LOAD EXAMINATION OF VEHICLE-BODY OF REINFORCED CYLINDRICAL SHELL IN CASE OF KINEMATIC LOAD

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Abstract

The slightly conic shell outrigger tail of helicopters can be modelled in strength calculation as a cylindrical shell reinforced longitudinally and laterally. The outrigger is connected to the body by bolted joint. Because of the inevitable production inaccuracy the flange serving joint is not of perfectly plane, thus the drawing of the coupling bolts forces local deformation on the structure. This effect causes relatively great load in stiffeners and sheet fields connected to the flange that weakens quickly having the flange. Size deviations causing kinematic load can be regarded as random variables of normal distribution and independent pair-wise, according to experience. Thus their effect can be taken into account already in course of design. In the knowledge of the operation load of the structure and alloved load the production tolerance and mounting technology ensuring suitable strength reserve can be prescribed.

I. Introduction

The stress-control calculation of structures is generally performed on the assumption that the structure is force- and stress-relieved under no-load condition.

Due to the inevitable geometrical inaccuracies resulting from manufacturing, the elements of the structure will probably not perfectly fit itself, hence some of the elements should undergo deformation during their assemblage.

In this paper, a method is introduced briefly which is suitable for the estimation of stresses arising during the assemblage of reinforced cylindrical shell structures.

The outrigger tail of helicopters is generally a slender, gently conical light-weight structure built up of thin flanges /longitudinal stiffeners/, rings /transverse stiffening frames/ and thin-wall covers. The tail is generally connected to the fuselage /body/ by means of connecting rings with bolted joints.

Owing to the inevitable production inaccuraces there are smaller or larger clearances between the matching surfaces, hence the tightening of set-screws forces some elements to undergo local deformation. Let this forced deformation be called briefly "kinematic load".

For the sake of a better survey, the mechanical model of the structure in this case is built up as a strongly idealized one

- the shell is considered to be a circular cylinder,
- there is no opening or local stiffening or reinforcement, respectively, on the examined section of the structure,
- the rings are enclosed frames of uniform dimensions and are spaced along the length at equal intervals. They can be modelled ideally stiff under the load applied to their own plane /as diaphragms/ or if it is necessary, their real elastic properties can be taken into account.
- They can be modelled as ideally flexible under the load applied vertically to their own plane.
- The cover is thin, so it can take over only membrane-forces.
- The flanges of constant cross-section are spaced along the perimeter at equal intervals, and they are suitable for taking over only normal forces.
- Let it be supposed that the kinematic load itself does not involve either any plastic deformation of the structure or the corrugation of covers, i.e. the principle of superposition can be applied without any restriction.
- The end points of flanges coupled to the connecting rings are fastened by a fixed hinged-joint, while their opposite end is free.

The enumerated idealization do not affect the mechanical principle of the problem but, first of all, they serve to the pupose of clearer representation of the phenomenon. /In a given case, the calculations can be performed even with much less idealization taken into account at all./

The calculating operations will be considerably simplified further on if instead of the entire outrigger tail /length of 10-12 frame intervals/, only a shorter section of it, e.g. a section of four frame-intervals in length is taken under close examination. If this shorter section is considered to be supported at both ends, then the model is more rigid, while if it is considered to be free at one end, then it is more flexible than the real structure. In this way, the lower and upper bounds of the stresses resulting from the given kinematic load are obtained, while the calculating operations are diminished.

The bounds can be reduced in length by increasing the number of the reckoned frame-intervals.

Determination of the resultant stresses of the structure from a given /deterministic/ kinematic load

The problem is solved by means of the force method. The model of the structure is shown in Fig.1. Let it be assumed that the number of flanges is z_1, and the number of rings is $z_2 > 3$.

The degree of redundancy of the model is:

$$n' = (z_2-1)\cdot(z_1-3) + 3z_2 ,$$

if it is free at one end, and

$$n'' = (z_2-1)\cdot(z_1-3) + z_1 + 3z_2$$

if it is supported at both ends.

The number of the dimensional inaccuracies involving kinematic loads is „k", wich is not necessarily identical with the degree of redundancy.

If the statically "redundant" external constraints and internal connections are eliminated in principle, then the statically determinate basic system is yielded.

