SOME ASPECTS OF THE RELIABILITY ANALYSIS OF AIRCRAFT STRUCTURES*

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Abstract

This paper briefly introduces some researches about the reliability analysis of aircraft structures proceeding in Northwestern Polytechnical University (NPU). A set of fundamental equations with some numerical examples to compute the failure rate of aircraft structures in service is presented. It takes the factors into consideration as complete as possible.

From the numerical results the influences of various factors on reliability of aircraft structures are estimated and the current criteria of aircraft fatigue design are evaluated. It is also pointed out that the 10^{-7} level of failure probability (per hour) with 95% confidence recommended in Engineering Sciences Data of RAE is unreasonable.

For the data treatment of some important factors, i.e. the initial life, the probability of detection, the distribution of initial length of flaw and the distribution of K_{1c}, some new methods are developed with some new results obtained. Among them are an estimation method of the parent distribution parameters of initial life from the laboratory test as well as the detection at airports with the results of incomplete life and "ultracomplete" life, and an modified method using the F-distribution to be able to estimate simply the flaw detection probability curve. Some large samples have been tested by the authors in order to obtain more confident distribution patterns of initial crack length and of fracture toughness, being used to evaluate the existing results in literature.

The Fundamental Equations for Reliability Analysis

The reliability models developed at NPU by the authors have been presented in some previous papers(1-4). For a single critical point structure, the model considers the following factors: the static strength of structure, the initial crack, the initiation and propagation and unstability of fatigue crack, the residual strength of structure, the statistical distribution of load, the periods of overhaul, the accident damage, the communication of damage among fleets etc.

The main symbols used are as follows:

$d(n_s)$	Probability rate of detecting cracks at n_s
$F_S(s)$	Exceeding probability of external load $F_S(R_S)\times\bar{N} = F_0(n,x,z)$ or $F_a(n,x,y)$
$H(n_s)$	Probability of failure up to n_s
l	The value of L, $l = g(n,z)$ or $h(n,y)$
L	Crack length
n	Life, $n = N/\bar{N}$
n_i	Life at i-th inspection (overhaul)
n_s	Current life
N	Number of load applications
$\bar{N}$	Reference life
$p(\ )$	Probability density function (PDF)
$p_D(l)$	Probability rate of detecting crack length l
P_{Di}	Probability of detecting cracks at i-th inspection, $\bar{P}_{Di} = 1 - P_{Di}$
$P_D(l)$	Probability of detecting crack l
$r(n_s)$	Risk of failure at n_s
$r^*(n_s)$	Probability rate of failure at n_s
$R(n_s)$	Survival probability up to n_s
R_s	Residual strength (and its value)
s	The value of S
S	External load
$x\ y\ z$	The values of ξ, η, ζ respectively
α	The percent of elements containing initial crack in parent population
α_K	The percent of unexpected cracks at life n_K
δ	The symbol denoting ignorable term
ζ	Life of crack initiation
η	Initial crack length
ξ	Residual strength parameter

For the case with continuous (per flight) and intervallic (overhaul) inspections, the survival probability, the risk function, and the probability function of failure are respectively:

$$R(n_S) = (1-\alpha)\iint\limits_{n_0(x,z)=n_S}^{\infty} e^{-\int_0^{n_s}(F_0(n,x,z)+p_D(g(n,z)))dn} \times \prod_{i=1}^{K}\bar{P}_{Di}(g(n_i,z))\,p_1(x)\,p_3(z)\,dx\,dz$$

$$+\ \alpha\iint\limits_{n_a(x,y)=n_S}^{\infty} e^{-\int_0^{n_s}(F_a(n,x,y)+p_D(h(n,y)))dn} \times \prod_{i=1}^{K}\bar{P}_{Di}(h(n_i,y))\,p_1(x)\,p_2(y)\,dx\,dy \qquad (n_K<n_S<n_{K+1}) \tag{1}$$

