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Abstract

A system of non-linear differential equations describing quite general flow phenomena is presented and discussed from a kinematic point of view. Reducing the dynamic variables to one scalar pressure function and a velocity vector this approach is based on a concept of viscosity diverging from the usual definition. It does not proceed on local shear stresses but on the assumption that fluid friction depends on irreversible processes within the flow field. The equations admit discontinuities in shear velocity as well as compressibility effects. In the limit of incompressibility agreement with the EULER-equations is obtained. The numerical solution by an explicit finite difference method results in the simulation of unsteady flow fields and energy distributions. For example the flow over a step within an insulated system, around a wing profile with a flap and a circular cylinder has been investigated.

I. Introduction

This theoretical study of unsteady viscous flows is based on model equations, presented by the author in [1]. By means of these equations, complex fluid motions such as vortex formation and separation in a wake were simulated numerically in rather good qualitative agreement with visual and experimental evaluations. Characteristic of this approach is a concept of viscosity diverging from the classical one. Accordingly, shear viscosity as the relation between the tensors of stress and strain loses its principal significance, being restricted to boundary layer calculations from a more pragmatic point of view. Viscosity in this sense is less understood as a property of substance but as an irreversible phenomenon of the flow field in its entirety.

In a formal manner the model equations can be derived from the Navier-Stokes-equations and the energy-equation such that two requirements are met: on the one hand the number of the scalar variables 'pressure, density and temperature' must be reduced to only one by means of a barotropic approximation, and on the other hand the shear viscosity term $\mu \Delta \vec{v}$ is neglected unlike the bulk viscosity $\zeta \text{grad div } \vec{v}$. Though this second assumption has been proposed by VON NEUMANN-RICHTMYER in 1950 (see ROACHE [2]) in connection with artificial viscosity, it has to be considered unusual at least. Bulk viscosity is said to be small in general, vanishing altogether in incompressible and monoatomic fluids. There is, however, some uncertainty about the realistic contribution of bulk viscosity, because periods of relaxation and integrals of collision are difficult to measure. As known from the literature

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ture, the bulk viscosity coefficient can take high values well enough. Particularly if high frequencies are involved - such as in the case of sound waves - the bulk viscosity coefficient can exceed considerably that of shear viscosity - see LANDAU-LIFSHITZ [3]. Rapid changes in the variables of state by compression and dilatation effect disturbances of the thermodynamic equilibrium so that innermolecular processes take place resulting in energy dissipation. Nevertheless it is unsatisfactory to substantiate the influence of bulk viscosity based on conventional theory, for high temperatures, MACH-numbers or frequencies are unalterable.

It is possible that better arguments for this approach result from an extreme point of view, which questions the necessity of the fundamental shear stress model at all. That is why this is called a kinematic approach. If shear stresses - and consequently forces in general - and mass are banished from the basic equations of motion, the dynamic variables may reduce to kinematic quantities such as a velocity vector and one scalar variable representing internal energy per mass or heat or a 'pressure function'. Moreover this approach satisfies the demand of irreversibility to avoid thermodynamic equations of state strictly valid only for equilibrium. An inconsistency with the conservation of mass is not seen; the equation of continuity can be ensured - only with the restriction that it be a necessary but insufficient condition of motion. Details of this interpretation will be discussed after the model equations have been presented and illustrated by examples showing an amazing simplicity and universality.

II. The concept of viscosity

In fluid mechanics the concept of viscosity is closely associated with the existence of shear stresses. In view of this essential fact the well-proved NEWTON-STOKES hypothesis of a linear relation between the shear stress components and the gradients of the velocity components is of secondary importance. The existence of shear stresses, effective on an element of volume 'large enough to contain many molecules but small enough to be used as an element of integration' (see TRUESDELL/TOUPIN [4] and their criticism) is an axiom. Nevertheless it is not quite clear whether the shear stresses are the 'cause' of friction in a fundamental sense. Even PRANDTL, father of the boundary layer theory using the concept of shear stresses, reduces turbulent and 'true' friction ultimately to the transport of momentum by molecular motion [5]. LAGALLY [6] goes beyond this. For him shear stress in a boundary layer is a 'consequence' of friction. He says: 'The assumption that vortices occur

in the boundary layer due to friction against the wall surface leads to a mathematical discrepancy. On the contrary, it must be assumed that a vortex layer present at the wall causes friction, which, in order that the assumption be in agreement with the actual physical phenomena, cannot be regarded as negligible.' Accordingly it is a question of cause and effect. Does the friction by shear stresses imply vortex layers or does a vortex layer produce shear stresses ?

Perhaps an indication of the founding mechanism may result from the theory of 'fractals', originating from MANDELBROT [7]. In this sense 'roughness' of a wall should be a quality in principle, according to which smooth surfaces of solid bodies are fictional - produced ultimately by length scale averaging. The physical reality behind this would be the 'fractal' - microscopic or macroscopic - structure of matter responsible for the formation of vortices in front of or behind obstacles, whatever order of magnitude. Of course, the reference to the idea of 'fractals' is inconclusive. Perhaps disturbances inherent to real flows will have similar effects on boundary layers.

After all strain and rotation phenomena could be considered from this point of view as a product of normal stresses and body shapes only, while shear stresses inside a fluid seem to reduce to a more or less 'useful model'. These reflections suggest that bulk viscosity be thought of as the essential quantity representing the effect of normal stresses in compressible fluids.

A kinematic view goes beyond this, eliminating the concept of stress altogether and retaining pure geometric relations between velocity components and internal energy. Possibly the main advantage will be to recognize discontinuities in shear velocities, because a tear-off of the flow behind an edge for example does not result automatically in infinite shear stresses as $\tau \equiv du/dy$ would necessitate. This would explain the high inner mobility and instability of fluids leading to problems in defining steady state conditions. In contrast to the damping nature of the LAPLACE-operator, the vector-operator 'grad div' is destabilising. A more physical interpretation of bulk viscosity would lead to questions of relaxation, dissipation or other irreversible dynamic processes.