In the place of the eliminated redundant connections, loading self-equilibrating and unit value couples of forces are assumed successively, and from these, the generalized stresses of each section within the properly divided structure are determined. It follows from the properties of the model that the stresses arising in the flange-sections will be a normal force changing linearly due to the unit loads, while the stresses in the cover-sections will be a shear-flow constant for each element, and the stresses arising in the rings will be a variable bending moment describable by means of a trigonometric function. /This latter can be approximated by means of piece-wise quadratic parabola-sections./

The different stresses are arranged in the unit load matrix $\underline{\underline{B}}$.

The data characteristic of the elasticity of each section are given in spring-matrix $\underline{\underline{R}}$. /See Fig.2./

The set of compatibility equations in the form of a matrix is:

$\underline{\underline{D}}\,\underline{\underline{y}} = \underline{\underline{\varphi}}$, where

$\underline{\underline{D}} = \underline{\underline{B}}^t\,\underline{\underline{R}}\,\underline{\underline{B}}$, the so called coefficient matrix /nxn/

$\underline{\underline{y}}$ is the column-vector of the connection-forces to be determined /n/

$\underline{\underline{\varphi}}$ is the column-vector of the kinematic load /n/.

Kinematic load vector is generated by a properly chosen geometric transforming matrix from column-vector $\underline{\underline{\varepsilon}}$ /now considered deterministic/ containing dimensional inaccuracies k in number:

$$\underline{\underline{\varphi}} = \underline{\underline{L}}\,\underline{\underline{\varepsilon}}$$

/Matrix $\underline{\underline{L}}$ consists of rows n and columns k; the physical sense /i=1,2,...,n; j=1,2,...,k/ of its element ℓ_{ij} is: the displacement occuring in the place and direction of the i^{th} unknown connection-force under the influence of eliminating the j^{th} dimensional inaccuracy in the basic system./

Accordingly, the solution of the set of compatibility equations is the following:

$$\underline{\underline{y}} = \underline{\underline{D}}^{-1}\,\underline{\underline{\varphi}} = \underline{\underline{D}}^{-1}\,\underline{\underline{L}}\,\underline{\underline{\varepsilon}}$$

The stresses arising in each section as caused by the elimination of dimensional inaccuracies given in column-vector $\underline{\underline{\varepsilon}}$ /i.e. tightening the coupling-bolts/ are yielded as the elements of the column-vector:

$$\underline{\underline{M}} = \underline{\underline{B}}\,\underline{\underline{\varphi}} = \underline{\underline{B}}\,\underline{\underline{D}}^{-1}\,\underline{\underline{L}}\,\underline{\underline{\varepsilon}}$$

/Construction of $\underline{\underline{M}}$ is similar to that of any columns of $\underline{\underline{B}}$./

It is to the pupose to concentrate the quantities independent from the concrete values of dimensional inaccuracies into a single matrix:

$$\underline{\underline{H}} = \underline{\underline{B}}\,\underline{\underline{D}}^{-1}\,\underline{\underline{L}} ,$$

so the stresses can be calculated by means of the formula:

$$\underline{\underline{M}} = \underline{\underline{H}}\,\underline{\underline{\varepsilon}}$$

Note that the calculations can be performed also by the assumption of a less idealized model /approximating the reality better, e.g. by means of the finite element method/, and its result can be written similarly, as the product of a correspondent matrix $\underline{\underline{H}}$ and column-vector $\underline{\underline{\varepsilon}}$.

Determination of the stresses resulting from random kinematic loads

According to experience, the production inaccuracies /i.e. the elements of column-vector $\underline{\underline{\varepsilon}}$ / can be considered as random variables of normal distribution and independent pair-wise. In this case, the stresses coming from kinematic load will also be random variables of normal distribution, which can be characterized satisfactorily by their expected value and dispersion at any point of the structure.

On the basis of these two characteristics, even the maximum stresses of each critical structural element can be estimated.

The expected value of the dimensional inaccuracies are given in column-vector:

$$\underline{\underline{\bar{\varepsilon}}}^t = [\varepsilon_1,\ \varepsilon_2,\ \dots\ \varepsilon_j,\ \dots\ \varepsilon_k]$$

With the pair-wise independence taken as a basis, the dispersions of the dimensional inaccuracies can be arranged in diagonal matrix like this:

$$\hat{\underline{\underline{\varepsilon}}} = \langle \hat{\varepsilon}_1, \hat{\varepsilon}_2, \ldots, \hat{\varepsilon}_j, \ldots, \hat{\varepsilon}_k \rangle$$

The expected value of the stresses is:

$$\bar{\underline{\underline{M}}} = \underline{\underline{H}}\, \bar{\underline{\underline{\varepsilon}}}$$

