TIME-DELAY COMPENSATION IN ACTIVE CONTROL ALGORITHMS

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Abstract

In this paper a direct digital control law algorithm is proposed which in addition to providing tight non-interacting tracking behaviour and excellent disturbance-rejection characteristics, provides for the compensation of finite-time delays in implementation. The control-law algorithm is defined and system stability is proved in the case of multi-input multi-output linear systems. The theory is synthetic and leads directly to the determination of the appropriate controller matrices.

The theory is illustrated by the presentation of simulation results in which the aircraft is represented by an analogue computer and the digital control system is implemented on a microprocessor. In the simulation study the controller is required to effect fuselage pitch pointing and vertical translation manoeuvres for the analogue computer representation of the YF-16 aircraft. It is shown that tight non-interacting control is achievable even when the control implemented is delayed by 0.1 seconds.

1. Introduction

The general results of Bradshaw and Porter [1],[2] for discrete-time tracking systems indicate that tight non-interacting control is, in general, achievable by the implementation of fast-sampling error-actuated controllers. Indeed, the efficiency and effectiveness of such controllers has been demonstrated by their application to the YF-16 aircraft where they are required to effect fuselage pitch pointing and vertical translation manoeuvres [3]. Implicit in these controllers, however, was the assumption that the computational time delay is small compared with the sampling period. In some cases, this assumption may not be valid and it is necessary to compensate for the time delay. Indeed, if no such compensation is provided the resulting tracking system will have either very poor performance or will be unstable.

The general results of Bradshaw and Porter [1],[2] have been extended to allow for a computational time delay of one sampling period if the control algorithm is appropriately modified. The resulting control algorithms are simple to implement and provide tight non-interacting control. Their efficiency and effectiveness have been demonstrated by Porter, Bradshaw, Garis, and Woodhead [4] in the presentation of the results of a laboratory microprocessor implementation in which the controllers are required to effect fuselage pitch pointing and vertical translation manoeuvres in the case of an analogue computer representation of the YF-16 aircraft.

In many situations, the time delay is greater than one sampling period, as shown by Butler et al [5] and McCruer [6]. In this paper, the results for the single sampling period time delay have been extended by modifying the control methodology to accommodate multiple-period delays. The resulting control algorithms provide the necessary tight non-interacting control and their effectiveness is demonstrated through the presentation of the results of a laboratory microprocessor implementation. Once again, the controllers are required to effect fuselage pitch pointing and vertical translation manoeuvres for an analogue computer simulation of the YF-16 aircraft.

2. Discrete-Time Tracking Systems with Finite Time-Delay Compensation

2.1 System Configuration

In general, high-performance discrete-time tracking systems with finite time-delay compensation consist of linear multivariable plants governed on the continuous-time set $T = (0,+\infty]$ by state, output, and measurement equations of the respective forms

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} , & A_{12} \\ A_{21} , & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u(t) \quad , \qquad (1)$$

$$y(t) = [C_1 \ , \ C_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad , \qquad (2)$$

and

$$w(t) = [F_1 \ , \ F_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad , \qquad (3)$$

together with fast-sampling error-actuated digital controllers governed on the discrete-time set $T_T = \{0,T,2T,...\}$ by control-law equations of the form

