COMPRESSOR RESPONSE TO PERIODIC TRANSIENTS

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Abstract

The theoretical prediction of the transient behavior of axial or centrifugal compressors requires the knowledge of its transfer function i.e. the transient pressure increase, function of inlet parameters : axial and angular velocities and their time derivatives.

There is no present evidence of the validity of the transfer functions proposed up to now in the literature. Therefore a systematic study was undertaken to investigate the transient behavior of a compressor.

In a first step, a sudden mass flow increase was imposed downstream of a test compressor and the change in mass flow and pressure rise analysed. A theoretical transfer function was derived from the test results that take into account both the time wise pressure rise and velocity changes during the transients.

However the analysis of such tests appeared to be cumbersome and a second series of tests used the periodic modulation of the outlet section of the compressor. These tests give the frequency dependence of the transients and validated actually the transfer function previously obtained that is slightly more complicated that the conventional ones since it takes into account the time history of the flow.

The effect of test facility geometry on the amplitude and phase lag of pressure rise perturbation was also demonstrated by these tests.

Introduction

Aeronautical compressors, mainly those of military use, are often submitted to low or medium frequency pressure perturbations.

These perturbations can be simply due to interactions with the wakes issued from upstream stages or from stationary or unsteady inlet flow maldistributions.

Only a few theoretical or experimental results are reported on this problem [1], [2]. Therefore ONERA has undertaken a systematic research on the response of a rotating axial cascade to periodic transients.

Although apparently very simple, this research has shown many difficulties and the present paper is more a progress report than a final paper.

Test facility

The facility used for this research is an open circuit compressor made of a high hub-to-tip ratio rotor and stator (outer diameter 0.465 m, blade height 0.01 m).

Basically, it would have been interesting to study the simple rotor-stator combination, without any inlet or outlet duct. For obvious reasons, it was necessary to keep an upstream duct with a quite large settling chamber as well as an outlet duct (figure 1).

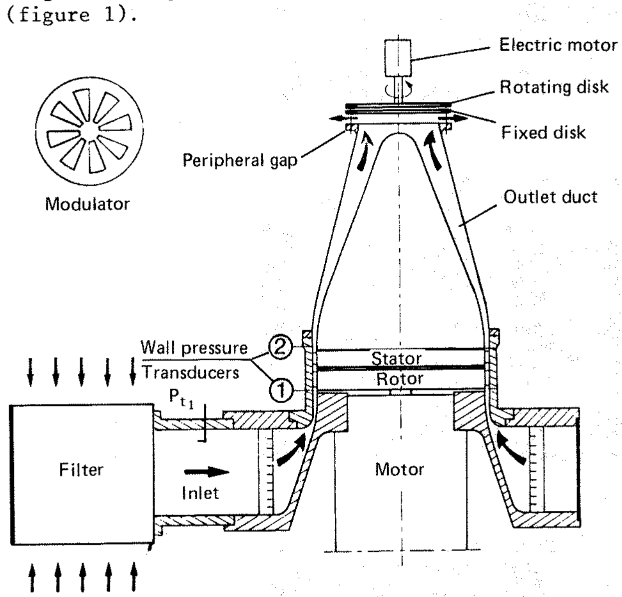

Fig. 1 — Schematic view of the test compressor.

Therefore some studies had to be made to determine the effect of each of these ducts on the transient response of the compressor.

In order to avoid compressibility effects, low speed tests only were performed. Further, such an operating point of the pressure rise - mass flow characteristic line of the compressor was chosen, that any surge hazard was avoided.

The periodic transients were obtained by means of a rotating disk with either 16 - 32 or 64 triangular holes. This disk rotates downstream of an identical fixed disk. The axial gap between the fixed disk and the exit section of the outlet duct is large enough to avoid surging even when all the holes are fully closed.

Well calibrated pressure transducers record the wall pressure fluctuations upstream of the rotor (station 1) and downstream of the stator (station 2).

The pressure signals are either recorded on magnetic tape, printed on a polaroid picture or directly processed on a phase - amplitude computer.

Analysis of the transient pressure data

Examples of polaroid pictures are shown on figure 2. The pressure signals are apparently periodic, but certainly not of the sine type. Definition of the maximum amplitude of each signal and that of the phase angle is difficult.