While, instead of the dispersion of stresses, there are column-vectors k in number separately, each of them represents dispersion of stresses calculable from the dimensional inaccuracy-dispersion, i.e.:

$$\hat{\underline{\underline{M}}} = \underline{\underline{H}}\, \hat{\underline{\underline{\varepsilon}}} = \left[\hat{\underline{\underline{M}}}_1, \hat{\underline{\underline{M}}}_2, \ldots, \hat{M}_j, \ldots, \hat{M}_k \right] .$$

Consequently, the j^{th} column of matrix $\underline{\underline{M}}$ yields the dispersion of stresses calculated from the j^{th} inaccuracy-dispersion in each element of the structure provided that the other dimensional inaccuracy-dispersions are equal to zero.

The simultaneous reckoning of the total effect of all the dimensional inaccuracy-dispersions can take place at any element of the structure so that the row of matrix $\underline{\underline{M}}$ as belonging to the selected element, is looked for, and then the square-root of the quadratic sum of the matrix elements to be found in this row is calculated:

$$\hat{M}_i = \sqrt{(\hat{M}_{1,i})^2 + (\hat{M}_{2,i})^2 + \ldots + (\hat{M}_{k,i})^2} =$$

$$= \mathrm{sqrt}\left[\sum_{j=1}^{k} (h_{i,j}\, \hat{\varepsilon}_j)^2 \right]$$

$i = 1, 2, \ldots, p$

The stress-dispersion of all the structural elements, with all the dimensional inaccuracy-dispersions being taken into account, is yielded by the column-vector of the square-roots of the principal diagonal elements of a matrix:

$(\hat{\underline{\underline{M}}})^2 = \hat{\underline{\underline{M}}}\, \hat{\underline{\underline{M}}}^T = \underline{\underline{H}}\, \hat{\underline{\underline{\varepsilon}}}\, \hat{\underline{\underline{\varepsilon}}}^t\, \underline{\underline{H}}^t$, like this:

$$\underline{\underline{M}}_s = \mathrm{sqrt}\left\{ \underline{\underline{H}}\, \hat{\underline{\underline{\varepsilon}}}\, \hat{\underline{\underline{\varepsilon}}}^t\, \underline{\underline{H}}^t \right\}$$

Stresses in any selected element are found in the confidence-interval determined by the expected value and the triple dispersion to a probab of 99,7%, hence the relationship applicable to the stress-control calculations is:

$$|M_{i,o}| \geq |\bar{M}_i| + 3\, |\hat{M}_{i,s}| \qquad i = 1,2,\ldots,p$$

where

$M_{i,o}$ is the permissible stress of the element.

If this inequality is not true for any section of the structure, then the stresses resulting from kinematic load should be reduced.

In principle, there are two possibilities of solving this problem:

a./ Reduction in stiffness of the critical elements by modifying the structure /this involves the variation of the elements of matrix $\underline{\underline{H}}$ in the formulae/. This solution in generally involves reduction in the "operating" load-capacity, so it cannot be carried out in most of the cases.

b./ By means of changing the technology of manufacturing and assemblage, the elements of matrices $\bar{\underline{\underline{\varepsilon}}}$ and $\hat{\underline{\underline{\varepsilon}}}$ can be reduced.

Owing to the fact that improvement in technology will increase the manufacturing costs, it is important that tolerances are made closer only to a justified extent. On the basis of the foregoing, this task might be performed as well, even by means of the trial- and error method: the variation in technology leaves the elements of matrix $\underline{\underline{H}}$ unchanged, hence $\bar{\underline{\underline{M}}}$ and $\hat{\underline{\underline{M}}}$ can be simply re-calculated by means of matrices $\bar{\underline{\underline{\varepsilon}}}^*$ and $\hat{\underline{\underline{\varepsilon}}}^*$ characteristic of the modified processes of manufacturing and assemblage.

With the calculations reiterated several times, the solution of the problem, as demanding the least possible extra expenses, can be found.

Effect of dimensional deviations of unknown type of distribution and non- *in*-dependent types

Such a case occurs, e.g. when each element of the structure is manufactured by machines of identical type, and the dimensions of these do not follow the normal distribution.

Let it be assumed that the measured dimensional errors of elements N in number are known. With the use of a suitable method, an approximate empirical distribution-function can be derived from the former ones /the more accurate the calculations are, the greater is figure N/ but it is better to do the calculations by means of empirical moments which can be handled easier numerically.