$$s(kT) = f(K_0 e(kT)+K_1 z(kT)) \qquad (4)$$

and computational time-delay compensation equations of the form

$$r(kT) = s(kT) - \sum_{i-1}^{m} \gamma_i r\{(k-i)T\} \qquad (5)$$

where

$$e(kT) = v(kT) - w(kT) \quad , \qquad (6)$$

and

$$z(kT) = z(o) + T \sum_{j=0}^{j=k-1} e(jT) \quad . \tag{7}$$

In equations (1) to (7), $x_1(t) \in R^{n-\ell}$, $x_2(t) \in R^{\ell}$, $u(t) \in R^{\ell}$, $y(t) \in R^{\ell}$, $w(t) \in R^{\ell}$, $A_{11} \in R^{(n-\ell)x(n-\ell)}$ $A_{12} \in R^{(n-\ell)x\ell}$, $A_{21} \in R^{\ell x(n-\ell)}$, $A_{22} \in R^{\ell x \ell}$, $C_1 \in R^{\ell x(n-\ell)}$, $C_2 \in R^{\ell x \ell}$, $F_1 \in R^{\ell x(n-\ell)}$, $F_2 \in R^{\ell x \ell}$, rank $C_2B_2 < \ell$, rank $F_2B_2 = \ell$, $r(kT) \in R^{\ell}$, $s(kT) \in R^{\ell}$, $v(kT) \in R^{\ell}$, $e(kT) \in R^{\ell}$, $z(kT) \in R^{\ell}$, $K_o \in R^{\ell x \ell}$, $K_1 \in R^{\ell x \ell}$, $f = 1/T$ is the sampling frequency, and $\gamma_i (i=1,2,\ldots, m) \in R$ are the delay compensation parameters. Since the computational time delay is m sampling periods the digital controller is required to generate the control input vector

$$u(t) = r\{(k-m)T\}, \; t \in [kT,(k+1)T), \; kT \in T_T \; , \tag{8}$$

so as to cause the output vector $y(t)$ to track any constant command input vector $v(t)$ on T_T in the sense that

$$\lim_{k\to\infty}\{v(kT)-y(kT)\} = 0 \tag{9}$$

as a consequence of the fact that the error vector $e(t) = v(t) - w(t)$ assumes the steady-state value

$$\lim_{k\to\infty} e(kT) = \lim_{k\to\infty}\{v(kT)-w(kT)\} = 0 \tag{10}$$

for arbitrary initial conditions. In case

$$[F_1 \; , \; F_2] = [C_1+MA_{11} \; , \; C_2+MA_{12}] \quad , \tag{11}$$

it is evident from equations (2), (3), and (11) that the vector

$$w(t) - y(t) = [MA_{11} \; , \; MA_{12}] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{12}$$

of extra measurements is such that $v(kT)$ and $y(kT)$ satisfy the tracking condition (9) for any $M \in R^{\ell x(n-\ell)}$ if $e(kT)$ satisfies the steady-state condition (10), since equation (1) clearly implies that

$$\lim_{t\to\infty}[A_{11} \; , \; A_{12}] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = 0 \tag{13}$$

in any steady state. However, the condition that rank $F_2B_2 = \ell$ requires that C_2 and A_{12} are such that M can be chosen so that

$$\text{rank } F_2 = \text{rank}(C_2+MA_{12}) = \ell \quad . \tag{14}$$

If the control input vectors are stored in the manner of Koepcke [7] by introducing the extra state variables

$$q_i\{(k+1)T\} = q_{i+1}(kT) \quad (i=1,2,\ldots,m-1)$$

and

$$q_m\{(k+1)T\} = Tr(kT) \quad , \tag{15}$$

then it is evident from equations (1) to (7) and (15) that such discrete-time tracking systems are governed on T_T by state and output equations of the respective forms

$$\begin{bmatrix} z\{(k+1)T\} \\ x_1\{(k+1)T\} \\ x_2\{(k+1)T\} \\ q_1\{(k+1)T\} \\ q_2\{(k+1)T\} \\ \cdots\cdots \\ \cdots\cdots \\ q_m\{(k+1)T\} \end{bmatrix} = \begin{bmatrix} I_\ell, & -TF_1, & -TF_2, & 0, & 0, & 0, & \ldots, & 0 \\ 0, & \Phi_{11}, & \Phi_{12}, & f\Psi_1, & 0, & 0, & \ldots, & 0 \\ 0, & \Phi_{21}, & \Phi_{22}, & f\Psi_2, & 0, & 0, & \ldots, & 0 \\ 0, & 0, & 0, & 0, & I_\ell, & 0, & \ldots, & 0 \\ 0, & 0, & 0, & 0, & 0, & I_\ell, & \ldots, & 0 \\ \cdots & & & & & & & \cdots \\ \cdots & & & & & & & \cdots \\ K_1, & -K_oF_1, & -K_oF_2, & -\gamma_m I_\ell, & -\gamma_{m-1}I_\ell, & -\gamma_{m-2}I_\ell, & \ldots, & -\gamma_1 I_\ell \end{bmatrix}$$

$$x \begin{bmatrix} z(kT) \\ x_1(kT) \\ x_2(kT) \\ q_1(kT) \\ q_2(kT) \\ \cdots \\ \cdots \\ q_m(kT) \end{bmatrix} + \begin{bmatrix} TI_\ell \\ 0 \\ 0 \\ 0 \\ 0 \\ \cdots \\ \cdots \\ K_o \end{bmatrix} v(kT) \tag{16}$$

