A GENERAL FORMULATION FOR THE AEROELASTIC DIVERGENCE OF COMPOSITE SWEPTFORWARD WING STRUCTURES

L. Librescu*
Professor
Virginia Polytechnic Institute and State University
Department of Engineering Science and Mechanics
Blacksburg, VA 24061 USA

J. Simovich
Principal Engineer
Israel Aircraft Industry
Structural Division
Lod, Israel

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Abstract

This paper is devoted to the formulation of a simple algorithm allowing the determination, in a closed form, of the divergence instability of advanced composite swept (back and forward) wing structures. The warping restraint effect is incorporated into the analysis and its influence on the associated static instability condition is put into evidence. In this sense, it is shown that, in contrast to the case of conventional metalic wings, when the warping restraint effect has** a stabilizing influence only, in the case of anisotropic composite wing structures, its influence becomes more complex. The behaviour in divergence of swept composite type wings in which framework the warping restraint effect is taken into consideration constitutes the principal goal of this study. The numerical examples illustrate the complex role played by the warping restraint effect on the divergence instability of composite wings.

1. Introduction

The expanded utilization of laminated anisotropic composites in the modern aircraft industry has led, during the last few years, to the development of a new concept in the design of aeronautical and space structures . This concept known as aeroelastic tailoring is defined as the technology applied to flight vehicle structures (and to other devices experiencing aeroelastic instability phenomena as e.g., turbine and helicopter blades), in which framework the exotic characteristics of advanced filamentary composite materials are properly used in order to enhance their aeroelastic response characteristics. One of the best and most efficient applications of this concept is constituted by the forward swept wing (FSW) aircraft. As is well known, in the case of swept-back wings, the bending deformations tend to reduce the local angle of attack (and implicitely the aerodynamic load), while in the case of a swept-forward wing the bending deformations have an opposite effect, tending to increase the local angle of attack.

The above mentioned effects, referred to as wash-out and wash-in, respectively, result either in an increase of the aeroelastic divergence speed or, in the second instance, in a low static instability speed. In the fundamental works[1,2] devoted to the study of the divergence instability of swept metalic wings it was shown that this behaviour precludes practically the consideration of FSW aircraft as a possible option, in spite of its potential advantages of aerodynamic and performance nature. In an attempt to alleviate this instability phenomenon jeopardizing the free employment of FSW aircraft, the studies prompted by Krone[3] continued by Weisshaar[4-7] and followed in a series of other theoretical and experimental research works,[8-13] have revealed, that a composite forward-swept wing can be aeroelastically tailored† to overcome this adverse static instability phenomenon.

Concerning the general framework in which this problem was modeled and the character of the solutions obtained thereof, it should be pointed out that: i) the concept of a simple anisotropic composite plate-beam model originated in the studies[15,4-7] was used throughout these investigations and that, ii) the divergence instability solutions for composite swept wings encompass both closed form solutions[4,6-8] and numerical ones.[10,13] As concerns the first mentioned class of solutions, they are either exact††[4-6] or approximate ones.[8] It should also be mentioned that, as far as the author of this paper is aware, with the exception of,[13] the free warping assumption for the wing twist has been unanimously adopted in the treatment of this problem. However, the results obtained in[16,13] related to the divergence instability, as well as the ones derived in[17-21] reveal the great importance of the axial warping restraint effect (WRE) on the behaviour of cantilevered type structures. The goal of this paper is two fold: i) to develop a simple algorithm allowing the determination in an explicit form of the static instability conditions for swept composite type wings, the warping restraint effect being incorporated and ii) to elucidate its implications in the aeroelastic divergence problem of composite swept wings.

The aeroelastic tailoring concept applied to composite lifting surfaces in general and to forward-swept wings in special was discussed recently and thoroughly analyzed in a series of

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*On leave from Tel-Aviv University, Faculty of Engineering, Tel-Aviv, Israel.

**This stabilizing influence is strongly manifested in the case of small AR wings and declines for moderate AR wings.

†The aeroelastic tailoring concept was also applied in the supersonic panel flutter problems. For a state of the art of this topic see Chapter 1 of [14].

††Here the term exact is used in the sense that the divergence instability solution was obtained in the framework of some assumptions initially stipulated. This means that no other assumptions beyond the initial ones were used in the treatment of the problem.

excellent survey-papers, where the state of the art of the problem is presented. In this sense the reader is referred to references 22-24.

2. Analytical Developments

2.1 Preliminaries. Basic Assumptions

Consider the case of a swept (back or forward) wing structure, idealized as a box-beam whose upper and lower faces are constructed of laminated composite materials. As in references [5,6], we shall postulate also the existence of a reference axis (RA), coinciding with the y axis and located in the reference plane of the box-beam and at mid-distance between its front and rear edges.

The angle of sweep (considered positive for swept back and negative for swept-forward wings) is measured in the plane x-y of the wing from the direction normal to the airflow to the reference axis. The wing is considered clamped normal to this RA, its effective length ℓ being measured along this axis. All parameters associated to the wing sections, such as chord, location of the aerodynamic center, etc. are based on sections normal to the RA (see Fig. 1).

The material of each constituent lamina is assumed to be orthotropic. Without contrary mentions the orthotropicity angle $\theta_{(j)}$ of each laminae is measured in the counterclockwise direction starting from the rearward normal to the y axis. In addition to these assumptions, we shall postulate that the chordwise deformation, as well as the wing distortions are negligibly small.

2.2 The Governing Equations

For the sake of completeness, and definition of the involved quantities, a short derivation of the equations governing the aeroelastic equilibrium of a composite wing structure is presented in the Appendix. In their deduction, the above mentioned assumptions are appropriately used. For the static case considered in the paper, the aerodynamic terms L and T intervening in the equilibrium equations and representing the lift and the aerodynamic torsional moment (per unit length), respectively are expressed as:

$$L(y) = q_n c a_o \theta_{eff}; \quad T(y) = q_n c e a_o \theta_{eff} \qquad (1)$$

where $q_n (\equiv \rho_o/2\ V_n^2) = q\cos^2\Lambda$ denotes the dynamic pressure component normal to the leading edge; a_o $(\equiv 2\pi AR/(AR + 4\cos\Lambda))$ denotes the lift curve slope coefficient, where AR denotes the wing aspect ratio. (For additional refinements allowing to incorporate compressibility effects see References [25, 26]); $\theta_{eff} = \theta - Z_o'\tan\Lambda$ where θ denotes the twist† about the RA while $Z_o'(\equiv dZ_o/dy)$ denotes the bending slope of the RA measured along this axis. With all these in view, the equations governing the static aeroelastic equilibrium of non-uniform composite swept wings reduce to:

$$(\mathcal{D}_{22}Z_o'')'' - (\mathcal{D}_{26}\theta')'' = qca_o\cos^2\Lambda(\theta - Z_o'\tan\Lambda)$$

$$(\frac{c^3}{12}D_{22}\theta'')'' - (\mathcal{D}_{66}\theta)' + (\mathcal{D}_{26}Z_o'')' = qcea_o\cos^2\Lambda(\theta - Z_o'\tan\Lambda) \qquad (2)$$

These equations are to be supplemented by the appropriate boundary conditions (A.14) and (A.15).