Arrange the dimensional errors known from measurements in Table $\underline{\underline{T}}$:

$$\underline{\underline{T}} = \begin{bmatrix} \varepsilon_{1,1} & \varepsilon_{1,2} \cdots & \varepsilon_{1,N} \\ \varepsilon_{2,1} & \varepsilon_{2,2} & \varepsilon_{2,N} \\ \vdots & \vdots & \vdots \\ \varepsilon_{k,1} & \varepsilon_{k,2} & \varepsilon_{k,N} \end{bmatrix}$$

The empirical estimate of the expected value of vector $\underline{\underline{\varepsilon}}$ is as follows:

$$\bar{\underline{\underline{\varepsilon}}} = \frac{1}{N}\, \underline{\underline{T}}\, \underline{\underline{e}} \quad , \text{ where}$$

$\underline{\underline{e}}^t = [1, 1, \ldots, 1]$, sum-forming vector of element k .

The definition of the uncorrected covariance-matrix of random variable $\underline{\underline{\varepsilon}}$ is:

$$[\hat{\underline{\underline{\varepsilon}}}^2] \cong \overline{\underline{\underline{\varepsilon}}\,\underline{\underline{\varepsilon}}^t} - \bar{\underline{\underline{\varepsilon}}}\,\bar{\underline{\underline{\varepsilon}}}^t$$

This can be written in the following form by means of the measurement data:

$$[\hat{\underline{\underline{\varepsilon}}}^2] \cong \frac{1}{N}\,\underline{\underline{T}}\,\underline{\underline{T}}^t - \frac{1}{N^2}\,(\underline{\underline{T}}\,\underline{\underline{e}})^t = \frac{1}{N}\,\underline{\underline{T}}\,\underline{\underline{Q}}\,\underline{\underline{T}}^t ,$$

where

$$\underline{\underline{Q}} = \underline{\underline{E}} - \frac{1}{N}\,\underline{\underline{e}}\,\underline{\underline{e}}^t \quad \text{and}$$

$\underline{\underline{E}}$ is an identity matrix of proper dimension.

The expected value and dispersion of the stresses can be determined by means of the train of thoughts dealt with in the foregoing:

$$\bar{\underline{\underline{M}}} = \frac{1}{N}\,\underline{\underline{H}}\,\underline{\underline{T}}\,\underline{\underline{e}}$$

is the expected value and

$$\hat{\underline{\underline{M}}}^2 = \frac{1}{N}\,\underline{\underline{H}}\,\underline{\underline{T}}\,\underline{\underline{Q}}\,\underline{\underline{T}}^t\,\underline{\underline{H}}^t$$

is the covariance-matrix.

The stress-dispersion of each element can be obtained in the form of:

$$\underline{\underline{M}}_s = \text{sqrt}\ (\hat{\underline{\underline{M}}}^2\,\underline{\underline{e}})$$

The probability of the chance that the actual stresses of a selected element deviate from $\bar{M}$ to an extent of $\lambda\hat{M}$, can be estimated in this case only on the basis of Csebisev-inequality, or its sharpened versions including higher momenta.

The Csebisev-inequality:

$$P\ (\ |M_i - \bar{M}_i| < \lambda\hat{M}_i) \geq 1 - \frac{1}{\lambda^2}$$

If similarly to the criterion used with normal distributions, $\lambda = 3$, then the value yielded is:

$$P = 0{,}888$$

Consequently, inequality

$$|M_{i\ max}| \leq |\bar{M}_i| + 3\,|\hat{M}_i|$$

is true only to an extent of 89% of probability.

If $\lambda = 4$ is valid, there is still 7% probability of

$$|M_{i\ max}| \geq |\bar{M}_i| + 4\,|\hat{M}_i| .$$

The reliability of estimation can be increased by the application of higher momenta.

<u>A numerical example</u>

As an example of the application of this method we introduce briefly the results of calculations concerning about a shell structure with 12 longitudinal stiffeners and 4 ring.

Geometrical data of the structure:

Diameter of rings: $D = 52o$ mm
Distance of rings: $l = 4oo$ mm
Thickness of cover panels: $v = o{,}6$ mm
Cross-section area of flanges $A = 25o\ \text{mm}^2$
Inertia of the ring cross-sections: $I = 8{,}33 \cdot 1o^4\ \text{mm}^4$
Material of all the elements: aluminium alloy: Young-modulus: $E = 7 \cdot 1o^4\ \frac{N}{mm^2}$,
shear-modulus: $G = 2{,}8 \cdot 1o^4 \frac{N}{mm^2}$.