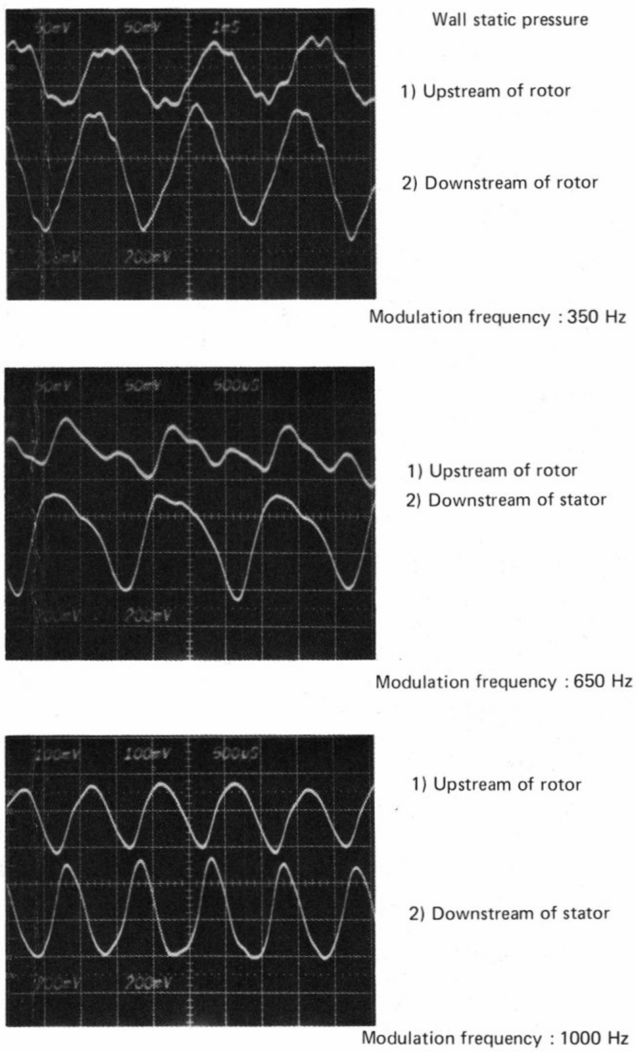

Fig. 2 – Examples of pressure measurements.

On figure 3, we have shown the variation with frequency in the 100 - 1000 Hz range, of the amplitude P1M and P2M of the wall pressure fluctuations P1(t) and P2(t) for three different test facility configurations.

The basic test 'A' was performed at N = 3000 rpm with the shortest possible inlet and outlet ducts (inlet Li = 1.00 m, Ai = 0.038 sq m ; outlet Lo = 0.70 m, Ao = 0.038 sq m).

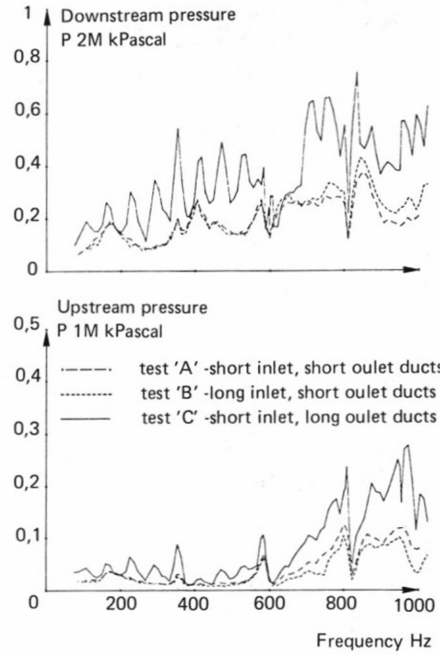

Fig 3 – Variation of the wall pressure fluctuations with frequency (three different compressor geometries).

Test 'B' performed with the same outlet duct, had a longer inlet duct (L inlet = 4.15 m). The amplitudes of the pressure fluctuations measured for these two tests are practically the same.

Test 'C' was performed with a short inlet duct and a longer outlet duct (L outlet = 2.30 m). Both pressure fluctuations P1M and P2M were deeply modified by this change in geometry. Not only the overall amplitude level became higher but a low frequency modulation became visible.

