A COMPARISON OF AN EXPLICIT AND AN IMPLICIT TIME MARCHING METHOD FOR CALCULATING INVISCID INTERNAL FLOWS

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Abstract

In this paper MacCormack's 1969 explicit numerical procedure will be compared with his 1981 implicit scheme. Both methods are applied to two-dimensional inviscid internal flow near Mach number one. A pseudo-unsteady formulation of the governing equations is used, although only the steady-state solution is of interest. Consequently the energy equation can be replaced by an algebraic expression. In the case of the implicit algorithm the time step size has been chosen to be so large that the Courant number is greater than unity in all spatial directions. Thus all boundary conditions, also those at the entrance and exit region have to be given in an implicit formulation. The numerical efficiency has been measured by comparisons of the CPU-time at a conventional CDC-Cyber 76 computer and a CRAY1. Whereas in the case of the explicit procedure the time step size is limited by the CFL-condition it has been found out that there exists also a restriction for the implicit numerical scheme because of accuracy requirements and wiggles originated by the treatment of the boundary conditions. As a result it will be shown that a fully vectorized version of the explicit MacCormack procedure is slightly more efficient than a vectorized implicit algorithm for the calculation of inviscid internal flow.

1. Introduction

In recent years time marching procedures have been developed for integrating the Euler- and Navier-Stokes equations. Because the time derivatives are retained the schemes remain of a hyperbolic type in the whole domain whereas the governing equations for a steady-state flowfield change from an elliptic type if the Mach number is less than unity to a hyperbolic one for supersonic flow regions[1]. Therefore time marching methods have mainly been applied for calculating transonic flowfields.

In the 1960's explicit time marching methods had been developed[(2,3)]. As there is no matrix inversion required explicit procedures are compact and straightforward to be computerized. Another property of the explicit schemes is the simplicity of implementing boundary conditions. Thus there exist many references about the application of explicit time-dependent finite difference schemes for calculating steady-state flowfields in turbomachines of any kind[4,5,6].

The main disadvantage of explicit procedures is a very restrictive stability criterion known as CFL-condition[7]. A physical meaning of the criterion can be given if one considers the cone built by the bicharacteristics which start from the interested point in the x,y - plane at time level $(t+\Delta t)$. The time step size Δt is bounded by the condition that the cone must intersect the x,y - plane at the former time level t in the space between the neighboring mesh points, see fig.1. Therefore the step size valid for the whole computational domain is determined by the minimum mesh spacing. Especially for typical meshes for solving the Navier-Stokes equations where strong velocity gradients across the boundary layer cause a fine mesh spacing the restricted time step size can lead to very large CPU-times. In many cases the time increment becomes of about two orders of magnitude less than that required for solving the Euler equations[7].

To overcome the problems concerned with restricted time increments hybrid schemes have been developed. MacCormack[8] suggested to combine the best properties of explicit, implicit and characteristic methods. In the paper referred to the fluid flow equations are split into several parts, each of them being treated in a different manner. For multidimensional problems the computer running time can be reduced up to two orders of magnitude.

In other investigations further attempts have been made for solving transonic flowfields in a more efficient manner. Because of the requirement for fine mesh spacing as e.g. in the neighborhood of the stagnation points of blunt bodies multiple grid techniques[9] have been developed. Time-splitting[10] offers advantages in viscous flow problems where the spatial increment normal to the wall has to be chosen considerably smaller than in streamwise direction. Nevertheless, the structure of those codes is very complicated, and the computerization of these algorithms needs a large amount of efforts. Therefore, since the 1970's more and more implicit procedures have been used which are based on Yanenko's method of fractional steps[11] or alternating direction implicit (ADI-) schemes first given by Douglas et al[12]. In both methods the time consuming matrix inversion is reduced to a series of small bandwidth matrix inversion problems in each spatial direction. Beam and Warming[13] developed an ADI-sequence where block tridiagonal matrices have to be inverted along each line of the computational grid. MacCormack extended his 1969 explicit method by adding an implicit operator [14]. As the MacCormack scheme consists of a predictor and corrector step only block bidiagonal matrices have to be handled with. The implicit scheme has been applied for calculating external flowfields [15] as well as viscous internal flows[16]. In the last mentioned case the implicit operator has been only applied perpendicular to the wall whereas in streamwise direction the explicit procedure with a Courant number less than unity has been retained.

