SIMPLIFIED DESIGN OF COMPOSITE MATERIALS

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ABSTRACT

An internally consistent theory of micro- and macromechanics has been formulated to link materials, geometric and environmental parameters that affect the stiffness and strength of structural elements made of multidirectional composite laminates. Design simplifications are presented and other unique features of this integrated framework are discussed. The framework has been implemented on a personal computer in a spreadsheet format and validated using organic fiber-matrix composite materials data.

INTRODUCTION

The analysis of stress and deformation of anisotropic materials, and the study of micromechanics (composite properties as functions of the materials and geometric properties of the fiber and the matrix) are well developed sciences. The process of design, on the other hand, is more art, especially with composite materials. The approach to design presented here is a step towards making design more of a science. This discussion is limited to isothermal, static strength behavior. The micromechanical variables are linked to the strength and deformation of simple, statically determinate structural elements. In addition, simplification of complex analytical relationships is recommended so that the phenomenal performance provided by modern composite materials can be made available to every materials scientist and structural designer. Because of the large number of design parameters, simplification will permit more iterations and more optimal results than the complete theory. The objective is to capture 90 percent of the results with 10 percent of the effort, rather than the other way around.

There is no fundamental difference between composite materials and conventional materials, as far as governing equations are concerned. For anisotropic materials, numerically more complicated stress-strain relations are required than for isotropic materials. One of the primary goals of design is to determine the required thickness of a material subjected to the anticipated loads. The isotropic design process is shown in Figure 1. Given "Load 1", the "Thickness 1" required can be determined based on deformation or strength. Similarly "Thickness 2" can be determined for "Load 2", and so on. The controlling load in an isotropic design is simply the load that calls for the largest thickness.

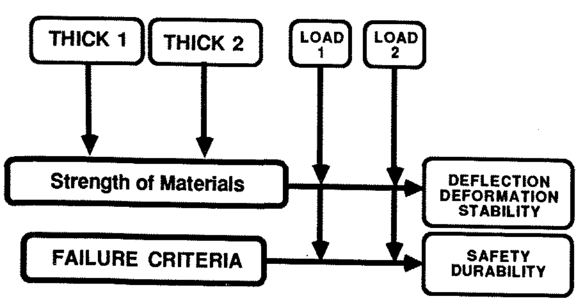

Figure 1. Flowchart of the isotropic design process, whereby the largest thickness indicates the controlling load.

For anisotropic design, the process must be modified, as shown in Figure 2. The controlling load among all possible loads cannot be selected based on the largest thickness or number of plies in a multidirectional laminate. The ply orientation or the layup must be included. The controlling load is in fact not self-evident. The so-called secondary load may turn out to be controlling. Furthermore, complex loads must be treated as a unit. They cannot be treated as unrelated, non-interacting loads. And, if multiple loading condition exists, it must be considered early in the design process.

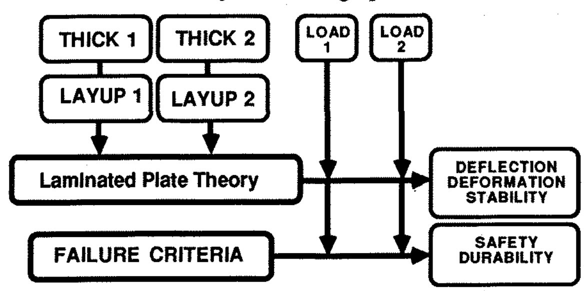

Figure 2. Flowchart of the anisotropic design process, whereby both thickness and layup indicate the controlling load.

Archaic Tools

The pioneers of composite materials proposed simplified models to aid design. There is nothing wrong with simple tools. But if they are misleading, even if not incorrect, they should not be used. Three tools can be considered archaic.

Netting analysis, credited to Mr. Richard Young of the Young Development Company in the USA, is illustrated in Figure 3. A complex stress resultant can be resolved into two normal stresses and one shear stress, all of which are sized independently. By netting analysis, the total number of plies required to carry the complex stresses is simply the sum of plies required for the individual stresses. The assumption that plies can carry stress along the fibers is not a bad one. However, the uncoupling of the plies to carry individual stresses is incorrect because plies in a laminate are bonded together and cannot deform independently. Netting analysis is most frequently used for the strength determination of filament-wound pressure vessels. It is not reliable.