In the following, the equations (2) will be converted to an integral form, having an energetic meaning. Towards this end, the equations (2) and (3) will be multiplied by $Z_o dy$ and θdy, respectively and integrated over $[0,\ell]$. (In the terminology of the functional analysis, the above mentioned operation is referred to as the scalar multiplication [27]). Introduction of non-dimensional deflection $\check{Z}$ $(\equiv Z_o/\ell)$ and spanwise coordinate η $(\equiv y/\ell)$, followed by the partial-integration of the obtained equations, whenever possible, result in the equilibrium equations expressed in integral form as well as in some terms to be evaluated at η = 0, 1 and which vanish by virtue of the boundary conditions (A.15) and (A.16). Under this modified form the equations are:

$$\int_0^1 \mathcal{D}_{22}(\check{Z}_{,11})^2 d\eta - \int_0^1 \mathcal{D}_{26}\check{Z}_{,11}\theta_{,1} d\eta - q_n a_o \ell^3 [\int_0^1 c\theta\check{Z} d\eta - \tan\Lambda \int_0^1 c\check{Z}\check{Z}_{,1} d\eta] = 0$$

$$(12\ell^2)^{-1}\int_0^1 D_{22}c^3(\theta_{,11})^2 d\eta + \int_0^1 \mathcal{D}_{66}(\theta_{,1})^2 d\eta - \int_0^1 \mathcal{D}_{26}\check{Z}_{,11}\theta_{,1} d\eta - q_n a_o \ell^2 [\int_0^1 ce\theta^2 d\eta - \tan\Lambda \int_0^1 ce\theta\check{Z}_{,1} d\eta] = 0 \qquad (3)$$

Under this form, the equations (3) express a balance of energies, each term playing a certain role in this balance. The lowest positive value of the dynamic pressure q for which the elastic stored energies equal the ones furnished by the airstream, corresponds to he wing divergence speed. In other terms, the minimum value of q for which the balance of energies as governed by the equations (3), vanishes identically, corresponds to the divergence instability conditons. The above equations may be used for the approach of the static instability of variable and uniform composite swept wings, as well.

2.3 The Case of Geometrically Similar Cross-Section Wings

In this case, the geometrical and structural parameters of the composite wing may be expressed as (see e.g., [1,6]):

†The twisting angle θ is not to be confused with the orthotropicity angle $\theta_{(j)}$.

$$c(\eta) = \xi c_R \quad , \quad e(\eta) = \xi e_R;$$
$$\mathcal{D}_{22}(\eta) = \xi^4 \mathcal{D}_{22}^R, \quad \mathcal{D}_{66}(\eta) = \xi^4 \mathcal{D}_{66}^R \qquad (4)$$
$$\mathcal{D}_{26}(\eta) = \xi^4 \mathcal{D}_{26}^R \quad , \quad D_{22}(\eta) = \xi^4 D_{22}^R$$

where

$$\xi = 1 - (1 - f)\eta \quad \text{while } f(\equiv c_T/c_R)$$

denotes the wing taper ratio ($0 < f \leq 1$). Here the superscript (or subscript) R affecting a certain quantity identifies its affiliation to the wing root section.

By virtue of (4), the equations (3) may be reduced to:

$$\int_0^1 \xi^4(\tilde{Z}_{,11})^2 d\eta - K \int_0^1 \xi^4 \tilde{Z}_{,11}\theta_{,1} d\eta$$
$$- a_1\left[\int_0^1 \xi\theta\tilde{Z} d\eta - \tan\Lambda \int_0^1 \xi\tilde{Z}\tilde{Z}_{,1} d\eta\right] = 0.$$
$$S_R \int_0^1 \xi^7(\theta_{,11})^2 d\eta + \int_0^1 \xi^4(\theta_{,1})^2 d\eta \qquad (5)$$
$$- G \int_0^1 \xi^4 \tilde{Z}_{,11}\theta_{,1} d\eta$$
$$- a_2\left[\int_0^1 \xi^2\theta^2 d\eta - \tan\Lambda \int_0^1 \xi^2\theta\tilde{Z}_{,1} d\eta\right] = 0.$$

where

$$K \equiv \mathcal{D}_{26}^R/\mathcal{D}_{22}^R \quad ; \quad G \equiv \mathcal{D}_{26}^R/\mathcal{D}_{66}^R \qquad (6)$$

define the non-dimensional structural bending-torsion coupling parameters (shown in reference [4] to satisfy the condition $KG < 1$, where KG defines the cross-coupling stiffness parameter),

$$S \equiv \frac{c_r^3}{12\ell^2} \frac{D_{22}^R}{\mathcal{D}_{66}^R} \qquad (7)$$

stands for the warping stiffness parameter:

$$a_1 = q_n c_r \ell^3 a_0/\mathcal{D}_{22}^R \qquad (8)$$
$$a_2 = q_n c_r \ell^2 a_0/\mathcal{D}_{66}^R$$

define two speed parmeters associated to the bending and torsional degrees of freedom, respectively.

Concerning $\theta(\eta)$ and $\tilde{Z}(\eta)$, they will be expressed in the form:[28]

$$\theta(\eta) = \zeta_\theta f(\eta)$$
$$\tilde{Z}(\eta) = \zeta_Z W(\eta) \qquad (9)$$

where $f(\eta)$ and $W(\eta)$ stand for the normalized torsional and bending deflection functions, (assumed to satisfy the boundary conditions ((A.14) and (A.15)), ζ_θ and ζ_Z playing the role of generalized coordinates. Insertion of (9) in (5) and invoking the standard requirement of non-triviality of the solution of the obtained homogeneous equations, result in the divergence condition expressed in determinantal form as:

$$\begin{vmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{22} \end{vmatrix} = 0. \qquad (10)$$

where

$$A_{11} = \int_0^1 \xi^4(W_{,11})^2 d\eta + a_1 \tan\Lambda \int_0^1 \xi W W_{,1} d\eta$$
$$A_{12} = K \int_0^1 \xi^4 W_{,11} f_{,1} d\eta + a_1 \int_0^1 \xi W f d\eta$$
$$A_{22} = S_R \int_0^1 \xi^7(f_{,11})^2 d\eta + \int_0^1 \xi^4(f_{,1})^2 d\eta$$
$$- a_2 \int_0^1 \xi^2\theta^2 d\eta \qquad (11)$$
$$A_{21} = G \int_0^1 \xi^4 W_{,11} f_{,1} d\eta - a_2 \tan\Lambda \int_0^1 \xi^2 f W_{,1} d\eta$$

Equation (10) considered together with (11) may be converted into the form:

$$a_1^2 L_{11} + a_1 L_{12} + L_{13} = 0 \qquad (12)$$

Its lowest positive root corresponds to the divergence speed of swept composite type-wings.