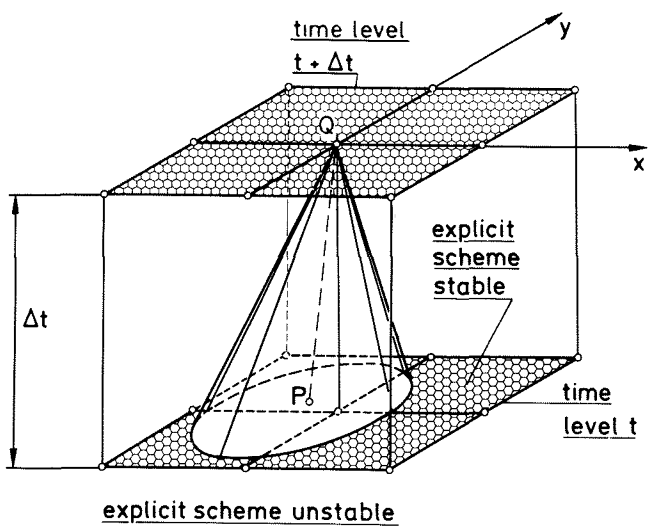

Figure 1. Symplified explanation of the physical meaning of the CFL-stability criterion
P: location of fluid particle at time t
Q: location of fluid particle at time t + Δt

As a difference to the requirements for solving viscous flow problems there is no need for choosing computational meshes with very fine spacing normal to solid boundaries in inviscid internal flowfield applications because of the lack of viscous terms. Hybrid schemes or partially explicit-implicit schemes as used in ref.(16) do not offer any advantages because of the overall uniform mesh spacing. Therefore the implicit operator has to be applied to all spatial directions and it can only be more efficient than an explicit scheme if the increase of the CPU-time per step due to the matrix inversion can be compensated by introducing larger time increments. As a consequence the Courant number becomes larger than unity also in streamwise direction and, thus, all boundary conditions have to be given in an implicit formulation. Recently, Wiedermann[17] examined the efficiencies of both MacCormack's explicit and implicit schemes for calculating two-dimensional inviscid flowfields in a converging-diverging nozzle. He applied the implicit MacCormack scheme to a pseudo-unsteady formulation of the governing equations. In the present contribution the authors report about their additional experiences in this topic including CPU-time comparisons of fully vectorized versions of the schemes at a CRAY1 computer.

2. Calculation of flowfields in Laval-nozzles

The sample calculations used in the comparison concern the steady-state transonic flowfield of an inviscid fluid in a converging-diverging nozzle. In a former publication the authors reported about the influence of different throat geometries on the flowfield and the discharge coefficient of high-loaded nozzles[18]. The calculation was carried out with MacCormack's explicit scheme[3] and good agreement with experimental data was demonstrated. Now the results shall be used as reference data for comparison purposes of the explicit and implicit MacCormack schemes.

2.1 The governing equations

In favor of a consistent formulation of the boundary conditions along the nozzle contour a boundary fitted coordinate system has been chosen, see fig.2. In a conservative formulation the governing equations for an adiabatic inviscid flowfield are given by

$$\frac{\partial \vec{U}}{\partial t} + \frac{\partial \vec{F}}{\partial \xi} + \frac{\partial \vec{G}}{\partial \eta} + \frac{\varepsilon}{r}\vec{H} = \vec{0} \tag{1}$$

with the matrices

$$\vec{U} = J^{-1}\begin{bmatrix}\rho \\ \rho u_x \\ \rho u_y\end{bmatrix} ; \quad \vec{F} = J^{-1}\frac{\partial \xi}{\partial x}\begin{bmatrix}\rho u_x \\ \rho u_x^2 + p \\ \rho u_x u_y\end{bmatrix}$$

$$\vec{G} = J^{-1}\begin{bmatrix}\rho u_\eta \\ \rho u_x u_\eta + p\dfrac{\partial \eta}{\partial x} \\ \rho u_y u_\eta + p\dfrac{\partial \eta}{\partial y}\end{bmatrix} ; \quad \vec{H} = J^{-1}\begin{bmatrix}\rho u_y \\ \rho u_y u_x \\ \rho u_y^2\end{bmatrix} \tag{2}$$

where p and ρ denote the static pressure and density, whereas u_x and u_y are the velocity components in a cartesian reference frame. The different rows of the matrices contain the continuity equation and the two components of the momentum equation. $\partial \xi/\partial x$, $\partial \eta/\partial x$ and $\partial \eta/\partial y$ are the non-vanishing metric derivatives and J is the Jacobian of the boundary-fitted coordinate system given by

$$J = \frac{d\xi}{dx}\frac{\partial \eta}{\partial y} \tag{3}$$

u_η means the contravariant velocity component perpendicular to the lines η = const. ε is set to zero for plane flow and becomes unity for axisymmetric flowfields.