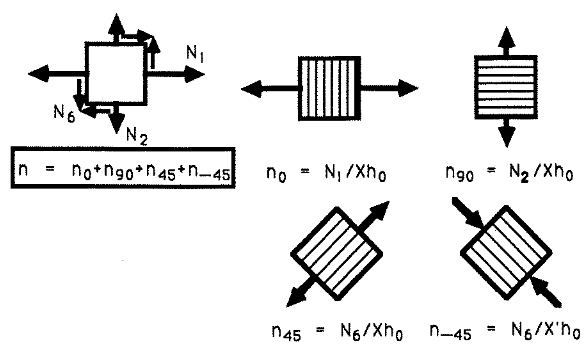

Figure 3. Illustration of netting analysis using a $\pi/4$* laminate approach, whereby a combined stress state is broken into components and attributed a number of plies, n, based on the applied stress components, N_i, longitudinal fiber tensile strength, X, and laminate thickness, h_0.

* A $\pi/4$ laminate, also referred to as a quadri-directional, is one with an arbitrary number of unidirectional plies arranged every $\pi/4$ or 45 degrees; for example, in the -45, 0, 45, and 90 degree directions.

A second archaic tool is the shear lag analysis as applied to the model illustrated in Figure 4. It is assumed that composite materials can be represented by a short fiber imbedded in an infinite domain of matrix material. When shear lag analysis is applied, it can be concluded that the strength of a short fiber will approach that of a continuous fiber composite. The result of this model leads to concepts of critical length, critical volume, interfacial strength, and fiber pull-out as a mode of failure in a composite materials. Shear lag analysis cannot account, however, for fiber discontinuities in matrix/fiber composites with high fiber volume fractions where interaction among fibers must be included. In this case, concepts of critical length, critical volume, and other materials approaches in composite materials will be, in fact, difficult to define or to substantiate.

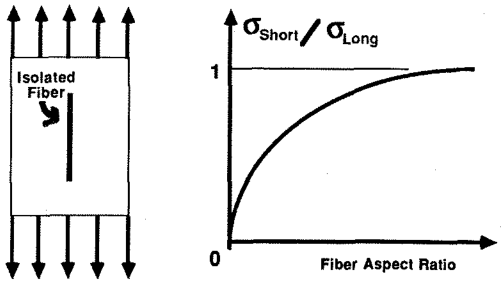

Figure 4. Illustration of shear lag analysis.

Finally, carpet plots, an example of which is shown in Figure 5, are still in use in the USA. They are graphical solutions for determining the uniaxial strengths of $\pi/4$ laminates. The curves are ambiguously generated. The basis for the calculated strength is rarely defined. The transition of uniaxial strength from unidirectional to multidirectional laminates is often resolved by arbitrary trunction of theory or data. It is never clear if the strength of a laminate is based on first- or last-ply-failure. Carpet plots incorrectly assume that a biaxial state of stress can be resolved into linearly superimposable components of stress in the manner of netting analysis. Furthermore, the incorrect results obtained from design by carpet plots are inconsistent and either over- or underestimate the number of plies required.

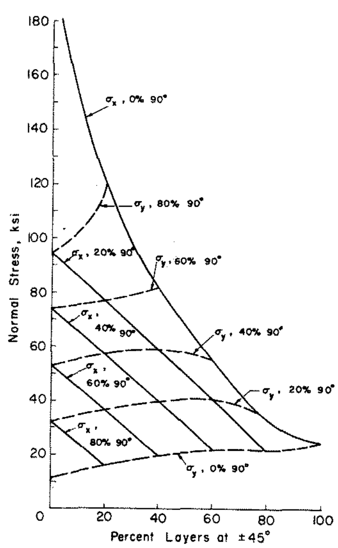

Figure 5. Typical example of a carpet plot for strength analysis.

