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Improving the Efficiency of Smaller

Transport Aircraft

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IMPROVING THE EFFICIENCY OF SMALLER TRANSPORT AIRCRAFT

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ABSTRACT

In this Guggenheim Lecture we return to an old theme, the high-altitude airplane. Considered apart from its propulsive system we find the airplane itself adapted to higher flight altitudes than those in current use. Scaling on the assumption of constant aircraft density indicates that this conclusion applies most importantly to smaller transport aircraft. Climb to 60,000 ft could save time and energy for trips as short as 500 miles.

The paper concludes with a discussion of the effect of winglets on aircraft efficiency. It is found that a 10% reduction of induced drag below that of a comparable elliptic wing can be achieved either by horizontal or vertical wing tip extensions.

INTRODUCTION

The fuel consumption of most vehicles increases disproportionately when we try to increase their speed by simply installing more horsepower. The airplane differs from earthbound vehicles in this respect, however, since by climbing to high altitudes it can increase its speed without necessarily increasing its fuel consumption per mile of flight. Thus a 747 cruising in the thin air at 40,000 ft can achieve about the same specific fuel economy as a small automobile while traveling 10 times as fast.

In 1929, V. B. Korvin Kroukovsky, one of America's most distinguished aeronautical engineers, published a series of articles in the magazine Aviation entitled "The High Altitude Airplane." In that discussion he pointed out that high economical cruising speeds could be obtained by an ordinary low-speed airplane if its engine could be adapted to flight at very high altitudes. I like to illustrate Korvin Kroukovsky's concept by applying it to a small airplane such as a Piper Cub.[1] At sea level the Cub might cruise at 100 mph and at a lift/drag ratio of 10:1. Let us suppose the Cub to be equipped with an engine capable of supplying one-tenth of its weight in thrust at any altitude. Figure 1 shows the progressive increase of cruising speed with altitude. At 40,000 ft the speed will have increased to 200 mph and at 70,000 the Cub will be traveling more than 400 mph. One is inclined to ask whether the fabric wings can withstand such a speed, or whether there will be difficulty moving the controls. Of course the indicated airspeed has remained at 100 mph so that the air forces are essentially unchanged. The relative damping of angular motions, however, will have been reduced to one-fourth its sea level value and this effect might be noticeable. Since the drag is still one-tenth the weight, the energy expended per mile of flight remains the same as at sea level. Thus our super engine has provided us with a 400-mph airplane that can land in a cow pasture.

At this point we are inclined to ask: Why climb all the way up to 70,000 ft for a short trip,

say 500 miles? A simple calculation shows that it would in fact save time to climb to 80,000 ft for a trip of 500 miles (fig. 2). If the Cub were equipped with a jet engine, climbing to a high altitude would also save fuel. It is worth noting here that our earliest jetliner, the DH Comet, needed to climb to 35,000 ft to reach a 200-mile alternate airport with its reserve fuel.

In Korvin Kroukovsky's day the power required to get to high altitudes was not available. Now, however, the super engine required for our Cub is here; it is the gas turbine. In older piston propeller airplanes, the power plant amounted to 20% or more of the weight of the airplane, and would hardly function at 30,000 ft. The turbofan engines of a modern large jet account for hardly more than 5 to 6% of the gross weight and are capable of supplying cruise thrust at 40,000 ft and more. Figure 3, adapted from reference 2, shows the rapid increase in power available since the advent of the gas turbine. With its four turbofan engines, the 747 has a lighter power loading than the Pitts Special.

STUDIES FOR A 40-PASSENGER TRANSPORT

It is customary in so-called "optimization studies" to consider the airplane and its engine as an interacting unit, as indeed they are. Nevertheless, it is of interest to consider what course the design of the airplane itself might take if we could obtain whatever engine was needed to suit our requirements. Following this idea, we will, I think, find that the airplane, even at its maximum density, is really adapted to flight at much higher altitudes than are now current.

Figure 4 shows a typical result of some studies of a small 40-passenger transport. In these studies I have assumed a certain fixed weight, which includes the payload, and have supposed that the wing area is increased progressively to adapt the airplane to progressively higher altitudes. The higher altitude means, of course, that the engines must be larger, the frontal area of the engines in fact increasing directly with the wing area. The larger wing and larger engines result in an increase of structure weight, so that the ratio of gross weight to payload would seem less favorable. In spite of this, however, the efficiency of the airplane in passenger miles per gallon increases continuously up to altitudes higher than those currently used.

