MODELS FOR THE MOTOR STATE OF VSCF AIRCRAFT ELECTRICAL POWER SYSTEM

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Abstract

Mathematical models for the motor state of AC-AC variable speed constant frequency (VSCF) power system are established in this paper. Two computing methods, a method finding the analytic solution of the state transition matrix e^{At} with computer and a combining method of 0.618 optimization method and Runge-Kutta method, are proposed. These methods reduce computation time more effectively.

I. Introduction

Modern aircraft electrical power systems are under Voltage direct current power system (UVDC), constant speed constant frequency alternating current power system (CSCF) and variable speed constant frequency power system (VSCF). High voltage direct current power system (HVDC) will be used in future advanced aircraft.

The UVDC system is used only in some middle or small aircrafts with low performances. CSCF power system is used widely at present and its performances are being improved further, but the constant speed driver (CSD) of the system is not easy to maintain and the electrical power quality of CSCF system is not available for future advanced aircraft.

The development in rare earth permanent magnetic material, power semiconductor devices and microprocessor provides possibility for development VSCF power system. At present, VSCF system is used for generating electrical power only. But this system can soon operate both in generator state and motor state for starting aircraft engine.

If HVDC system is to be adopted, the electrical distribution system and some electrical equipments of aircraft must be reformed. The system can't be applied rapidly. But VSCF power can replace directly CSCF power system. In this sense, to research VSCF system, especially to research thoroughly motor state of the system, is very necessary.

At present there are two fundamental types of VSCF system,AC-AC system and DC-link system. DC-link system has two kinds: the one is composed of a electromagnetic brushless alternator, a rectifier and an inverter; the other is composed of a permanent magnetic alternator, a controlled rectifier and an inverter. Both of them are used for generating electrical power now. AC-AC system is composed of a generator and a cycloconverter. Because the reversibility of the permanent magnetic electrical machine and cycloconverter is better than the DC-link system, this kind VSCF system can operate both in generator state and in motor state.

When AC-AC system operates in generator state, the alternator converts mechanical energy into variable frequency (>1200 Hz) a.c. electrical energy. Then the a.c. power is transformed into constant frequency (400Hz) a.c. power by cycloconverter. In starting state the alternator operates as a motor and is fed by airfield constant frequency electrical power source or aircraft a.c. power source. The converter transforms this power into variable frequency (0-1000Hz) a.c. power which provides electrical machine to start aircraft engine. The power circuit of this system operating in motor state is as same as one operating in generator state. But the direction of power flow is opposite.

Three phase machine VSCF system operating in motor state is composed of two reactors, a converter, a synchronous machine and a position sensor of the machine rotor as shown in Fig.1.

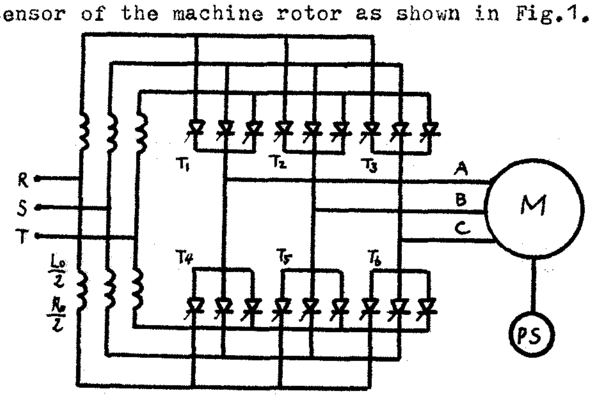


Fig.1 The power circuit of three phase machine system.

There are two commutation cases of the thyristor in the system, the one corresponds to the machine operating in high speed and the other in low speed.

When the machine operates in high speed the converter is commutated by the induced emf of the synchronous machine and call this mode load commutation. If the motor operates at low speed the thyristor is commutated by the source voltage and it is called source commutation. When the motor speed is very low and the load of the motor is light the fireing angle of thyristor group is near 90 electrical degrees and the current in armature of the motor is discontinuous.

So the analysis for motor state is composed of an analysis for high speed operation, an analysis of low speed operation at current continuous state and an analysis of very low speed operation at discontinuous current state. In the paper only the former two parts are discussed.

II. Mathematical Model for the System at High Speed

Equivalent Circuit

If only two groups of thyristors are conducting (assume T_1 and T_5), the system power circuit can be redrawn as shown in Fig. 2 Evidently this is a three phase rectifier circuit. If the leakage inductance of the power source is neglected and the current of the machine is continuous, the rectifier output voltage is obtained.

$$E_0=2.34\ V_m\cos\alpha \qquad (1)$$

where V_m phase voltage of the power source in volt (rms);

α fireing angle of the rectifier circuit in degree.

