CASE STUDIES OF THE EFFECTS ON NON-LINEARITIES ON THE ACCURACY OF GAS TURBINE CONTROL

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#### Introduction

Control theory is built very largely on a foundation of linear system analysis with extensions to cover the effects of non-linearities on stability in a noise free system. Similarly the effects of signal noise are generally studied for linear systems. These two extensions of basic linear theory are logical and necessary attempts to look at these real world effects on conclusions based on linear theory. However very little work has been done to look at the combination of non-linearity and signal noise on control systems. In practical systems this situation is the norm rather than the exception.

In this paper a set of three case studies have been used to illustrate the loss of accuracy which arises when non-linearity and signal noise effects are combined. This effect does not arise when only non-linearity or signal noise are considered in analysing the system. It is a phenomenon which will only occur in the presence of both effects. The full mathematical analysis of a system containing non-linearities and signal noise would be extremely difficult and no attempt has been made by the author to do this. However some simple methods can be used to illustrate the cause of the problems and evaluate various solutions.

## List of Symbols

ALT	Aircraft Altitude
EPR	Engine Exhaust Pressure Ratio
EPRCMD	Demanded engine EPR
EPRERR	Error between Demanded and Actual
	EPR
g(s)	General Laplace transfer function
Im	Fuel Valve Torque Motor Current
Mn	Aircraft Mach Number
N2	High Pressure Engine Rotor Speed
P2	Engine Intake Pressure
P8	Engine Exhaust Pressure
T2	Engine Intake Temperature
WF	Engine Fuel Flow rate
HW	Select Highest Input
LW	Select Lowest Input
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## Gas Turbine Control Example

The case studies presented are all drawn from recent experiences in the development of microprocessor based gas-turbine engine fuel control systems. Two case studies arise due to non-linearities in the control logic implemented in the microprocessor and have been solved by modification to the logic. The third case study arises due to non-linearity in the fuel system. To illustrate the problems and clarify the circumstances in which they arose they have been translated into a simple general engine control scheme which illustrates all three problems in the one system. The system consists of a conventional civil aircraft turbo-fan engine illustrated in Fig 1. Two control modes are used in this example though a real engine control will typically have at least ten different modes. These two modes are:-

- Range thrust control from idle to maximum take-off power. The engine thrust is estimated by measuring the exhaust pressure (P8) and intake pressure (P2). The ratio P8/P2 is commonly referred to as EPR and is used as the main thrust setting parameter for the engine. The pilot sets his required thrust level (EPRCMD) by advancing the throttle lever. The angle of the lever (TLA) is measured by the engine controller and this angle together with other measurements such as Altitude (ALT), Aircraft Mach Number (Mn) and intake temperature (T2) are used to compute the required thrust and hence EPRCMD. The engine EPR is then regulated by comparing EPRCMD with measured EPR and from this a closed loop control function adjusts the engine fuel flow (WF) to achieve the required thrust.
- (ii) The engine is itself subject to various other limitations and two have been illustrated in this example. These are the upper and lower HP speed (N2) limits. The lower limit is required since aircraft pumps and generators are driven from this engine shaft by means of an external gearbox. If engine speed falls below a minimum level the essential electrical and

hydraulic power for aircraft systems is lost. The upper limit is required because the engine HP shaft is stressed to a maximum normal operating limit and any excedance of this limit requires engine maintenance action. These limits are incorporated with the range thrust control in the microprocessor. An N2 speed measurement is obtained from the engine electrical generator which also powers the control system. The N2 errors and the range thrust control errors each generate a demanded change of fuel flow. The fuel metering unit is driven by a torque motor. The relationship between torque motor current and fuel flow is fixed and this is used in the microprocessor to convert the demanded fuel flow to drive current. The control logic used for implementing the two modes in the microprocessor is illustrated in Fig

The fuel system consists of several units. A Centrifugal pump and a gear pump provide high pressure fuel. A spill valve which returns excess fuel to the gear pump inlet. A pressure drop control maintains constant pressure drop across the metering valve. Finally a fuel metering valve which moves in response to a torque motor to increase the metering area and thus change the fuel flow to the engine.