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$$r^*(n_S)=(1-\alpha)\iint\limits_{n_0(x,z)=n_S}^{\infty} K_0(n_0,x,z)\prod_{i=1}^{K}\bar{P}_{Di}(g(n_i,z))\,p_1(x)\,p_3(z)\,dx\,dz + \alpha\iint\limits_{n_a(x,y)=n_S}^{\infty} K_a(n_S,x,y)\prod_{i=1}^{K}\bar{P}_{Di}(h(n_i,y))\,p_1(x)\,p_2(y)\,dx\,dy+\delta r^*(n_S)$$

$$(n_K<n_S<n_{K+1}) \tag{2}$$

where

$$K_0(n_S,x,z)=F_0(n_S,x,z)e^{-\int_0^{n_S}(F_0(n,x,z)+p_D(g((n,z)))dn}$$

$$K_a(n_S,x,y)=F_a(n_S,x,y)e^{-\int_0^{n_S}(F_a(n,x,y)+p_D(h(n,y)))dn}$$

$$H(n_S)=\sum_{j=1}^{K-1}H_j+\triangle H_K(n_S)+\delta H(n_S) \qquad (n_K<n_S<n_{K+1}) \tag{3}$$

where

$$H_j=\int_{n_j}^{n_{j+1}}r^*(n_S)dn_S$$

$$\approx(1-\alpha)\iint_0^{\infty}\int_{n_j}^{n_{j+1}}K_0(n,x,z)dn\prod_{i=1}^{j}\bar{P}_{Di}(g(n_i,z))\,p_1(x)\,p_3(z)\,dx\,dz$$

$$+\alpha\iint_0^{\infty}\int_{n_j}^{n_{j+1}}K_a(n,x,y)dn\prod_{i=1}^{j}\bar{P}_{Di}(h(n_i,y))\,p_1(x)\,p_2(y)\,dx\,dy$$

$$\triangle H_K(n_S)=\int_{n_K}^{n_S}r^*(n_S)dn_S$$

$$\approx(1-\alpha)\iint_0^{\infty}\int_{n_K}^{n_S}K_0(n,x,z)dn\prod_{i=1}^{K}\bar{P}_{Di}(g(n_i,z))\,p_1(x)\,p_3(z)\,dx\,dz$$

$$+\alpha\iint_0^{\infty}\int_{n_K}^{n_S}K_a(n,x,y)dn\prod_{i=1}^{K}\bar{P}_{Di}(h(n_i,y))\,p_1(x)\,p_2(y)\,dx\,dy$$

This model is in principle belong to the same approach as that developed at ARL by Payne et al[5-8], Hooke[9-12], and Ford [13-15]. All these models consider the relation between the strength degradation and crack length as well as the variation in structural behaviours, but there are still some important different aspects among them. For examples, the risk of failure $F_S^*(R_S)$ and the probability rate of detecting crack $p_D(l)$ for a differential element in formulas (1) - (3) are taken as

$$F_S^*(R_S)=ln\frac{1}{1-F_S(R_S)}$$

$$p_D(l)=\frac{1}{\triangle n_1}\ln\frac{1}{1-P_D(l)}$$

instead of $F_S(R_S)$ and $P_D(l)/\triangle n_1$ respectively. That is because otherwise, the integral kernel function will not be zero when $F_S(R_S)$ or $P_D(l)$ equals 100%, that is obviously not in conformity to the true.

Besides, the NPU method takes the unexpected damage in manufacturing or in service into consideration. The importance of considering that factor can be shown by the fact that the accidents caused by unexpected damage in 1970's promoted the damage tolerance design principle for aircraft structures. The NPU method also consider the influence of communication of damage among fleets. Provided that the case after any damage is denoted by subscript 2 to be distinguished from the virgin case denoted by subscript 1, the probability rate of failure is

$$[R_1(n_S)]^{m-1}r_1^*(n_S)+(m-1)\int_0^{n_S}[R_1(n)]^{m-2}[r_1^*(n)+d_1(n)]\times$$

$$[(1-\alpha)\iint\limits_{n_0(x,z)=n_S}^{\infty}F_{02}(n_S,x,z)\,e^{-\int_0^{n}(F_{01}(n,x,z)+p_{D1}(g(n,z)))dn}$$

$$\times\, e^{-\int_n^{n_S}(F_{02}(n,x,z)+p_{D2}(g(n,z)))dn}\prod_{i=1}^{K_n}\bar{P}_{D1i}(g(n_{1i},z))\times$$