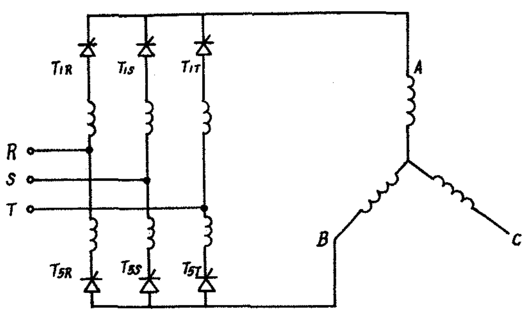


Fig. 2. Three phase rectifier circuit, thyristor groups T_1 and T_5 are conducting

Eo is applied to the phase winding A and B of the machine. If T_5 turns off and T_6 turns on, but the angle α does not change, same voltage Eo is applied to windings A and C of the motor.

If ripple components of the rectifier output voltage Eo and the effect of power source voltage to commutation of the thyristor are neglected, the circuit of Fig.1 may be equivalent to that as shown in Fig. 3.

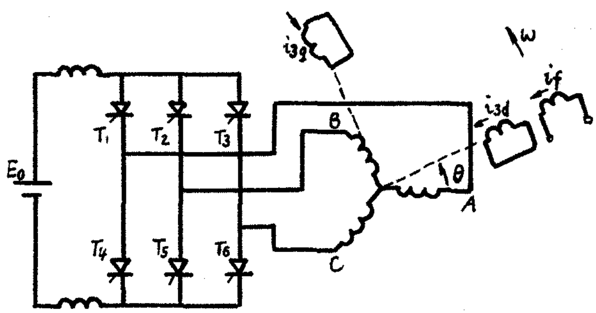
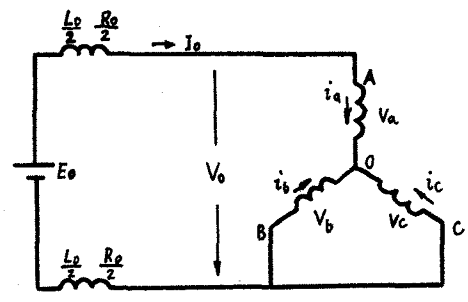


Fig. 3. The equivalent power circuit of the system

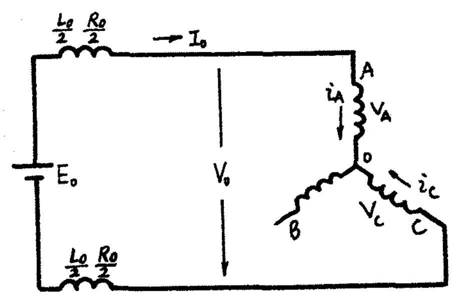
Modes of the System

When thyristor group T_6 turns on, T_5 doesn't turn off immediately, because there is inductance in the system, the main of which is the leakage inductance of the motor windings. In this case current flows through three armature windings of the machine. This operating mode is called commutation mode.

By the effect of the emf of the synchronous motor the current in winding B decreases to zero and current in winding C increases to a value which is equal to the current of winding B before commutation. After current in winding B reduces to zero, thyristor group T_5 turns off and current flows only through two armature windings A and C of the motor. This operating mode is called single mode. The equivalent circuits of the system in commutation mode and in single mode are shown in Fig. 4(a) and 4(b) respectively.



a. commutation mode



b. Single mode

Fig. 4. Two modes of the system in high speed

When the system is operating, the two modes, commutation mode and single mode, change over alternately. The sum of duration of the commutation mode t_1 and the single mode t_2 is constant if the system operates in steady state, that is

$$T=t_1 + t_2 = \text{const} \qquad (2)$$

where T is known as computation period.

Equation of the System

In order to turn off the thyristor and perform load commutation, the instant to start commutation must be in advance of the natural commutation point. The advanced angle of the inverter fireing signal is denoted by γ . This fireing signal is called γ signal.

For simplifing the analysis, the step by step α-β coordinate system may be used[1] . This coordinate system is stationary to the armature windings except for the instant of commutation start. If the rotor has revolved 60 electrical degrees and new commutation starts, the coordinate system is rotated 60 electrical degrees along the rotating direction of the machine immediately. Using the α-β coordinate system, the system states in the present computation period are equal to the states in the next computation period, i.e. the system states in present 60 electrical degrees of the machine rotor is as same as that in the next 60 electrical degrees if the machine is operating in steady state. The characteristic of the system is called symmetry on 60 electrical degrees of the machine.