Similar studies of the effect of increasing aspect ratio (fig. 5) at the expense of increased structural weight indicate that gains are possible in this direction. To utilize the benefit of higher aspect ratio, we should operate the airplane at a higher lift coefficient, and since we like to fly fast, our supercritical airfoils should be designed for higher lift coefficients. Again the results of my studies indicate that the ratio of payload to gross weight of an airplane is not a good measure of efficiency.

200-PASSENGER TRANSPORT REDUCED TO 12 PASSENGERS

Figure 6 shows a plot of weight versus a linear dimension for a number of airplanes, old and new. In the early days the airplane was of necessity designed to become airborne with a minimum of power, leading to a large wing area and a light, flimsy structure. With the advent of the gas turbine, the problem has become that of containing the payload, the structure, and the fuel in the smallest volume possible. At that point the graph begins to show a weight proportional to the cube of the linear dimension, a constant density which we may reasonably suppose to be the maximum density, leaving a minimum of empty space. The density of the airplane turns out to be about 10 lb/ft^3, about the same as a loaded railway car.

Scaling at constant density means that the well-known "square cube" law will apply. Thus if we multiply every linear dimension by a scale factor k the following relations will apply:

Wing area . . . . k^2
Weight of all components . . . . . . k^3
Structural stress level . . . . . . k
Air density at cruise altitude . . . k
Reynolds number . . . . k^2
Landing speed . . . . $\sqrt{k}$
Runway length . . . . k

The Reynolds number changes faster than the linear dimension because of the change in flight altitude.

Let us apply these scale relations to an efficient large transport, such as the Boeing 767, reducing it to the size and payload of a smaller 12-passenger airplane such as the Cessna Citation (fig. 7). In table 1 we compare the scaled-down S767 with the actual 12-passenger jet. The S767 turns out to be about 37% the size of the 767 and the component weights about 5%.

Most noteworthy is the fact that the S767 requires only 4500 lb of fuel for the allotted range while the actually smaller airplane uses more than 8000 lb. What are the reasons for this large difference? One would suppose, and it has been supposed in the past, that the square cube law would operate in favor of the smaller airplane. Thus the stress level (except for the fuselage pressurization stress) in the S767 is only 37% of that in the large airplane. In spite of this the empty weight of the S767 is nearly the same as that of the Citation. One important difference is the large wing area and light wing loading of the S767 when compared to the Citation. To fly at the same cruise lift coefficient as the 200-passenger 767, the smaller version must fly at an altitude of 55,000-60,000 ft.

In making the comparison I have assumed that the scaled engines retain the same efficiency as the full-size engines, and this of course cannot be quite true because of the reduced Reynolds number. It is interesting that the engine frontal area should scale directly with the wing area; that is, disproportionately larger engines should not be needed for the higher altitude if the same temperature rise can be produced in the rarified air. The S767 engines, however, are larger than the engines powering the Citation.

The conversion efficiency of the engine can be best understood in terms of the following formula for the range

$$R = R_{SP}\eta L/D \ln W_{max}/W_{min}$$

Here R_{SP} is the specific range corresponding to the chemical energy content of the fuel (2700 miles for kerosene), η is the conversion efficiency of the power plant, and the other terms are self-explanatory. According to published figures the conversion efficiency of the small Citation engines is about 24% at Mach 0.8 while the large engines of the 767 show nearly 30%. This difference can account for about one-third of the excess fuel needed by the small airplane.

Figure 8 shows the variation of conversion efficiency and thrust for a small turbofan engine such as that used in the Citation. It is noted that while the thrust falls off dramatically with altitude, the efficiency of the engine remains rather high. The gain in efficiency with Mach number could easily outweigh the small drop in η with altitude. One would willingly accept a compromise in the thrust and efficiency at sea level if it could lead to better performance at high altitude.

Our scale relations do not take into account the change of Reynolds number and this could be responsible in part for the reduced efficiency of the smaller airplane. Actually, the Reynolds number varies with k^2 rather than with k directly since the smaller airplane must fly at a higher altitude. Thus the Reynolds number of the full-size 767 is 30 to 40 million, while the S767 operates at about 5 million. At 5 million there exists the possibility of considerable laminar flow; hence, the drag coefficient of the smaller airplane need not be significantly higher than that of the full-size version. Quite possibly the major loss occasioned by the lower Reynolds number will occur in the engine, since the Reynolds number of the turbine and compressor blades is already rather low. Adapting the small engine to operate efficiently at higher altitudes may require a change to wider blades and special airfoil sections.

