PROPOSED CONTROL OF COMPRESSOR STALL BY PRESSURE PERTURBATION AND BLADE DESIGN

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Abstract

A method is proposed for suppressing stall and stall flutter in an axial flow compressor by injecting upstream perturbations. Evidence from another field shows that the addition of particular noise perturbations to a bifurcating system can greatly reduce the entropy increase and yield power spectra with several pronounced frequency peaks. We consider problems in achieving downstream pertubations having a spectrum consistent with stability requirements under perturbations for the non-linear blade dynamics. The requirements may be modified by blade design parameters. The controlled injection of turbulence into the upstream flow has been shown to have the desired effect of suppressing laminar separation. Stall suppression devices for fans based on the recirculation of turbulent tip region fluid are successful but are not well understood. The suggestions of this paper should provide a hypothetical basis for the design of these devices and provision of a modified endogenous recirculation procedure.

1. Introduction

The problems of predicting stall and stall flutter in axial flow compressors (1,2) are still not satisfactorily solved for practical situations. The most usual approach to the problems involves consideration of the physics of the separation process. In this paper an alternative procedure is proposed which involves one of the basic phenomena which can trigger stall and excite stall flutter - the perturbations from upstream wakes. Stall is to be suppressed through a rescaling of re-injected vortex structures by digital computer control of a physical feedback channel. Some of the principles developed may explain the extraordinarily stable operating range observed in certain axial flow fans and offer a means of extending that feature to axial flow compressors.

2. Large Range of Scales of Length and Time in a Compressor - The Renormalisation Problem

A prime source of difficulty in overcoming turbulence is the large range of scales involved in length, energy and time. It is one of a group of problems in physics in which all these scales are equally important. The difficult problems in turbulence theory are to do with essential non-linearity, with strong departure from absolute statistical equilibrium plus the excited degrees of freedom with these wide ranges of length and time scales at high Reynolds' Numbers.

The chaotic nature of turbulence tends to separate any two fluid elements which are initially close. As a result there is a tendency to stretch initial vorticity distributions into longer and longer and thinner vortex ribbons, until viscosity stops the thinning. As the cross section decreases the fluid in the ribbon must spin harder (Kelvin's circulation theorem) and the combination of increasing spin and thinning means an increase in enstrophy (3) and a transfer of energy from lower to higher wavenumbers in its Fourier representation, E(k), of the turbulence. The interactions between wavenumbers, k, differing by as much as two octaves can contribute strongly to the inertial-range energy transfer. The foregoing leads to the idea of the energy cascade being considered as a type of diffusion process in wavenumber (k) space. Kolmogorov's original hypothesis (4) implied that in this process all detailed statistical information about the source of energy in the large spatial scales is lost and the only macroscopic parameter controlling the cascading process is the cascade rate ε (dissipation rate per unit-mass) which satisfies

$$\varepsilon / \nu = 2\int_0^\infty k^2 E(k) dk, \tag{1}$$

ν being the viscosity. In 1962 when data on small-scale intermittency became available, Kolmogorov (5) and Oboukhov (6) modified the original (1941) theory, which involved only ε and ν, to include L_0, the length scale of the largest eddies in the flow and proposed that the cascading of excitations from large to small scales takes place in finite logarithmic steps to wavenumber or scale size. The number of steps from scale L_0 to a domain of size r is measured by $\ln(L_0/r)$. An implication is that a systematic increase of intermittency in the cascading chain means that the vorticity becomes concentrated in an increasingly sparse collection of intensive ribbons, as stated earlier.

There is a peculiar experimentally observed dependence of the kinetic energy dissipation rate into heat through viscosity. As viscosity $\rightarrow 0$, ε appears to approach a non-zero limit which is independent of viscosity. This behaviour seems to be associated with the thinning and intensification of the vortex ribbons which, on average, compensates the lessened dissipation that would otherwise accompany the reduction in viscosity. A formal procedure which has been successful in helping the understanding of such phenomena associated with non-equilibrium Navier-Stokes systems is the often powerful method of "renormalisation". This involves the use of renormalised perturbation series which generalise those used in quantum field theory. Although this has serious deficiencies it does have the advantage of describing in a natural way the physical phenomena described earlier: that small scales of turbulence react on large scales, like a dynamical or eddy viscosity which augments the molecular viscosity ν. Therefore we are prompted to introduce (inject) small scales of turbulence upstream in an axial flow compressor to increase the effective local viscosity over vibrating blades downstream - in a necessarily carefully controllable manner.