As suggested by Veuillot and Viviand[19] the energy equation is replaced by an algebraic expression if the total enthalpy is assumed to be constant within the whole domain and if only the steady-state solution is of interest. For a perfect gas one obtains

$$h_t = \frac{\gamma}{\gamma - 1}\frac{p_t}{\rho_t} = \frac{\gamma}{\gamma - 1}\frac{p}{\rho} + \frac{\vec{u}\cdot\vec{u}}{2} \tag{4}$$

where γ is the ratio of specific heats and h_t the stagnation enthalpy.

At the entrance local isentropic conditions and the flow angle are given. The boundary conditions at the exit region depend on whether supersonic or subsonic motion occurs. In the first case no boundary condition is allowed to be prescribed and all the variables are obtained by applying a linear extrapolation. For subsonic outlet flow the static pressure has to be fixed.

At the center line symmetry conditions have to be fulfilled. Finally along the solid boundary the flow angle must coincide with the slope of the contour line.

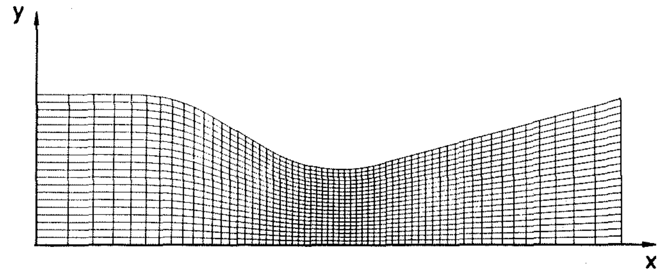

Figure 2. Boundary fitted computational mesh for 2D-nozzle flow

2.2 The explicit MacCormack scheme

As already mentioned the MacCormack integration scheme consists of a predictor and a corrector step[3]. Applied to the vector form eq. (1) one gets for the

predictor step

$$\begin{aligned}\Delta U^n_{i,j} = &-(F^n_{i+1,j} - F^n_{i,j})\cdot \Delta t/\Delta\xi \\ &-(G^n_{i,j+1} - G^n_{i,j})\cdot \Delta t/\Delta\eta - \Delta t\frac{\varepsilon}{r}H^n_{i,j} \\ \overline{U^{n+1}_{i,j}} = &\; U^n_{i,j} + \Delta U^n_{i,j}\end{aligned} \tag{5}$$

and for the

corrector step

$$\begin{aligned}\Delta\overline{U^{n+1}_{i,j}} = &-(\overline{F^{n+1}_{i,j}} - \overline{F^{n+1}_{i-1,j}})\cdot \Delta t/\Delta\xi \\ &-(\overline{G^{n+1}_{i,j}} - \overline{G^{n+1}_{i,j-1}})\cdot \Delta t/\Delta\eta - \Delta t\frac{\varepsilon}{r}\overline{H^{n+1}_{i,j}} \\ U^{n+1}_{i,j} = &\;\frac{1}{2}(U^n_{i,j} + \overline{U^{n+1}_{i,j}} + \Delta\overline{U^{n+1}_{i,j}})\end{aligned} \tag{6}$$

The subscripts i and j mark the location of the discrete mesh points in the x,y - plane whereas n denotes the time level. Increments are indicated by the symbol "Δ".

At the boundaries special one-sided difference operators are introduced for the free flow variables.

2.3 The implicit MacCormack scheme

MacCormack's 1981 published difference scheme[14] is an "add-on" to his earlier explicit method. An implicit operator is incorporated into the original algorithm to provide the capability of taking larger steps than are allowed by the explicit stability condition.

The first step to be done is a linearization of eq.(1) with respect to the components of the vector $\vec{U}$. After having introduced Jacobian matrices $\underline{A}$ and $\underline{B}$, defined by

$$\underline{A} = \frac{\partial \overline{F}}{\partial \overline{U}} \; ; \underline{B} = \frac{\partial \overline{G}}{\partial \overline{U}} \; ; \underline{A}^{*} = \frac{\partial \overline{H}}{\partial \overline{U}} \tag{7}$$

eq.(1) can be modified into a relation similar to the wave equation:

$$\frac{\partial \overline{U}}{\partial t} + \underline{A}(\overline{U}) \frac{\partial \overline{U}}{\partial \xi} + \underline{B}(\overline{U}) \frac{\partial \overline{U}}{\partial \eta} + \frac{\varepsilon}{r} \underline{A}^{*}(\overline{U}) \cdot \overline{U} = \overline{0} \tag{8}$$

As the Jacobian matrices $\underline{A}$ and $\underline{B}$ are no diagonal matrices the obtained linearized scalar equations are coupled. That means that all unknown flow variables have to be evaluated simultanously and, as a more difficult effect, all the boundary conditions have to be inserted into the complete matrix.