From Fig. 4 the KVL of the circuit is

$$Eo=(Ro+LoP)Io+Vo \tag{3}$$

Where Ro, Lo the resistance (Ω) and inductance (H) of the reactor in the circuit respectively;
Io current flowing through the reactor (A);
Vo voltage applied to the machine armature (V).

From Fig. 4(a) We get some equations in commutation mode.

$$V_o=V_a-V_c=V_a-V_b \; ; \tag{4}$$
$$V_b=V_c \; ; \tag{5}$$
$$I_o=i_a=-(i_b+i_c) \tag{6}$$

Transform these equation which are in ABC coordinate system into α-β coordinate system, obtain

$$V_{1\alpha}=\sqrt{\tfrac{2}{3}}E_o-\sqrt{\tfrac{2}{3}}(R_o+L_oP)i_{1\alpha} \tag{7}$$

$$V_{1\beta}=0 \tag{8}$$

Assume field current I_f of the machine is constant, the following expressions are valid.

$$i_{2\alpha}=I_f\cos\theta \tag{9}$$

$$i_{2\beta}=I_f\sin\theta$$

the matrix form of equations (9) is as follows

$$\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}P & \dot\theta\\ -\dot\theta & P\end{bmatrix}\begin{bmatrix}i_{2\alpha}\\ i_{2\beta}\end{bmatrix} \tag{10}$$

Because the damper windings of the machine are shortted, we get

$$\begin{bmatrix}V_{3\alpha}\\V_{3\beta}\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}i_{3\alpha}\\ i_{3\beta}\end{bmatrix} \tag{11}$$

Equations (7),(8),(10) and (11) may be written in matrix form

$$V = RI + LpI \tag{12}$$

where

$$V=\mathrm{Col}\begin{bmatrix}V_{1\alpha} & V_{1\beta} & V_{2\alpha} & V_{2\beta} & V_{3\alpha} & V_{3\beta} & E_o\end{bmatrix}$$

$$I=\mathrm{Col}\begin{bmatrix}i_{1\alpha} & i_{1\beta} & i_{2\alpha} & i_{2\beta} & i_{3\alpha} & i_{3\beta} & E_o\end{bmatrix}$$

$$R=\begin{bmatrix}-\tfrac{2}{3}R_0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\tfrac{2}{3}}\\ & & & 0_{5\times7} & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

$$L=\begin{bmatrix}-\tfrac{2}{3}L_0 & 0 & 0 & 0 & 0 & 0 & 0\\ & & & 0_{6\times7} & & & \end{bmatrix}$$

From appendix I the impedance matrix Z of synchronous machine can be divided into two parts.

$$Z = Z_1 + Z_2 p \tag{13}$$

If add a state variable Eo into vectors I and V then rewritten the matrices Z_1 and Z_2 , the equation of the machine is as follows.

$$V = Z_1 I + Z_2 pI \tag{14}$$

Combining equation (12) with (14) we get the equation of VSCF system operating in commutation mode. That is

$$PI=(Z_1-L)^{-1}(R-Z_2)I=A_1 I \tag{15}$$

where $A_1=(Z_1-L)^{-1}(R-Z_2)$ coefficient matrix of the system.

From Fig. 4(b), the equations of the system operating at single mode are as follows.

$$V_o=V_a-V_c \tag{16}$$
$$I_o=i_a=-i_c \; , \; i_b=0 \tag{17}$$

In the α-β coordinate system, that is

$$\sqrt{3}\,V_{1\alpha}+V_{1\beta}=-\tfrac{2}{\sqrt{3}}(R_o+L_oP)i_{1\alpha}+\sqrt{2}\,E_o \tag{18}$$

The external circuit equation in matrix form is similar to equation (12). So the equation of VSCF system operating in single mode is as follows

$$PI = A_2 I \tag{19}$$

where A_2 7x7 coefficient matrix

If the system begins to transfer from commutation mode into single mode at $t=t_1$, i.e. current i_b just reduces to zero and i_c just decreases to $-i_a$ in the time, the transfer condition expressing in α-β coordinate system is as follows

$$i_{1\alpha}(t_1)=\sqrt{3}\,i_{1\beta}(t_1) \tag{20}$$

Equations (15), (19) and (20) are the mathematical model for the system operating at high speed.