However, dividing for each of these three tests, P1M by P2M we obtained a single curve, function of frequency only (fig. 4). For a given speed of rotation and at a given mean point, 'the response of a compressor to a downstream pressure and mass flow modulation is function of the frequency and indepedent of the duct downstream of the compressor'.

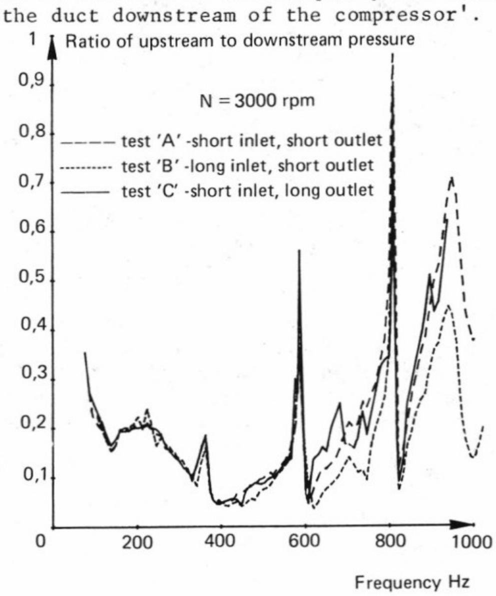

Fig. 4 – Variation of the pressure ratio with frequency (three different compressor geometries).

This conclusion is obvious from theoretical considerations and helps the theoretical analysis of the transients.

The effect of rotor speed of rotation on the inlet and outlet pressure fluctuations are shown on fig. 5 (short inlet and long outlet ducts). The non-reduced data (fig. 5a) show a strong increase of the fluctuation level as the rotor velocity increases (N = 3000 rpm for test 'C' and 5000 rpm for test 'D').

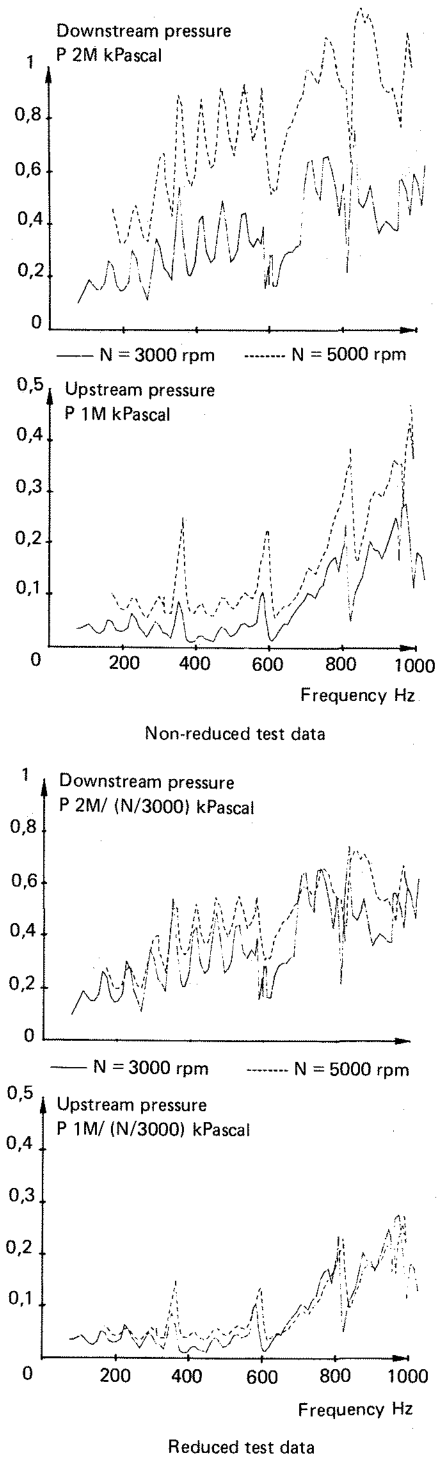

Fig. 5 – Influence of speed of rotation.

If these data are reduced by dividing the amplitude of the pressure fluctuations by the factor N/3000, practically the same fluctuation levels are obtained in both tests (fig. 5b). This conclusion shows that the pressure fluctuations behave as small amplitude perturbations.