The control must be on a time scale in harmony with the theory on which the method is based. It must also provide wakes having a k-spectrum consistent with the stable operating range, for the non-linear dynamics of blade motion, as will be presented in Section 3.

Kraichnan (7) has shown that, if the contribution of local interactions in wavenumber is emphasized, the effective step size of the cascading process is reduced. Therefore if the build-up of intermittency depends on the effective number of logarithmic steps then it is affected and controllable (see Fig. 1) as is the positive exponent μ in the expression (8,9) arising from the modified form of $\varepsilon(k)$ for such steps:

$$\varepsilon(k) = C\varepsilon^{2/3}k^{-5/3}\,(kL_o)^{-\mu}\,, \quad \mu > 0. \tag{2}$$

(Interpretation of μ as giving more detailed features of non-linear interactions can be found in Ref. 10 in relationship to intermittency in developed turbulence.) In two dimensions the enstrophy is given by

$$\oint_{flow} (\text{vorticity})^2\, dk.$$

If we consider the Fourier representation of enstrophy (per unit volume) $\Sigma_\alpha k_\alpha y_\alpha^2$, where k_α is the wave vector for mode y_α which we wish to control, then if we change the enstrophy by adding vorticity we change y_α and $A_{\alpha\beta\gamma}$ in the incompressible Navier-Stokes equation written in the following form (7):

$$(\frac{d}{dt} + \nu_\alpha)y_\alpha = \Sigma_{\beta\alpha} A_{\alpha\beta\gamma} y_\beta y_\gamma. \tag{3}$$

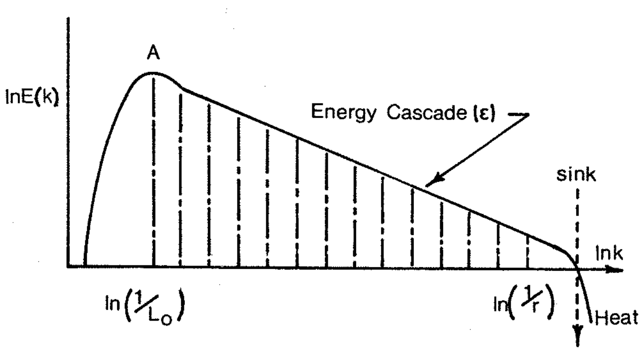

Figure 1. Distribution of turbulent energy among different scales. Energy supplied at scales of order L from external source E. Transferred to smaller scales at mean rate ε per unit mass. If a change of scaling is introduced the cascade rate and the number of steps are changed.

But the $A_{\alpha\beta\gamma}$, ... must satisfy the corresponding equation for enstrophy conservation

$$k_\alpha^2 A_{\alpha\beta\gamma} + k_\beta^2 A_{\beta\gamma\alpha} + k_\gamma^2 A_{\gamma\alpha\beta} \equiv 0 \tag{4}$$

(for two dimensions). Because of the conservation relation (4) the transport of kinetic energy in two-dimensional turbulence is toward lower, instead of higher, wave numbers (3). A change in enstrophy from injected or recirculated upstream wakes can change the wavenumber distribution and, in view of the change in local interaction of wavenumber contributions, offer a means of altering the downstream development of turbulence, resulting in concomitant beneficial behaviour such as the suppression of stall.