### Case Study 1

# Effect of Highest/Lowest Wins Function with Noisy Inputs

The basic control is Proportional plus Integral (P+I) and has zero steady state regulating error, this is an essential requirement for aircraft operation, particularly at take-off conditions where thrust shortage could lead to a disaster and excess thrust severely reduces engine life as well as increasing fuel consumption.

However the engine may be required to operate very close to its HP speed limit (N2MAX) at these conditions. During engine development testing this occurs regularly because operation at or near engine limits is required for airworthiness certification. With this type of control logic problems were encountered during engine development where inability to achieve EPRCMD coupled with less stable control was noticed. The problem was investigated by using a data logging facility which allowed parameters in the microprocessor to be recorded for every

computation cycle. This revealed that the problem was arising only when the engine power was such that both the EPR and N2MAX controls were operating with the mode switching randomly between these modes. The N2 signal has very good signal to noise ratio (generally better than 2000:1) however the EPR control is more noisy. The P8 signal is approximately 1.6 times the P2 signal value at high engine powers and the measured EPR even with high accuracy vibrating cylinder transducers has a signal to noise ratio of about 200:1. This was worse when operating in an engine test cell since pressure fluctuation in the cell produced a signal to noise ratio of 50:1. With the two controls having similar demanded fuel flow, the noise on EPR, caused mode switching to N2 control to occur when the EPR value went low. However this did not occur when the EPR value went high. This assymetry resulted in the observed poor control stability (repeated random mode switching) and a steady state error in the control such that both EPRERR and N2HERR had positive mean values. The latter was more serious because this would lead to significant shortage of engine thrust.

#### Problem Analysis

The problem has been analysed by means of a very simple system as illustrated in Fig 3 in which the following assumptions were made:-

- (i) One feedback signal was effectively noise free.
- (ii) The other signal contained gaussian white nose of variance  $^2$ .
- (iii) The worst case was when the control would have equal probability of being in either mode i.e. when the demand D1 and D2 are the same.

For convenience let D1 = D2 = 0

Now let (t) gaussian white noise with = 0.12

$$E1 = -Y - (t)$$
  
 $Ex = -Y$ 

A more general analysis is possible for any particular condition using conventional probability theory. However for the special case chosen, the analysis is much simpler. The integrator input Y is considered to consist of the sum of two signals:-

$$EL = -Y + (t)$$

Where (t) = 
$$\begin{pmatrix} 0 & \text{for } (t) < 0 \\ (- & (t) & \text{for } (t) > 0 \end{pmatrix}$$

And the probability density function of

(t) is:-

$$f(x) = \frac{1}{/2} \exp \frac{(-x^2)}{(2^2)}$$
 - the guassian distribution

The probability of (t)  $\neq$  0 is  $\frac{1}{2}$  and the expected value of (t) is

E((t)) = 
$$\frac{1}{2}$$
 exp  $\frac{(-X^2)}{(2^2)}$  x dx + 0dx

$$E((t)) = -\frac{1}{2}$$

But because of the integration in the plant, the expected value of EL must be zero

$$E2 = \frac{}{/2} = 0.399$$

Hence the offset error due to noise is 40% of the standard deviation of that noise. To demonstrate this a simple simulation of such a control system has been used. The time response of this system when subject to noise of unity variance is illustrated in Fig 4.

#### Chosen Solution

Two approaches to solving this problem were considered. The first is the conventional and more obvious one of digitally filtering the EPR signal to reduce the noise amplitude and hence reduce the bias. However with simple digital filter algorithms (upto second order) the amount of attenuation required would have introduced significant additional phase lag in the EPR control loop. This in turn would have required a reduction of the control gains and hence the bandwidth and response of this loop would have deteriorated. The alternative and chosen solution was to leave the basic control loop unchanged but to add an extra integral term to remove the bias. It would have limited authority, low gain and would integrate a heavily filtered error signal. The logic chosen is shown in Fig 3. This is more complex than simply filtering the error signals. However the loop bandwidth was unchanged and the logic successfully removed the bias effect. The value of the filter time constant and authority of the bias integrator were determined from the analysis. By knowing the noise characteristics, a filter lag was chosen which would make this noise negligible. The integrator authority was chosen to be greater than the maximum bias value.