$$\times\prod_{j=1}^{O_n}\bar{P}_{D2j}(g(n_{2j},z))\,p_1(x)\,p_3(z)\,dx\,dz+$$

$$+\alpha\iint\limits_{n_a(x,y)=n_S}^{\infty}F_{a2}(n_S,x,y)\,e^{-\int_0^{n}(F_{a1}(n,x,y)+p_{D1}(h(n,y)))dn}$$

$$\times\, e^{-\int_n^{n_S}F_{a2}(n,x,y)+p_{D2}(h(n,y)))dn}\prod_{i=1}^{K_n}\bar{P}_{D1i}(h(n_{1i},y))\times$$

$$\times\prod_{j=1}^{O_n}\bar{P}_{D2j}(h(n_{2j},y))\,p_1(x)\,p_2(y)\,dx\,dy]dn$$

$$(n_{1K_n}<n<n_{1(K_n+1)},\ n_{2j}=n+\triangle n_{2j},\ \triangle n_{2O_n}<n_S-n_u<\triangle n_{2(O_n+1)}) \tag{4}$$

Some Conclusions from Numerical Results

1. The unexpected damage has a decisive influence to the reliability function, but unfortunately the factor α itself can be hardly estimated by a routine sample test.

2. To increase the initial crack life is one of the most important devices to reduce the probability of failure as shown in Fig.1 taken from one example.

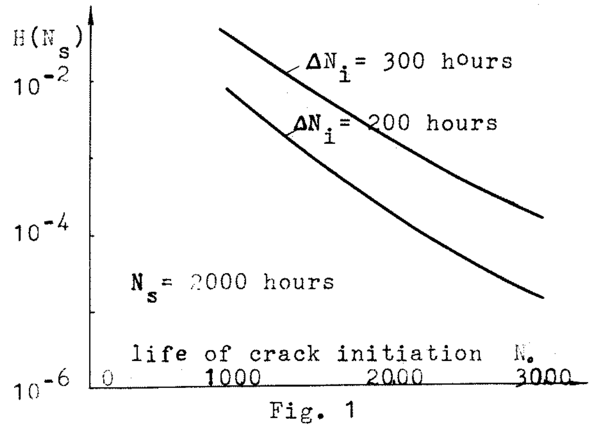

Fig. 1

3. The inspection period and the detective probability are other important factors affecting the probability of failure as shown in Fig.2 and Fig.3, especially when $\alpha \neq 0$, the affect will be more important.

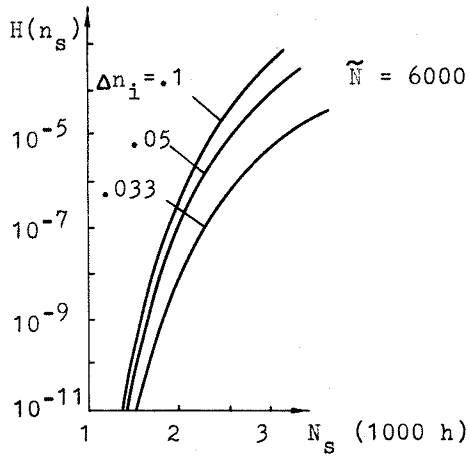

Fig. 2

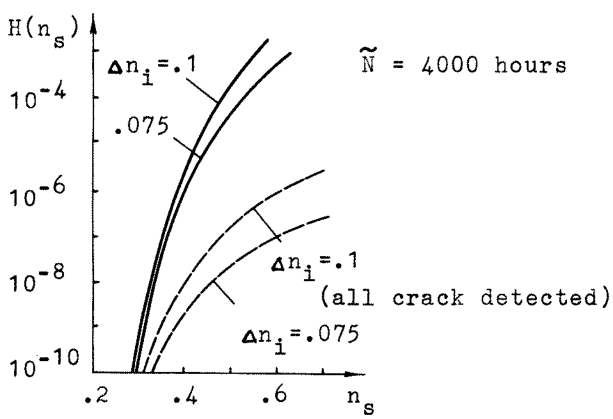

Fig. 3

4. The influence of the variations of K_{1c} is not negligible as shown in Table 1 and Table 2.

Table 1 $H(n_i) - H(n_{i-1})$

$(\bar{K}_{1c} = 252\ kg/mm^{3/2})$

i	standard deviation of K_{1c}		
	7	20	23.5
3	0.75×10^{-6}	0.93×10^{-5}	0.25×10^{-4}
5	0.30×10^{-5}	0.30×10^{-4}	0.73×10^{-4}
7	0.89×10^{-5}	0.82×10^{-4}	0.19×10^{-3}
9	0.27×10^{-4}	0.21×10^{-3}	0.48×10^{-3}