As a next step the Jacobian matrices have to be diagonalized by the matrices $\underline{L}_A$ and $\underline{L}_B$ to yield the form

$$\underline{A} = \underline{L}_A \underline{D}_A \underline{L}_A^{-1} \; ; \underline{B} = \underline{L}_B \underline{D}_B \underline{L}_B^{-1} \tag{9}$$

The diagonal matrices $\underline{D}_A$ and $\underline{D}_B$ contain the eigenvalues λ_i of $\underline{A}$ and $\underline{B}$. As the energy equation is replaced by an algebraic expression the matrices consist of three rows and three columns. $\underline{L}_A$ and $\underline{L}_B$ are constructed of the left hand eigenvectors belonging to the eigenvalues λ_1, λ_2 , and λ_3 as columns. After having replaced the eigenvalues through the expressions

$$\begin{aligned} \Lambda_A &= \max(|\lambda|_A - \frac{1}{2}\frac{\Delta t}{\Delta \xi} ; 0) \\ \Lambda_B &= \max(|\lambda|_B - \frac{1}{2}\frac{\Delta t}{\Delta \eta} ; 0) \end{aligned} \tag{10}$$

one obtains the new matrices

$$|\underline{A}| = \underline{L}_A \underline{\Lambda}_A \underline{L}_A^{-1} \; ; |\underline{B}| = \underline{L}_B \underline{\Lambda}_B \underline{L}_B^{-1} \tag{11}$$

relating to the Jacobian matrices. Because $\underline{\Lambda}_A$ and $\underline{\Lambda}_B$ contain non-negative elements the modified matrices $|\underline{A}|$ and $|\underline{B}|$ are positiv definite.

If one applies these procedures to eq. (8) the implicit MacCormack-scheme gives finally:

predictor step:

$\Delta U_{i,j}^{n}$ calcul. with eq. (5)

$$\begin{aligned} &(\underline{E} + \frac{\Delta t}{\Delta \xi} |\underline{A}|_{i,j}^{n}) \, \delta U_{i,j}^{\overline{\widetilde{n+1}}} = \Delta U_{i,j}^{n} + \frac{\Delta t}{\Delta \xi} |\underline{A}|_{i+1,j}^{n} \delta U_{i+1,j}^{\overline{\widetilde{n+1}}} \\ &(\underline{E} + \frac{\Delta t}{\Delta \eta} |\underline{B}|_{i,j}^{n}) \, \delta U_{i,j}^{\overline{n+1}} = \delta U_{i,j}^{\overline{\widetilde{n+1}}} + \frac{\Delta t}{\Delta \eta} |\underline{B}|_{i,j+1}^{n} \delta U_{i,j+1}^{\overline{n+1}} \\ &U_{i,j}^{\overline{n+1}} = U_{i,j}^{n} + \delta U_{i,j}^{\overline{n+1}} \end{aligned} \tag{12}$$

corrector step:

$\Delta U_{i,j}^{\overline{n+1}}$ calcul. with eq. (6)

$$\begin{aligned} &(\underline{E} + \frac{\Delta t}{\Delta \xi} |\underline{A}|_{i,j}^{\overline{n+1}}) \, \delta U_{i,j}^{\widetilde{n+1}} = \Delta U_{i,j}^{\overline{n+1}} + \frac{\Delta t}{\Delta \xi} |\underline{A}|_{i-1,j}^{\overline{n+1}} \delta U_{i-1,j}^{\widetilde{n+1}} \\ &(\underline{E} + \frac{\Delta t}{\Delta \eta} |\underline{B}|_{i,j}^{\overline{n+1}}) \delta U_{i,j}^{n+1} = \delta U_{i,j}^{\widetilde{n+1}} + \frac{\Delta t}{\Delta \eta} |\underline{B}|_{i,j-1}^{\overline{n+1}} \delta U_{i,j-1}^{n+1} \\ &U_{i,j}^{n+1} = \frac{1}{2}(U_{i,j}^{n} + U_{i,j}^{\overline{n+1}} + \delta U_{i,j}^{n+1}) \end{aligned} \tag{13}$$

with $\underline{E}$ as the identity matrix. As one can see only block bidiagonal matrices have to be inverted in each spatial direction. Following the ADI-sequence the integration is split into different operators in x- and y-direction. Relating to the forward and backward difference operators the sweep direction for solving the set of linear equations has to be changed in the predictor and the corrector step. A special procedure suggested by MacCormack[14] has been applied. The explicit scheme has to fulfill a stability criterion (given by MacCormack[10])

$$\Delta t = \frac{1}{\frac{|u_x|}{\Delta x} + \frac{|u_y|}{\Delta y} + a\sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2}}} \tag{14}$$

with a being the speed of sound. By adding eqs.(12) and (13) the complete scheme remains unconditionally stable for larger time increments.