3. Frequency Range Limits for Stability of the Blade Dynamics

A common type of motion exhibited by fluttering compressor blades is a bending vibration and the equation of motion, at a representative point on the blade, close to stall is of the form:

$$\ddot{y}(t) + \frac{\omega\delta_L}{\pi}\dot{y}(t) + \omega^2 y(t) = F[\dot{y}(t-\tau)]$$
$$= \frac{2ab}{\overline{m}} f[\alpha(t-\tau)]. \tag{5}$$

In the above, F is the dynamical force coefficient given by the slope of the lift curve, as in Fig. 2, and has the form $f(\alpha) = -k\alpha + L\alpha^2 - M\alpha^3$, where ω is the natural bending mode frequency, δ_L the log decrement of the blade in still air, $\overline{m}$ the blade mass per unit span and b the blade semi-chord. The effective angle of attack α for a tapered, twisted blade, is

$$\alpha = \alpha_i + \dot{y}/V - [\theta + n_o b\dot{\theta}/V]$$

where n_o is a constant ~ 1.5 at the 1/4 chord point and $\theta(t)$ is the small torsional motion of the blade due to its tapered-twisted form which gives a small orthogonal displacement to the main bending displacement. The phase lag τ is the lag of aerodynamic force behind the blade motion. Experimental work on blading (ref. 11) confirms previous industrial observations and shows that τ is a function of the reduced frequency k_b and not a simple function of the slope of the static lift curve.

Our previous studies (12) incorporated all of the foregoing into an investigation of the stability of blade oscillations near the stall in the presence of upstream wakes. Third and higher-order non-linear differential systems can exhibit chaotic behaviour and, as such, Eq.(5) is characteristically sensitive to initial conditions with all the peculiarities of strange attractors, bifurcations, and frequency doubling on the path to chaos. The stability analysis required a digital study in the presence of external perturbations using the Dual Input Describing Function Method (DIDF) applied to a non-linear feedback system to which the difference-differential equation is equivalent. Computer generated stability diagrams were obtained using an analysis for the amplitude-frequency spectrum of downstream pressure fluctuations, arising from the wakes of a row of

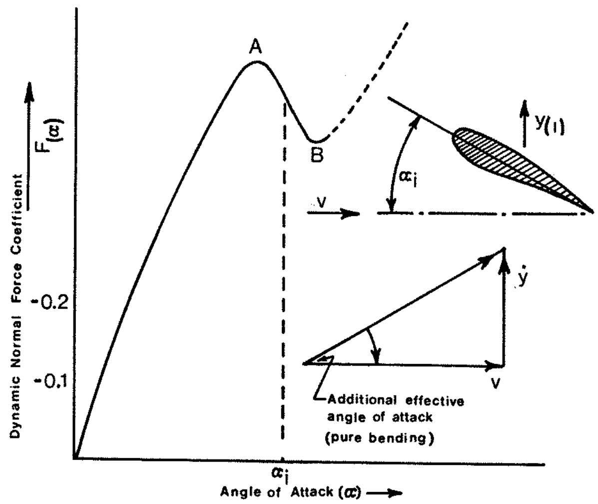

Figure 2. Dynamic lift curve of blade with operation into the stall region.

moving upstream blades, as a function of the phase lag $\omega\tau(k_b)$. Revised computation of such stability diagrams has since been possible with far better computing facilities and some inadequacies in earlier results have been revealed. We also note that particular mathematical techniques (13) for non-linear stability analysis can be used to predict the onset of oscillations but these methods do not appear to be useful for predicting the onset or nature of more complex time-dependent phenomena including non-periodic motion. The DIDF method appears necessary for our purposes.

Figure 3 shows the type of stability diagram produced by this method. An important characteristic of these results is that this Nyquist incremental open loop gain locus rotates clockwise as the phase lag angle $\omega\tau(k_b)$ increases. The rotation is accompanied by small radial oscillations but the amount of rotation is clearly observable. As the rotation increases for a stable system it eventually includes the Nyquist stability point (-1, jo) over a range of $\omega\tau$ before reverting to stability as $\omega\tau$ increases further.