and

$$y(kT) = [0,C_1,C_2,0,0,\ldots,0] \begin{bmatrix} z(kT) \\ x_1(kT) \\ x_2(kT) \\ q_1(kT) \\ q_2(kT) \\ \cdots \\ \cdots \\ q_m(kT) \end{bmatrix} \tag{17}$$

where

$$\begin{bmatrix} \Phi_{11} \; , & \Phi_{12} \\ \Phi_{21} \; , & \Phi_{22} \end{bmatrix} = \exp\left\{ \begin{bmatrix} A_{11} \; , & A_{12} \\ A_{21} \; , & A_{22} \end{bmatrix} T \right\} \tag{18}$$

and

$$\begin{bmatrix}\Psi_1\\ \Psi_2\end{bmatrix} = \int_o^T \exp\left\{\begin{bmatrix}A_{11}, & A_{12}\\ A_{21}, & A_{22}\end{bmatrix}t\right\}\begin{bmatrix}0\\ B_2\end{bmatrix}dt \quad . \tag{19}$$

2.2 System Analysis

The transfer function matrix relating the plant output vector to the command input vector of the closed-loop discrete-time tracking system governed by equations (16) and (17) is clearly

$$G(\lambda) = [0, C_1, C_2, 0, 0, \ldots, 0]$$

$$\times\begin{bmatrix}\lambda I_\ell - I_\ell, & TF_1, & TF_2, & 0, & 0, & \ldots, & 0\\ 0, & \lambda I_{n-\ell}-\Phi_{11}, & -\Phi_{22}, & -f\Psi_1, & 0, & \ldots, & 0\\ 0, & -\Phi_{21}, & \lambda I_\ell - \Phi_{22}, & -f\Psi_2, & 0, & \ldots, & 0\\ 0, & 0, & 0, & \lambda I_\ell, & -I_\ell, & \ldots, & 0\\ 0, & 0, & 0, & 0, & \lambda I_\ell, & \ldots, & 0\\ \ldots & & & & & & \\ \ldots & & & & & & \\ -K_1, & K_oF_1, & K_oF_2, & \gamma_m I_\ell, & \gamma_{m-1}I_\ell, & \ldots, & \lambda I_\ell + \gamma_1 I_\ell\end{bmatrix}$$

$$\times\begin{bmatrix}TI_\ell\\ 0\\ 0\\ 0\\ 0\\ \ldots\\ \ldots\\ K_o\end{bmatrix} \tag{20}$$

and the fast-sampling tracking characteristics of this system can accordingly be elucidated by the singular perturbation analysis of transfer function matrices. Indeed, since it follows from equations (18) and (19) that

$$\lim_{f\to\infty} f\begin{bmatrix}\Phi_{11}-I_{n-\ell}, & \Phi_{12}\\ \Phi_{21} & ,\Phi_{22}-I_\ell\end{bmatrix} = \begin{bmatrix}A_{11}, A_{12}\\ A_{21}, A_{22}\end{bmatrix} \tag{21}$$

and

$$\lim_{f\to\infty} f\begin{bmatrix}\Psi_1\\ \Psi_2\end{bmatrix} = \begin{bmatrix}0\\ B_2\end{bmatrix} \quad , \tag{22}$$

these results indicate that as $f \to \infty$ the transfer function matrix $G(\lambda)$ assumes the asymptotic form

$$\Gamma(\lambda) = \tilde{\Gamma}(\lambda) + \hat{\Gamma}(\lambda) \tag{23}$$

where the 'slow' transfer function matrix

$$\tilde{\Gamma}(\lambda) = (C_1 - C_2F_2^{-1}F_1)(\lambda I_{n-\ell} - I_{n-\ell} - TA_{11} + TA_{12}F_2^{-1}F_1)^{-1}TA_{12}F_2^{-1} \tag{24}$$

and the 'fast' transfer function matrix

$$\hat{\Gamma}(\lambda) = C_2F_2^{-1}[\lambda^m(\lambda-1)I_\ell + \lambda^{m-1}(\lambda-1)\gamma_1 I_\ell + \lambda^{m-2}(\lambda-1)\gamma_2 I_\ell + \ldots + \lambda(\lambda-1)\gamma_{m-1}I_\ell + (\lambda-1)\gamma_m I_\ell + F_2B_2K_o]^{-1}F_2B_2K_o \quad . \tag{25}$$