III. Mathematics Model for the Motor State at low Speed

Characteristics in Motor at low Speed and Special Rotation Speed

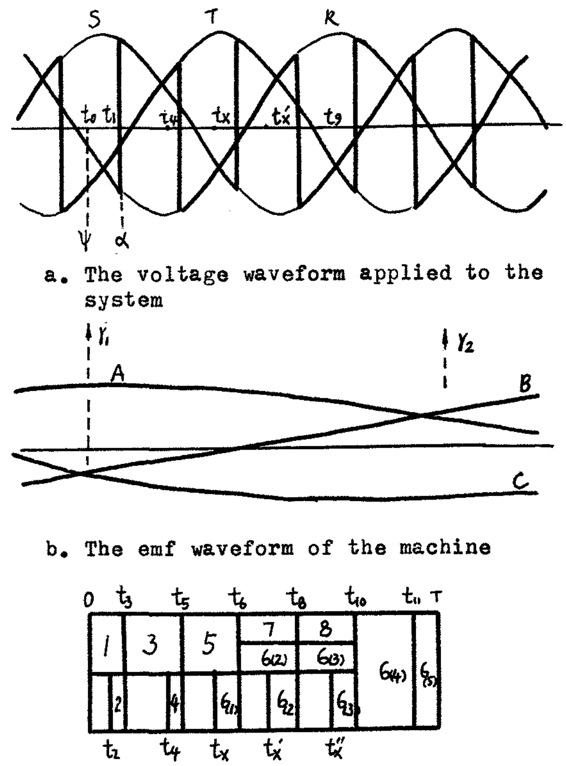
When AC-AC System is in motor operation at low speed, the thyristor is turned off by the source voltage. Bacause the source voltage and parameter of the machine change simultaneously and does not dependent each other, in general case the system losts the symmetrical characteristic on 60 electrical degrees of the machine. But if the relation between the rotating speed of the machine ω and the frequency of the source voltage ω_e satisfies $\omega=\omega_e/2K$ (Where K is positive integer) and the source voltage coordinate system is transformed properly, the system gets the symmatrical characteristic on 60 electrical degrees of the machine again. These rotating speeds are called special rotation speeds. If the maximum speed of the low speed range isn't large, a general rotating speed range is replaced by a special rotation speed does not cause large error. So analysis for low speed operation may change into analysis for special rotation speeds.

Analysis on Commutation and the transfer conditions

When the system operates at low speed with continuous current, viewing from the machine side, there exist still two cases: the current flows through three phase windings and through two phase windings. In general, machine commutation can't finish within a 1/6 period of the source voltage (i.e. an interval between two triggles of the thyristors). Because within the interval the converter will complete some source commutations, analysis for the system is more complicated.

The machine commutation of the system at low speed may be analysed with the help of Fig. 5. Where α is fireing angle of silicon controlled rectifier, γ_1 is a rotor position signal sent out from the position sensor of the machine, γ_2 is the next signal of the sensor. ψ is phase angle of the source voltage when rotor position signal γ_1 is sent out. There exist two cases: $\psi<\alpha$ and $\psi\geq\alpha$. First we discuss the former case, it is shown in Fig.5.

Assume that, before γ_1 signal sent out, thyristors T_{1R} and T_{5T} are conducting and a current is flowing through windings A and B as shown in Fig.6(a). At t=0 γ_1 signal is sent out and thyristor T_{6T} starts to conduct. A current in winding C increases from zero as shown in Fig. 6(b). This state is called mode 1. At the moment $t=t_1$ the system reaches natural commutation point of phases T and R of the source voltage. After this point source voltage E_{TR} reverses and the current in phase winding C decreases. If, in the interval from t_1 to t_3 , current i_c reduced to zero, a current ~~only~~ flows through phase windings A and B again as shown in Fig. 6(c). We call this mode mode 2. Whether the case $i_c=0$ appears or not in the interval from t_1 to t_3 , at the moment $t=t_3$ thyristor T_{1S} turns on and thyristor T_{1R} turns off and the source phase R is replaced by phase S. Source voltage E_{ST} is positive and current i_c begins to increase again as shown in Fig. 6(d). This is mode 3. After the natural commutation point of source phase voltages Es and E_T, voltage E_{ST} becomes negative and i_c decreases again. If, before $t=t_5$ (this is the next natural commutation point of phases T and R of the source), i_c reduces to zero the system enters into mode 4 as shown in Fig. 6(e).