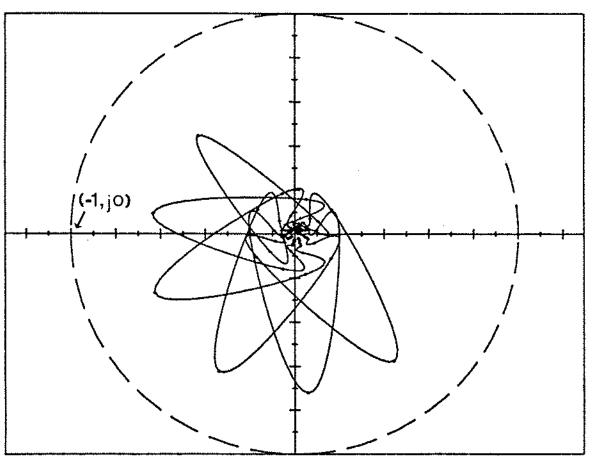

Figure 3. Typical incremental open loop gain loci (stability diagrams). Loop rotates as phase lag τ changes and instability results if curve encloses the (-1, jo) point.

In this way stable ranges of operation can be determined in terms of the reduced frequency k_b. Also, changes in the lift curve brought about by blade design through the parameters K, L and M, $\bar{m}$ and b in the function F, can change the entry and exit points (and therefore the k_b values) for the range of instability as the stability diagram rotates, thereby offering an avenue for modifying control requirements via blade design.

If the upstream perturbations can be controlled in such a way that their amplitude-frequency effects on the fluttering blade downstream are consistent with the stability diagram, maintaining the exclusion of the (-1, jo) point, then stable operation should be maintained. Changes of flow parameters at compressor inlet by a frequency lower than 1 Hz can be considered slow enough to preserve the characteristics. A change of pressure and of the phase of inlet/outlet pressures results from faster alterations.

The problem, therefore, is how the upstream injected or recirculated vortices can be introduced in the vicinity of upstream blades to give an entrainment of the resulting wakes which works in a beneficial manner downstream. The amplitude-wavenumber spectrum is to satisfy the conditions of maintaining exclusion of the (-1, jo) point in the stability diagram. The downstream control parameters are K and the blade design parameters k, L and M, $\bar{m}$ and b. The upstream parameters are those for the injection process which we will now discuss.

We refer to recent work (14) where the entropy increase of a chaotic chemical system can be rapidly reduced by introduction of perturbation "noise" and an emphasis on particular wavenumbers obtained in the power spectrum. This suggests that our objective may be achievable. Scaling of injections would be required in our case as explained in Section 1 and consideration given to the experimental data available on injection and mixing (15) in turbulent flow.

A possibly simpler application of the theory is proposed in which part of the incipient separation wake flow structure is re-circulated from downstream into the upstream flow via a rescaling process. The process is controlled by the physical dimensions of the feedback channel and the helicity, which is

$$\oint_{\text{flow}} \overrightarrow{\text{vorticity}} \cdot \overrightarrow{\text{velocity}}\ dk$$

in three dimensions, achievable from stator blades located in the return channel, prior to re-injection and mixing. These methods will be discussed in Section 6 after entrainment is considered.

4. Entrainment from Intermittency as a Possible Control Factor

Gollub and Benson (16) have found experimentally that two oscillations in multiply-periodic flow can become entrained in such a way that m cycles of one oscillation and n cycles of the other require the same time and that the onset of non-periodic motion is associated with the failure of entrainment. They suggest that non-periodic flow can be understood in terms of the interaction of coupled oscillators. The susceptibility of the phase-locked state to external perturbations and internal dynamic perturbation is, however, significant as we can now illustrate.

Consider the system of Figure 4 where the injected or recirculated flow is the input to the upstream oscillating blade system. There are three types of entrainment possible - stable, unstable and metastable. The difference between the metastable state and the stable state is shown in the simulated example of Fig. 5 using the techniques developed in Ref.(17) corresponding to two intermittency periods of injection such as would arise (14) in high pass filtering of fed-back turbulent velocity signals. In the upstream non-linear oscillating blade system the "natural frequency" may have a range such as in Fig. 6 due, for example, to changes in the aerodynamic lag τ (which is sensitive to k). If our fundamental injected frequency f is f_1 the entrainment remains in the metastable state represented by the range of the sloping characteristic shown. However if f moves to f_2 the system moves out of entrainment. Therefore, depending on the instantaneous value of f, the system will move in and out of entrainment i.e. unstable entrainment results. This occurs in some biological systems where useful techniques have been developed to study such behaviour (18).