In (12), the coefficients L_{ij} depend on the structural and geometrical characteristics of the composite wing and on the selected mode shape W and f. Their expression is given by:

$$L_{11} = A tg\Lambda\left[\int_0^1 \xi f W d\eta \int_0^1 \xi^2 f W_{,1} d\eta\right.$$
$$\left.- \int_0^1 \xi^2 f^2 d\eta \int_0^1 \xi W W_{,1} d\eta\right]$$
$$L_{12} = S \tan\Lambda \int_0^1 \xi^7(f_{,11})^2 d\eta \int_0^1 \xi W W_{,1} d\eta$$
$$+ \tan\Lambda \int_0^1 \xi^4(f_{,1})^2 d\eta \int_0^1 \xi W W_{,1} d\eta$$
$$- G \int_0^1 \xi^4 W_{,11} f_{,1} d\eta \int_0^1 \xi W f d\eta \qquad (13)$$
$$- A\left[\int_0^1 \xi^2 f^2 d\eta \int_0^1 \xi^4(W_{,11})^2 d\eta\right.$$
$$\left.- K\tan\Lambda \int_0^1 \xi^4 W_{,11} f_{,1} d\eta \int_0^1 \xi^2 f W_{,1} d\eta\right]$$
$$L_{13} = S \int_0^1 \xi^7(f_{,11})^2 d\eta \int_0^1 \xi^4(W_{,11})^2 d\eta$$
$$+ \int_0^1 \xi^4(f_{,1})^2 d\eta \int_0^1 \xi^4(W_{,11})^2 d\eta$$
$$- KG\left[\int_0^1 \xi^4 W_{,11} f_{,1} d\eta\right]^2.$$

In the deduction of (13), use was made of the relationship:

$$a_2 = a_1 A, \text{ where } A \equiv \frac{\mathcal{D}_{22}^R e_R}{\ell \mathcal{D}_{66}^R} \left(= \frac{e_r}{\ell}\frac{G}{K}\right) \qquad (14)$$

Concerning the solution of Eq. (12), it should be stressed that its accuracy depends essentially on the appropriate selection of modal functions $f(\eta)$ and $W(\eta)$.

2.4 Special Cases of the General Equation

Several special cases will be considered next. They result by the appropriate specialization of the general equation (12). These cases concern: a) the divergence of swept wings in pure bending. In this case it may be assumed that the torsional rigidity is very large, allowing to consider $\mathcal{D}_{66} \to \infty$. From (12), having in view (13), (6) and (7) one obtains:

$$(q_n)_D = - \frac{2 \int_0^1 \mathcal{D}_{22} (W,_{11})^2 \, d\eta}{\ell^3 a_0 \tan\Lambda \int_0^1 c(W^2),_1 d\eta} \tag{15}$$

This result coinciding with the one obtained in [26] shows that only a swept-forward wing ($\Lambda \to -\Lambda$) can diverge in bending. In this case the aeroelastic tailoring reduces to the increase, as much as possible, of the bending rigidity $\mathcal{D}_{22}$. However, as it may be inferred from (A.16), a symmetric laminate (for which the coupling rigidities $K_{ij} \to 0$) is preferable to an asymmetric one (for which $K_{ij} \neq 0$). For a uniform wing, by using for $W(\eta)$ the representation: $W(\eta) = (6\eta^2 - 4\eta^3 + \eta^4)/3$, one obtains readily that $\lambda_D = 6.40\, D_{22}/c\ell^3$. The difference occuring with respect to the exact one (whose coefficient is 6.33) is of only 1%. Here $\lambda \equiv q_n a_0 \tan\Lambda$ where Λ is to be considered a negative angle. It may be remarked that in the pure bending case the warping restraint effect is not present, as it is well evident.

b) Divergence in pure torsion. In this case it is assumed that the bending rigidity $\mathcal{D}_{22}$ is a large quantity while the warping rigidity parameter remains a finite quantity. Conversion of the Eq. (12) in terms of a_2, with the help of (8), and employment of (6), result in the following approximate expression for $(q_n)_D$:

$$(q_n)_D = \frac{1}{\ell^2 a_0} \frac{\int_0^1 \mathcal{D}_{66} (f,_1)^2 d\eta}{\int_0^1 ce\, f^2 \, d\eta} \times \left(1 + \frac{\int_0^1 D_{22}\, c^3 (f,_{11})^2 d\eta}{12\ell^2 \int_0^1 \mathcal{D}_{66} (f,_1)^2 \, d\eta}\right) \tag{16}$$

The last term in the bracket identifies the contribution of the warping restraint effect. Eq. (16) shows that the divergence instability in pure torsion may occur for straight wings only (in this case $(q_n)_D \to q_D$). Considering the case of uniform wings and selecting $f(\eta) = 2\eta - \eta^2$, one obtains:

$$q_D = \frac{2.5\, \mathcal{D}_{66}}{ce\, \ell^2 a_0} (1 + 3\, S). \tag{17}$$

Two remarks are in order: a) In the case of the free warping (corresponding to $S \to 0$), the exact solution predicts 2.47 instead of the coefficient 2.5 obtained here, and b) in contrast to the case of metalic wings where the warping parameter S is fixed by the isotropicity of the material, in this case, this parameter results as a function of ply orientation and stacking sequence. Therefore its value may be increased by using adequately the tailoring concept.

c) The case of uniform swept wings. The equation (12) of the divergence instability remains valid in this case, too. Its coefficients L_{ij} appropriate to this case are obtained by specializing (13) for $\xi \to 1$.

In such a manner the properties of the uniform wing will correspond to the ones defined at the wing root. Having in mind this fact, in the following the index R will be suppressed.