III. A formular approach

Equations of motion

The unsteady Navier-Stokes equations for a compressible medium subject to friction, ignoring volume forces and using the summation convention, are :

$$\rho \frac{dv_i}{dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \right) - \frac{2}{3} \frac{\partial}{\partial x_i} \mu \frac{\partial v_j}{\partial x_j} + \frac{\partial}{\partial x_i} \zeta \frac{\partial v_j}{\partial x_j}$$

μ = shear viscosity, ζ = bulk viscosity (1)

Separation into shear viscosity and bulk viscosity is not dealt with in a uniform manner. In the present paper, the term 'bulk viscosity' is limited to terms with the second viscosity coefficient ζ , i.e. in equation (1) to

$$\frac{\partial}{\partial x_i} \left[\zeta \frac{\partial v_j}{\partial x_j} \right]$$

This term is also defined as 'pressure viscosity', while 'bulk viscosity' then includes

$$-\frac{2}{3} \frac{\partial}{\partial x_i} \left[\mu \frac{\partial v_j}{\partial x_j} \right]$$

Equation (1) is made nondimensional in the conventional manner using a characteristic length and the incident flow quantities. In general, the viscosity coefficients are not field constants. The reference quantity can, for example, be μ_0 , so that with $\zeta = \mu_0 \zeta^*$ (* = nondimensional) only Re_μ appears or $\zeta = \zeta_0 \zeta^*$ is used as well as $\mu = \mu_0 \mu^*$ so that Re_ζ also appears as a second Reynolds number. Thus, considered from the nondimensional point of view, the shear viscosity term in (1) is of order $1/Re_\mu$ while bulk viscosity is of order $1/Re_\zeta$ or $\zeta_0 / (\mu_0 Re_\mu)$.

According to the assumption that remote from thermodynamic equilibrium - at high Reynolds numbers - the ratio of the instant flow viscosities $\zeta_0 / \mu_0 \gg 1$, shear viscosity can be neglected in a first approximation. Equation (1) then reduces in nondimensional form to :

$$\rho \frac{dv_i}{dt} = -\frac{\partial p}{\partial x_i} + \frac{\zeta_0}{\mu_0} \frac{1}{Re_\mu} \frac{\partial}{\partial x_i} \left[\zeta \frac{\partial v_j}{\partial x_j} \right]$$

Energy equation

Neglecting the terms associated with shear viscosity, only the quadratic term $\zeta (\text{div } \vec{v})^2$ of the dissipation function remains in the energy equation. The shortened energy equation is then, in dimensional form :

$$\rho \frac{du}{dt} = \frac{\partial}{\partial x_j} \left[\lambda \frac{\partial T}{\partial x_j} \right] - p \frac{\partial v_j}{\partial x_j} + \zeta \left[\frac{\partial v_j}{\partial x_j} \right]^2 \quad (2)$$

with internal energy u , temperature T , pressure p and heat conductivity λ . An equation for T or p then follows if an ideal gas is assumed. The following then apply :

$$p = R \rho T$$

and
$$du = c_v dT .$$

Using the PRANDTL number formed from the specific heat c_v

$$Pr = \mu_0 c_v / \lambda_0 ,$$

the ECKERT number

$$Ec = U_0^2 / c_v T_0$$

and the gas constant R, the nondimensional equation for temperature, for example, is found to be :

$$\rho \frac{dT}{dt} = \frac{1}{Pr \cdot Re_\mu} \frac{\partial}{\partial x_j} \left[\lambda \frac{\partial T}{\partial x_j} \right] - (\kappa - 1) \rho T \frac{\partial v_j}{\partial x_j} + \frac{\zeta_o}{\mu_o} \frac{Ec}{Re_\mu} \zeta \left[\frac{\partial v_j}{\partial x_j} \right]^2$$

In conjunction with the continuity equation, the ideal gas law and the equation of motion, there is a closed system of equations for the variables \tilde{v} , p , ρ and T , provided a functional connection or another relationship is known for the bulk viscosity.

IV. Model equations

Assuming an ideal gas and making the further assumption of barotropy, i.e. $\rho = \rho(p)$ and $\text{grad } \rho \ll \text{grad } p$, the momentum and energy equations can be reduced to four variables - the velocity \tilde{v} and a pressure function $\phi(\tilde{x}, t)$.

Using the kinematic coefficients

$$\nu = \frac{\mu}{\rho}, \quad \Theta = \frac{\zeta}{\rho} \quad \text{and} \quad \Lambda = \frac{\lambda}{\rho}$$

which can now be considered as material constants, equations (1) and (2) give :

$$\frac{d\tilde{v}}{dt} = \Theta \text{ grad div } \tilde{v} - \text{grad } \phi \quad (3)$$

$$\frac{d\phi}{dt} = \varepsilon \text{ div grad } \phi - \phi \text{ div } \tilde{v} + \Theta (\text{div } \tilde{v})^2 \quad (4)$$

where $\varepsilon = \Lambda/R$. (the nondimensional proportionality factor $\kappa - 1$ is omitted here in order to make the arrangement of the equations clear)

Further reduction of equation (4) occurs if $(\text{div } \tilde{v})^2$ is neglected since it is small in a higher order compared with $\text{div } \tilde{v}$. After combination of the term $\phi \text{ div } \tilde{v}$ with the convective term $\tilde{v} \cdot \text{grad } \phi$, the following is obtained

$$\frac{\partial \phi}{\partial t} + \text{div } \phi \tilde{v} = \varepsilon \Delta \phi \quad (5)$$

i.e. a relationship which can be understood as the conservation equation for the quantity ϕ , extended by a diffusion term.