Recent evidence (19) from chemical dynamics of the Belousov-Zhabotinskii reaction shows that the addition of particular noise perturbation to a bifurcating chaotic system can greatly reduce the entropy increase and yield power spectra with several pronounced frequency peaks. This new noise effect on chaos in one-dimensional mappings achieves transition from chaotic to ordered behaviour induced by particular external noise; this is in contrast to maps of the logistic type where external noise induces chaotic behaviour from periodic behaviour. If a suitable Markov map, equivalent to the Belousov-Zhabotinskii chemical "noise" injection experiment, can be found for the compressor problem then the proposal would be worth developing. Because of the difficulties of determining and then controlling such phenomena in our non-linear system, the alternative of using recirculated and rescaled downstream flow in the blade tip region, where separation starts, is now being considered. The map might be controllable by filtering to affect the intermittency (14). There may well be some elements of inherent vorticity and helicity which cannot be easily introduced into computer control of an exogenous flow yet may be important in establishing metastable entrainment if recirculated. This procedure does seem a possible candidate for emulating the B-Z noise injection experiment.

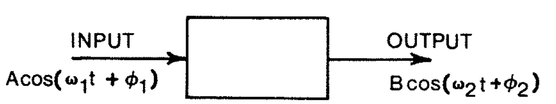

Figure 4. Entrainment of wakes from idealised insertion of sinusoidal input to upstream non-linear oscillating blade aeromechanical system. Resulting entrainment is stable if $\omega_1 = \omega_2$ and ψ_2 constant; metastable if $\omega_1 = \omega_2$ but B and ψ_2 may vary; unstable if $\omega_1 \neq \omega_2$ necessarily and B, ψ_2 are unstable.

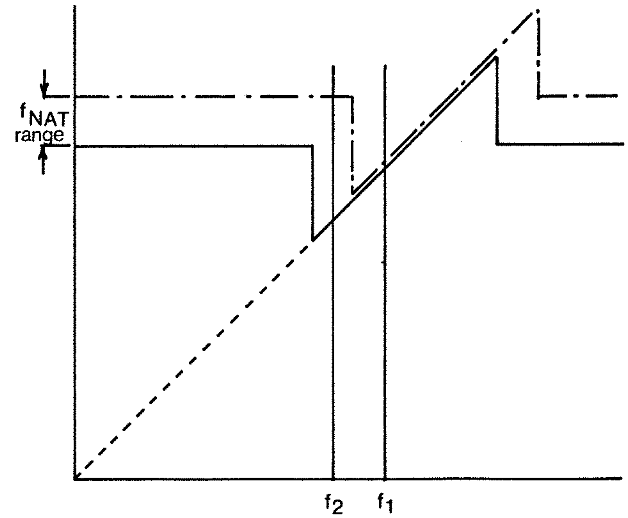

Figure 5. Relationship between stimulus frequency from intermittency and natural frequency.