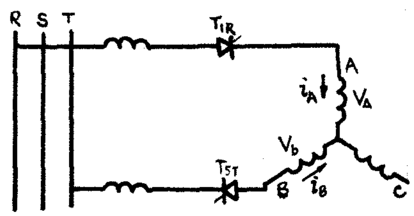
a. The voltage waveform applied to the system

b. The emf waveform of the machine

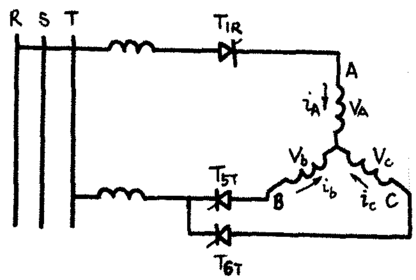
c. The mode sequence of the system

Fig.5 The souce commutation of the system in low speed

If in the interval i_c doesn't reduce to zero the system transfers immediately into mode 5 as shown in Fig. 6(f). From Fig.5 the source voltage E_{TR} is positive in the manner. The voltage is applied to the circuit which consists of two reactors, two thyristors and two phase windings of the machine. It causes the current in phase winding C to increase and the current in phase winding B to decrease. If the current in phase winding B reduces to zero in the interval of 1/6 period of the source voltage that interval starts at the moment $t=t_5$, the machine commutation finishes and the system enters into mode 6 as shown in Fig. 6(g). Otherwise, at the moment $t=t_6$ the source commutation happens again and the system transfers into mode 7, i.e. auxiliary commutation mode 1 as shown in Fig. 6(h). In this mode, because E_{TR} is positive for a long time, i_b continues to decrease. If $i_b=0$ within 1/6 period of the source voltage, the machine commutation succeeds and the system enters into mode 6. Otherwise, at the moment $t=t_8$ the source commutation starts again and the system enters into mode 8. i.e. auxiliary commutation mode 2 as shown in Fig. 6(i). If i_b can't reduce to zero yet within the interval from t_8 to t_{10} , the machine commutation break down . In reverse the machine commutation succeeds and the system enters into mode 6. Mode 6 corresponds to the interval from the time the machine commutation finishes to the time the next machine commutation starts. Because source commutation within this interval runs many times (e.g. n times), mode 6 contains n submodes. The number of submode depends on the loads and rotation speed of the machine. Transfer conditions and transfer time between modes are specified in every subfigure of Fig.6

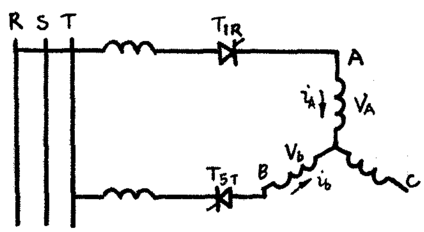


a. The circuit before machine commutation



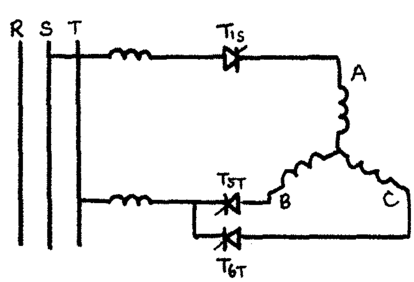
b. Mode 1.

If $t=t_2<t_3$, $i_c=0$, from mode 1 to mode 2 at $t=t_2$, otherwise, from mode 1 to mode 3 at $t=t_3$. $t_3=\frac{\alpha-\psi}{\omega_e}$.



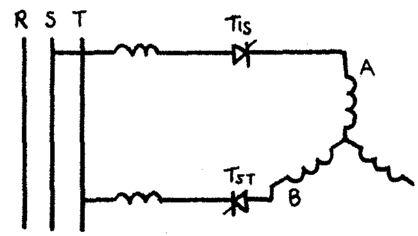
c. Mode 2.

From mode 2 to mode 3 at $t=t_3$,



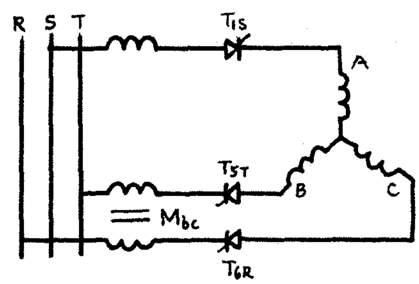
d. Mode 3.