It follows from equation (24) that the 'slow' modes Z_s of the tracking system correspond as $f \to \infty$ to the poles $Z_1 \cup Z_2$ of $\Gamma(\lambda)$ where

$$Z_1 = \{\lambda \in C : |\lambda I_\ell - I_\ell + TK_o^{-1}K_1| = 0\} \tag{26}$$

and

$$Z_2 = \{\lambda \in C : |\lambda I_{n-\ell} - I_{n-\ell} - TA_{11} + TA_{12}F_2^{-1}F_1| = 0\}, \tag{27}$$

and from equation (25) that the 'fast' modes Z_f of the tracking system correspond as $f \to \infty$ to the poles Z_3 of $\hat{\Gamma}(\lambda)$ where

$$Z_3(\lambda) = \{\lambda \in C : |\lambda^m(\lambda-1)I_\ell + \lambda^{m-1}(\lambda-1)\gamma_1 I_\ell + \lambda^{m-2}(\lambda-1)\gamma_2 I_\ell + \ldots + \lambda(\lambda-1)\gamma_{m-1}I_\ell + (\lambda-1)\gamma_m I_\ell + F_2B_2K_o| = 0\} \quad . \tag{28}$$

It is evident from these results that the computational time delay has no affect on the 'slow' modes as $f \to \infty$ but that the 'fast' modes are crucially affected as $f \to \infty$. Indeed, in case no time-delay compensation is used so that $\gamma_i = 0$, it is evident from equation (28) that at least some of the 'fast' modes will be unstable.

2.3 System Synthesis

It is evident from equations (11), (13), (16), and (17) that tracking will occur in the sense of equation (9) provided only that

$$Z_s \cup Z_f \subset \mathcal{D}^- \tag{29}$$

where $\mathcal{D}^-$ is the open unit disc. In view of equations (26), (27), and (28), the 'slow' and 'fast' modes will satisfy the tracking requirement (29) for sufficiently small sampling periods if the time-delay compensation parameters γ_i $(i=1,2,\ldots,m)$ and the controller matrices K_o, K_1, and M are chosen such that $Z_1 \subset \mathcal{D}^-$, $Z_2 \subset \mathcal{D}^-$, and $Z_3 \subset \mathcal{D}^-$. Indeed, if full time-delay compensation is employed, $\gamma_i > 0$ $(i=1,\ldots,m)$ and stability of the 'fast' modes is assured by requiring that

$$F_2B_2K_o = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_\ell) \tag{30}$$

where $1-\sigma_j \in R \cap \mathcal{D}^-$ $(j=1,2,\ldots,\ell)$. Moreover, if in addition M is chosen such that both $\Gamma(\lambda)$ and $\hat{\Gamma}(\lambda)$ are diagonal transfer function matrices, then it is evident that increasingly non-interacting tracking will occur as $f \to \infty$.

3. Direct Digital Flight-Mode Control Systems with Finite Time-Delay Compensation

3.1 Vertical Translation Manoeuvre

In the vertical translation manoeuvre, the linearised longitudinal dynamics of the YF-16 aircraft flying at a Mach number of 0.8 at sea level are governed on $\mathcal{T}$ by state and output equations of the respective forms [3]

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 , & 1 , & 0 \\ 0 , & -2.068 , & 10.029 \\ 0 , & 0.985 , & -2.155 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 , & 0 \\ -35.44 , & -5.124 \\ -0.238 , & -0.308 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \quad (31)$$

and

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 , & 0 , & 0 \\ 0 , & 0 , & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad (32)$$

where $x_1(t)$ is the change in pitch angle, $x_2(t)$ is the rate of change of pitch angle, $x_3(t)$ is the change in angle of attack, $u_1(t)$ is the elevator deflection, and $u_2(t)$ is the flaperon deflection. Hence, in the case $\{\sigma_1,\sigma_2\} = \{0.4,0.4\}$ $\gamma_i = 1/6 (i=1,..,5)$, $K_0^{-1}K_1 = \mathrm{diag}\{\rho_1,\rho_2\} = \mathrm{diag}\{2.5,2.5\}$, and