#### Case Study 2

Effect of Gain Scheduling with Noisy Parameters

In the control logic shown in Fig 2 the EPR control gains are shown as being functions of EPR and P2. It is necessary to provide some form of gain scheduling because the engine dynamics vary very considerably with both power level and flight condition. To a first approximation the engine transfer function is a first order lag:-

$$\frac{EPR (s)}{WF (s)} = \frac{K}{1+ s}$$

There are additional high order terms which effectively introduce significant phase lag. In moving from one extreme of the flight envelope (Sea Level with high forward speed) to another (high altitude and minimum forward speed) the gain will vary by an order of magnitude.

Similarly the time constant varies with intake pressure but it also varies with engine power level. A variation of upto 20:1 may occur in time constant when the power is varied from lowest idle condition to take-off power. This variation is particularly marked at the idle end of the power range. It can be seen from the combination of these effects that the time constant may vary by upto 200:1 over the full operating envelope though in practice due to increasing idle power at high altitude a 50:1 variation is typical.

In order to achieve fast and stable closed loop response at all conditions it is essential to vary both proportional and integral gains as flight condition and engine power level vary. There are many ways of doing this, however in the example EPR has been used to indicate power level and P2 to indicate flight condition. In practice this scheme resulted in similar problems to the first Case Study but this time at low engine powers. The thrust and engine speed was seen to be oscillating and the mean EPR did not match EPRCMD. A similar approach to understanding this problem was used as before. Recordings of primary parameters at every computational cycle were taken and subsequent analysis showed that the gains of the EPR control were varying significantly due to EPR signal noise. Close to idle power the engine dynamics vary rapidly and consequently the gains varied rapidly with EPR.

A simplified example of the observed problem has been used to illustrate the behaviour. The control loop block diagram is shown in Fig 5. Gaussian white noise was added with standard deviation of 0.003 pressure ratios. The resulting time

responses of such are shown in Fig 6. The parameter EACT is the true system error (signal noise removed) and shows the oscillatory behaviour and offset mentioned before. The large variations in gains are also shown.

#### Chosen Solution

Initially it was decided that noise on EPR acting through the gain scheduling was causing the problem. This was surmised from the variation in gain established by recording these at every computation cycle. The solution chosen at this stage was to put a first order filter on the EPR used in the gain scheduling algorithm.

The subsequent more detailed analysis confirms that this will improve the situation if the filter cut-off frequency is low enough since this will markedly reduce the gain variation. However simpler solution has been used widely in Rolls-Royce with no problem to date. This is to use the demand signal (EPRCMD) to schedule the gain. This implies that the gains change only when aircraft inputs (primarily TLA) are changed and these are not closely coupled to engine variation. No problems have been experienced due to rapid large changes of gain when large "snap" throttle movements are made. Another solution is to use the common downstream integrator as in the first system. This effectively removes the direct coupling between gain changes and the controller output since the error is differentiated, multiplied by the gain and then integrated.

#### Case Study 3

## Noise Input to the Fuel Metering Valve

This problem is similar to that in Case Study 2. It arose in the fuel metering system. Gear pump tooth ripple at low speeds combined with a pipework resonance caused significant pressure variation at inlet to the fuel metering valve, spill valve and pressure drop control systems. The pressure drop control bandwidth is intentionally lower (and is incapable of being as high) as the gear tooth ripple frequency.

The pressure drop control thus maintained a mean metering valve pressure drop. However the high frequency variation at a particular engine and pump speed was about ±50%. The flow through the metering valve is proportional to area and square root of pressure drop. Hence if the pressure drop is

P(t) = P0 + P1 sin wt Where w is the pump ripple frequency

and the area A is constant

Then Flow  $WF(t) = A.K. /P0 + P1 \sin wt$ Where K is dependent on fuel density

And the mean flow is:-

 $WF = A.K. /PO + P1 \sin wt dwt$ 

This has been numerically integrated for P1 = 0.5 and yields P0

WF = 0.983A.K./P0

This shows that the metered fuel flow drops when the valve is subject to pressure ripple. The most significant aspect of this particular problem was that digitally sampled instrumentation of the metering valve position and pressure drop showed no sign of any error in the system.