The energy equation (5) can be obtained as well by proceeding from a generalized Fourier equation for a moving medium. It reads :

$$\frac{dT}{dt} = a\Delta T + \frac{q_v}{c\rho} \quad (6)$$

with temperature T , temperature coefficient a , specific heat c , density ρ and inner sources of heat q_v . Setting these sources per mass proportional to $-\phi \cdot \text{div } \tilde{v}$ produces (5). In this way no further assumptions like that of ideal gas are needed.

It should be noted that equation (5) has a form complementary to that of the equation of motion (3). This becomes clear if the system of equations is reduced to an arrangement showing the essentials - taking no account of dimensional correctness, parameters and material property quantities. The arrangement indicates a feedback between gradients and sources. Using the further simplification of $\phi \text{ div } \tilde{v} \Rightarrow \text{div } \tilde{v}$, it becomes :

$$\frac{d\phi}{dt} = \text{div} (\text{grad } \phi - \tilde{v})$$

$$\frac{d\tilde{v}}{dt} = \text{grad} (\text{div } \tilde{v} - \phi)$$

The expressions in parentheses contain, on the one hand, the vector field composed of the acceleration and velocity fields and, on the other, the scalar field consisting of the potential from the acceleration and source strengths of the velocity field. The physical interpretation is that the space/time changes in the potential and velocity fields are equal to the sources and gradients of the 'true' fields, which are determined by the potentials and state of movement.

The complete system of equations is now :

$$\frac{d\phi}{dt} = \varepsilon \text{ div grad } \phi - \phi \text{ div } \tilde{v} \quad (*)$$

$$\frac{d\tilde{v}}{dt} = \Theta \text{ grad div } \tilde{v} - \text{grad } \phi$$

with the material properties ε, Θ to be determined by experiment.

The continuity equation is not required in this model. It can, however, be used to integrate the density over the velocity field, thus ensuring the conservation of mass.

The boundary conditions, which are dealt with in more detail below comprise the assumption of insulated walls with $\partial \phi / \partial n = 0$ (n = direction normal to surfaces) and the no-slip condition $\tilde{v}_w = 0$, which can be satisfied by virtue of the second-order terms.

The numerical method used is very simple. The model differential equations (*) are approximated with reference to the spacial coordinates in a central finite difference method (second order) for the first and second derivatives. The development with time occurs explicitly in a single-step finite difference method (first order). The step widths in space and time are selected such that the Courant-Friedrich-Levy condition is satisfied. A dependence of the computed results on the step widths has not been found.

V. Reduction of scalar variables

The approximation of barotropy was used in [1] to reduce the number of scalar variables to only one. Instead of pressure, density and temperature coupled by an equation of state, a 'pressure function' equivalent p/ρ was introduced. The reason for this simplification might be physical. Quantities of state like temperature are strictly defined only for thermodynamic equilibrium. To define them for irreversible processes like fluid motion is doubtful. Thus GOLDSTEIN [8] points out that the physical meaning of thermodynamic pressure p in the general relation

$$p_{ij} = \delta_{ij} (-p + \zeta \operatorname{div} \tilde{v} - \frac{2}{3} \mu \operatorname{div} \tilde{v}) + \mu e_{ij} \quad (7)$$

e_{ij} = components of the rate-of-strain tensor

is not clear. Except for thermodynamic equilibrium, p is not identified with an average normal pressure $p_{ii}/3$ or something similar. For this reason GOLDSTEIN proposes to neglect (7) and to favor the internal energy or heat as the primary variable independent from thermodynamic equilibrium contrary to the various modes of energy, i.e. translational, rotational and vibrational energy.

With the introduction of internal energy or the equivalent pressure function ϕ here, reversibility is however not banished totally from dynamics. GOLDSTEIN himself emphasizes: 'We want, if we can, to use the usual equilibrium reversible thermodynamics'. This is not quite clear. To use equations of state for non-equilibrium is contradictory. Nevertheless GOLDSTEIN defines by means of density ρ and an equation of state hypothetic 'pressures' and 'temperatures', conceding that the physical meaning of these calculated quantities is questionable at least.

This inconsistency in principle can be avoided only if the basic system of the equations of motion do not contain equations of state at all. Consequently this leads to only one scalar variable in non-equilibrium.

VI. Boundary conditions

Solid walls

For reasons of simplification only the velocity component u in x -direction may be considered. Then the second order derivative

$\frac{\partial^2 u}{\partial y^2}$ characterising shear viscosity shall be replaced comparatively by the bulk viscosity

$$\text{term } \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial x} \right) \equiv \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial y} \right)$$

The latter term is obviously dependent on the shape of solid surfaces. In parallel flow along

plane walls, $\frac{\partial}{\partial x} \left(\frac{\partial u}{\partial y} \right)$ approximates 0, even if $\frac{\partial u}{\partial y}$

will not be small at the wall because of the no-slip condition $u(\text{wall}) = 0$.

Thus, as confirmed by calculations, the actual choice of the tangential wall condition is rather insignificant. As the x -derivative vanishes in such parallel flows along surfaces, a more or less slip-flow is obtained in any

case. On the contrary an effect of $\frac{\partial^2 u}{\partial x \partial y}$

will be evident, if edges, nooks or other uneven contours exist. At these exposed regions even a slight compressibility of a fluid will have consequences - retarding the flow near walls and stimulating vortex formation. In this sense shear viscosity could be regarded as a time-space-averaged microscopic bulk viscosity.

Free boundaries

Free boundaries characterise open systems exchanging energy, momentum and mass with their environment. In numerical experiments these boundary conditions have to be prescribed though they are unknown in principle. Moreover, the assumptions of steady state conditions - e.g. at the influx and outlet of a simulated channel - will generally lead to an unrealistic evolution of the inner flow field. In modern theory of non-linear systems the sensitivity to small changes in the initial or boundary conditions is well known, which is what we notice here. In numerical experiments the interaction of the field with the boundaries is so intense that a steady flow initiated by free boundary conditions could be produced only approximately. Either the friction is too weak resulting in an acceleration of the flow or too strong with the consequence of rather stable vortices blocking up the channel or similar effects. For this reason insulated closed systems are now taken into consideration, after initial calculations have been performed with a global, constant pressure gradient as boundary condition tolerating an acceleration of the flow.