$$M = \begin{bmatrix} 0.25 \\ 0 \end{bmatrix} \quad (33)$$

it follows from equations (3), (5), (8), and (30) that the corresponding transducers and fast-sampling error-actuated digital controllers with finite time-delay compensation are governed on $\mathcal{T}$ and $\mathcal{T}_T$ by the respective measurement and control-law equations

$$\begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} = \begin{bmatrix} 1 , & 0.25 , & 0 \\ 0 , & 0 , & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad (34)$$

and

$$\begin{bmatrix} r_1(kT) \\ r_2(kT) \end{bmatrix} = f\left\{ \begin{bmatrix} -0.051 , & 0.211 \\ 0.039 , & -1.462 \end{bmatrix} \begin{bmatrix} e_1(kT) \\ e_2(kT) \end{bmatrix} + \begin{bmatrix} -0.127 , & 0.528 \\ 0.098 , & -3.655 \end{bmatrix} \begin{bmatrix} z_1(kT) \\ z_2(kT) \end{bmatrix} \right\} - \frac{1}{6} \sum_{i=1}^{5} \begin{bmatrix} u_1\{(k-i)T\} \\ u_2\{(k-i)T\} \end{bmatrix} . \quad (35)$$

It is then evident from equations (26), (27), and (28) that $\mathcal{Z}_1 = \{1-2.5T, 1-2.5T\}$, $\mathcal{Z}_2 = \{1-4T\}$, and $\mathcal{Z}_3 = \{0.155\pm0.735i, -0.581\pm0.377i, 0.883\pm0.317i\}$. It is also evident from equations (23), (24), and (25) that the asymptotic transfer function matrix assumes the diagonal form

$$\Gamma(\lambda) = \begin{bmatrix} \frac{4T}{\lambda-1+4T} , & 0 \\ 0 , & \frac{30}{30\lambda^6-25\lambda^5+7} \end{bmatrix} \quad (36)$$

and therefore that the direct digital flight-mode control system for the vertical translation manoeuvre of the aircraft, with a time delay of five sampling periods in the implementation of the control action, will exhibit increasingly non-interacting tracking behaviour as $f \to \infty$ when the piecewise-constant control input vector $[u_1(t), u_2(t)]^T = [r_1(kT), r_2(kT)]^T$, $t \in [kT,(k+1)T)$, $kT \in \mathcal{T}_T$, is generated by the fast-sampling digital controller governed on $\mathcal{T}_T$ by equation (35).

3.2 <u>Fuselage Pitch Pointing Manoeuvre</u>

In the fuselage pitch pointing manoeuvre, the linearised longitudinal dynamics of the YF-16 aircraft flying at a Mach number of 0.8 at sea level are governed on $\mathcal{T}$ by state and output equations of the respective forms [3]

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 , & 1 , & 0 \\ 0 , & -2.068 , & 10.029 \\ 0 , & 0.985 , & -2.155 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 , & 0 \\ -35.44 , & -5.124 \\ -0.238 , & -0.308 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \quad (37)$$

and

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 , & 0 , & 0 \\ 1 , & 0 , & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad (38)$$

where $x_1(t)$ is the change in pitch angle, $x_2(t)$ is the rate of change of pitch angle, $x_3(t)$ is the change in angle of attack, $u_1(t)$ is the elevator deflection, and $u_2(t)$ is the flaperon deflection. Hence, in the case $\{\sigma_1,\sigma_2\} = \{0.4,0.4\}$, $\gamma_i = 1/6 (i=1,...,5)$, $K_0^{-1}K_1 = \mathrm{diag}\ \{\rho_1,\rho_2\} = \mathrm{diag}\{2.5, 2.5\}$, and

$$M = \begin{bmatrix} 0.25 \\ 0 \end{bmatrix} , \quad (39)$$

it follows from equations (3), (5), (8), and (30) that the corresponding transducers and fast-sampling error-actuated digital controllers with finite time-delay compensation are governed on $\mathcal{T}$ and $\mathcal{T}_T$ by the respective measurement and control-law equations

$$\begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} = \begin{bmatrix} 1 , & 0.25 , & 0 \\ 1 , & 0 , & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad (40)$$