'The small amplitude pressure fluctuations induced by a downstream pressure modulation are proportional to the rotor velocity, hence to the mean inlet velocity'.

Therefore, if a downstream pressure perturbation is imposed, there exists a unique transfer function P1M/P2M for a given operating point.

This conclusion is confirmed by the consideration of the phase lag φ 1-2 between inlet and outlet pressure signals (fig. 6) corresponding to tests 'A', 'B' and 'C'.

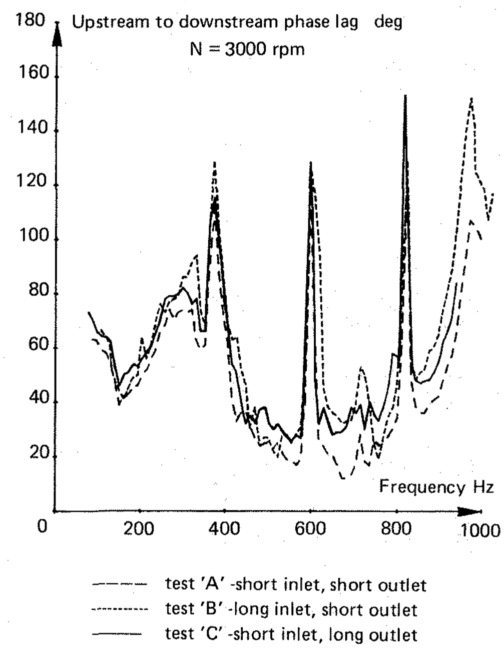

Fig. 6 – Variation of the phase lag with frequency. (three different compressor geometries).

Theoretical analysis of the response of a compressor to a downstream pressure modulation

A block diagram used for the theoretical model of the test compressor is shown on figure 7 :

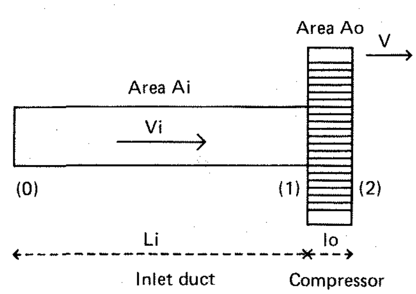

Fig. 7 – Block diagramme of the theoretical model.

The model is made of :

(i) a constant section inlet duct of length Li and area Ai ; constant total pressure is assumed at the entrance section ;

(ii) an one -dimensional compressor of axial length lc and annular passage area Ac. The steady state pressure-mass flow characteristic

$$\bar{P}_2 - \bar{P}_1 = f(\bar{V})$$

is known.

(iii) a periodic pressure fluctuation represented by the complex number

$$(1) \qquad P_2' = \tilde{P}_2 \, e^{i\omega t}$$

where $i = \sqrt{-1}$ and $\omega = 2\Pi f$ (f is the frequency) is imposed downstream, in the stator exit plane (the effect of the outlet duct is not taken into account).

Transfer function of the compressor

Since all perturbations are of small amplitude, the linearized momentum and continuity equations are used :

$$(2) \begin{cases} \dfrac{\partial v'}{\partial t} + \bar{V}\dfrac{\partial v'}{\partial z} + \dfrac{1}{\bar{\rho}}\dfrac{\partial p'}{\partial z} = \Gamma' \\ \bar{\rho}\,\bar{a}^2 \dfrac{\partial v'}{\partial z} + \dfrac{\partial p'}{\partial t} + \bar{V}\dfrac{\partial p'}{\partial z} = 0 \end{cases}$$

where v' and p' are the axial velocity and the pressure perturbations, Γ' the acceleration of the fluid due to body forces, $\bar{V}$, $\bar{\rho}$ and $\bar{a}$ the mean values of axial velocity, density and sound velocity (for small amplitude perturbations $p' = \rho' \bar{a}^2$).

All the perturbations terms will be assumed to be of the same type

$$\begin{cases} v' = \tilde{V}(z)\, e^{i\omega t} \\ p' = \tilde{P}(z)\, e^{i\omega t} \\ \Gamma' = \tilde{\Gamma}(z)\, e^{i\omega t} \end{cases}$$

where $\tilde{V}$, $\tilde{p}$, $\tilde{\Gamma}$ are complex functions of z.