An important point is a proper treatment of the implicit boundary conditions. As most of them in compressible fluid flow are non-linear relations between the direct flow variables u_x, u_y, p and ϱ a linearization with respect to the components of the vector $\vec{U}$ (see eq. (2)) has to be completed for implementing the boundary conditions into the linear set of equations (12) and (13). As an example the treatment at the entrance region will be shown. Assuming local isentropic flow there is a linearized relation between $\delta\varrho^{n+1}$ and $\delta\varrho u^{n+1}$ if the energy equation, eq. (4), and the equation for vanishing of the crosswise velocity component is used:

$$[(\gamma+1)g^{\gamma}-2g]^{n}\delta g^{n+1}+\left[\frac{\gamma-1}{\gamma}\,g u_x\right]^{n}\delta g u_x^{\,n+1}=0 \qquad (15)$$

All variables are normalized with the stagnation point values.

As at the edges the block bidiagonal structure of the matrices cannot be retained special algorithms have to be found for evaluating the flow variables at the boundaries.

3. Comparison of flowfields

Calculations have been carried out for plane flowfields in order to compare the results obtained by different numerical methods and on different computers. To characterize the nozzle geometry the radius of the longitudinal curvature at the throat is related to the half-width and gives the dimensionless number R_v. As being shown earlier[18] R_v has been proved to be the most essential parameter influencing the flowfield and the discharge coefficient of a nozzle. Geometries with R_v = 2. and R_v = .625 have been considered.

At first supersonic condition has been assumed at the nozzle outlet. The field of lines of constant Mach numbers is plotted in fig. 3. The computational mesh consists of 61*21 grid nodes. The results obtained with MacCormack's implicit scheme are in good agreement with the data calculated with the explicit scheme. The time increment has been chosen 10 times as large as the time step size allowed for the explicit scheme. In this case the maximum Courant number was 7.5. If the time increment is further increased the deviation between the Mach lines obtained by the use of different time step sizes will grow because factorization errors inherent to ADI-schemes begin to influence the results. At last instabilities occur, first in the throat region,