These three archaic tools have been in use for many years. They have confused new and old workers in composite materials. The best way to combat ill-defined concepts is to present internally consistent approaches. It is important to rely on established governing equations and to explicitly distinguish empiricism from analysis. Towards this aim, an integrated approach to analysis and design is described herein: classical laminated plate theory provides the analytic foundation; to that is added a simplified micromechanics and other shortcuts in order to make the design process readily adaptable to personal computers, user-friendly and fast.

Laminated Plate Theory

Figure 6 illustrates laminated plate theory. The strain and average stress across the laminate thickness are assumed to be linear. The normalized stress-strain relations in terms of the in-plane and flexural stresses and strains are

$$
\begin{aligned}
\{\sigma^o\} &= [A^*]\{\varepsilon^o\} + [B^*]\{\varepsilon^f\} \\
\{\sigma^f\} &= 3\,[B^*]\{\varepsilon^o\} + [D^*]\{\varepsilon^f\} \\
\{\varepsilon^o\} &= [a^*]\{\sigma^o\} + [b^*]\{\sigma^f\}/3 \\
\{\varepsilon^f\} &= [b^*]^T\{\sigma^o\} + [d^*]\{\sigma^f\}
\end{aligned}
\qquad (1)
$$

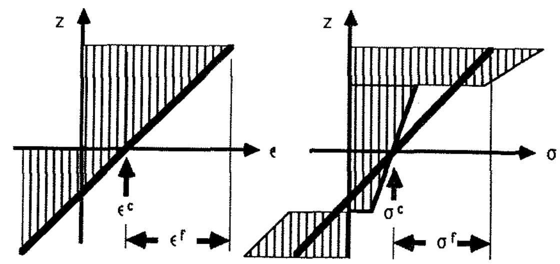

Figure 6. Illustration of in-plane and flexural average laminate strain, ε^o and ε^f, respectively, and in-plane and flexural average laminate stress, σ^o and σ^f, respectively, across a general/unsymmetric laminate (note that ply stresses remain piece-wise linear).

SIMPLIFIED DESIGN

Our proposed design methodology relies on the following:

- repeating sub-laminates,
- thin wall construction,
- stress partitioning method,
- power-law hygrothermal relations,
- residual stress due to lamination, and
- matrix degradation model to predict last-ply-failure.

The above tools are then easily implemented as demonstrated by integrated micro-macromechanics analysis.

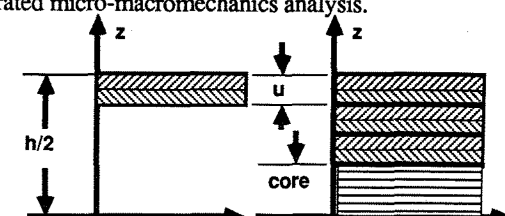

Figure 7. Illustration showing relation between sub-laminate and total laminate.

Repeating Sub-Laminates

The layup or ply orientation of a laminate can be considerably simplified if a laminate is constructed using repeating sub-laminates as shown in Figure 7. The stiffness of the sub-laminate can be easily converted to that of the total laminate, using the parallel axis theorem. As shown in Figure 8, the stiffness components corresponding to the original reference axis $z = 0$ can be transfered to the new axis $z' = 0$. In general, for a shift in the midplane:

$$[A'] = [A]$$
$$[B'] = [B] + d\,[A] \qquad (2)$$
$$[D'] = [D] + 2\,d\,[B] + d^2\,[A]$$

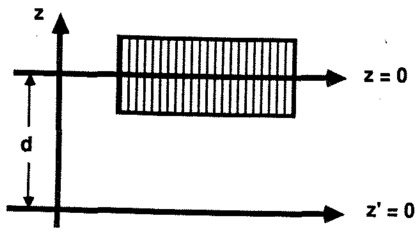

Figure 8. Original and new midplanes for the parallel axis theorem.

So, for a symmetric laminate constructed with a number, r, of repeated sub-laminates, the stiffness constants, A^o, B^o, D^o, of a sub-laminate translate into the stiffness constants, A, B, D, of the laminate as follows:

$$[A] = 2r\,[A^o]$$
$$[B] = 0 \qquad (3)$$
$$[D] = 2r\,\{[D^o] - (r-1)u[B^o] + (r-1)(2r-1)u^2[A^o]/6\}$$

The use of sub-laminates gives rise to many advantages. In addition to a simple laminate code, eliminating complex laminates results in reduced cost of lamination (modular construction) and less lamination error. Improved laminate toughness results because the best possible splicing is automatically insured. Optimization of the sub-laminate is fast because the number of plies is small. (The use of 8-ply sub-laminates is recommended.)