If $t=t_4<t_5$, $i_c=0$, from mode 3 to mode 4 at $t=t_4$, otherwise from mode 3 to mode 5 at $t=t_5$, $t_5=t_3+\pi/3\omega_e$.



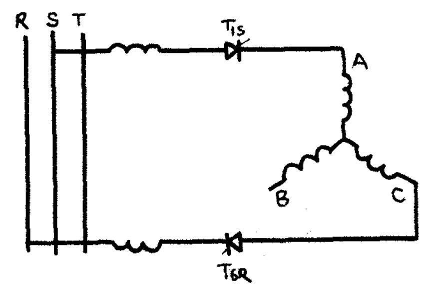
e. Mode 4.

From mode 4 to mode 5 at $t=t_5$.



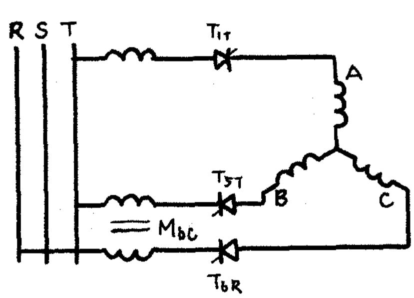
f. Mode 5.

If $t=t_x < t_6$, $i_b=0$, from mode 5 to mode 6 at $t=t_x$, otherwise from mode 5 to mode 7 at $t=t_6$, $t_6=t_5+\pi/3\omega_e$.



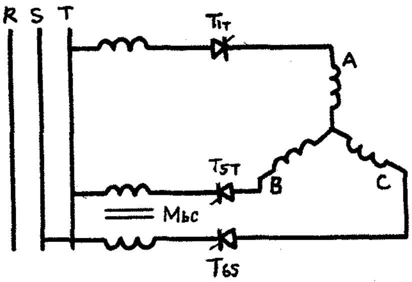
g. Mode 6.

From one submode to next submode at every 1/6 source voltage period, at $t=T=\pi/3\omega$. the system goes to the next computation period.



h. Mode 7.

If $t=t_x' < t_8$, $i_b=0$, from mode 7 to mode 6, otherwise from mode 7 to mode 8 at $t=t_8$, $t_8=t_6+\pi/3\omega_e$.



i. Mode 8.

If $t=t_x'' < t_{10}$, $i_b=0$, from mode 8 to mode 6 at $t=t_x''$. otherwise the machine commutation breaks down. $t_{10}=t_8+\pi/3\omega_e$.

Fig.6 The modes and mode transfer conditions of the system in low speed, $\psi<\alpha$.

If $\psi\geqslant\alpha$ mode 1 and 2 don't appear and the first mode of the system is mode 3. Fig.5(c) sum up the operating modes and transfer conditions from one mode to the next, where 1,2,3... 8 are numbers of mode that can appear in operation of machine. The letters in the round brackets of 6(1), 6(2),... are the number of submode and t_2 , t_3 ... are the mode transfer moments.

Transfer from one mode to the other is decided by two factors: the frequency of source voltage and the current in phase winding of the machine. In Fig. 5(c) the letters above the table is the moments of transfer caused by the first factor. Because the frequency of the source voltage ω_e is constant, the source commutation moments are definite. The letters below the table are moments of transfer caused by the second factor. They aren't definite and dependent upon the loads and initial conditions of the system. In the equations they are unknowns.

At low speed operation the number of mode of the system is not definite. In Fig. 7(a) there are eight modes which is the maximum. In Fig. 7(b) there are only three modes which is the minimum.

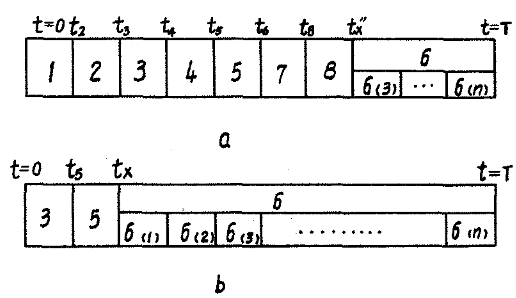


Fig.7 The maximum (a) and minimum (b) mode number of the system at low speed.

Equation of the System

From Fig. 6(b), the equation of mode 1 is as follows.

$$(E_R-E_T)-\frac{1}{2}(R_0+L_0P)(i_a-i_b-i_c)=V_a-V_b \quad (21)$$

$$i_a=-(i_b+i_c) \quad (22)$$

$$V_b=V_c \quad (23)$$

where E_R , E_S and E_T are phase voltage of the source.