Equations (2) can then be rewritten

$$(3) \begin{cases} i\omega\tilde{V} + \bar{V}\dfrac{d\tilde{V}}{dz} + \dfrac{1}{\bar{\rho}}\dfrac{d\tilde{P}}{dz} = \tilde{\Gamma} \\ \bar{\rho}\,\bar{a}^2\dfrac{d\tilde{V}}{dz} + i\omega\tilde{P} + \bar{V}\dfrac{d\tilde{P}}{dz} = 0 \end{cases}$$

Solution of system (3) is classical. The outlet pressure perturbation term is related to inlet pressure and velocity perturbations by :

$$(4) \qquad \tilde{P}_2 = \frac{i\bar{\rho}\bar{a}\tilde{\Gamma}}{2\omega}\left(e^{-i\theta_+} - e^{i\theta_-}\right) + \frac{1}{2}\left(\tilde{P}_1 + \bar{\rho}\bar{a}\tilde{V}_1\right)e^{-i\theta_+} + \frac{1}{2}\left(\tilde{P}_1 - \bar{\rho}\bar{a}\tilde{V}_1\right)e^{i\theta_-}$$

where $\theta_+ = \dfrac{\omega\, l_c}{\bar{a}\,(1+\bar{M})}$, $\theta_- = \dfrac{\omega\, l_c}{\bar{a}\,(1-\bar{M})}$, $\bar{M} = \dfrac{\bar{V}}{\bar{a}}$

A similar expression can be obtained for the outlet velocity also.

The transfer function $\tilde{P}_1/\tilde{P}_2$ corresponding to an imposed downstream pressure modulation depends on the induced inlet perturbations and also on the acceleration term $\tilde{\Gamma}$ due to the transient body forces.

Relation between the velocity and the pressure perturbation terms at the rotor inlet

If there would be no inlet duct, a configuration we could not realize on our test facility, a constant total pressure, independent of time, would give the relation between $\tilde{V}_1$ and $\tilde{P}_1$. This relation is modified in our case by acoustic effects in the inlet duct, at the inlet section of which :

$$\tilde{P}_{i_0} + \bar{\rho}\,\bar{V}_i\,\tilde{V}_{i_0} = 0$$

where : $\bar{V}_i = \sigma\,\bar{V}$

is the mean velocity in the inlet duct ($\sigma = A_c/A_i$).

Using relations similar to (4), but with no body force terms, we obtain the following relation between $\tilde{P}_{i1}$ and $\tilde{V}_{i1}$.

$$(5) \qquad \tilde{P}_{i1} = \bar{\rho}\,\bar{a}\,\tilde{V}_{i1}\,\frac{1 - \sigma\bar{M} - (1+\sigma\bar{M})\,e^{i(\theta_{i+}+\theta_{i-})}}{1 - \sigma\bar{M} + (1+\sigma\bar{M})\,e^{i(\theta_{i+}+\theta_{i-})}}$$

$$\theta_{i+} = \frac{\omega L_i}{(1+\bar{M}\sigma)} \qquad \theta_{i-} = \frac{\omega L_i}{(1-\bar{M}\sigma)}$$

The conditions

$$\begin{cases} \tilde{P}_{i1} + \bar{\rho}\,\bar{a}\,\sigma\bar{M}\,\tilde{V}_{i1} = \tilde{P}_1 + \bar{\rho}\,\bar{a}\,\bar{M}\,\tilde{V}_1 \\ \tilde{V}_{i1} = \sigma\,\tilde{V}_1 \end{cases}$$

at the inlet duct-rotor interface give the relation between velocity and pressure perturbations at the rotor inlet :

$$(6) \qquad \tilde{P}_1 = \bar{\rho}\,\bar{a}\,\tilde{V}_1\left[\sigma\,\frac{1-\sigma\bar{M}-(1+\sigma\bar{M})\,e^{i(\theta_{i+}+\theta_{i-})}}{1-\sigma\bar{M}+(1+\sigma\bar{M})\,e^{i(\theta_{i+}+\theta_{i-})}} - \bar{M}(1-\sigma^2)\right]$$

Transient body force model

Correct modelling of the transient acceleration term is of prime importance.