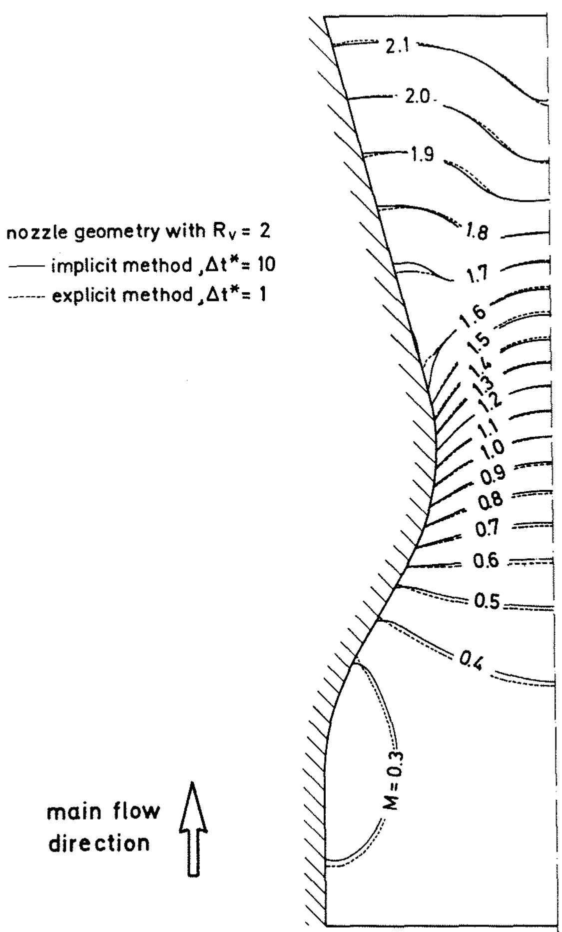

Figure 3. Lines of constant Mach number M

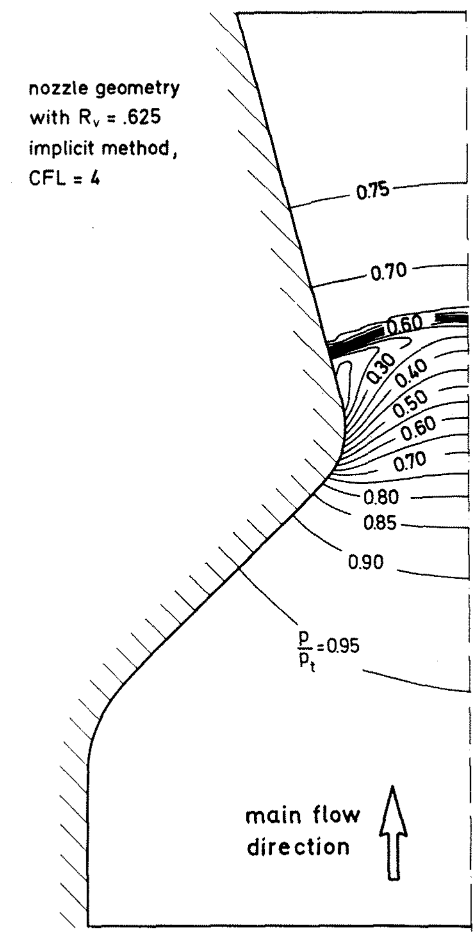

Figure 4. Isobars p/p_t, obtained with the implicit MacCormack scheme

obviously caused by the present treatment of the boundary conditions along the wall.

In a further example the flowfield in a plane nozzle with R_V = .625 will be discussed. Now the static pressure at the exit is prescribed in such a manner that a shock occurs in the nozzle (fig.4). If the implicit procedure is applied (Δt^* = 5; Courant number = 4). the discontinuity can be clearly seen in the region where the pressure gradient is very strong and many isobars p/p_t can be found. The best way in comparing the different two schemes for that case shows a graph in which the Mach number distributions along the solid wall and the center line are given (fig.5). Both schemes reproduce the shock with a steep gradient. With the first appearance of the shock wave during the computation small oscillations occur which can be stabilized by adding an artificial damping term in the implicit part of the scheme. By increasing the time increments firstly wiggles begin to grow nearby the discontinuity. At a certain limit instabilities destroy the result. Therefore it can be concluded that even in the case of inviscid internal flow the time step size remains bounded because of the factorization errors and occuring instabilities due to the formulation of the boundary conditions.

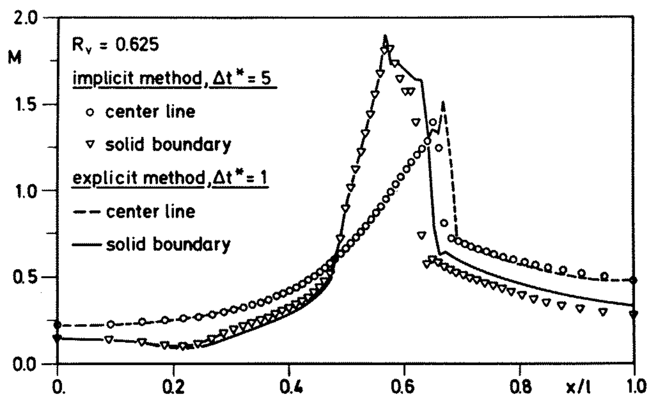

Figure 5. Mach number distribution
l: length of the nozzle

4. Convergence histories of the schemes

In order to discuss the efficiencies of both numerical schemes the square of the differences of the density between two adjacent time steps at every mesh point was summed up. After the normalization by its maximum value the square root has been calculated and yields

$$RMS = \sqrt{\frac{\sum_{i,j}(\rho_{i,j}^{n+1} - \rho_{i,j}^{n})^2}{\sum_{i,j}(\rho_{i,j}^{1} - \rho_{i,j}^{0})^2}} \qquad (16)$$

The symbol RMS was chosen with respect to the definitions in earlier investigations as e.g. by Chakravarthy[20] or Casier et.al.[21].

In fig.6 $\log_{10}$(RMS) has been plotted versus the number of time steps. In all cases the steady-state solution is approached asymptotically in form of a damped oscillation. A rather coarse mesh consisting of 31*7 grid nodes was chosen for calculating the flowfield in a nozzle with R_V = 2. At the outlet supersonic flow condition occurs. A dimensionless number Δt^* is introduced as a parameter and denotes the time step size normalized with the increment which had been applied for the explicit difference scheme. For the explicit scheme only 75% of the time step size allowed by the CFL-condition had been found to be the most efficient increment. Here the actual Courant number is given by multiplying Δt^* with the factor of 0.5. Fig. 6 shows that the required number of iteration steps can be reduced considerably if the implicit part has been added to the original scheme.