My own conclusion from the above study is that significant improvements in the efficiency of smaller transports can be achieved by engines designed specifically to provide thrust at very high altitudes. The high-altitude airplane has a larger wing area, can fly at a higher lift to drag ratio, and can land on shorter runways. Since the power available at sea level presents no problem for such an airplane, a compromise of the sea level engine performance to gain at high altitude should be acceptable.

EFFECT OF WINGLETS ON THE DRAG OF IDEAL WING SHAPES

It has been known for many years that vertical fins or end plates at the wing tips can significantly reduce vortex drag. According to Richard T. Whitcomb, F. W. Lanchester obtained a patent on the idea several years before the Wright brothers' first flight. Past work[3] has shown considerable improvement over earlier designs and raises the question whether such vertical extensions should

be part of the basic design of a wing intended for maximum efficiency.

In general, the vortex drag of a wing is decreased by extending the wing dimensions both vertically and horizontally. In Munk's solution of the problem of minimum drag, the dimensions of the wing were supposed to be given as is the total lift. Munk reduced the problem to that of conformal mapping of the vortex trace of the wing in a two-dimensional flow. Minimum drag occurred when the vortex trace moved downward as a rigid body. For the case of a planar wing, the elliptic load curve resulted. In 1933 Prandtl[4] sought to improve Munk's solution by considering the wing structure weight in more detail. Prandtl assumed that the wing weight would depend, not simply on the dimensions, but on an integrated or average of the bending moments along the span. By considering a family of wings of varying span, but having the same structure weight, Prandtl was able to obtain a 10% reduction of the induced drag when compared to an equivalent elliptic wing. Figure 9 shows a wing obtained by Prandtl's method compared with its equivalent elliptic wing.

More recently T. A. Lasinsky and I at Ames Research Center[5] have extended Prandtl's method to wings having winglets. I shall not repeat the details of the calculation here, but the method of solving the variational problem may be of interest.

Figure 10 shows a wing in front view (supposed to be given), together with three equations representing the quantities to be held stationary in the variational problem. The first equation sets the variation of the total lift to zero and the second indicates the variation of the particular structural quantity to be held fixed. The quantities m represent the contributions of the lifting elements $\delta \ell_n$, etc., to the particular structural parameter considered. Thus, in the case of a flat wing having a fixed bending moment at the wing root, the m_n are simply y_n. In other cases they are somewhat more complex, but are easily determined. The last equation represents the variation of the induced drag. Since the equations must hold for all variations and all positions of the $\delta \ell$'s, we have for the general solution

$$W/2V = A \cos \theta + Bm(s, \theta)$$

The determination of the load distribution is then a standard problem in wing theory. In our study we considered only wings with vertical winglets.

Calculations of drag, load distribution, and structural parameters remain unchanged if the wing system is inverted. The load on the downward-projecting fin will then be directed outward. As explained in reference 6, tip fins producing an outward lift can add significantly to the weathercock stability of the airplane. The usual upwardly projecting fins which lift inward subtract from the weathercock stability. Moreover, the conjunction of high-velocity regions on wing and winglet is avoided by the downward fin.

Figure 11 illustrates a typical result of the calculation for wings having 20% semispan winglets. The load distributions on the winglets are projected horizontally so that they appear as extensions of the wing span-load distribution. Wings of varying span, with and without winglets, are compared with a basic elliptic wing supposed according to our criterion to have the same total lift and the same spar weight. Wing A, having 0.9 the span of the comparable elliptic wing has the induced drag D_i equal to that of the elliptic wing D_{ie}. The more highly tapered wing B has a span equal to that of the elliptic wing and has about 10% less drag because of the winglet.

Figure 12 summarizes the results for all wings. It is seen from this figure that there is a "bottom line" for the induced drag which can be reached either by a 10% horizontal tip extension or by a 15% vertical tip, or winglet. The results show a slight preference for the horizontal tip extension because of its smaller wetted area. However, if the overall wingspan is limited by some other consideration, the winglet may be preferred.