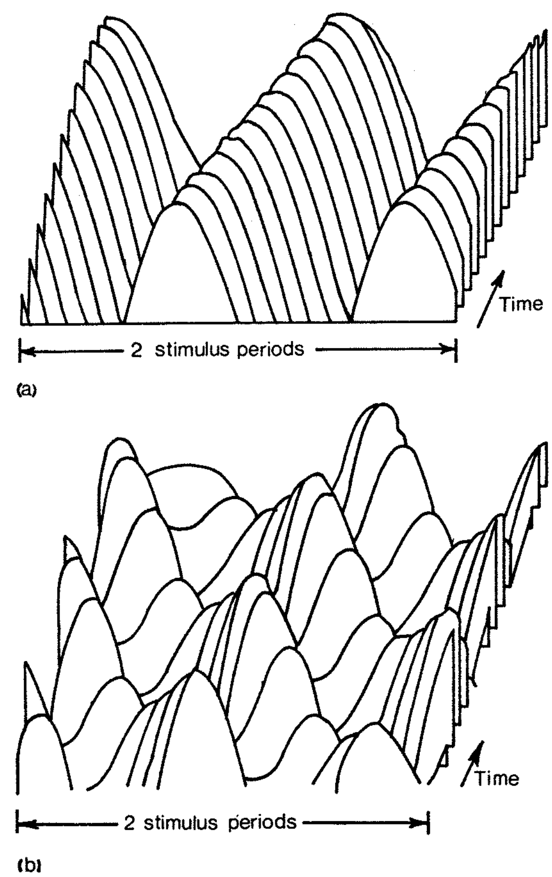

Figure 6 Stable (a) and metastable (b) entrainment (based on Fig. 3 representation of upstream blades and wakes) for simulated stimulation by controllable intermittency at time periods S_1 and S_2. In (b) both amplitude and phase vary with time, resulting in similar characteristics in the wake progressing downstream.

5. Reducing Entropy Increase by Adding Fluctuations in a Bifurcating Chaotic System

The role of fluctuations, although small, assumes a critical role near points of bifurcation, as explained by Prigogine (20), because there the fluctuations drive the average. This is the basis of Prigogine's original concept of "order through fluctuations". It is worth noting in this regard that injection of noise into a bifurcating system has been reported as producing earlier chaotic behaviour (21). However, added perturbations of a particular stochastic type to the B-Z chemical reaction has produced increased order whereas the opposite occurs for a logistic type disturbance based on the usual $x_{n+1} = \lambda x_n (1-x_n)$ type of map which produces frequency doubling.

Work reported on noise (22) is clarifying the connection between the frequency doubling route to chaos and the renormalisation-group approach to phase transitions in solid-state physics and the connection may be informative in the following ways. Because extended scaling laws apply in the frequency doubling system, the noise there serves a role similar to an external magnetic field in a magnetic phase transition. Huberman et al (23) have considered the motion of a particle in a spatially periodic potential and found this system to have chaotic solutions which reach chaos by frequency doublings. They believe that the striking rises in noise observed in Josephson - junction oscillators may be due to frequency doubling behaviour.

This can be related in our work to the important connection pointed out by Kraichnan (7) between the failure of standard renormalised perturbation theory to preserve Galilean invariance (i.e. inability to handle problems in which a wide range of scales is excited) and quantum field theory. He illustrates this with the case of a quantum-mechanical particle in a random potential where the wavelengths of the wavefunctions become vanishingly small compared with spatial scales of the potential and scattering cross-sections vanish in every order of line- or vertex-renormalised perturbation expansions. In turbulence the related result is an incorrect inertial range for E(k). Kraichnan points out how an acceptable WKBJ limit can be recovered by reworking the expansions to transform away the unwanted phase shifts; a generalised Schroedinger field is used for both equal and unequal time arguments in analogy to his procedure presented (24) for turbulence which utilizes a generalised velocity field.

Furthermore a basis for nuclear scattering potentials of quantum particles has recently been traced (25) to a functional differential equation, the solution of which indicates that all conventionally used potential shapes are composed of small discrete length elements forming an envelope of a basic quasi-periodic discrete potential $\phi(E, L, \Delta l)$. As a result, the scattering and absorption appear to include a functional dependence on a very small length scale $\Delta l(E)$. One might therefore ask, after Kraichnan's comments, if this contains an analogy for basic turbulence physics where the inertial range of the E(k,t) energy spectrum may be related not only to the quantum field analogy of a particle in a random potential but more deeply to a quasi-periodic potential with discrete length (or time), $\phi(E, t, \Delta t)$ in which randomness arises from ΔE. $\Delta t \approx h$ (Planck's constant) in accordance with the Heisenberg Uncertainty Principle.