VII. Insulated systems

The choice of insulated systems has several advantages. To remove free boundaries is only one aspect. More important is the fact that a concept of viscosity, as mentioned above, characterised by irreversibility requires insulated systems in principle. For computing purposes it is useful then to proceed from two fictitious reservoirs separated by a diaphragm which bursts at time $t=0$. For $t<0$ we have thermodynamic equilibrium inside the separate reservoirs. With equations of state valid for equilibrium, initial values of pressure p , temperature T and density ρ can be defined. The quantities at rest - e.g. of the first reservoir - are the reference variables.

The pressure function ϕ_0 at rest may be identified now as temperature, 'kinematic' pressure p_0/ρ_0 or velocity of sound analogous to $a_0^2 = (\kappa p_0/\rho_0)$ with the ratio of specific heat $\kappa = c_p/c_v$.

If $\phi_0^I = (p_0/\rho_0)^I$ is chosen the essential dimensional reference quantity related to reservoir I and L is a characteristic length of the system, the model equations can be written in the nondimensional form :

$$\frac{d\phi}{dt} = \frac{\varepsilon}{Lv\sqrt{\phi_0^I}} \operatorname{div} \operatorname{grad} \phi - \phi \operatorname{div} \tilde{v}$$

$$\frac{d\tilde{v}}{dt} = \frac{\theta}{Lv\sqrt{\phi_0^I}} \operatorname{grad} \operatorname{div} \tilde{v} - \operatorname{grad} \phi$$

with the relations to the physical quantities, marked by +

$$\tilde{x}^+ = L \cdot \tilde{x}, \quad t^+ = \frac{L}{\sqrt{\phi_0^I}} \cdot t, \quad \tilde{v}^+ = \sqrt{\phi_0^I} \tilde{v}, \quad \phi^+ = \phi_0^I \phi$$

More reasonable, however, is to introduce a characteristic time τ of the system as a second reference quantity. This time τ may be the period of eigen-oscillations of the closed system. Then the relations to the dimensional quantities (+) read :

$$t^+ = \tau t, \quad \tilde{v}^+ = \frac{L}{\tau} \tilde{v}, \quad \phi^+ = \left(\frac{L}{\tau}\right)^2 \phi$$

Moreover this approach corresponds to the view of friction depending on global parameters of the flow field system.

VIII. Test calculations

The flow over a step

Initial computational results concerning the flow over a backward-facing step, investigated by a great number of scientists (see TROPEA[9]) were reported in [1]. Though qualitatively satisfying, a quantitative comparison with experiments must fail, because the flow was accelerating due to a constant global pressure gradient in the simulated channel. Now the problem of unknown free boundary conditions has been discussed resulting in a restriction to insulated systems. Consequently an insulated 3-dimensional system was simulated numerically. Figure 1 shows a general view of two reservoirs with different initial conditions (high-pressure function on the left) connected by a channel containing a step. At time $t=0$ an imaginary diaphragm bursts at the influx of the channel accelerating the flow to a maximum mean velocity (related to the small cross section in front of the step).

Of course, the main flow parameter as the reattachment length and the wall pressure coefficient behind the step depend on the instantaneous state of the flow, but concrete, realistic conditions can be defined now for a later comparison with experiments starting from a state of rest.

Figure 2 shows the reattachment length, made nondimensional by the step height, as a function of a mean (nondimensional) velocity

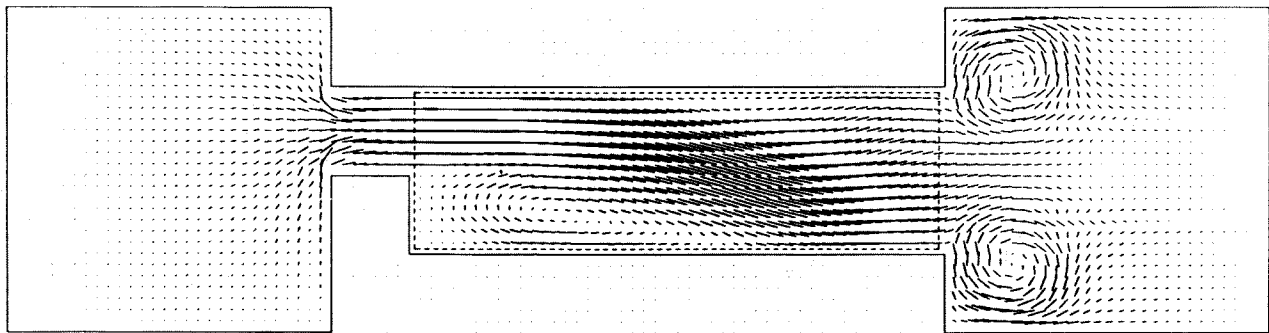


Fig. 1 Flow simulation over a step in a closed system (velocity vectors)

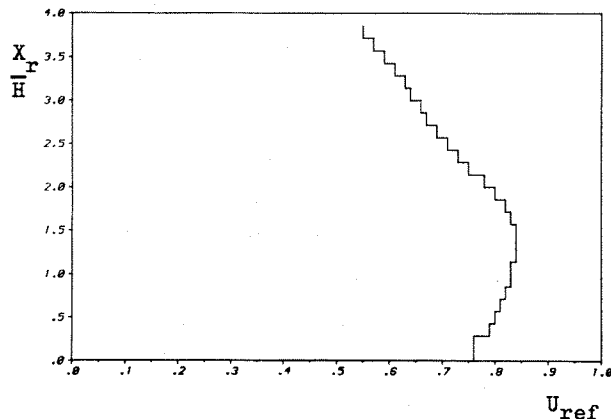


Fig. 2 Reattachment length as a function X_r/H (H =step height) of a ref. velocity U_{ref}