Transfer the values E_S , E_T and E_R which are in ABC coordinate system into α-β coordinate system and the new values are denoted by e_1 and e_2 , that is

$$\begin{bmatrix} E_S \\ E_T \\ E_R \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

We obtain

$$V_{1\alpha}=-\frac{2}{\sqrt{3}}e_2-\frac{2}{3}(R_0-L_0P)i_{1\alpha} \quad (24)$$

$$V_{1\beta}=0 \quad (25)$$

Assume

$$V=col\left[V_{1\alpha} \;\; V_{1\beta} \;\; 0 \;\; 0 \;\; 0 \;\; 0\right]$$

$$I=col\left[i_{1\alpha} \;\; i_{1\beta} \;\; i_{2\alpha} \;\; i_{2\beta} \;\; i_{3\alpha} \;\; i_{3\beta}\right]$$

$$E=col\left[e_1 \;\; e_2\right]$$

then get the matrix form of the external circuit expression

$$V=C_1PI+C_2I+C_3E \quad (26)$$

$$C_1=\begin{bmatrix} -\frac{2}{3}L_0 & 0 & 0 & 0 & 0 & 0 \\ & & O_{5\times6} & & & \end{bmatrix}$$

$$C_2=\begin{bmatrix} -\frac{2}{3}R_0 & 0 & 0 & 0 & 0 & 0 \\ & & O_{5\times6} & & & \end{bmatrix}$$

$$C_3=\begin{bmatrix} 0 & -\frac{2}{\sqrt{3}} \\ O_{5\times2} & \end{bmatrix}$$

The equation of the machine is as follows

$$V=(Z_1+Z_2P)I \quad (27)$$

Combining (26) and (27), get the equation of mode 1,

$$PI=M_1I \quad (28)$$

where

$$I=col\left[i_{1\alpha} \;\; i_{1\beta} \;\; i_{2\alpha} \;\; i_{2\beta} \;\; i_{3\alpha} \;\; i_{3\beta} \;\; e_1 \;\; e_2\right]$$

$$M_1 = \begin{bmatrix} -(Z_1-C_1)^{-1}(Z_2-C_2) & (Z_1-C_1)^{-1}C_3 \\ 0_{2\times6} & \begin{bmatrix} 0 & -\omega_e \\ \omega_e & 0 \end{bmatrix} \end{bmatrix}$$

The system equations of other modes are similar to the equation (28) of model 1. The general form is as follows.

$$PI = M_k I \tag{29}$$

where subscript $k=2,3,\ldots,8$. M_k is similar to M_1 and is also a 8x8 matrix.

The elements of matrix M_k depend on the number of phase winding of the machine with current flowing and the phases of source. The equations (28) and (29) of the system and the transform conditions are mathematics models of the system at low speed.

IV Six-Phase or Nine-Phase System

The electrical machine using in VSCF system is six-phase or nine-phase synchronous machine. Six-phase machine is often used for 30 or 40KVA generator system and nine-phase machine is for 90 KVA system or so, Using these systems, output voltage waveform of the generator system is better than that of the three phase system and the output filter of the system is more smaller.

The equivalent power circuit of six-phase system at high speed is shown in Fig.8. This is a simplified diagram in which a group of three-thyristors is replaced by one thyristor. The armature winding of six-phase machine is composed of two 3-phase windings. If magneto-motive forces of the two 3-phase windings are in phase, the sum of them is maximum and the torque of the machine is also the largest torque. So the thyristors gate signals must be subject to the relation as shown in Fig.9 and two 3-phase systems start commutation in the same time. On this manner the six-phase system operating in motor state is a 3-phase system in nature (But it is a six-phase system in generator state).

If two 3-phase systems start commutation at different time, one commutation has completed then the other starts commutation. The commutation overlap of the system is reduced. The gate signal scheme of the system is shown in Fig.10. For six-phase machine the space angle between two 3-phase windings is 180 electrical degrees. If $t_1 \neq 0$, the commutation angle of one 3-phase system is not equal to the another, the former is larger and the later is smaller. The overload ability of the machine is equal to the three phase machine approximately. Thus six-phase machine is no better than the three phase machine if they are operating in motor state.

If two 3-phase windings of the six-phase system start commutation at the same time, the mathematics model of this system is equal to the three phase system. If commutation of the two groups of 3-phase winding is not at the same time, the order of state equation of six-phase system operating at high speed is nine and at low speed is ten.