The two distinct cases, corresponding to the inclusion ($S \neq 0$) and rejection ($S = 0$) of WRE will be considered separately. Associated to these two cases, the boundary conditions at $\eta = 0,1$ are in number of four and three, respectively (see Eqs. (A.14), (A.15) and (A.17), (A.18)). The modal functions, appropriate to these cases will be considered in the form: For the free warping case:

$$f(\eta) = 3\eta - 3\eta^2 + \eta^3$$
$$W(\eta) = (6\eta^2 - 4\eta^3 + \eta^4)/3 \tag{18}$$

For the restrained warping case:

$$f(\eta) = 10\eta^2 - 20\eta^3 + 15\eta^4 - 4\eta^5$$
$$W(\eta) = (6\eta^2 - 4\eta^3 + \eta^4)/3 \tag{19}$$

Employment in (12) and (13), (specialized for $\xi \to 1$), of the representations (18) and (19) for f and W, result in the following expressions for q ($\equiv q_D$) in divergence:

$$q_D = \frac{2.8(1 - KG)}{a_0 c\ell^3 \cos^2\Lambda} \times \left\{\mathcal{D}_{22}/\left[\frac{1 - K\tan\Lambda}{(\ell/e)(\mathcal{D}_{66}/\mathcal{D}_{22})} - 0.4375\,(\tan\Lambda - G)\right]\right\} \tag{20}$$

(for the free warping case),

$$q_D = \frac{2.392(1.4 - KG) + 72.333S}{a_0 c\ell^3 \cos^2\Lambda} \times \left\{\mathcal{D}_{22}/\left[\frac{1.19347 - K\tan\Lambda}{(\ell/e)(\mathcal{D}_{66}/\mathcal{D}_{22})} - 0.45712\,(1.144\tan\Lambda - G) - 11.3S\tan\Lambda\right]\right\}. \tag{21}$$

(for the restrained warping case).

The expression (20) is similar to the one obtained in [6, Eq. 27], by neglecting the warping restraint effect. In spite of the slight differences in the coefficients (2.8 vs. 2.47 and 0.4375 vs. 0.39 - the last ones belonging to Weisshaar's expression of q_D), the numerical applications reveal a difference of only (1-2)% in the values of q_D, the ones obtainable from [6] being slightly more conservative than the present ones. For the case of composite unswept wings ($\Lambda \to 0$), when only the structural coupling is present, the employment of the following consistent representation for the modal functions:

$$f(\eta) = 2\eta - \eta^2 \quad ; \quad W(\eta) = (3\eta^2 - \eta^3)/2$$

results in a reduction of the differences of the two coefficients (i.e., 2.5 vs. 2.47 and 0.4166 vs. 0.39), and implicitely of the predicted critical values of q. It may be concluded that the presents analysis may furnish rather accurate solutions when compared with the available exact ones (derived for the free warping case).

In addition, the present approach could provide quantitative results concerning the influence of WRE on the divergence instability characteristics of composite swept wings. In this respect it should be pointed out that the incorporation of WRE generates a series of qualitative and quantitative differences with respect to the free warping counterpart. While the qualitative differences consists in the difference in the order of the equations governing the instability in divergence of composite swept wings, and as a result, in the difference of the associated number of boundary conditions to be fulfilled, the qualitative ones are not of less importance. These differences involving the values of the divergence speed could be of the order of (20-30)%. In this context it should be underlined that: i) in contrast to the case of metalic unswept wings where the stabilizing influence of the warping restraint effects for small-aspect ratio wings declines for large or moderate AR wings, (see [16]), in the case of composite swept wings, its influence may be strong also in the case of large AR wings, and ii) in contrast to the case of metalic swept wings in which context the warping restraint effect has solely a stabilizing character, in the case of composite wings its influence could be also destabilizing.

In the same context the concept of the divergence-free sweep angle prompted by Weisshaar[5,6] could be useful in this case, too. The angle Λ_{cr} is defined as the minimum (positive) and maximum (negative) sweep angle above and below which, respectively,the divergence instability becomes impossible. Their expressions in each case are obtainable from (20) and (21), resulting in:

$$\tan\Lambda_{cr} = \frac{G + 2.286(e/\ell)(G/K)}{1 + 2.286(e/\ell)G} \qquad (22)$$

(for free warping case)

and

$$\tan\Lambda_{cr} = \frac{G + 2.6108(e/\ell)(G/K)}{1.144 + 2.1876(e/\ell)G + 24.72Se/(\ell K)} \qquad (23)$$

(for restrained warping case).

In addition to the conclusion substantiated in references [5,6] (and which could be re-obtained from Eqs. (26) and (27)), according to which, in the case of swept composite wings there is possible to determine such sweep angles in the forward swept range for which the divergence instability is precluded at any flight speed, another one may be established. This one, resulting from Eq. (23) concerns the fact that WRE may contribute either to the increasing or decreasing of Λ_{cr}. However, as it may readily be seen from Eq. (23), this influence could be controlled through the appropriate tailoring. In this sense, the sign of the cross-coupling parameter K may play an important role in this aeroelastic tailoring control.

3. Numerical Illustrations

The accuracy of the present approach will be tested by comparing the results obtainable from the Eq. (20) with their counterparts derived in [4,6] in the framework of a free warping model. Towards this goal we shall consider the wing configuration analyzed in [4,6]. It consists of N = 20 plies of the same thickness (t = 0.12") and the same material (boron-epoxy). Its elastic constants are: $E_1 = 32.5 \times 10^6$ psi; $E_2 = 3.2 \times 10^6$ psi; $G_{12} = 1.05 \times 10^6$ psi; $\nu_{12} = 0.36$. It is assumed that the angle of the ply orientation $\theta_{(i)}$ is equal for all the constituent layers, and also that the wing is of constant chord, with AR = 6, e = 0.1c; $a_0 = 5$.

Table 1 displays several values of $(q_n|_D$ obtained on the basis of the present approach (Eq. (20)) and of the approach developed in [4,6]. These values are identified as $(q_n|_D$ and $(\tilde{q}_n|_D)$, respectively.

Table 1

$\Lambda = -30°$

θ	0°	30°	60°	90°
$(q_n)_D \times 10^{-3}$	278.265	227.270	230.687	1530.564
$(\tilde{q}_n)_D \times 10^{-3}$	272.301	225.060	226.486	1316.314
Relative Error %	2.2	.43	1.85	1.1

Table 1 Cont'd.

θ	120°	150°	180°
$(q_n)_D \times 10^{-3}$	-	572.070	278.26
$(\tilde{q}_n)_D \times 10^{-3}$	-	565.283	272.301
Relative Error %		1.2	2.2

In Figures (2)-(4), the variation of the ratio q_n^*/q_n vs AR is depicted, where q_n^* and q_n denote the q_D of divergence obtained by incorporating WRE Eq. (21) and by disregarding it (Eq. 20), respectively.

The configuration as well as the structural characteristics, previously considered, are adopted in these instances, too. Figures (2)-(4) reveal the stabilizing influence of WRE, which, in contrast to the conventional metalic wings, is present in the case of high AR wings, too. For special ply orientations (as e.g., $\theta = 60°$, WRE appears to have a strong beneficial influence even for high AR wings.