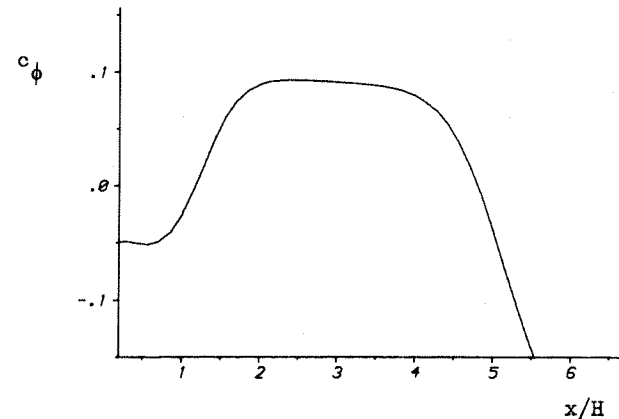


Fig. 3 c_ϕ as a function of distance x/H behind the step

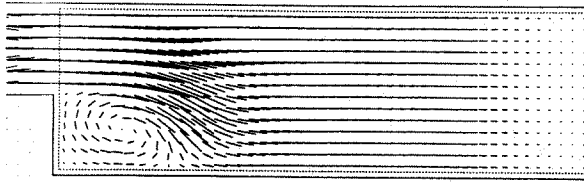


Fig. 4 Velocity field behind the step

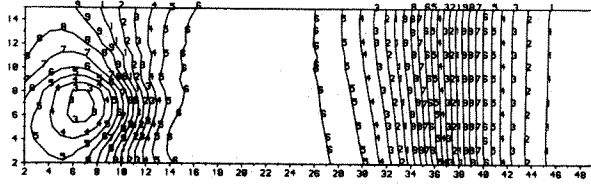


Fig. 5 Isolines of ϕ (correspond. Fig.4)

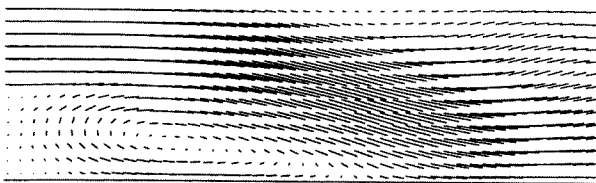


Fig. 6 Velocity field (see Fig.1)

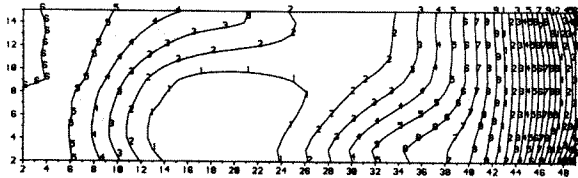


Fig. 7 Isolines of ϕ (correspond. Fig.6)

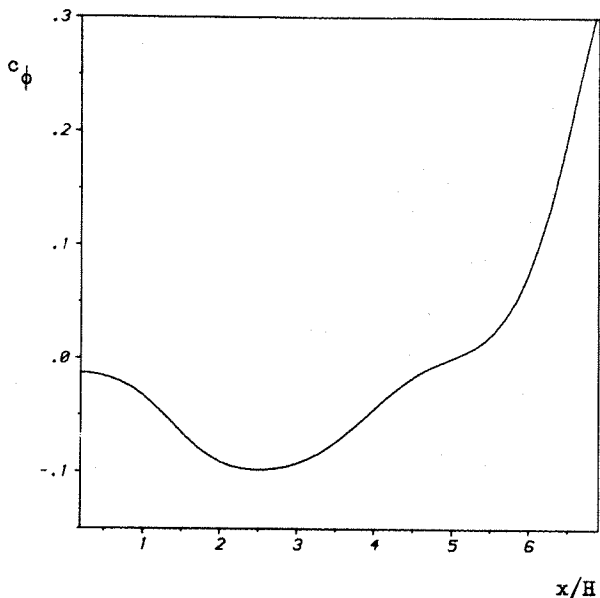


Fig. 8 Wall pressure function c_ϕ (corresponding Fig. 6,7)

at the influx of the channel. It is seen that the reattachment length is just increasing though the mean velocity has already exceeded its maximum. The interaction of the entire flow field shows how difficult it is to obtain approximately steady flow conditions in numerical experiments as well. In principle theorists can at best simulate what an experimental engineer can do in reality - but nothing more. Therefore free boundary conditions should be avoided if possible.

Figures 3-5 refer to the above-mentioned maximum mean velocity where the flow behind the step is considered to be quasi-steady. Here the characteristic increase of the coefficient of wall pressure c_p can be found, e.g. as measured by TANI [10].

The comparative coefficient

$$c_\phi = (\phi_{\text{wall}} - \phi_{\text{ref}}) / \phi_{\text{ref}}$$

with ϕ_{ref} = the mean pressure function at the channel influx is reproduced in Figure 3. Figure 4 shows the flow simulation of the section behind the step and Figure 5 the lines of constant pressure function (called 'isolines' in the following). The closed circular lines represent the vortex behind the step as a region of small pressure function. Figure 6 corresponds to the flow of Figure 1.