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2. Armstrong, F. W.: The Aero Engine and its Progress; Fifty Years after Griffith. Aeronaut. J., Br. Royal Aeronaut. Soc., Dec. 1976.

3. Whitcomb, Richard T.: A Design Approach and Selected Wind Tunnel Results at High Subsonic Speeds for Wing Tip Mounted Winglets. NASA TN D-8260, 1976.

4. Prandtl, L.: Über Tragflügel des Kleinsten Induzierten Widerstandes. Zeitschrift für Flugtechnik und Motorluftschiffahrt 24 Jg. 1933. (Reprinted in Tollmien, W.; Schlichting, H.; and Görtler, H., eds.: Gesammelte Abhandlungen. Springer-Verlag, 1961.)

5. Jones, R. T. and Lasinski, T. A.: Effects of Winglets on the Induced Drag of Edial Wing Shapes. NASA TM-81230, 1980.

6. Jones, Robert T.: Notes on the Stability and Control of Tailless Airplanes. NACA TN-837, 1941.

TABLE 1.- SCALED 767 COMPARED WITH CITATION 111

	767	S767	Cit. 111
Payload, lb	48,000	2,400	2,400
No. of passengers	200 +	10 + 2	10 + 2
Range, n. mi.	3,000	3,000	2,500
Gross weight, lb	318,000	15,900	19,500
Empty weight, lb	180,000	9,000	9,100
Fuel weight, lb	90,000	4,500	8,000
Mach no.	0.8	0.8	0.75
Cruise altitude, ft	35-39,000	56-61,000	41,000
Wingspan, ft	156	58	53
Wing area, ft^2	3,000	420	312
Fuselage diameter, ft	16.5	6	5' 10"
Approach velocity, knots	154	90	---
Landing run	---	0.368*767	---
Stress level, psi	10,000 (nom)	3,680	---
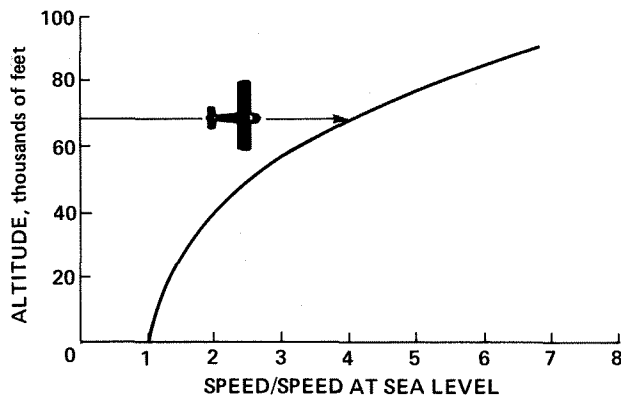
FIGURE 1. Increase of Cruising Speed with Altitude at Constant Thrust
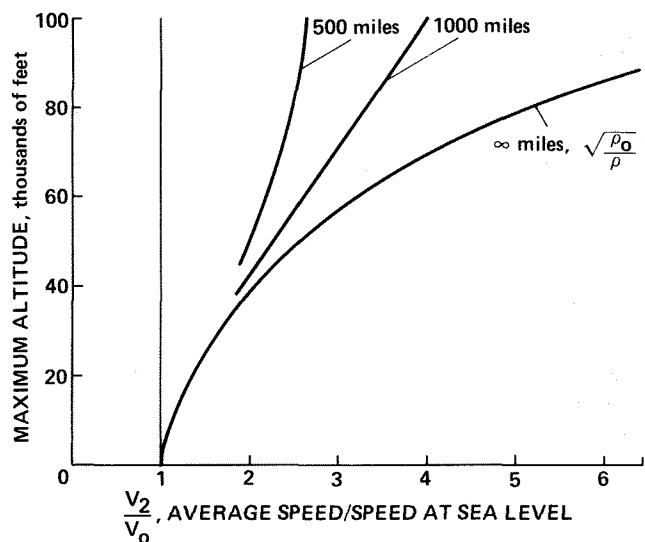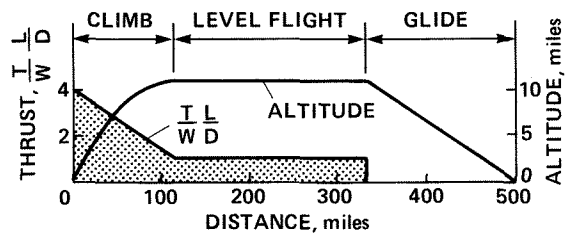
FIGURE 2. Increase of Average Speed with Altitude for 500-Mile Trip