However, as it may be shown, the WRE considered in the context of the composite type wings could be also detrimental. This is shown in the case of a wing structure in which the plies of a graphite-epoxy material, are arranged in the sequence $(90°/-45°/45°/0)_S$.

The boron-epoxy material is characterized by the constants: $E_1 = 30 \times 10^6$ psi, $E_2 = 0.7 \times 10^5$ psi; $G_{12} = 0.375 \times 10^6$ psi; $\nu_{12} = 0.25$; $t = 0.005"$.

We consider the wing of AR = 18 the remaining constants being the same as in the previous example. In this instance we obtain for the non-dimensional coupling parameters K and G the values: K = -0.13296; G = -0.20782. On the basis of Eqs. (20) and (21) one obtains:

$$q_n^* = \left(\frac{48.6\,\mathcal{D}_{22}}{c\ell^3}\right) < q_n = \left(\frac{63.7\,\mathcal{D}_{22}}{c\ell^3}\right)$$

which shows that the WRE could be also detrimental.

These numerical findings enforce the conclusion that in the case ofcomposite wings the warping restraint effect is to be taken into consideration whenever their divergence instability is investigated.

Appendix

The wing structure is idealized as a laminated composite flat plate whose constituent laminae are characterized by different orthotropicity angles and different material and thickness properties. The interface plane between the contiguous layers (r) and (r + 1), (1 < r < N where N denotes the total number of constituent layers), will be selected as the reference plane of the composite structure.

The points of the reference plane (defined by z = 0) will be referred to a Cartesian system of coordinates (x,y) (see Fig. 1), where the upward z-coordinate is considered perpendicular to the (x,y) plane. Adopting the Kirchoff assumptions for the composite plate as a whole, which yields

$$V_1 = v_1 - z\partial V_3/\partial x_1 \;;\; V_2 = v_2 - z\partial V_3/\partial x_2$$
$$V_3 = v_3 \qquad (A.1)$$

where

$$V_i \equiv V_i(x,y,z) \;;\; v_i \equiv v_i(x,y) = V_i|_{z=0};$$

representing the deflection v_3 in the form:

$$v_3(x,y) = Z_0(y) - x\theta(y) \qquad (A.2)$$

where $Z_0 \equiv Z_0(y)$ and $\theta \equiv \theta(y)$ denote the deflection associated to the reference axis points and the twist around this axis respectively and postulating that the chordwise sections of the wing are rigid which involves:

$$v_1 \to 0 \text{ and } v_2 \to v_2(y),$$

all these result in the following expressions for the 2-D strain components:

$$\varepsilon_{11} = 0 \;,\; \varepsilon_{12} = 0 \;,\; \varepsilon_{22} = v_2' \qquad (A.3)$$

$$\kappa_{11} = 0 \;,\; \kappa_{12} = 2\theta' \;,\; \kappa_{22} = -(Z_0'' - x\theta'')$$

where $\partial()/\partial y \equiv ()'$, while ε_{ij} and κ_{ij} denote the stretching and the bending strain components, respectively. In order to obtain the equations governing the static equilibrium of the composite wing and the associated boundary conditions, (B.C.) the virtual work principle of the 3-D elasticity theory will be applied. Towards this end the stationarity condition of the functional

$$J_{3-D} \equiv U - \int_{\Omega_T} T_i V_i \, d\Omega \qquad (A.4)$$

which is to be converted in terms of quantities depending on the y-coordinate only, are to be determined. In (A.4) U denotes the strain energy of the 3-D body while T_i denotes the external loads applied on $\Omega_T \in \Omega$. By selecting the reference axis located in the reference plane at the mid-distance between its front and rear edges, by performing the integration in the expression of U across the thickness of the composite plate and in the chordwise direction, and by defining the anisotropic stiffness quantities:

$$C_{ij} = \sum_{k=1}^{N} (\bar{Q}_{ij})_{(k)}(h_{(k)} - h_{(k-1)})$$

$$K_{ij} = \frac{1}{2}\Big[\sum_{k=1}^{r} (\bar{Q}_{ij})_{(k)}(h^2_{(k)} - h^2_{(k-1)})$$
$$- \sum_{k=r+1}^{N} (\bar{Q}_{ij})_{(k)}(h^2_{(k)} - h^2_{(k-1)})\Big], \qquad (A.5)$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} (\bar{Q}_{ij})_{(k)}(h^3_{(k)} - h^3_{(k-1)}),$$

$$(i,j = 1,2,6)$$

which correspond to stretching, bending-stretching coupling and bending rigidities, respectively (see [14]), and where $h_{(k)}$ denotes the distance along the z-coordinate from the reference plane until the top face of the kth layer; $(\bar{Q}_{ij})_{(k)}$ denote the reduced orthotropic moduli of the kth layer referred to the off-axis system (represented by the (x,y) coordinates). With all these in view, the 1-D counterpart of (A.4) writes

$$J_{1-D} = U_C + U_K + U_D - \int_0^{\ell} (Yv_2 + LZ_0$$
$$- T\theta)dy \qquad (A.6)$$

where

$$U_C \equiv \frac{1}{2}\int_0^{\ell} cC_{22}(v_2')^2 dy$$

$$U_K \equiv \int_0^{\ell} c(2K_{26}v_2'\theta' - K_{22}v'Z_0'')dy$$

$$U_D \equiv \frac{1}{2}\int_0^{\ell} (c[D_{22}(Z_0'')^2 + 4D_{66}(\theta')^2$$

$$- 4D_{26}\theta' Z_o'')dy \quad (A.7)$$

denote the strain energies associated to the stretching, bending-stretching coupling and bending, respectively, while Y, L and T denote spanwise, the lift and torsional aerodynamic moment, respectively.

From the stationarity condition

$$\delta J_{1-D} = 0 \quad (A.8)$$

by taking the variation of the involved quantities; by performing the integration by parts whenever possible; by collecting terms and by setting the coefficients of δZ_o, $\delta\theta$ and δv_2 in (A.6) (considered in conjunction with (A.7) to be zero, one obtains: i) the equations governing the static equilibrium of non-uniform composite wings given by:

$$\delta v_2: \quad (cC_{22}v_2' - cK_{22}Z_o'' + 2cK_{26}\theta')' = -Y \quad (A.9)$$

$$\delta\theta: \quad (c^3D_{22}\theta_2'')'' - 4(cD_{66}\theta')'$$

$$+ 2(cD_{26}Z'')' - 2(cK_{26}v_2')' + T = 0$$

$$\delta Z_o: \quad (cD_{22}Z_o'')'' - 2(cD_{26}\theta')''$$

$$- (cK_{22}v_2')'' = L$$

and ii) the consistent B.C.:

$$v_2 = Z_o = Z_o' = \theta = \theta' = 0 \text{ at } y = 0 \quad (A.10)$$

and

$$cC_{22}v_2' - cK_{22}Z_o'' + 2cK_{26}\theta' = 0$$

$$cD_{22}Z_o'' - 2cD_{26}\theta' - cK_{22}v_2' = 0$$

$$(cD_{22}Z_o'' - 2cD_{26}\theta' - cK_{22}v_2')' = 0 \quad (A.11)$$

$$- (c^3D_{22}\theta''/12)' + 4cD_{66}\theta' - 2cD_{26}Z_o''$$

$$+ 2cK_{26}v_2' = 0$$

$$c^3D_{22}\theta''/12 = 0$$

at $y = \ell$.