Thin Wall Construction

A thin wall construction, illustrated in Figure 9, is one that has thin facing laminates relative to the total depth of the construction. The applied stress resultants and bending moments can be resolved into stress resultants applied to the top and bottom faces. Such simple relations are also shown in Figure 9.

$$\{N+\} = \{N/2 + M/h\} \qquad (4)$$
$$\{N-\} = \{N/2 - M/h\}$$

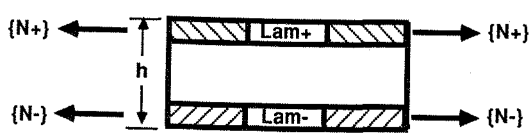

Figure 9. Illustration of thin wall construction.

The stiffness of the unsymmetric construction is also expressed in terms of the in-plane stiffness of the top and bottom faces in simple formulas

$$[A] = [A^+ + A^-]$$
$$[B] = h^2/4\,[A^+ + A^-] \qquad (5)$$
$$[D] = h/2\,[A^+ - A^-]$$

Similar relations for the compliance of the construction in terms of the in-plane compliance of the faces exist. Only 3x3 matrices are needed, which are of course much easier to invert than 6x6 matrices. Thin wall constructions are a valid simplifying approximation for most applications of composite materials.

Stress Partitioning Method

The stress partitioning method is a simplified micromechanics analysis of the transverse stiffness and shear modulus of a unidirectional ply. With a semi-empirical definition of the stress partitioning parameters, defined as the ratio of the average matrix and fiber stresses

$$\eta_j = (\sigma_m)_j/(\sigma_f)_j\,, \quad j = y, s \qquad (6)$$

the transverse stiffness and shear modulus can be derived using modified rule-of-mixtures relations

$$(1 + v_j^*)/E_y = 1/E_f + v_j^*/E_m \qquad (7)$$
$$(1 + v_j^*)/E_s = 1/G_f + v_j^*/G_m$$

where $v_j^* = \eta_j\, v_m/v_f\,, \quad j = y, s\,.$

The detailed derivation of these micromechanics relations can be found in *Introduction to Composite Materials* by Tsai and Hahn, Technomic (1980).

These micromechanics formulas are among the simplest in the literature. Other formulas for the same elastic moduli are more complex. Owing to the unmeasurable fiber constants under the transverse and shear loadings, and the varying degrees of idealization of the micromechanics formulas, it can be concluded that the only realistic use of micromechanics for the purpose of design is for sensitivity assessment of the material and geometric variables of the constituents. Therefore, the use of micromechanics should be only to assess the variation of properties from a set of baseline data. The stress partitioning parameters are treated as empirical constants from which the missing fiber moduli are "back-calculated", rather than the usual practice of determining ply properties from constituent data.

This use of micromechanics can be extended to the empirical fitting of hygrothermal data, the matrix modulus degradation model for the last-ply-failure prediction, and the time-temperature correspondence principle for time-dependent properties.

Hygrothermal Properties

Temperature and moisture dependent properties of unidirectional composite materials are typically measured at various combinations of quasi-static temperatures and moisture contents. Normally, there should be at least four temperatures (sub-zero, room temperature and two elevated temperatures), and three moisture contents (dry, 0.005 and 0.01, by weight). There are 12 combinations. For each hygrothermal combination, at least four stiffness and five strength measurements are needed. As all independent constants, this results in over 100 data points.