and

$$\begin{bmatrix} r_1(kT) \\ r_2(kT) \end{bmatrix} = f\left\{\begin{bmatrix} -0.051 & , & 0.211 \\ 0.039 & , & -1.462 \end{bmatrix}\begin{bmatrix} e_1(kT) \\ e_2(kT) \end{bmatrix} + \begin{bmatrix} -0.127 & , & 0.528 \\ 0.098 & , & -3.655 \end{bmatrix}\begin{bmatrix} z_1(kT) \\ z_2(kT) \end{bmatrix}\right\} - \frac{1}{6}\sum_{i=1}^{5}\begin{bmatrix} u_1\{(k-i)T\} \\ u_2\{(k-i)T\} \end{bmatrix} \quad . \tag{41}$$

It is then evident from equations (26), (27), and (28) that $Z_1 = \{1-2.5T, 1-2.5T\}$, $Z_2 = \{1-4T\}$, and $Z_3 = \{0.115 \pm 0.735i, -0.581 \pm 0.377i, 0.883 \pm 0.317i\}$. It is also evident from equations (23), (24), and (25) that the asymptotic transfer function matrix assumes the diagonal form

$$\Gamma(\lambda) = \begin{bmatrix} \frac{4T}{\lambda-1+4T} & , & 0 \\ 0 & , & \frac{30}{30\lambda^6-25\lambda^5+7} \end{bmatrix} \tag{42}$$

and therefore that the direct digital flight-mode control system for the fuselage pitch pointing manouevre of the aircraft, with a time delay of five sampling periods in the implementation of the control action, will exhibit increasingly non-interacting tracking behaviour as $f \to \infty$ when the piecewise-constant control input vector $[u_1(t), u_2(t)]^T = [r_1(kT), r_2(kT)]^T$ $t \in [kT, (k+1)T)$, $kT \in T_T$, is generated by the fast-sampling digital controller governed on T_T by equation (41).

4. Laboratory Microprocessor Implementation Studies

4.1 Apparatus

The laboratory apparatus for the study of the microprocessor implementation of such controllers consists of an EAI 180 analogue computer for the simulation of the linearised model of the plant (given in equations (31) and (32) or (37) and (38)), a microprocessor system based on the M68000 MPU for the computation of the controller equations, a data-acquisition system for data handling and processing between the controller and the plant and a variable hardware interrupt unit for accurate cycle timing.

The microprocessor system is based upon the Apollo 68000 Stand-Alone Computer module which provides the user with the means of efficiently interfacing the MPU to peripheral devices through an 8255 PPI and an 8250 VARI or directly via the bus, depending on the data format required. All user memory, 4K RAM, is contained within this module. The high-speed nature of the MPU, operating at 10 MHz, along with the sophisticated M68000 instruction set, enables efficient on-chip execution of the arithmetic manipulations.

The data-acquisition system consists of a high-level A/D module and an analogue output module arranged in a memory-mapped configuration. Both these modules have been custom designed for optimum speed, efficiency and compatibility with the processing module and contain the necessary latches, amplifiers, filters, and sample-and-hold and decoding devices.

4.2 Vertical Translation Manoeuvre

The simulated behaviour of the YF-16 aircraft in the vertical translation mode when controlled in accordance with equation (35) is shown in Figure 1 when the command input vector is 'ramped up' in 2 sec to the steady value $[v_1(t), v_2(t)]^T = [0,2]^T$ deg. In these tests, a sampling period of 0.02 sec is used and it is evident that high-accuracy non-interacting tracking behaviour is achieved despite the time delay of five sampling periods in the control action. Furthermore, it is apparent from Figure 1 that the vertical translation manoeuvre is effected without the use of excessive transient control surface deflections.

4.3 Fuselage Pitch Pointing Manoeuvre

The simulated behaviour of the YF-16 aircraft in the fuselage pitch pointing mode when controlled in accordance with equation (41) is shown in Figure 2 when the command input vector is 'ramped up' in 2 sec to the steady value $[v_1(t), v_2(t)]^T = [2,0]^T$ deg. In these tests, a sampling period of 0.02 sec is used and it is evident that high accuracy non-interacting tracking behaviour is achieved despite the time delay of five sampling periods in the control action. Furthermore, it is apparent from Figure 2 that the fuselage pitch pointing manoeuvre is effected without the use of excessive transient control surface deflections.