6d
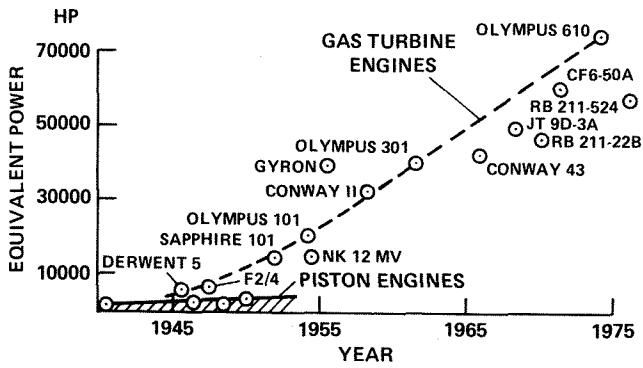
FIGURE 3. Increase of Power with Gas Turbine Engines
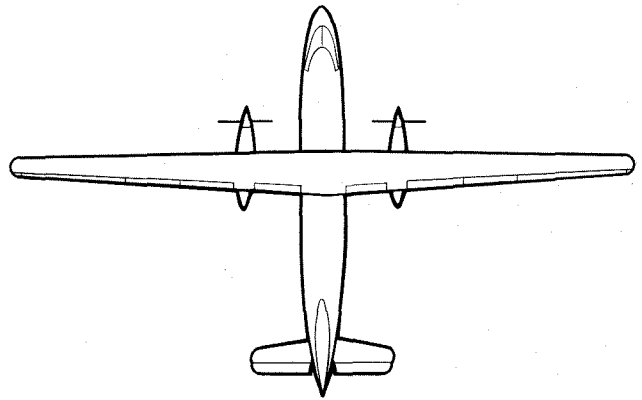
FIGURE 5. Fuel-Efficient 50-Passenger Transport
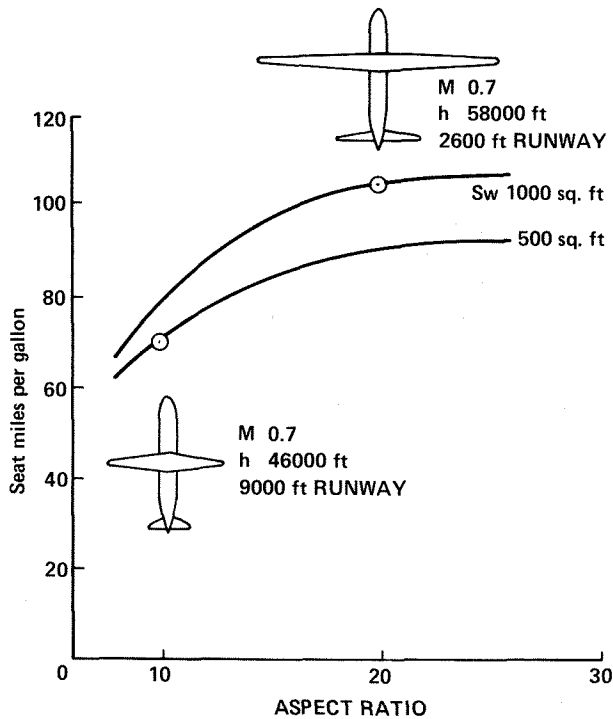
FIGURE 4. Effect of Wing Area, Aspect Ratio, and Flight Altitude on Fuel Economy (750-mile range, 40 passengers)
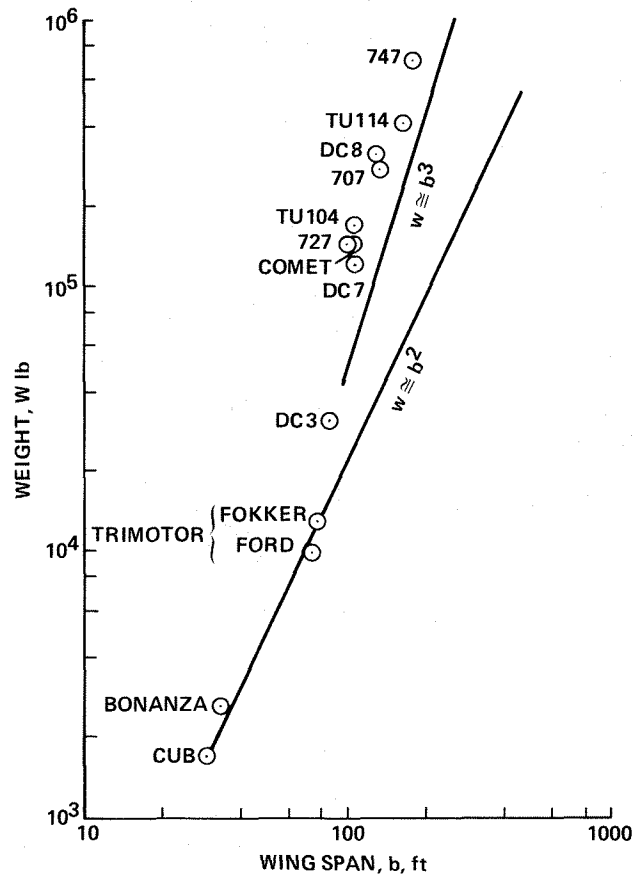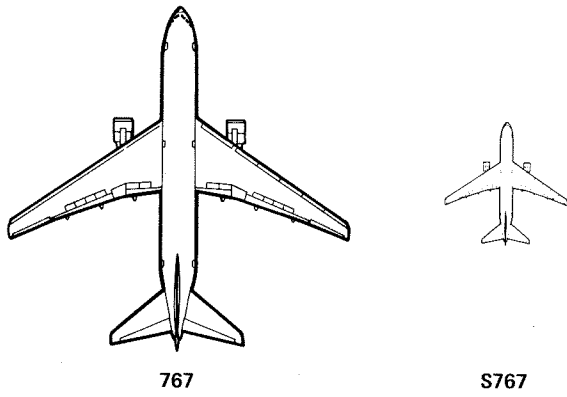