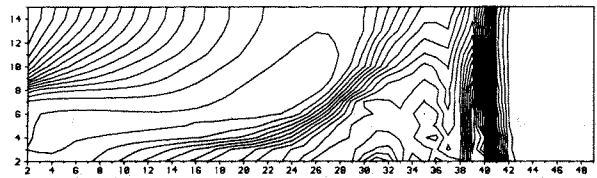


Fig. 9 Shock isolines

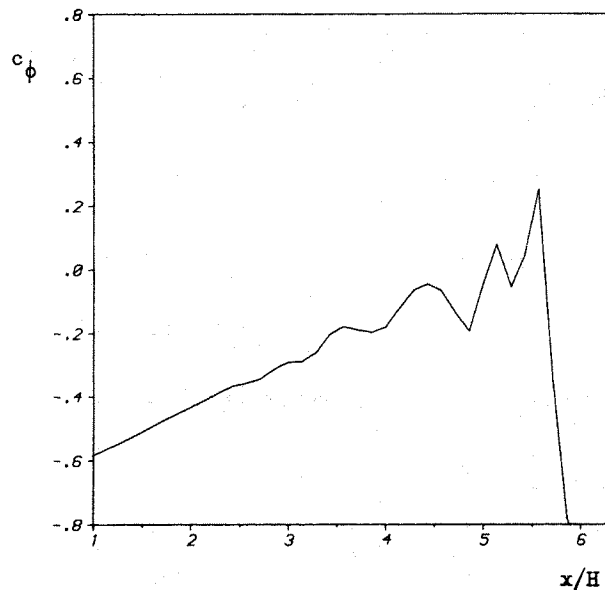


Fig. 10 Wall pressure function c_ϕ (corresponding Fig. 9)

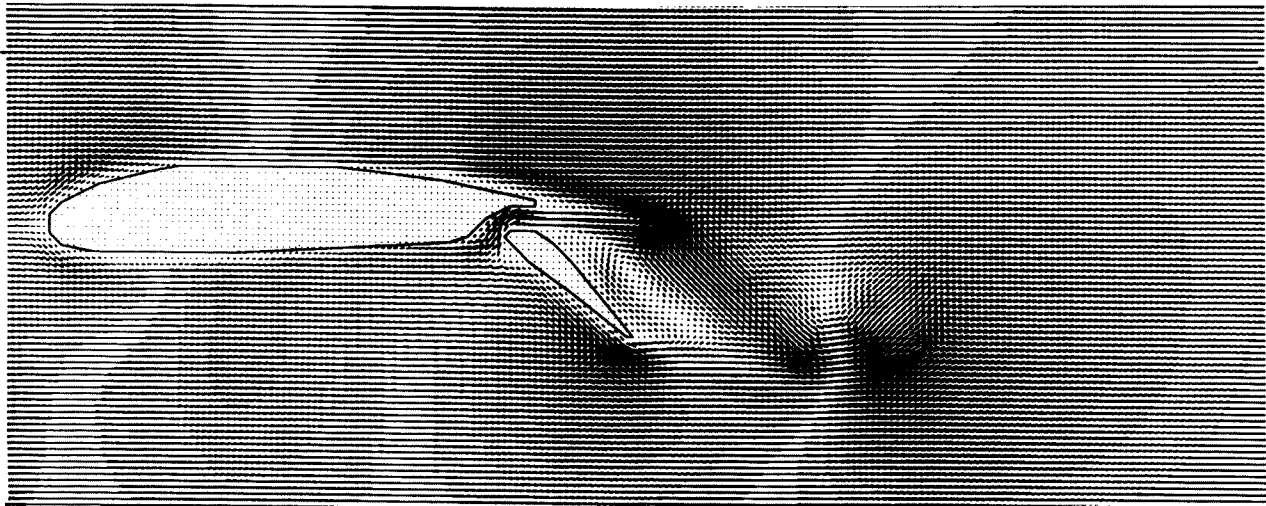


Fig. 11 Velocity field simulation

Here an increase of ϕ at the right-hand side of the flow section is legible from the plot of isolines (Figure 7) and of coefficient c_ϕ of the wall pressure function (Figure 8).

If ϕ_0^I at rest is increased by a factor 10, a strong shock is running into the channel. Behind the shock front (Figure 9) a splitting of the shock and fluctuations of c_ϕ at the wall (Figure 10) are observed.

Airfoil with flap

In comparison to a wing profile measured and calculated by STEINBACH (DFVLR) on the basis of potential and boundary layer theory, first qualitative results are obtained. Figure 11 gives a general view of the profile configuration, computed by equations (*). The time-independent method of STEINBACH is rather successful except for the wake behind the small flap. Here experimental results indicate reverse flow forming a vortex. Figure 12 shows the time-averaged coefficient c_p measured by STEINBACH.

By means of the model equations the test calculations yield an unsteady vortex flow (with time-independent free boundary conditions) illustrated by the simulated velocity field and the isolines of the function ϕ behind the flap (Fig.13-16). The time-shifted isoline plots show the drift of the vortices quite clearly as well as the emerging and fading away of regions of high and low pressure functions. To a certain degree Figure 13 agrees with the measurements of STEINBACH (Fig. 12).

Circular cylinder

The simulation of a starting flow around a circular cylinder (Figure 17) shall demonstrate that the mechanism of developing symmetric vortices may be different from a drifting single vortex or vortex street. The isolines

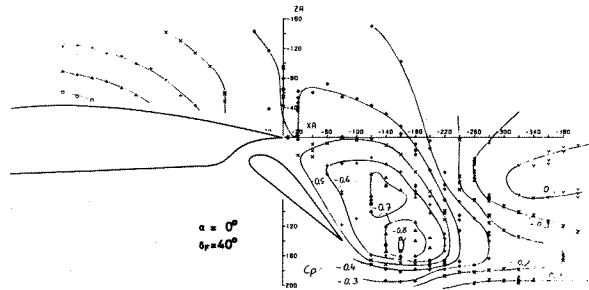


Fig. 12 c_p measured by STEINBACH, $Re=3.0 \cdot 10^6$

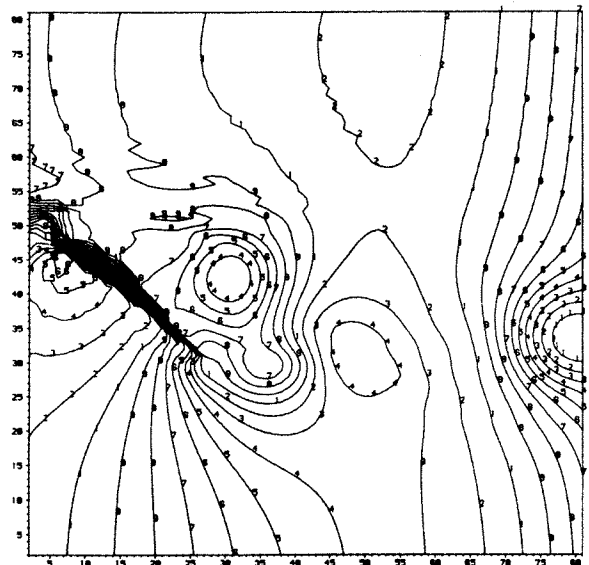


Fig. 13 ϕ -isolines (time-dependent)