Earlier series of tests [3] with a step variation of outlet area, have shown that the conventional relaxation type equation for transients [4]:

$$\tau\,\frac{d\Gamma'}{dt} + \Gamma' = \Gamma_{ss}$$

is not sufficient for the prediction of high frequency phenomena and a 'memory' type term has to be added.

The modified relaxation equation, that was proposed after analysis of the test data obtained during the earlier tests :

$$(7) \qquad \tau\,\frac{d\Gamma'}{dt} + \Gamma' = \Gamma_{ss}' + C\,\frac{d\tilde{P}_1'}{dt}$$

give also a correct description of the present experimental results :

τ is an acoustical time lag, corresponding to the travel of sound waves through the rotor and the stator ($\tau = l_c/\bar{a}$) ;

Γ_{ss}' the 'steady state' flow acceleration due to the transient velocity perturbation v_1'

$$\Gamma_{ss}' = \frac{d(\bar{P}_2 - \bar{P}_1)}{\rho\, d\bar{V}}\left(1-\bar{M}^2\right)\frac{v_1'}{l_c}$$

where $d(\bar{P}_2 - \bar{P}_1)/d\bar{V}$ is the slope of the steady state characteristic ; this expression is compatible with equations (2) and dp'/dt is closely related to the acceleration of the fluid particles at the rotor entrance.

For the modulated perturbation case one obtains :

$$\Gamma' = \frac{\dfrac{d(\bar{P}_2 - \bar{P}_1)}{\rho\, d\bar{V}}(1-\bar{M}^2)\dfrac{\tilde{V}_1'}{\ell_c} + i\omega C \tilde{P}_1'}{1 + i\omega\tau} \quad (8)$$

This expression will be used for the comparison of the theoretical predictions to the experimental results.

Comparison theory - experiments

The representation of an actual compressor, its inlet and outlet ducts and the pressure modulator by means of an one dimensional block module needs some constant fitting.

Actually the only constant, the value of which has to be adapted to fit the test results is the 'memory' constant C for which the value

$$C = 15 \text{ sec/m}$$

gives the best theoretical representation of the test results.

Figure 8a compares the theoretical value of the ratio of the maximums of the pressure fluctuations, i.e. the modulus of the transfer function $\tilde{P}_1/\tilde{P}_2$ to the experimental transfer function already shown on figure 4.

In a similar way, figure 8b shows the comparison between the argument of the complex transfer function $\tilde{P}_1/\tilde{P}_2$ and the experimental phase lag shown on figure 6.

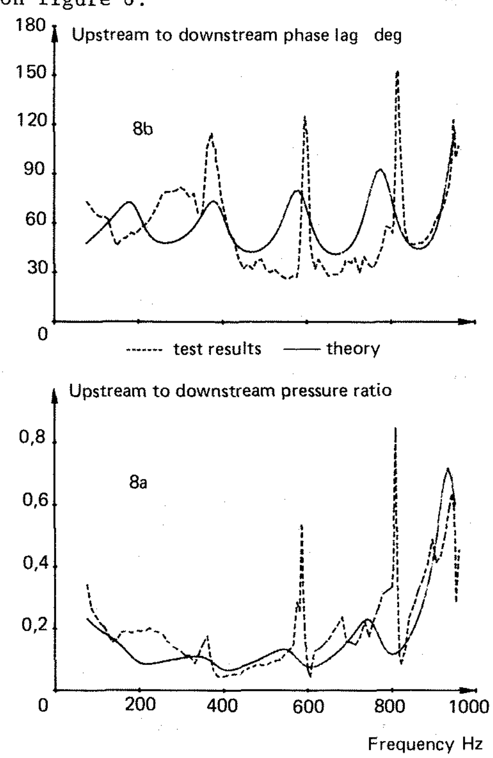

Fig. 8 – Comparison theory-experiments.

Conclusion

The series of tests made on an axial flow compressor with exit area modulation have shown the validity of the generalized relaxation type formulation of the transient body forces.

These tests, performed in the 100 to 1000 Hz frequency range, have also shown the importance of acoustic effects at higher frequencies, corresponding to the acoustic modes of the test facility.

Further tests will be necessary to give a better understanding of the physical meaning of the 'memory' term used in the generalized relaxation equation.

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