FIGURE 6. Weight Versus Linear Dimension

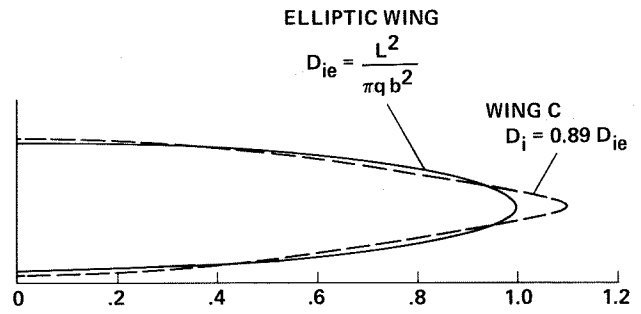6e

FIGURE 7. Scale Model of Boeing 767

ELLIPTIC WING

$$D_{ie} = \frac{L^2}{\pi q\, b^2}$$

WING C
$$D_i = 0.89\, D_{ie}$$

FIGURE 9. Planar Wings Having Equal Spar Weight
According to Prandtl's Criterion

MACH NO.
0.96
0.76
0.56

THRUST
M = 0.96

η

ALTITUDE, thousands of feet
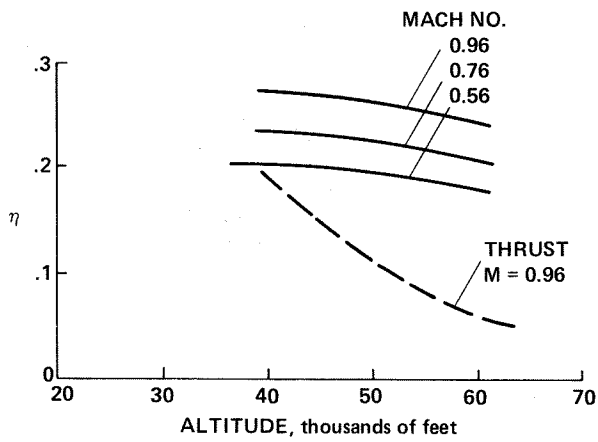
FIGURE 8. Conversion Efficiency and Thrust of
Small Turbofan Engine