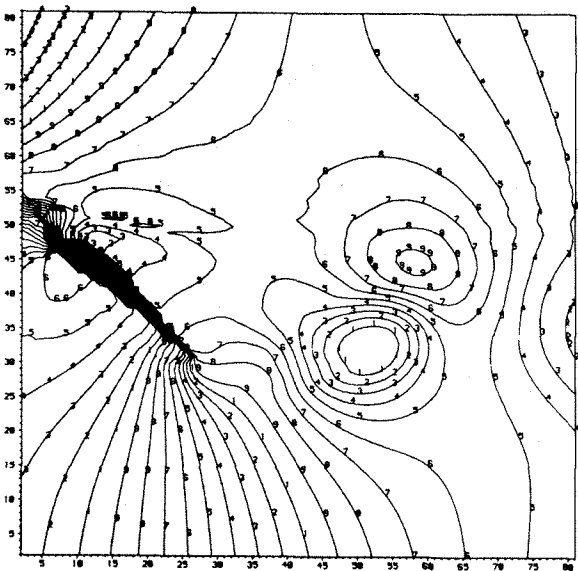
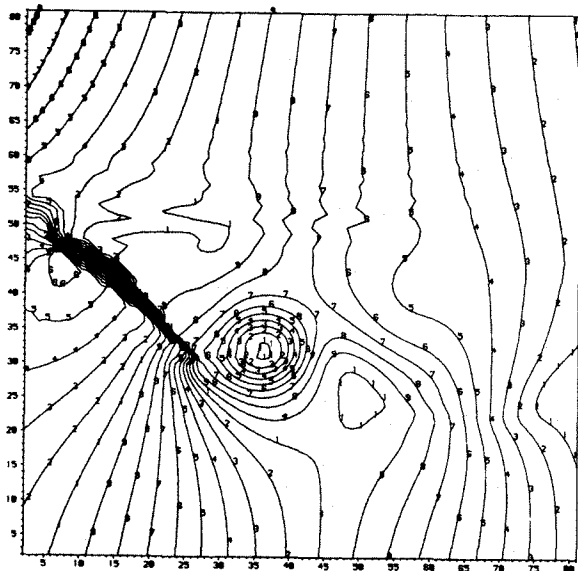
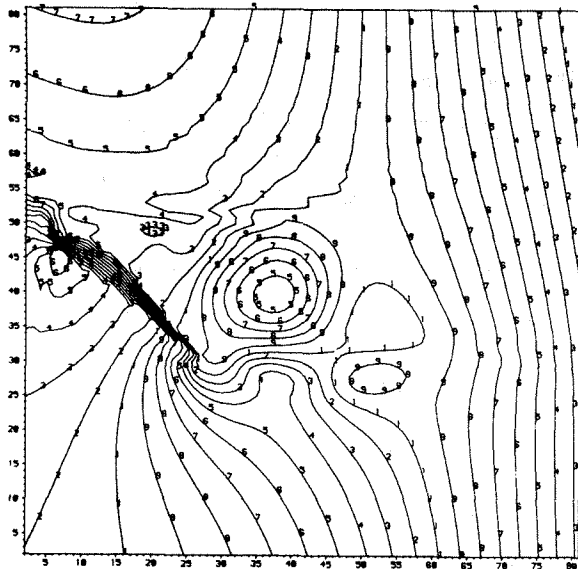


Fig. 14-16 Isolines of ϕ (time-dependent)

show only one region of low ϕ instead of two (Figure 18). This combination seems to be unstable.

If the responsible global pressure gradient is very low, a creeping flow without reverse flow is obtained.

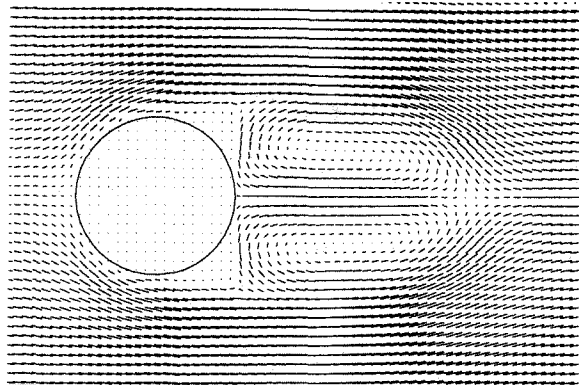


Fig. 17 Starting vortices behind a cylinder

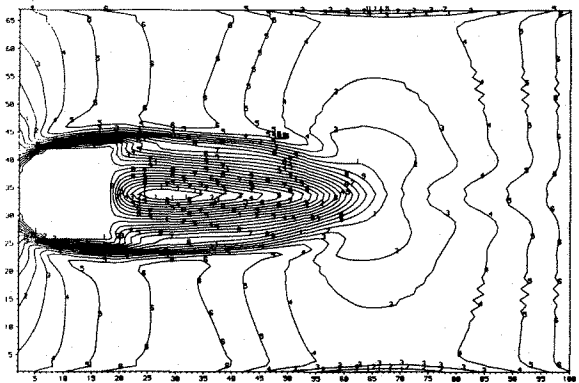


Fig. 18 Isolines of ϕ (correspond. Fig.17)

IX. Turbulence

Realising that equations (*) cover in principle different flow phenomena - from laminar flow to unsteady vortex separation - the relevancy of these equations to turbulence must be sought. Thinking of the universal character nonlinear dynamic systems may have, including steady, periodic and chaotic solutions, we cannot deny a priori this possibility. This premise is supported by the evidence of fluctuations of the velocity vector in critical zones like mixing layers, if a reference velocity exceeds a threshold value (Figure 19). Of course, it is a problem to separate physical from numerical effects here. As calculations have shown, there is a sudden increase in numerical instability reaching critical conditions. But the calculations seem to confirm as well, that minimising the time-step (intensifying the CFL-condition) does not completely prevent the development of fluctuations. Obviously there is a correlation between the decrease of vortices and the increase of fluctuations in vortex vicinity.