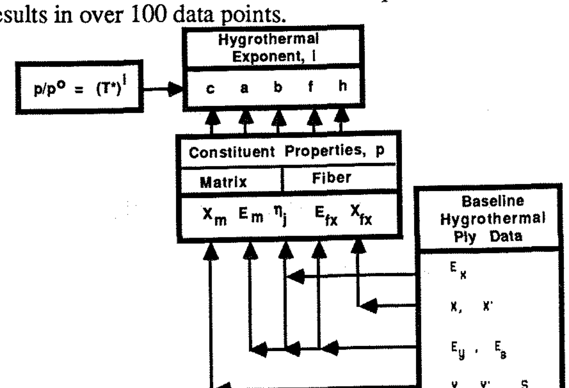

Figure 10. Back-calculation of the hygrothermal exponents, i, using micromechanics relations and the measured baseline ply data as functions of temperature and moisture.

Using micromechanics, the number of constants can be drastically reduced. This is shown schematically in Figure 10. From the baseline hygrothermal ply data, the implied constituent stiffnesses and strengths can be back-calculated. As a simple approximation, a power-law relation can be assumed for the hygrothermally induced changes in the ply data, based on a nondimensional temperature. Five exponents are back-calculated to curve-fit empirical data.

Moisture absorption is taken into account by the nondimensional temperature, the ratio of the difference between the operating temperature and the glass transition temperature of the organic matrix to the difference between room temperature (or some reference temperature) and the glass transition temperature. Moisture absorption results in a reduced the glass transition temperature, as modeled by a simple relation. Typical power-law hygrothermal relations are shown in Figure 11, where the hygrothermal exponent is arbitrarily chosen to be 0.3.

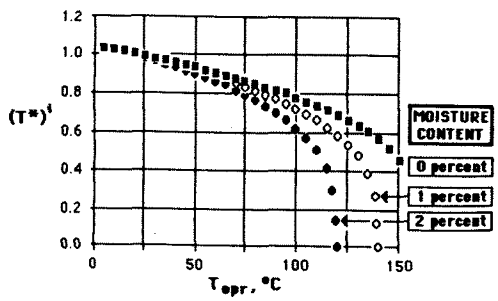

Figure 11 Effects of moisture content on the nondimensional temperature function, T*, for a constant exponent of i = 0.3.

Residual Stresses

Due to the difference between the longitudinal and transverse hygrothermal expansion coefficients of unidirectional plies, residual stresses are induced in the curing and hygrothermal conditioning of multidirectional laminates. It is straightforward to apply the linear theory of thermoelasticity to assess the nonmechanical strains and the residual strains resulting from the nonzero traction-free hygrothermal expansional strains. The relationships of these strains are illustrated in Figure 12 for a [0/90] laminate.

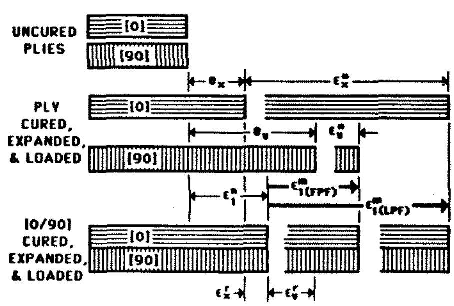

Figure 12. Illustration showing: for individual 0 and 90 degree plies, respectively, the free expansional strains, e_x, e_y, and the ultimate or failure strains, ε_x^*, ε_y^*; and therefore, for the [0/90] laminate, the nonmechanical strains, e_1^n, the residual strains, ε_x^r, ε_y^r, and the mechanical strains for first-ply-failure, $\varepsilon_1^m{}_{(FPF)}$, and last-ply-failure, $\varepsilon_1^m{}_{(LPF)}$.

In Figure 12, the failure strains of the unidirectional plies and the first-ply-failure (FPF) strain of a multidirectional laminate are also shown. The experimentally determined failure strains of the cured unidirectional ply are affected by the difference between the operating and the cure temperatures, and the moisture content. The residual stresses on the micromechanical level are important factors that affect the measured failure strain of the baseline ply. The difference between the constitutuent hygrothermal expansions is the source of the micromechanical stresses and may be significant.