The calculation was prepared in four variations:

A: supported structure on both ends with rigid rings
B: supported structure on both ends with elastic rings
C: free structure on one end with rigid rings
D: free structure on one end with elastic rings

The elements of $\underline{\underline{\varepsilon}}$ matrice are the mounting clearence sizes between the first ring and the support wich are measurable in the place and direction of the stiffeners. /i.e. k = 12 in each case/. Owning to the cyclic symmetry of the structure the final degree of redundancy in the above mentioned cases is: n = 21, 29, 14 and 22.

After determining the elements of the matrice $\underline{\underline{H}}$ let the effect of the "unit kinematic load" /i.e. $\underline{\underline{\varepsilon}}^t = [1, o, o, \ldots] \cdot 1o^{-1}$ mm be examined first.

The normal forces in flanges and the shear flows in cover-sections owning to this "unit kinematic load" are presented in Fig. 3. Those stresses weaken quickly leaving the first ring. The character of the functions is different, but the value of maximal stresses is in each case nearly of the same size, in spite of the fact, that the elastic ring is /as compared to the elasticity of other elements/ very soft.

Owning to the cyclic symmetry it is an allowable assumption that the expected value and dispersion of random kinematic load is equal in every supporting places. In this case the expected value of the stresses is all over zero. The dispersion of the stresses may be calculated simply:

$$\hat{\underline{\underline{M}}} = \underline{\underline{H}}\,\hat{\underline{\underline{\varepsilon}}} = \hat{\underline{\underline{\varepsilon}}}\,\underline{\underline{H}}\,\underline{\underline{E}} = \hat{\varepsilon}\,\underline{\underline{H}} \quad \text{, and}$$

$$\hat{M}_i = \hat{\varepsilon} \cdot \text{sqrt}\left(\Sigma h_{ij}^2\right)$$

In Fig. 4. the dispersion of stresses is presented when $\hat{\varepsilon} = o{,}1$ mm.

