuber's rule are closer to experimental results. The key to modified Neuber's rule is how to select a appropriate K_f value. Although modified Neuber's rule is an approximate estimation method and still needs further improvement, but it is a simple method and does not need a lot of test data, and this is widely applicable. In determining the local stress-strain of a notch root, the amount of work of modified Neuber's rule is far less than the amount of work of a good elasto-plastic finite element method, and we also should point out that the nominal stress method is still usefull in engineering.

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