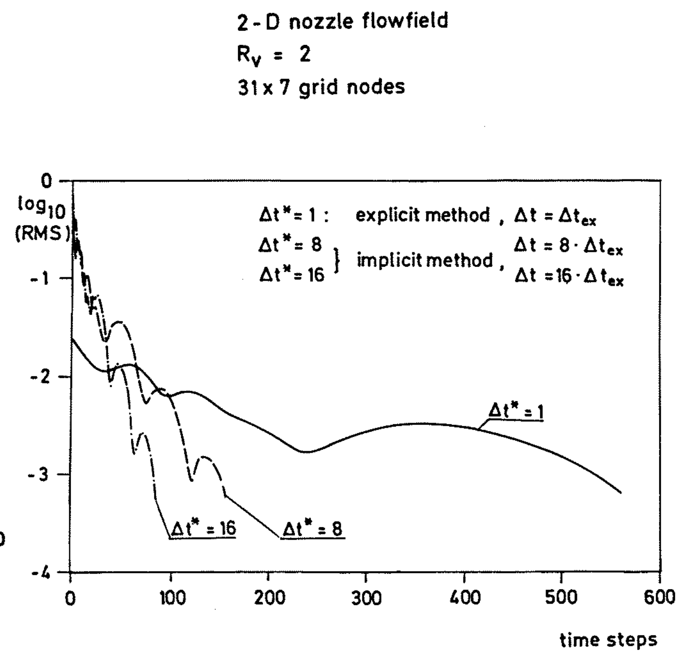

Figure 6. Convergence histories of MacCormack's schemes

However, the comparison of computer times required for these examples does not provide a saving by applying the implicit algorithm (table 1). Although a high level of vectorization has been achieved in both schemes the larger effort of a single implicit sweep cannot be compensated by larger time step sizes except the cases Δt^* > 16. As a remarkable result it can be demonstrated that there does not exist a linear relationship between time increment and the number of iteration cycles. For the case Δt^* = 20 there is even a slight increase of required steps.

That means that one can obtain an optimum value nearby the Courant number of 8. This kind of the convergence history had been also observed by White and Anderson[22] who applied MacCormack's implicit method to quasi-one-dimensional nozzle flows. As stated in the present paper they also found that there exists a limitation for the time step size due to the implicit treatment of the boundary conditions. The maximum Courant number obtained by them is of the same order of magnitude than that here in this investigation.

$R_v = 2$, 31x7grid nodes

$\Delta t^* = \Delta t/\Delta t_{ex}$	number of time steps	CPU - time (completely)
1	562	5.91
8	159	9.66
12	111	6.74
16	88	5.35
20	92	5.6

Table 1. Comparison of CPU-times at a CRAY1

In fig. 7 the residue time history is plotted for another example with a finer, 61*21 computational mesh. As shown above the number of iteration cycles can also be reduced considerably at higher Courant numbers. However, a look at the CPU-times gives a similar result as in the case of a coarse mesh (table 2). At a conventional CDC-CYBER 76 there is a saving of nearly half the computer running time of the explicit scheme. On the other hand the explicit scheme is more efficient at a CRAY. Although in both schemes a high level of vectorization has been attempted the efficiency of the explicit MacCormack scheme grows faster with respect to the length of the vectors. An analyzation of the flow trace dumped by the CRAY-processor led to the conclusion that mainly the special algorithms for evaluating the boundary conditions prevents a better vectorization at a CRAY.