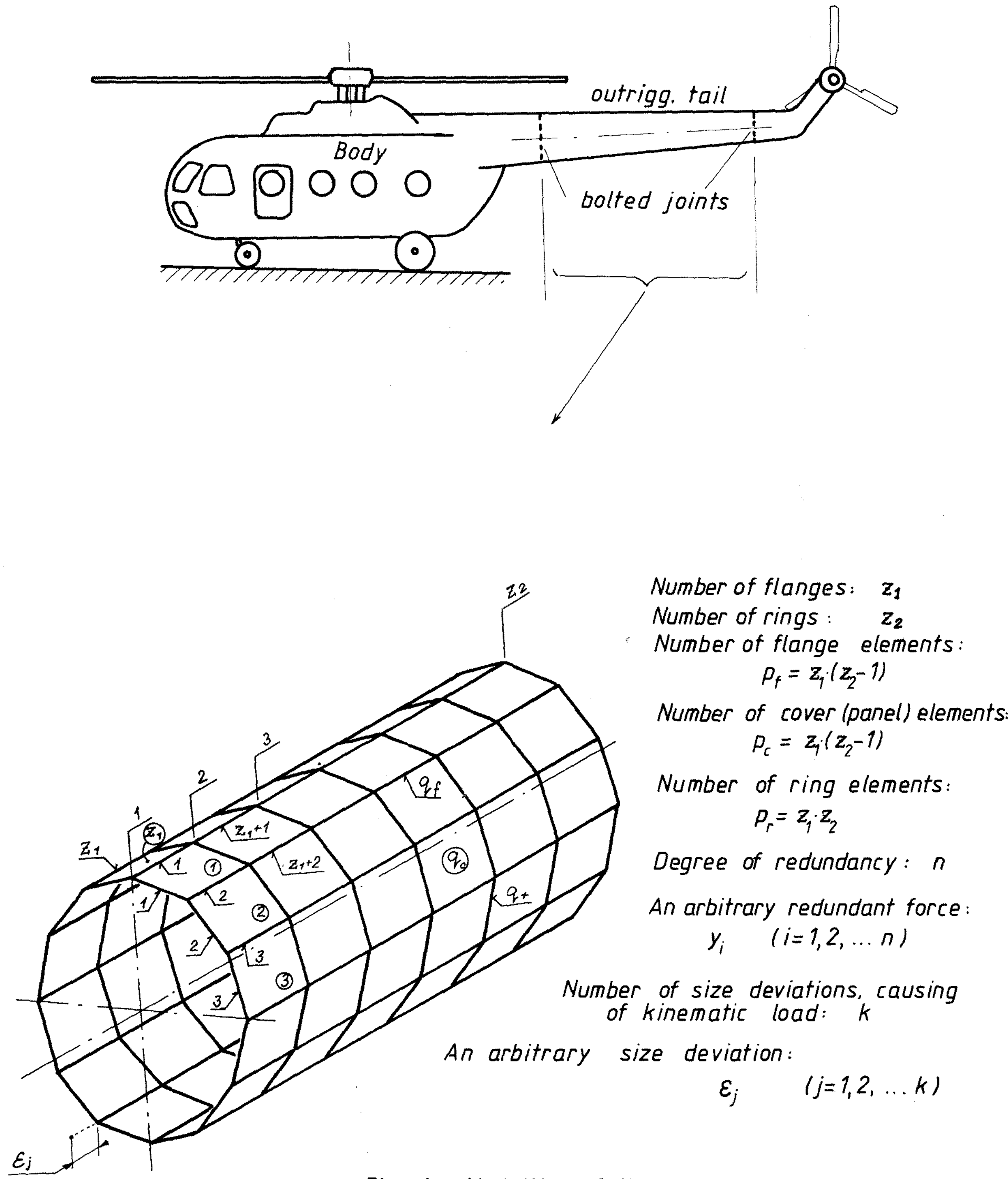
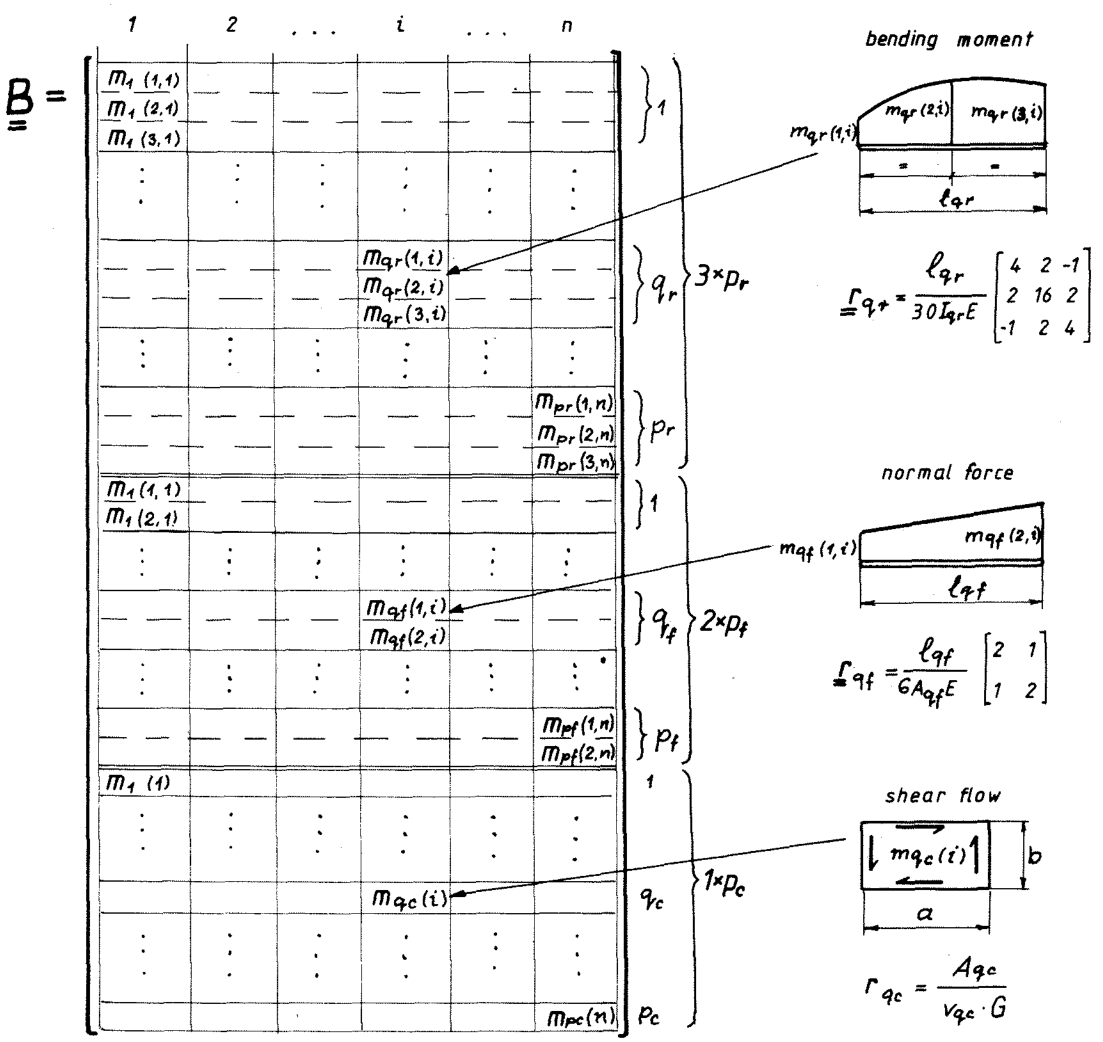