6. Application to Compressor Blading

Discontinuities, hysteresis phenomena and bifurcations in aerodynamic behaviour are often associated with the different rates of separation onset and re-attachment. The separated or un-separated state tends to persist, followed by an abrupt change, which may involve other associated flow changes such as transition from a laminar boundary layer to a turbulent layer. This behaviour becomes evident for a wing or compressor blade as the aerodynamic loading is varied. The nature and amplitude of turbulence in an oncoming flow can also produce strong non-linearities.

An example of the systematic variation in inlet flow turbulence level onto a compressor blade is given in Figure 7. These measurements given by Schlichting and Das (26), from cascade tests by Kiock, show the local boundary layer state over the suction surface of a NACA 65 series compressor blade as the inlet turbulence level is increased. For conditions of low inlet turbulence level, below 1%, there exists a laminar boundary layer up to the vicinity of 60% chord, followed by a long laminar separation bubble. The bubble has a length of about 15% chord and is terminated in the usual way by transition to turbulence causing re-attachment and the subsequent growth of a turbulent boundary layer.

As the turbulence level is increased transition occurs earlier over the bubble, reducing its length. At a critical turbulence level, in this case about 2.4%, the turbulence-induced transition occurs sufficiently early to suppress the separation bubble completely. It is interesting to note that the laminar layer is not stable under these conditions and transition moves abruptly to the leading edge. The boundary layer is turbulent over the entire blade surface for all inlet turbulence levels higher than 2.4%.

This is an example of an experiment which indicates that the injection of turbulence into the incoming

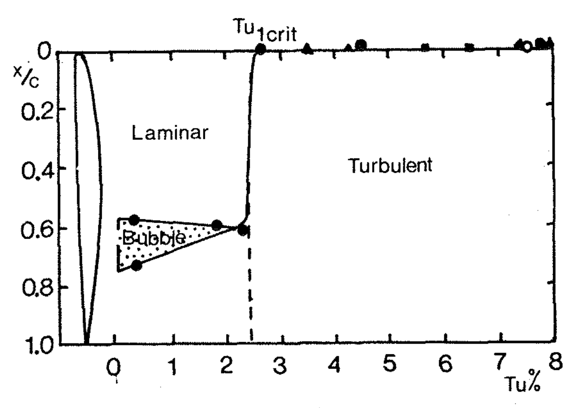

Figure 7. State of suction surface boundary layer on a compressor blade, as a function of free-stream turbulence level.

flow field of a blade row causes a difference in the mode of separation, and hence stall. In this experiment the turbulence was produced by an upstream grid of bars. An alternative turbulence source would be the reinjected and rescaled turbulent flow ducted from the downstream flow-field. The chopped-up downstream flow which is reinjected will, in general, have a smaller length scale than that previously existing in the low turbulence inlet field.

It is therefore proposed that the upstream injection of relatively fine-scale turbulence provides a mechanism for the control of separation behaviour of the blading.

One application in which the above mechanism may already play an important role is the suppression of undesirable abrupt stall behaviour in axial flow fans and compressors by the incorporation of a flow separator. Such devices of various configurations have been used in axial flow fans, initially in the Soviet Union (27), for some years. A related concept is the use of casing treatment in some aircraft engine compressors. The aerodynamic understanding, however, of these devices has remained obscure. One objective of this paper is to provide a new hypothesis for understanding the behaviour of stall suppressing devices.

An example of the flow characteristic of a single-stage axial compressor (ref. 28), both with and without a separator, is given in Figure 8. A compressor characteristic is often double-valued with pronounced hysteresis under high-loading conditions; in a multi-stage compressor this behaviour may result in a bifurcated performance map. An example of the hysteresis phenomenon is given in the dashed curve of Figure 8.