$\delta\ell_1$

$\delta\ell_3$

$\delta\ell_2$

$$\delta\ell_1 \cos\theta_1 + \delta\ell_2 \cos\theta_2 + \delta\ell_3 \cos\theta_3 = 0$$

$$\delta\ell_1 m_1 + \delta\ell_2 m_2 + \delta\ell_3 m_3 = 0$$
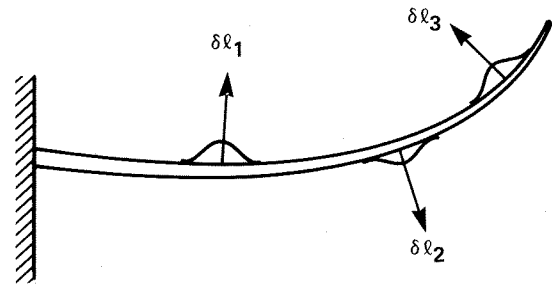
$$\delta\ell_1 \frac{w_1}{2V} + \delta\ell_2 \frac{w_2}{2V} + \delta\ell_3 \frac{w_3}{2V} = 0$$
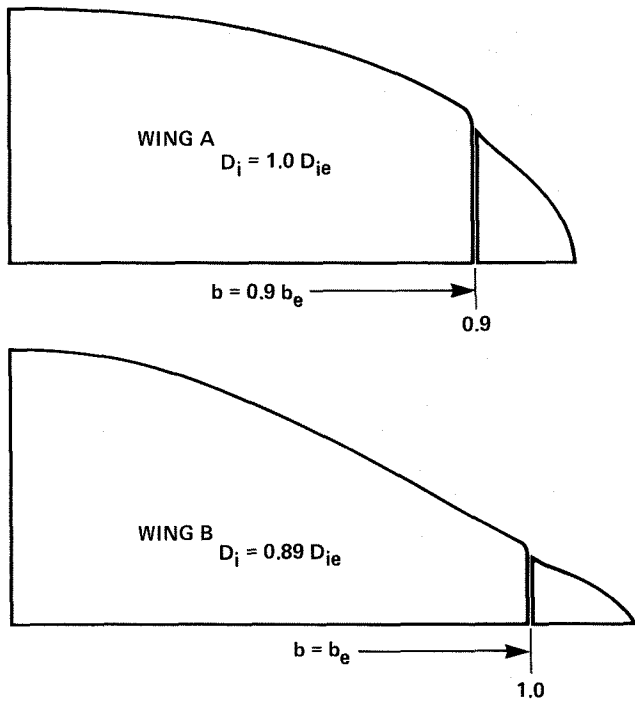
FIGURE 10. Variational Problem
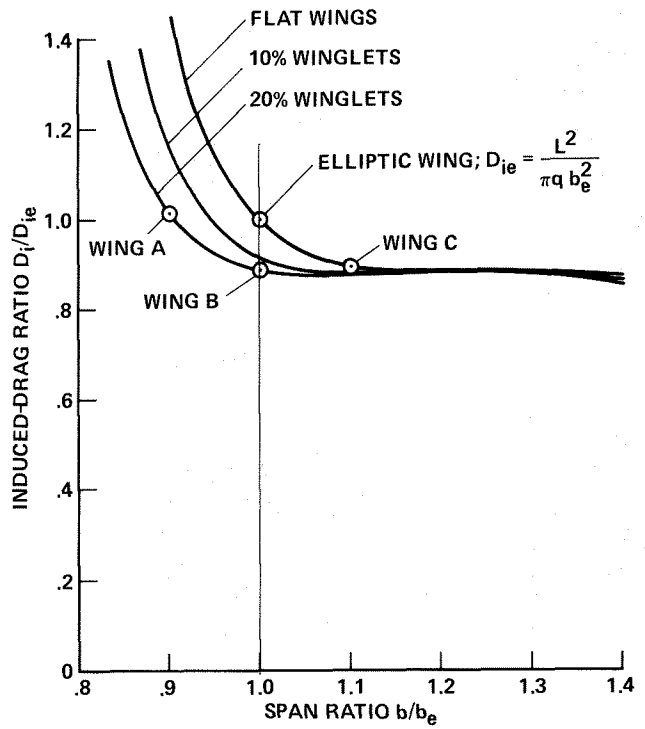
FIGURE 11. Load Curves for Wings with Winglets

FIGURE 12. Induced Drag of Wings Having the Same Integrated Moment $\bar{M}$