However, it may easily be seen that in the absence of the spanwise load Y, the ten order governing equations expressed in terms of v_2, Z_0 and θ may exactly be reduced to an eight order governing equation system expressed in terms of θ and Z_o, only. Towards this end, integration of $(A.9)_1$ (provided Y = 0) and comparison with $(A.11)_1$ yields:

$$cC_{22}v_2' - cK_{22}Z_o'' + 2cK_{26}\theta' = 0 \quad (A.12)$$

Substitution of v_2' as it results from (A.12) in the remaining equations $(A.9)_{2,3}$ yields the appropriate governing equations:

$$(\mathcal{D}_{22}Z_o'' - \mathcal{D}_{26}\theta')'' = L \quad (A.13)$$

$$(c^3D_{22}\theta''/12)'' - (\mathcal{D}_{66}\theta' - \mathcal{D}_{26}Z_o'')' = -T$$

In a similar way, the B.C. (A.10) and (A.11) modify as:

$$Z_o = Z_o' = \theta = \theta' = 0 \text{ at } y = 0 \quad (A.14)$$

and

$$\mathcal{D}_{22}Z_o'' - \mathcal{D}_{26}\theta' = 0$$

$$(\mathcal{D}_{22}Z_o'' - \mathcal{D}_{26}\theta')' = 0 \quad (A.15)$$

$$-(c^3D_{22}\theta''/12)' + \mathcal{D}_{66}\theta' - \mathcal{D}_{26}Z_o'' = 0$$

$$D_{22}c^3\theta''/12 = 0 \quad \text{at } y = \ell$$

In (A.13) and (A.15) $\mathcal{D}_{22}$, $\mathcal{D}_{26}$ and $\mathcal{D}_{66}$ stand for the coupling stiffness parameters expressed by:

$$\mathcal{D}_{22} \equiv c(D_{22} - K_{22}^2/C_{22});$$

$$\mathcal{D}_{26} \equiv 2c(D_{26} - K_{22}K_{26}/C_{22}) \quad (A.16)$$

$$\mathcal{D}_{66} \equiv 4c(D_{66} - K_{26}^2/C_{22})$$

As it may easily be observed, the term $c^3D_{22}\theta''''/12$ present in Eq. (A.13) identifies the warping restraint effect. When this effect is ignored, from (A.10), (A.11), (A.13) the pertinent governing equations result as:

$$(\mathcal{D}_{22}Z_o'' - \mathcal{D}_{26}\theta')'' = L$$

$$(\mathcal{D}_{66}\theta' - \mathcal{D}_{26}Z_o'')' = T \quad (A.17)$$

while the associated B.C. read:

$$Z_o = Z_o' = \theta = 0 \text{ at } y = 0 \quad (A.18)$$

and

$$\mathcal{D}_{22}Z_o'' - \mathcal{D}_{26}\theta' = 0$$

$$(\mathcal{D}_{22}Z_o'' - \mathcal{D}_{26}\theta')' = 0 \quad (A.19)$$

$$\mathcal{D}_{66}\theta' - \mathcal{D}_{26}Z_o'' = 0 \text{ at } y = \ell$$

(A.18) and (A.19) are consistent with the sixth order governing equation system (A.17). The equations (A.17) - (A.19) coincide to the ones obtained in [4] while Eqs. (A.13)-(A.15) specialized for the case of uniform wings are similar to the ones presented in [12]. At the same time, for the case of unswept metalic type wings, Eqs. (A.13)-(A.15) reduce to the ones obtained in [16] by using the theory of thin walled beams with warping inhibition.

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References

1. Diedrich, F. W., and Budiansky, B., "Divergence of Swept Wings," NACA Tech. Note. 1680 (1948).

2. Miles, J. W., "A Formulation of the Aeroelastic Problem for a Swept Wing," J. Aeronaut. Sci., 16, 477-490 (1949).

3. Krone, N. J., Jr., "Divergence Elimination with Advanced Composites," AIAA Paper 75-1009, Aug. 1975.

4. Weisshaar, T. A., "Aeroelastic Stability and Performance Characteristics of Aircraft with Advanced Composite Sweptforward Wing Structures," AFFDL-TR-78-116, Sept. 1978.

5. Weisshaar, T. A., "Forward Swept Wing. Static Aeroelasticity," AFFDL-TR-3087, June 1979.

6. Weisshaar, T. A., "Divergence of Forward Swept Composite Wings," Journal of Aircraft, Vol. 17, June 1980, pp. 442-448.

7. Weisshaar, T. A., Aeroelastic Tailoring of Forward Swept Composite Wings," Journal of Aircraft, Vol. 18, Aug. 1981, pp. 669-676.

8. Niblett, L. T., "Divergence and Flutter of Swept-Forward Wings with Cross Flexibilities," RAE-TR-80047, April 1980.

9. Sherrer, V. C., Hertz, T. J. and Shirk, M. H., "Wind Tunnel Demonstration of Aeroelastic Tailoring Applied to Forward Swept Wings," J. of Aircraft, Vol. 18, Nov. 1981, pp. 976-903.

10. Lehman, L. L., "A Hybrid State Vector Approach to Aeroelastic Analysis," AIAA Journal, Vol. 20, 10, Oct. 1982, pp. 1442-1449.

11. Hollowell, S. J. and Dugundji, J. "Aeroelastic Flutter and Divergence of Stiffness Coupled, Graphite/Epoxy, Cantilevered Plates," AIAA Paper 82-0722, May 1982.

12. Oyibo, G. A., "Generic Approach to Determine Optimum Aeroelastic Characteristics for Composite Forward-Swept-Wing Aircraft," AIAA Journal, Vol. 22, No. 1, Jan. 1984, pp. 117-123.

13. Lottati, I., "Flutter and Divergence Aeroelastic Characteristics for Composite Forward Swept Cantilevered Wing," J. of Aircraft, Vol. 22, 11, Nov. 1985, pp. 1001-1007.

14. Librescu, L., Elastostatics and Kinetics of Anisotropic and Heterogeneous Shell-Type Structures, Chapter 1 - Aeroelastic Stability of Anisotropic Multilayered Thin Panels, pp. 1-281, Noordhoff International Publishing, Leyden, The Netherlands, 1975.