5. Conclusion

A direct digital control law algorithm has been proposed which, in addition to providing tight non-interacting tracking behaviour and excellent disturbance rejection characteristics, provides for the compensation of finite-time delays in implementation. Stability of the closed-loop system has been proved in the case of multi-input multi-output linear systems.

The theory has been illustrated by the presentation of simulation results in which the aircraft was represented by an analogue computer and the digital control system was implemented on a microprocessor. It was thereby shown that tight non-interacting control is achievable even when the finite time delay in implementation amounts to several sampling periods.

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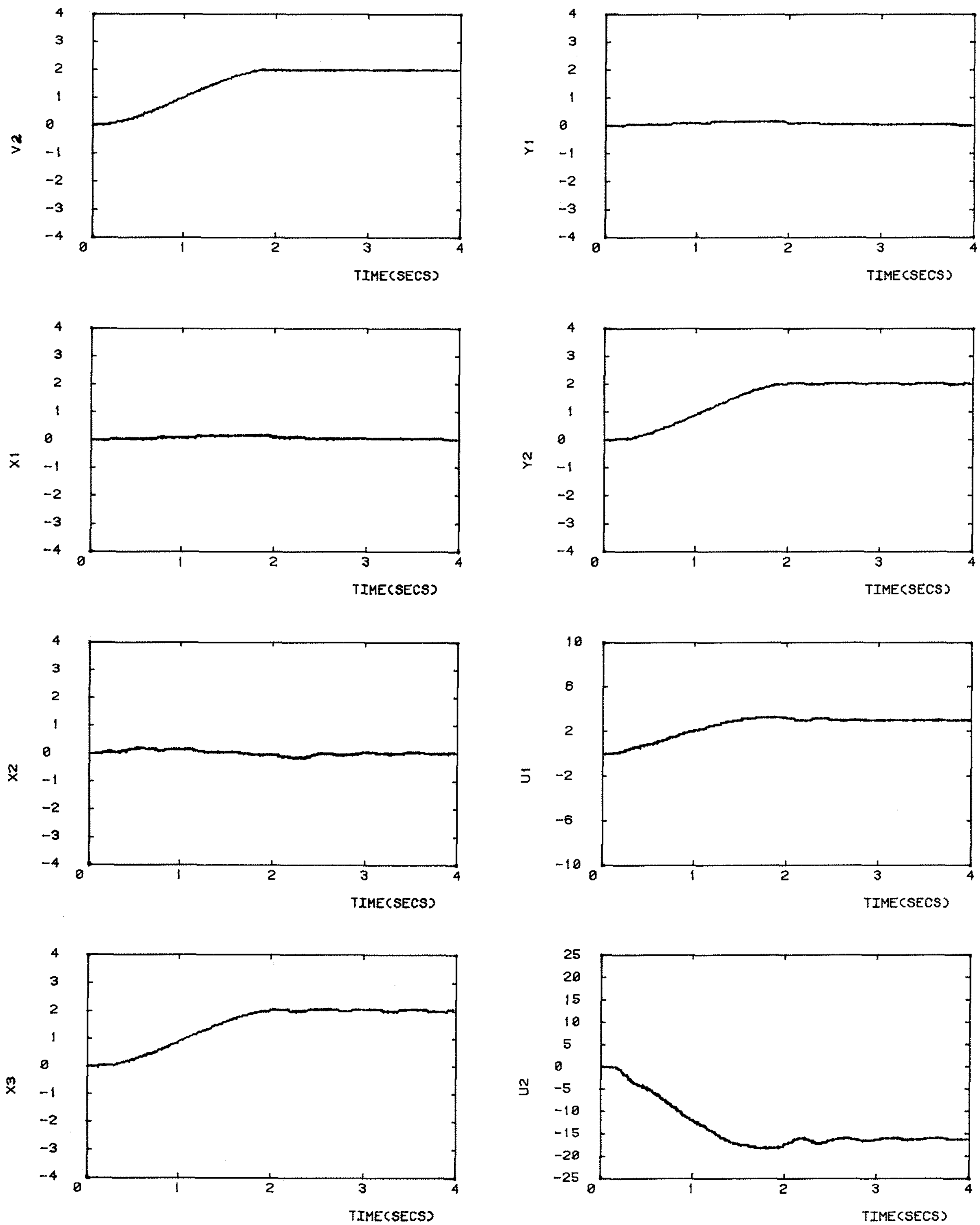