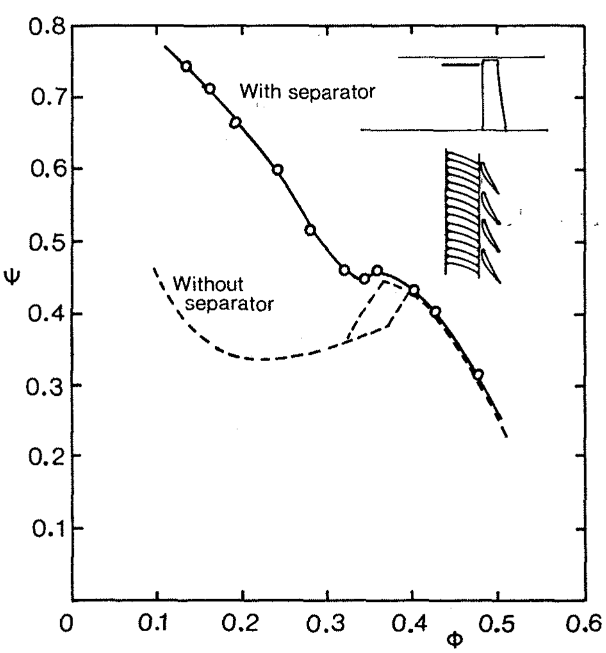

Figure 8. Pressure rise coefficient ψ - flow coefficient ϕ characteristic for axial compressor rotor. Dashed line (with hysteresis loop) is characteristic of conventional compressor. Full line indicates improved characteristic with flow separator present in the inlet region.

Inclusion of the separator device upstream of the compressor rotor results in the characteristic given by the full curve. The abrupt stall and hysteresis have been eliminated and the characteristic has a negative slope for almost all throttle settings. Axial fans of this type have a remarkable range of stable operation. The separator device, which in some designs is actually over the blade tip and within the casing, acts as a physical recirculation channel to duct the turbulent wake fluid from the tip region back into the inlet flow. The functions of control over helicity and length scale are built in using an additional row of stator vanes.

There is seen to be scope for digital control of flow within a recirculation channel to effect a desirable rescaling of turbulence. This presumes the thoretical determination of control functions. It seems that no practical calculations can yet be done which will accurately enumerate relevant statistical functions at high Reynolds' Numbers. However, the time to reach the viscous cut-off wavenumber k_o approaches a finite limit as $k_o \longrightarrow \infty$ ($\nu \longrightarrow 0$), so if we can increase ν we will increase the time required; thereby also allowing any entrainment from upstream to develop and provide an acceptable perturbation spectrum for maintenance of blade stability downstream. The characteristic frequency doubling time for breakdown of vortex structures of wavenumber k into structures of wavenumber $2k$ is $O(\varepsilon k^2)^{-1/3}$ so we have two parameters, ε and k, at our disposal for control. Control of ε is possible via the energy spectrum $E(k) = C\varepsilon^{2/3}k^{-5/3}$ for three dimensional turbulence because, by adding recirculated vortices, we are utilizing the small scale statistical dependence of intermittency on L_o, ε and r. The number of steps required for excitation to cascade from dimension L_o to r is $a \ln(L_o/r)$. Therefore we can control the number of steps via control of L_o/r in Fig. 1. If the recirculation of separated tip-end flow is passed through a channel of controllable cross section (to govern r) and the flow passed through a cascade of stator blades (to rescale the dimension of the vortex structures regarding L_o), then the foregoing objectives would seem achievable.

7. Discussion

A digital dynamic control operating on adjustable vanes in the recirculation channel is proposed. Adjustment of stator blades for improved stall margin is in use on many production engines. It would seem possible to integrate the control procedure with an existing digital engine control system such as HIDEC. At high angles of attack at high altitude, low speed conditions require large fan compressor stall margins because distorted inflow at such conditions can result in the stall of highly loaded blading. Preset margins to avoid this problem sacrifice engine performance at the more common flight conditions but HIDEC utilises sensed flight parameters to overcome this situation and allows an up-trim from the operating line of EPR versus airflow closer to the stall line without encroachment, thereby delivering extra thrust. Data from such a system could be incorporated (29) into the multi-task software for digital control of the recirculation channel described herein if the engine stall characteristics under different flight conditions were known. The use of advanced engine test computer monitoring with array processors (30) can provide this data base. New types of optimisation software may be required which are not currently available including an efficient model updating a procedure for use with biassed sensor readings and spline programs in which both the control parameters and state parameters appear. Similar needs have been expressed (31) in optimisation on-line of engine fuel consumption.

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