15. Housner, J. M. and Stein, M., "Flutter Analysis of Swept-Wing Subsonic Aircraft with Parameter Studies of Composite Wings," NASA TN-D7539, Sept. 1974.

16. Petre, A., Stanescu, C. and Librescu, L. "Aeroelastic Divergence of Multicell Wings Taking Their Fixing Restraints into Account," Revue de Mechanique Appliquee, Vol. 19, No. 6, 1961, pp. 689-698.

17. Reissner, E. and Stein, M., "Torsion and Transverse Bending of Cantilevered Plates," NACA TN 2369, June 1951.

18. Crawley, E. F. and Dugundji, J., "Frequency Determination and Non-Dimensionalization for Composite Cantilever Plates," J. of Sound and Vibration, Vol. 72, No. 1, 1980, pp. 1-10.

19. Jensen, D. W., Crawley, E. F. and Dugundji, J., "Vibration of Cantilevered Graphite/Epoxy Plates with Bending-Torsion Coupling, Journal of Reinforced Plastics and Composites, Vol. 1, July 1982, pp. 254-269.

20. Kaza, K. R. V., and Kielb, R. E., "Effects of Warping and Pretwist on Torsional Vibration of Rotating Beams," Journal of Applied Mechanics, Trans. of ASME, Vol. 51, Dec. 1984, pp. 913-920.

21. Oyibo, G. A., Berman, J. H., "Anisotropic Wing Aeroelastic Theories with Warping Effects," Second International Symposium on Aeroelasticity and Structural Dynamics, Aachen, FRG, April 1985.

22. Hertz, T. J., Shirk, M. H., Ricketts, R. H. and Weisshaar, T. A., "On the Track of Practical Forward-Swept Wings," Astronautics and Aeronautics, Vol. 20, Jan. 1982, pp. 40-52.

23. Weisshaar, T. A., "Tailoring for Aeroelastic Stability and Lateral Control Enhancement," Second International Symposium on Aeroelasticity and Structural Dynamics, Aachen, FRG, April 1985.

24. Shirk, M. H., Hertz, T. J. and Weisshaar, T. A., "Aeroelastic Tailoring - Theory, Practice and Promise," J. Aircraft, Vol. 23, 1, Jan. 1986, pp. 6-18.

25. Bisplinghoff, R. L., Ashley, H. and Halfman, R. L., Aeroelasticity, Adeson-Wesley, 1955.

26. Flax, H. H., "Aeroelasticity and Flutter in High Speed Problems of Aircraft and Experimental Methods," Vol. VIII High Speed Aerodynamics and Jet Propulsion, Eds. Donavan, H. F. and Lawrence, H. R., Princeton University Press, 1961, pp. 161-417.

27. Mikhlin, S. G., Variational Methods in Mathematical Physics, Pergamon Press, 1964.

28. Bland, S. R., "Illustration of Airfoil Shape Effect on Forward-Swept Wing Divergence," J. Aircraft, Vol. 17, 10, Oct. 1980, pp. 261-763.