Figure 1 Vertical Translation Manoeuvre (Time Delay 0.1 seconds)

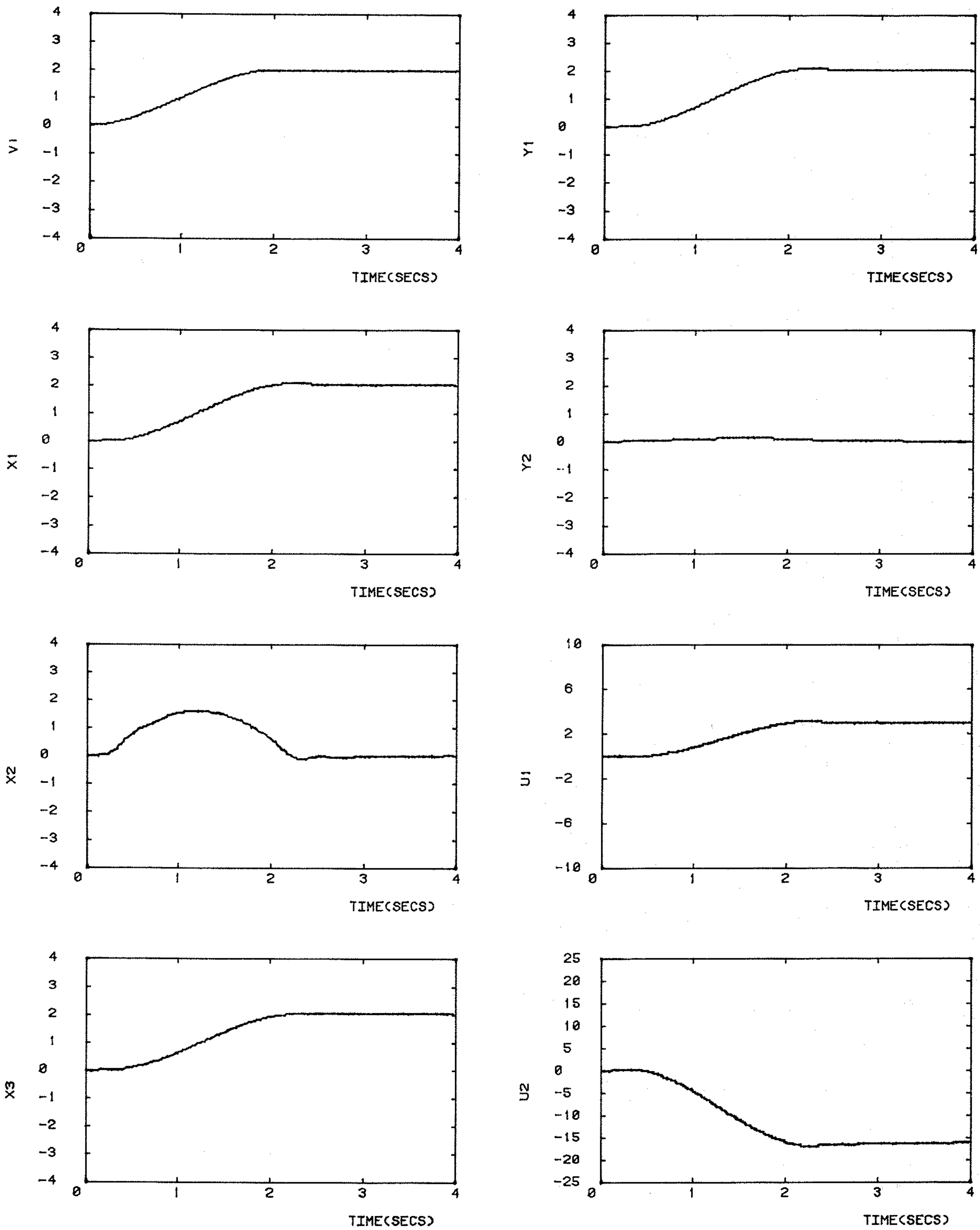

Figure 2 Fuselage Pitch Pointing Manoeuvre (Time Delay 0.1 seconds)