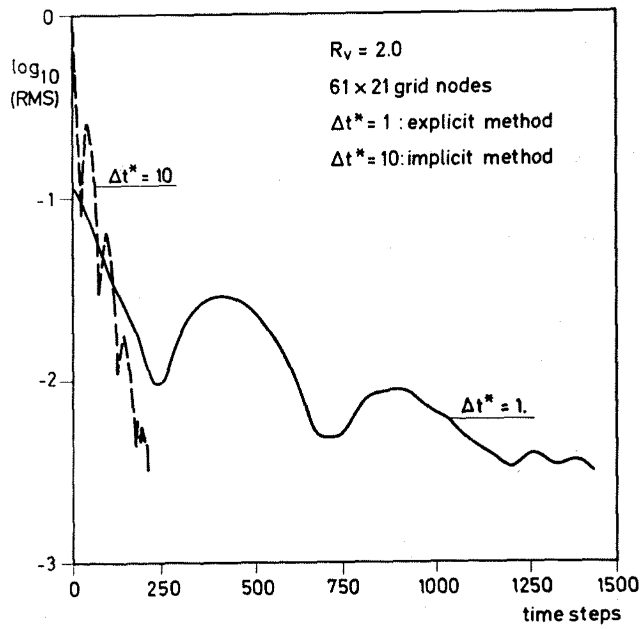

Figure 7. Convergence histories of MacCormack's schemes

In table 3 a comparison of the computational efforts for a single time step per mesh point is given. In both cases there is an immense saving of computer running time at a CRAY in comparison with the CPU-time at a CDC-CYBER 76. It can be also seen that for the CRAY1 the explicit scheme becomes more than twice as fast as the implicit scheme. The absolute values of the calculation times are similar to the figures obtained by Kordulla[23] who also compared the efficiencies of the MacCormack schemes at different computers. But opposite to the present paper his work is concerned with external viscous flow problems and he only applied the implicit operator normally to the wall, whereas the Courant number remained less than unity in streamwise direction.

At last the influence of the total number of mesh points on the required CPU-times per step per grid node shall be demonstrated, fig. 8. The CPU-time necessary for the implicit difference scheme is normalized with the corresponding value of an explicit method. The graph demonstrates the tendencies of the computational

numerical method	time step Δt	Number of time steps	CPU - time (CDC), sec.	CPU - time (CRAY 1), sec.
explicit	0.002	1437	368.2	16.56
implicit	0.02	210	217.2	26.83

Table 2. Comparison of CPU-times at a CDC-CYBER 76 and a CRAY1

61x21-grid nodes

effort with growing mesh point numbers. Whereas there is a negative slope in case of a conventional computer the relation between the running times of the implicit and explicit scheme grows considerably if using a vector computer. As mentioned above the special treatment of the boundary conditions by applying the additional implicit operator is mainly responsible for the unfavorable development of the vectorization level.

method	CDC	CRAY1
explicit	$20.0 \cdot 10^{-5}$	$0.9 \cdot 10^{-5}$
implicit	$81.0 \cdot 10^{-5}$	$10.0 \cdot 10^{-5}$

Table 3. Absolute CPU-time per time step and per mesh point.

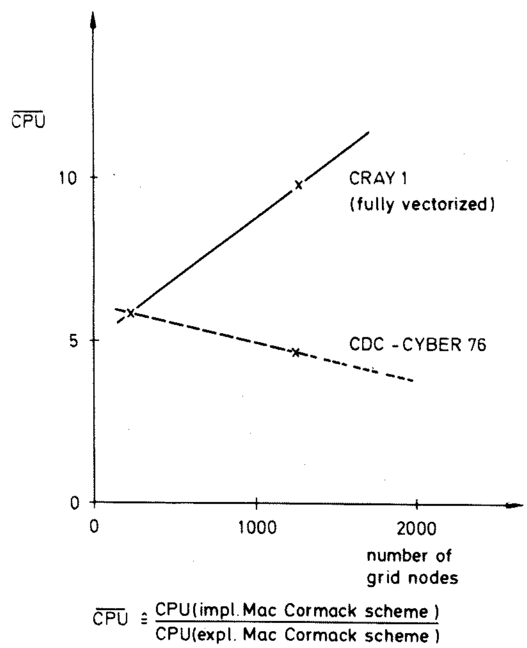

Figure 8. Relation between CPU-times of the implicit and the explicit MacCormack schemes versus number of time steps

5. Conclusions

In this paper a comparison between MacCormack's 1969 explicit scheme[3] and his 1981 implicit time marching method[14] has been carried out. Both methods have been applied to inviscid two-dimensional nozzle flow. In the case of the implicit algorithm the time step size has been chosen to be so large that the Courant number is larger than unity in all directions. Of course all boundary conditions, also those at the entrance and the nozzle outlet have to be formulated in an implicit manner. At the exit the flow might be alternatively supersonic or subsonic.

As a conclusion it has been found out that the time step size is also restricted in the implicit procedure mainly due to the following points:

1.) The introduced ADI-scheme yields to an additional approximate factorization error.

2.) Further errors are introduced by linearizing the nonlinear boundary conditions. These can cause convergence problems and even instabilities.

Comparisons of CPU-times at a conventional CDC-CYBER 76 and a CRAY lead to the conclusion that the explicit MacCormack scheme is slightly more efficient than the implicit scheme. As an important result it has been observed that the explicit scheme becomes more efficient if the number of mesh points is increased because the special handling of the implicit boundary conditions prevents a better vectorization. In addition the explicit scheme is easier to be computerized whereas the vectorization of the implicit operator is very elaborate especially due to the implementation of the boundary conditions. Thus, it can be concluded that the explicit scheme is more efficient for calculating two-dimensional inviscid internal flowfields.

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