Table 2 $r^*(n_s)$

l(mm)	Var. of $K_{1c} \neq 0$	Var. of $K_{1c} = 0$
0.72	0.15×10^{-7}	0.05×10^{-9}
0.8	0.15×10^{-6}	0.45×10^{-9}
0.9	0.13×10^{-5}	0.45×10^{-8}
1.0	0.15×10^{-4}	0.45×10^{-7}
1.1	0.49×10^{-4}	0.15×10^{-6}
1.2	0.24×10^{-3}	0.74×10^{-6}
1.3	0.80×10^{-3}	0.25×10^{-5}
1.4	0.22×10^{-2}	0.67×10^{-5}
1.5	0.48×10^{-2}	0.15×10^{-4}

5. Comparing the numerical results of three kinds of materials with the following properties

Material No.	$K_{1c}(kg/mm^{3/2})$	$\sigma_b(kg/mm^2)$
1	207	184
2	252	174
3	281	170

has shown that the third one has more advantage in reliability design.

6. To limit the maximum risk rate the intervals of inspection must be shorten as the life increases as shown in Table 3.

Table 3 $H(n_i) - H(n_{i-1})$

$(\bar{K}_{1c} = 252$, Dev. of $K_{1c} = 20)$

i	Period (hours)	
	100	50
3	0.93×10^{-5}	0.93×10^{-5}
5	0.13×10^{-4}	0.90×10^{-5}
7	0.23×10^{-4}	0.12×10^{-4}
9	0.39×10^{-4}	0.15×10^{-4}

The Tolerant Failure Probability

There are a variety of suggestions of the tolerant failure probability of aircraft structures in literature (refer to the author's recent paper[17]). The different standards are in conform with the experience, the models adopted, and the factors considered by the researchers. Here the authors will only point out that the

10^{-7}/h level of failure probability with 95% confidence recommended in reference[18] is unreasonable.

Generally analysing, provided that there are m different fleets, the i-th fleet includes n_i airplanes, and the probability of failure of the i-th fleet is p_i, the total failure frequency r/n of m fleets as a whole according to Poisson theorem is

$$\lim_{m \to \infty} P\left\{ \left| \frac{r}{n} - \frac{1}{n} \sum_{i=1}^{m} p_i n_i \right| < d \right\} = 1 \qquad (5)$$

where

r — total number of failure

n — total number of airplanes, $n = \sum_{i=1}^{m} n_i$

d — arbitrary small quantity

Further, we suppose

$$Dp_i \leq c$$
$$n_i \leq N$$
$$Mp_i = m'$$

where

D — standard deviation
M — expectation
c, N, m′ — constants

according to the ЧебыщеВ,П.Λ. Theorem,

$$\lim_{m \to \infty} P\left\{ \left| \frac{1}{m} \sum_{i=1}^{m} p_i n_i - \frac{1}{m} \sum_{i=1}^{m} M(p_i n_i) \right| < d \right\} = 1 \qquad (6)$$

or

$$\lim_{m \to \infty} P\left\{ \left| \frac{1}{n} \sum_{i=1}^{m} p_i n_i - \sum_{i=1}^{m} M(p_i \frac{n_i}{n}) \right| < \frac{d}{n/m} \right\} = 1 \qquad (7)$$

where n/m is the mean amount of airplanes per fleet and is limited. On the other hand

$$\sum_{i=1}^{m} M(p_i \frac{n_i}{n}) = (Mp)(\sum_{i=1}^{m} \frac{n_i}{n}) = Mp \qquad (8)$$