The residual stresses in Figure 12 are those due to the lamination process. The first-ply-failure (FPF) can be used as a basis of limit load, analogous to the onset of yielding in metals. Appropriate failure criteria based on ply stress or ply strain can be applied to determine the lowest mechanical load that causes ply failure. Laminates after FPF may have load carrying capability if the last-ply-failure (LPF) load is higher than the FPF. The determination of the LPF however has been a subject of uncertainty for many years. As mentioned earlier in this paper, neither netting analysis nor carpet plots provide a rational answer. It is a common design practice of filament-wound pressure vessels to use the laminated plate theory for the prediction of FPF, and netting analysis for that of LPF. The use of entirely different approaches for the strength analysis of the same structure is not internally consistent and is not justifiable.

Matrix Degradation Model

Instead of the common practice of degrading the transverse and shear components of the stiffness matrix of plies which have failed in a laminate after the FPF, the stiffness degradation can be applied to the matrix modulus. Because the failed plies have sparsely spaced cracks, the loss of the effective ply stiffness is nowhere catastrophic. Assigning the near zero values for the transverse and shear moduli of the cracked plies appears to be unnecessarily severe. The use of micromechanics, however, gives a reasonable guideline as to the degree of degradation of the ply stiffness after FPF. In Figure 13, the calculated degradation of the ply stiffnesses as functions of a 40 percent degradation of the matrix modulus is shown. The latter percentage is empirically determined based on the laminate stiffness after FPF. Based on limited data, the 40 percent value is confirmed to be reasonable. Although a common practice in the USA, it should not be smaller than 1 percent.

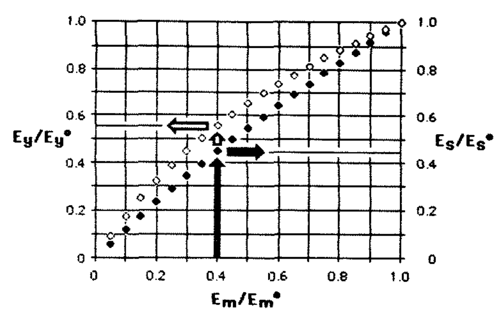

Figure 13. Graphical representation of micromechanics to compute relative reduction in transverse and shear moduli based on assumed relative reduction in matrix modulus to 0.4.

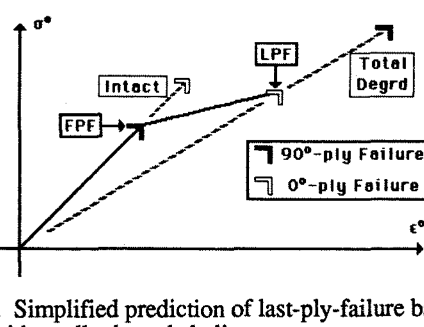

Figure 14. Simplified prediction of last-ply-failure based on the laminate with totally degraded plies.

In Figure 14, the calculation of the FPF and LPF of a [0/90] laminate is shown. The FPF is determined by using intact plies in the laminated plate theory; the LPF, degraded plies. The lowest load in the intact and degraded models determines the FPF and LPF, respectively. In Figure 14, the 90 degree ply is broken at FPF as expected. The 0 degree is broken at LPF, again as expected. This is easy to rationalize because the load required to cause the broken 90 degree ply to break again would appear to require a very high load. The mode of failure would be by longitudinal compression.

If the calculated LPF is lower than that of FPF, it can be assumed that there is no post-FPF load carrying capability; that is, the ultimate load is equal to the limit load for this laminate. For example, if a [0/90] laminate is subjected to a uniaxial compressive load, we can show that the FPF occurs in the 0 degree ply, and the LPF load is lower than the FPF load. There is then no post-FPF load; i.e., the FPF is the LPF. The 90 degree ply fails at higher loads in both intact and degraded matrix calculations.

Integrated Micro-Macromechanics Analysis

Current design practice is to use macro- or structural mechanics only. It is proposed to incorporate micromechanics as well, for a number of reasons. With micromechanics, the contributions of the constituents can be related as expected, but in a slightly different approach. As stated earlier, micromechanics is used for sensitivity study. Initially a set of baseline ply data is used to back-calculate the implied constituent properties. Henceforth, forward-calculation is applied to incorporate not only changes in the materials and geometric variables of the constituents, but also the hygrothermal dependent properties, and the matrix degradation model for the LPF prediction. The flow diagram in Figure 15 shows the proposed framework. This integrated micro-macromechanics framework (the Mic-Mac) provides the materials scientist and design engineer with a working tool that will do justice to composite materials.