A turbine flowmeter however showed the drop in fuel flow. This caused some considerable puzzlement until analogue pressure measurements were taken showing clearly the pump ripple magnitude. The problem was cured by reducing the pump ripple by pipework modification.

It is worth noting that many digital control systems also use functions similar in nature to the square root function for fuel flow through a valve. These are clearly subject to the same potential problem and these functions should be used very carefully if steady state error due to noise is to be avoided. One solution is to filter the signals to remove this noise but where this is undesirable one alternative is to always use a linear function on the measured signal. The coefficients of this function can be updated from a heavily filtered version of the signal (at a lower update rate if desired) and this is implemented by the algorithm below.

Input x; Output y

y = f(x)

The algorithm implementation is:-

x(s) = g(s) x(s) Where g(s) is a low pass

y = A0 + A1 x

Where  $A0 = f(x_1)$ 

and Al =  $\frac{df}{dx}$  (x<sub>1</sub>)

## Concluding Remarks

The combination of system non-linearity and measurement noise is the norm rather than the exception in practical control systems. The increasing use of digital control has lead to more intentional control system non-linear functions and an expectation of

greater accuracy in the system. These trends have highlighted the potential for non-linearity and noise in combination to give rise to loss of accuracy in gas turbine control systems. The case studies illustrate some ways in which the problem has arisen in the author's recent experience and that by careful analysis and design the problems can be avoided. The greatest problem is a general lack of awareness and understanding of this particular aspect of control system behaviour. It is not an aspect that is covered in most degree or advanced courses in control theory. The author has demonstrated that theory and simulation can be effectively used to analysis system designs and other analysis approaches can undoubtedly be used. The author would like to make a plea for greater effort by the acedemic world in research and teaching to develop this very important aspect of practical control system performance.

#### References

- DAZZO J J and Houpis CH 'Feedback Control System Analysis and Synthesis' McGraw Hill 1966
- Spiegel MR "Probability and Statistics" McGraw Hill 1975
- Winter J S, Corbin M K and Murphy LM
  "Description of TSIM2: A software
  package for computer aided design of
  flight control systems"
  Royal Aircraft Establishment report
  TR83007 1983.

Fig 1 Turbofan control system

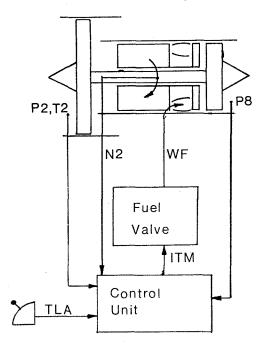


Fig 2 Two mode control system

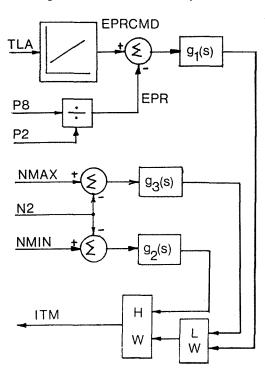


Fig 3 Simplified lowest wins problem

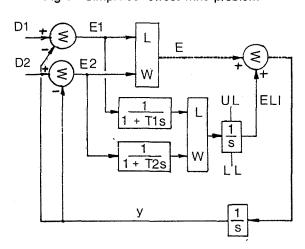


Fig 4 Time response for lowest wins problem

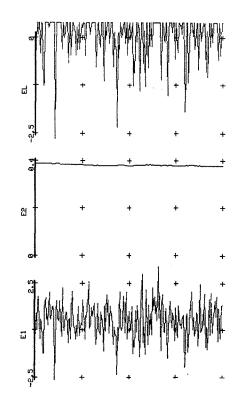


Fig 5 Gain scheduling control scheme

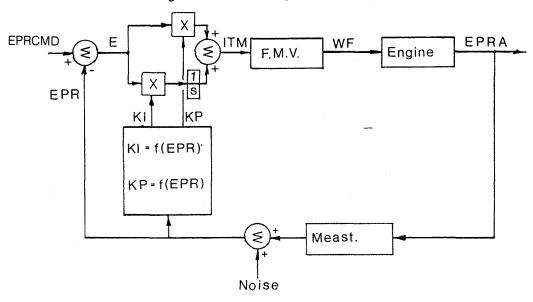


Fig 6 Time response of gain scheduling control scheme