Fig. 1. Modelling of the structure



$$\underline{\underline{R}}_r = \langle \underline{\underline{r}}_1, \cdots \underline{\underline{r}}_{qr}, \cdots \underline{\underline{r}}_{pr} \rangle$$

$$\underline{\underline{R}}_f = \langle \underline{\underline{r}}_1, \cdots \underline{\underline{r}}_{qf}, \cdots \underline{\underline{r}}_{pf} \rangle$$

$$\underline{\underline{R}}_c = \langle r_1, \cdots r_{qc}, \cdots r_{pc} \rangle$$

$$\underline{\underline{R}} = \langle \underline{\underline{R}}_r, \underline{\underline{R}}_f, \underline{\underline{R}}_c \rangle$$

I_{qr} – inertia of ring element's cross section

A_{qf} — area of flange -"-

v_{qc} — thickness of cover elements

$A_{qc} = a{\cdot}b$ — area -"-

E – Young-modulus of the material

G — Shear modulus -"-

Index of an arbitrary flange element: $q_f = 1, 2, \ldots p_f$

-"- cover -"- $q_c = 1, 2, \cdots p_c$

-"- ring -"- $q_r = 1, 2, \cdots p_r$

Fig. 2. Mounting of the $\underline{\underline{B}}$ and $\underline{\underline{R}}$ matrices