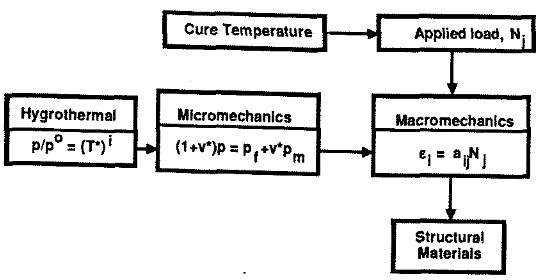

Figure 15. The integrated micro-macromechanics framework of composites analysis and design.

The easy accessability of computers enables archaic tools to be replaced by more reliable analysis. Since the advent of personal computers, spreadsheets have been a highly developed scheme. Thousands of cells can be linked by specified relationships. Computers can execute changes in data and interlocking relationships at amazing speed. Sophisticated logical decisions and integrated graphics are also available. Above all, spreadsheets are user-friendly. The use of a spreadsheet is like that of an automated camera with auto-focus: simply point and shoot. An instant answer can be gotten to "what if?"

The Mic-Mac has been implemented on a personal computer. No instruction is necessary, assuming a basic familiarity with the terminology. A Mic-Mac for simple structures like pressure vessels, beams, sandwich beams, beams made of thin tubing, and others requires a spreadsheet of 40x80 with about 1500 active components or cells. At the upper left-hand corner of a Mic-Mac spreadsheet is the control module, an example of which is shown in Figure 16. Key input and output data appear on the screen of the computer. For the Mic-Mac Vessel, macromechanics is packaged between rows 2 and 5. Micromechanics is located between rows 14 and 16. The structural mechanics is in the middle in rows 6 to 13. The spreadsheet accommodates many variables. In Figure 16, the sub-laminate layup, and the repeating index in the macromechanics section can be changed. Also, the length and diameter of the vessel, the applied axial load, internal or external pressure, and the torque can also be changed in the structural mechanics section. Finally, the micromechanical variables include the operating temperature, moisture content, fiber volume fraction, and the contituent stiffness and strength.

Vessel.exc

	1	2	3	4	5	6	7	8
1	MIC-MAC/CYLIN VESSEL: {[theta/#], . . .}T					Ply mat:	T3/N52[SI]	
2	{theta}	0.0	90.0	45.0	-45.0	[repeat]	h,#	h,E-3
3	[#plies]	1.0	0.0	0.0	0.0	8	8.0	1.0
4	R/FPF	0.08	#####	######	#####	limit	0.08036	104
5	R/LPF	0.08	#####	######	#####	ultimate	0.08063	104
6	size,m or in		ΔL,ΔD,mm or mil		[Loads]	<sigma>	<sig>lim	<sig>ult
7	[Length]	1.00	0.6077	F,MN,kip	0.00	250	20	20
8	[Diamtr]	1.00	48.157	P,MPa,ksi	1.00	500	40	40
9	Angle/twist,deg		0	Torque	0.00	0	0	0
10	/Q-iso	at limit	at ult	[Em/Em°]	0.30	<eps>E-3	<eps>lim	<eps>ult
11	stiffness	0.13	0.06	R*u/R*l	1.00	0.61	0.05	0.05
12	strength	0.11	0.07	[Rotate]	0.00	48.16	3.87	8.76
13						0.00	0.00	0.00
14		T opr	c,moist	vol/f	Em	Efx	Xm	Xfx
15	Baseline	22.0	0.005	0.70	3.40	259	40.0	2143
16	[Modifd]	22.0	0.005	0.70	3.40	259	40.0	2143

Figure 16. Control module of Mic-Mac/Vessel spreadsheet.

CONCLUSIONS

Based on a few design simplifications, an internally consistent theory of micro- and macromechanics has been formulated and implemented on a personal computer. The same framework can be extended to include not only the conventional design considerations such as the time-dependent properties, buckling and interlaminar stresses, but also unique factors of composite materials such as repairability, and the cost of materials and processing. In closing, archaic tools should not be used, in light of the first generation of design tools herein described.