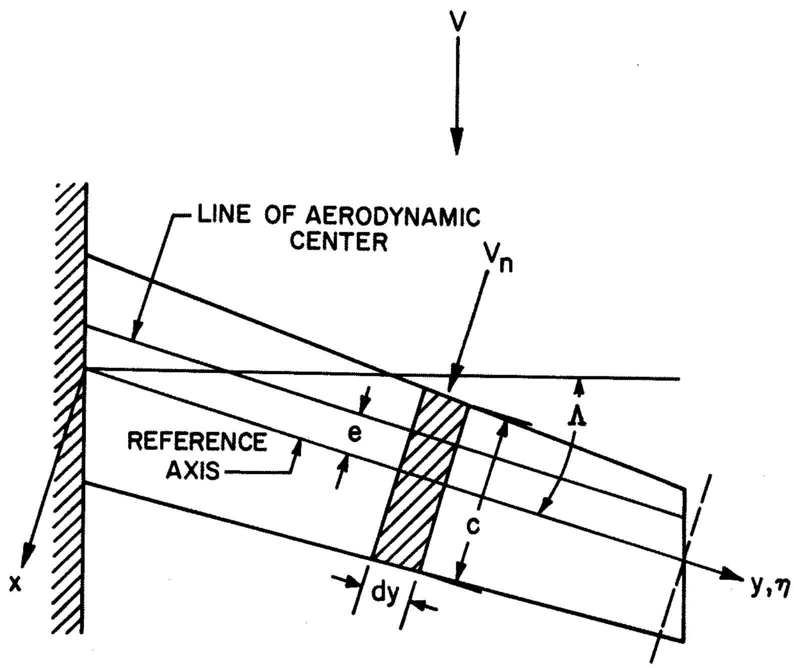

Figure 1 Geometry of the Swept-Wing Aircraft

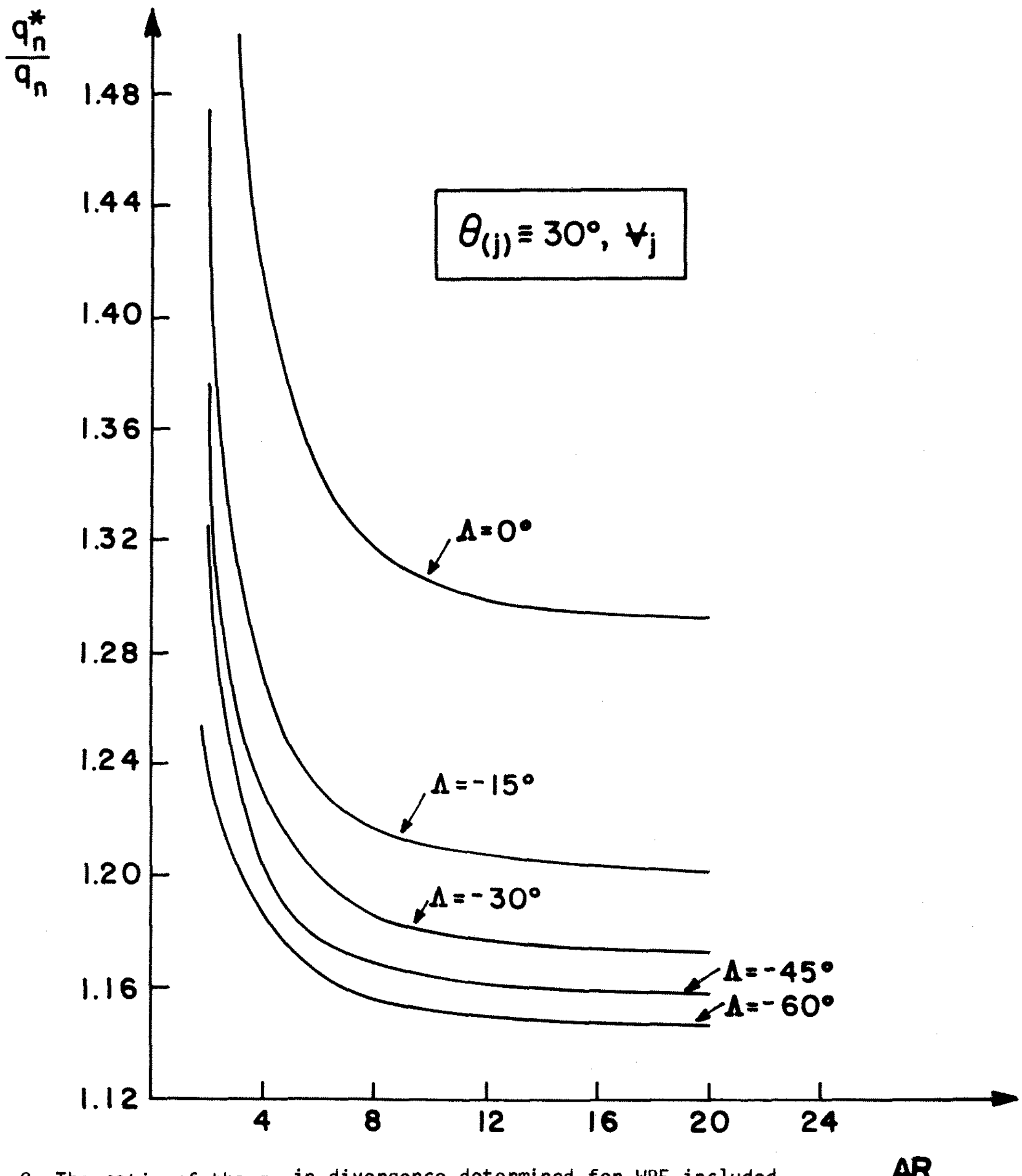

Figure 2 The ratio of the q_n in divergence determined for WRE included (q_n^*) and for free waring (q_n) versus AR for five different sweep angles and for a laminate with all fibers oriented at an angle $\theta_{(j)} \equiv \theta$ ($\forall j$) indicated on the figure.

Figure 3

The ratio of the q_n in divergence determined for WRE included (q_n^*) and for free waring (q_n) versus AR for five different sweep angles and for a laminate with all fibers oriented at an angle $\theta_{(j)} \equiv \theta$ ($\forall j$) indicated on the figure.

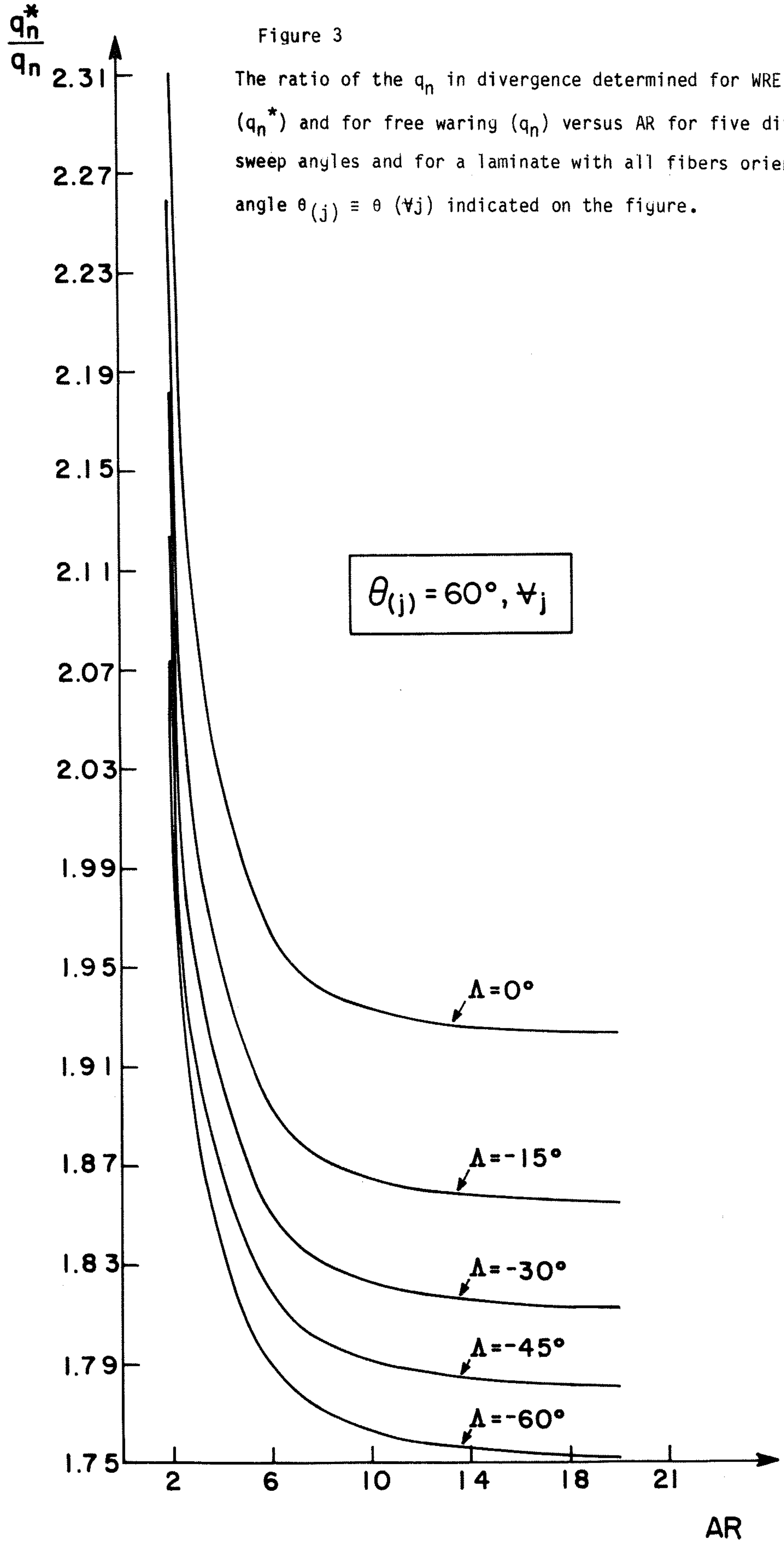

Figure 4

The ratio of the q_n in divergence determined for WRE included (q_n^*) and for free waring (q_n) versus AR for five different sweep angles and for a laminate with all fibers oriented at an angle $\theta_{(j)} \equiv \theta$ ($\forall j$) indicated on the figure.