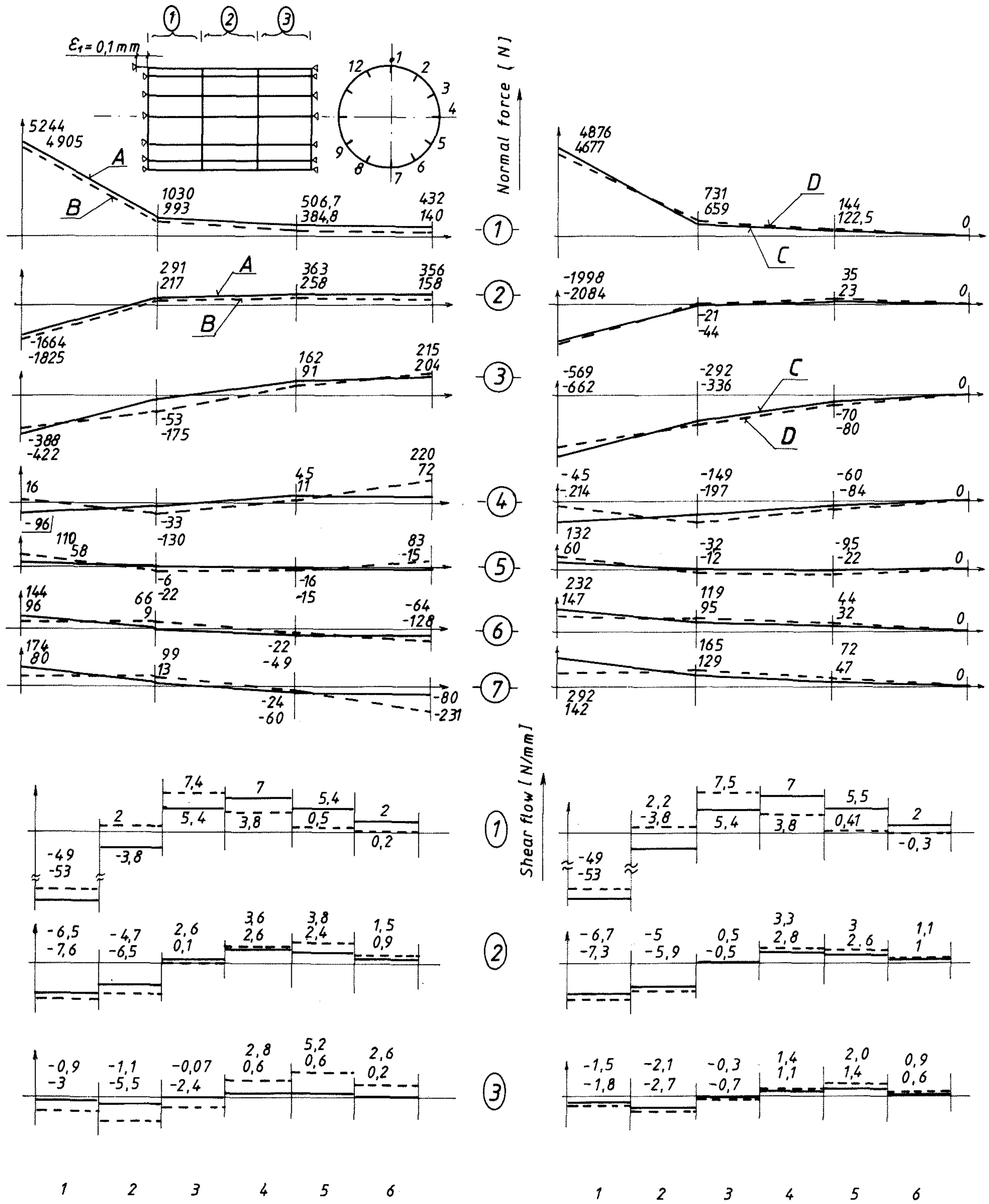


Fig. 3. Stresses owning to „unit kinematic load"

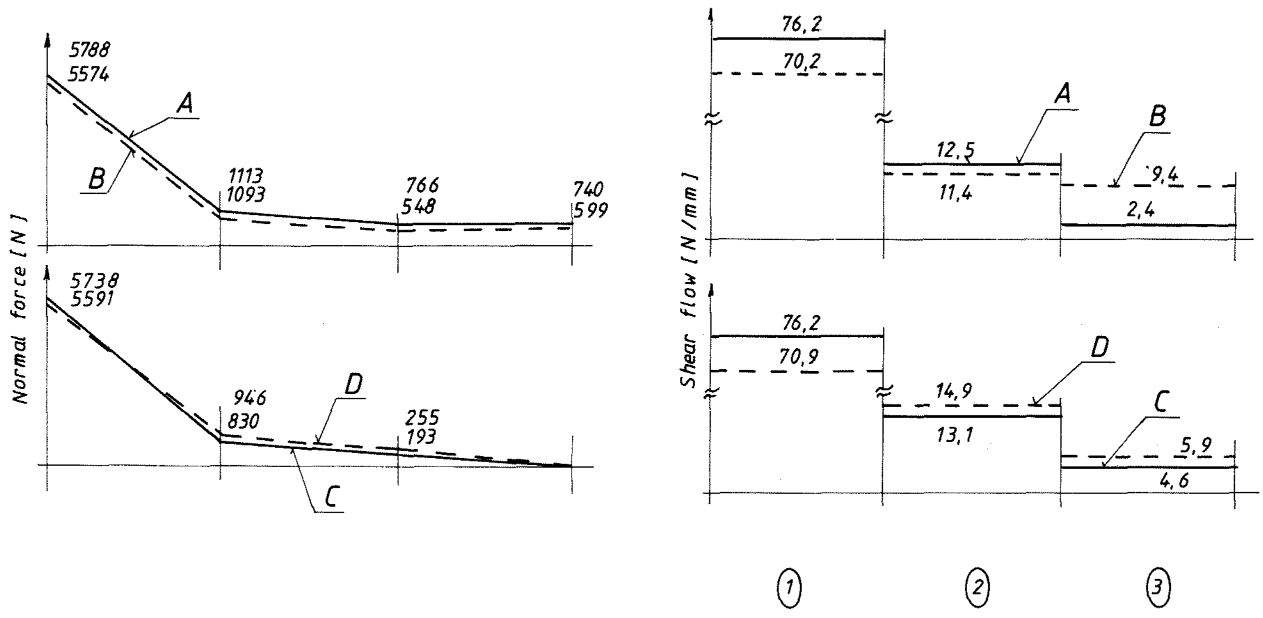


Fig. 4. Stress dispersions when $\hat{\mathcal{E}}=0{,}1mm$ in every supporting place