so that

$$\lim_{m \to \infty} P\left\{ \left| \frac{1}{n} \sum_{i=1}^{m} p_i n_i - Mp \right| < \frac{d}{n/m} \right\} = 1 \qquad (9)$$

and Mp can be used as an index of reliability analysis. Under the supposition of log-normal distribution for fatigue life, when we use the conventional estimation of safe life with two indexes, i.e. the survival probability p^* and the confident level β:

$$x_s = \bar{x} - ks \qquad (10)$$

then

$$p(\bar{x}, s) = \int_{\bar{x}-ks}^{\infty} f(x; a, \sigma) dx \qquad (11)$$

$$\beta = P\{p \geq p^*\} = P\{\bar{x} - ks < a - k_{p^*}\sigma\} \qquad (12)$$

$$p^* = \int_{a-k_{p^*}\sigma}^{\infty} f(x; a, \sigma) dx \qquad (13)$$

where the coefficient

$$k = k(n, p^*, \beta) \qquad (14)$$

can be found in the Handbook[19]

On the other hand

$$Mp = \int_0^1 - \frac{d\beta(y)}{dy} y \, dy = \int_0^1 \beta(y) \, dy$$

$$= \int_0^1 dy \int_{-\infty}^{-k_y \sqrt{n}} dz \int_z^{\infty} p_1(x_1) p_2(x_1 - z) dx_1$$

$$= \psi(n, k) = \psi(n, k(n, p^*, \beta)) \qquad (15)$$

where

k_y — y·100% percentile point of standard normal distribution

$$p_1(x_1) = \frac{1}{\sqrt{2\pi}} e^{-(x_1^2/2)}$$

$$p_2(x_2) = \begin{cases} \left(\frac{\sqrt{n-1}}{k\sqrt{n}}\right)^{n-1} \dfrac{2}{2^{\frac{n-1}{2}} \Gamma(\frac{n-1}{2})} x_2^{n-2} e^{-(\frac{n-1}{k^2 n} \frac{x_2^2}{2})} & (x_2 \geq 0) \\ 0 & (x_2 \leq 0) \end{cases}$$

Equation (15) denotes the relation between Mp and k,n. The numerical results are partly shown in Table 4.

Table 4

(p =0.999, = 0.95)

n	2	3	4	5	6	7	8
$\frac{1-Mp}{1-p^*}$	9.5	4.6	3.5	2.1	1.1	.72	.49
n	9	10	11	13	16	21	240
$\frac{1-Mp}{1-p^*}$	.44	.39	.35	.31	.26	.21	.50

It can be concluded that when p =.9999999,

β= 95%, $(1-Mp)/(1-p^*)$ is much larger than 1. p^* can not reflect Mp reasonably in that case. Generally speaking, if $p^* \gg \beta$, then $(1-Mp)/(1-p^*) \gg 1$, and the dual index criteria will give the results on the unsafe side. Only in some region of (p^*, β), $p^* \approx Mp$, the dual index criteria is reasonable.

Some Progress in Data Treatment of Factors for Reliability Analysis

NPU has some progress in the data treatment of some important factors for reliability analysis. They are essentially the initial life, the probability of detection, the distribution of K_{1c}, etc. Ref.[20] presented an estimation method of the parent distribution parameters of initial life from the laboratory test as well as the detection at airports with the results of incomplete life and "ultracomplete life".

As usually assume

$$x = \log N \sim N(a+bz, \sigma(z))$$

and denote

$$\sigma(z) = \sigma / \sqrt{M(z)}$$

the primary solutions can be found by

$$[A]\begin{bmatrix} a \\ b \\ -1 \end{bmatrix} - [B]\sigma + \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix}\sigma^2 = 0 \qquad (16)$$

where

$$[A] = \sum_n w\sqrt{M}\begin{bmatrix} 1 & z & x \\ z & z^2 & xz \\ x & xz & x^2 \end{bmatrix}$$

$$[B] = \sum_n q\, w\sqrt{M}\begin{bmatrix} 1 \\ z \\ x \end{bmatrix}$$

and the i-th iterative solutions use $[B]_i$ instead of $[B]$:

$$[B]_i = [B] + \sum_n q\sqrt{M}\begin{bmatrix} 1 \\ z \\ x \end{bmatrix}(N(qy)-0.8(1+qy)) \qquad (17)$$

where

- x — log-life
- z — load (or stress) level
- $y = (x-\mu)/\sigma$
- μ — expectation value of x, μ=a+bz
- σ — standard deviation of x
- $q_i = \begin{cases} 0 & \text{(complete)}(i=1-r) \\ 1 & \text{(incomplete)}(i=r+1-s) \\ -1 & \text{(ultracomplete)}(i=s+1-n) \end{cases}$
- $N(y)$ — Hill's function
- $w_i = 1$ (complete) or 0.8 (non-complete)

Reference (20) also presented the approximation of confident interval estimation. This method has been practically used with satisfactory results.

In NPU some large samples have been tested by the authors to obtain the more confident distribution of fracture toughness, and an modified method using the F-distribution to estimate simply the flaw detection probability curve is also developed. The size of samples to estimate the distribution of K_{1c} of 30 CrMnSiNi2A are 86 specimens and 28 specimens. The candidate distribution curves are normal, log-normal and two parameter Weibull distributions. Using χ^2, Smillnof and Shapiro-Wilk tests, the tests show normal and log-normal distributions are acceptable, but Weibull distribution is refused. The point estimations are shown in Table 5.

Table 5

n	$K_{1c}(kg/mm^{3/2})$	σ	σ_{log}
86	229.6	20	0.034
28	330.4	20	0.034

where the standard deviation σ is obtained by pool-estimation.

To estimate the flaw detection probability curve, three kinds of curves are candidate:

1. Yang-Trapp's power function[21]

$$P(D/a) = \begin{cases} \left(\frac{a-a_1}{a_2-a_1}\right)^m, & a_1 \leq a \leq a_2 \\ 0, & a < a_1 \\ 1, & a > a_2 \end{cases}$$

2. Davidson's exponent function[22]

$$P(D/a) = \begin{cases} 0, & a \leq a_0 \\ c_1(1-\exp(-c_2(a-a_0))), & a > a_0 \end{cases}$$

3. Heller-Stevens's Weibull distribution[23]

$$P(D/a) = 1 - \exp(-(\frac{a}{c})^b), \quad a > 0$$

The tests done by the authors[24] show the above three patterns are comparable. Some numerical results are shown in Table 6 and in Fig.4. Where "a" denotes the crack length instead of l, and P(D/a) denotes the flaw detection probability instead of $P_D(l)$ in the list of main symbols. The total number of cracks T_n and the number of detected cracks D_n are shown in Table 7.

Table 6

FUNCTION	YANG-TRAPP	DAVIDSON	WEIBULL
Estimation of parameters	$a_1=0.59$mm	$a_o=0.53$mm	$b=2.06$
	$a_2=1.82$mm	$c_1=0.99$	$c=0.87$mm
	$m=0.246$	$c_2=3.43\text{mm}^{-1}$	
Residual deviation	0.058	0.050	0.061

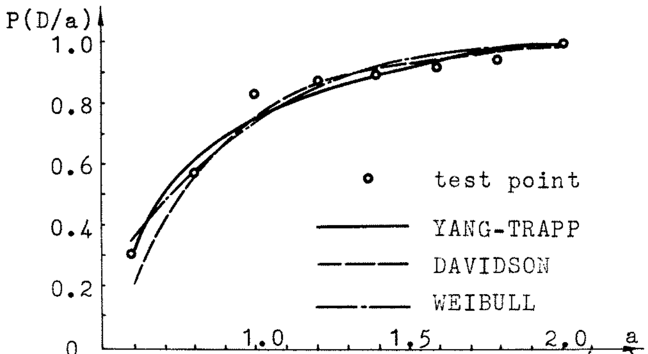

Fig. 4

Table 7

a (mm)	T_n	D_n
0.41 — 0.60	513	158
0.61 — 0.80	264	152
0.81 — 1.00	111	95
1.01 — 1.20	157	141
1.21 — 1.40	191	175
1.41 — 1.60	225	213
1.61 — 1.80	176	169
1.81 — 2.00	65	65

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