Another argument results from the charac-

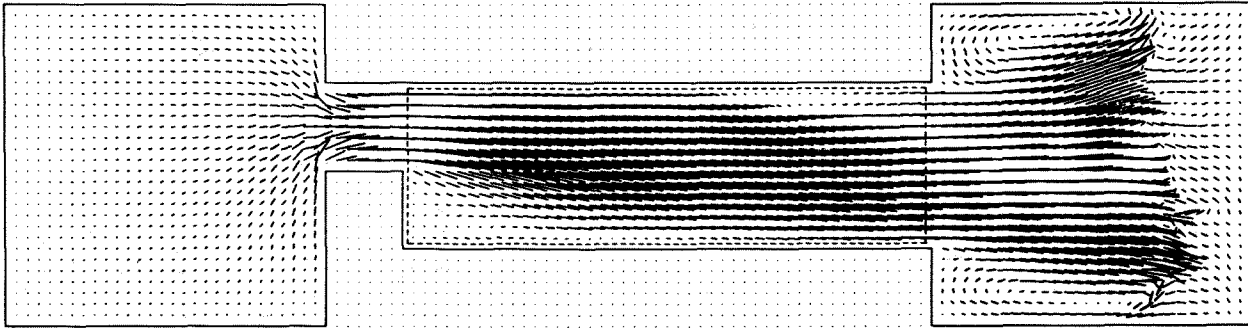


Fig. 19 Flow over a step with developing (turbulent ?) fluctuations in the mixing zone (flow just reflected at the right border)

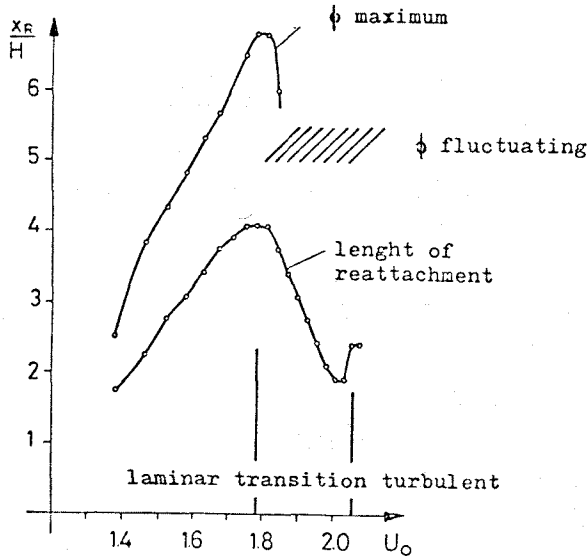


Fig. 20 Length of flow reattachment X_R/H as a function of a nondimensional reference velocity U_0

teristic dependence of the reattachment length behind a step on the mean flow velocity, as shown in Figure 20, that originates from [1]. Compared to measurements the range of velocity is however rather small because of the accelerating flow conditions in that calculation.

An interesting aspect results from Figures 13 f./18 which show the isolines of the pressure function. Remarkably, a fluctuation or breaking off of the isolines is noted in the regions of transition between closed isolines and those ending on solid walls. On the other hand the space between these neighbouring isolines is rather large, characterising a low gradient of ϕ . This seems to be a region of instability where branching of the field solutions may occur. Possibly the critical mixing zones correspond to zones of developing turbulence.

Outlook

This theoretical study results lastly in an elimination of concepts connected with 'an element of fluid of finite volume': the assump-

tion of local thermodynamic equilibrium, of causal shear stresses and of forces consequently. Mass appears as a fluid parameter only, hidden in the coefficients of substance ϵ, θ . For this reason this theory is a kinematic one, describing a fluid field by purely geometric relations between the internal heat (pressure function) and the velocity. Within the limit of incompressibility, agreement with the EULER-equations is reached. 'Viscosity' is understood as an irreversible phenomenon of the flow field in its entirety, enforcing the consideration of insulated systems. Equivalently the basic equations are dissipative.

A more general form of equations (*) can be obtained if the coefficients of substance ϵ, θ are factored out. Writing

$$\epsilon = a[L^2/t] \cdot \epsilon_{rel}$$

$$\theta = a[L^2/t] \cdot \theta_{rel}$$

with $\epsilon_{rel}, \theta_{rel}$ nondimensional, a of the dimension $(\text{length})^2/\text{time}$

the relations of material

$$\psi = \epsilon_{rel} \phi$$

$$\tilde{w} = \theta_{rel} \tilde{v}$$

can be separated. The dimensional quantity 'a' should be an appropriate constant of nature, analogous to the velocity of light c normally introduced into the Maxwell equations. Here a quantity of the proper order of magnitude and physical meaning would be

$$\hbar/m_e \approx 1.15 \text{ cm}^2/\text{sec}$$

(\hbar = Planck's constant/ 2π ; m_e = mass of electron).

On a molecular level friction would then be considered an exchange of energy quanta by interaction between shell electrons. Following this reasoning equations (*) read :

$$\frac{d\phi}{dt} = \frac{\hbar}{m_e} \Delta\psi - \phi \text{ div } \tilde{v}$$

$$\frac{d\tilde{v}}{dt} = \frac{\hbar}{m_e} \text{grad div } \tilde{w} - \text{grad } \phi$$

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