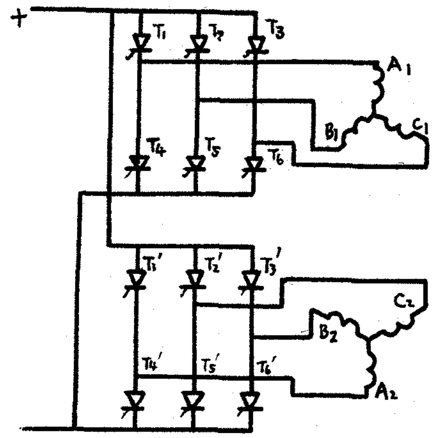


Fig.8 The simplified power circuit of six-phase system

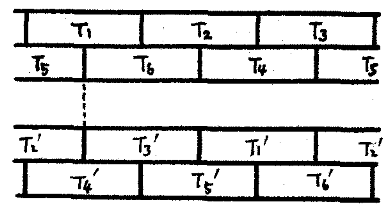


Fig.9 The conducting sequence of the thyristors, case 1.

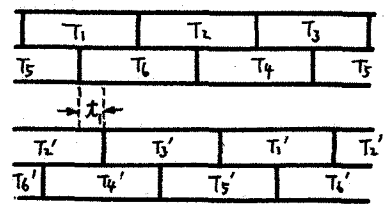


Fig.10 The conducting sequence of the thyristors, case 2.

The armature of nine-phase machine is composed of three groups of 3-phase winding. The angle between two 3-phase windings is 40 electrical degrees as shown in Fig.11. The figure is also a simplified diagram. The gate signal for nine-phase system operating at high speed is shown in Fig.12. In this manner the magnetomotive force of first 3-phase winding is behind the second 20 electrical degrees in space and the second is also behind the third 20 electrical degrees. The magnetomotive force of the nine-phase machine is a little lower than three times of any magnetomotive force of the three phase winding of the machine. The ripple of torque of the nine-phase machine is more little than that of the three phase machine and the overload ability of nine-phase machine is larger than that of the three phase system, becuase the interval between two moments two 3-phase windings start commutation respectively is 20 electrical degrees. The order of the equation of nine-phase system operating at high speed is eleven and at low speed is twelve. The nine-phase system is more complex than six-phase system.

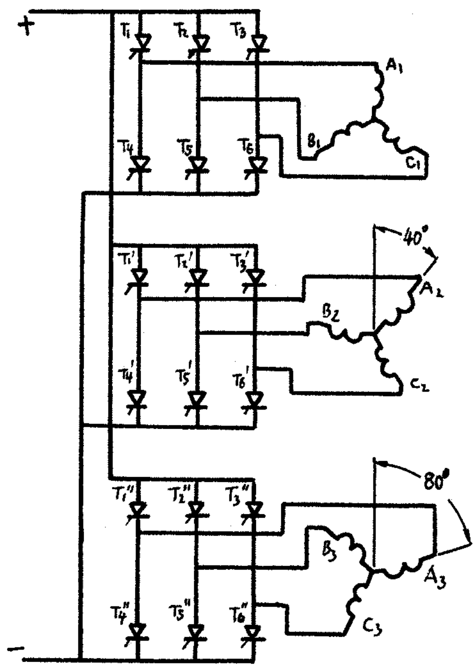


Fig.11. The simplified power circuit of nine-phase system.

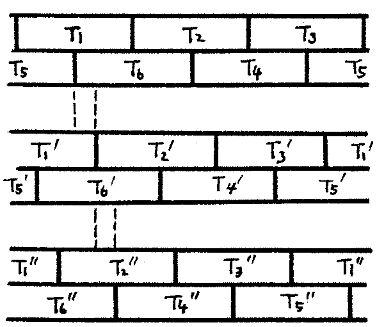


Fig.12. The conducting squence of the thyristors of nine-phase system

V. Solution of the Equation of the System

Solution for the System Equation with nonsalient machine

For nonsalient machine the elements of every coefficient matrix, i.e. matrices A_1 , A_2 or M_1, M_2 ..., are constant, so equations of the system are constant coefficient homogeneous differential equations and can be solved by the method of state transition matrix e^{At}.

When the machine operates at high speed the system has two modes. The solution of the system equation is

$$I(t) = e^{A_1 t} I(o) \qquad 0 \le t \le t_1 \tag{30}$$

$$I(t) = e^{A_2(t-t_1)} I(t_1) \qquad t_1 \le t \le \pi/3\omega \tag{31}$$

where I(o) is initial values of the system at t=0.

At steady state the state of the system at t=T is same as one at t=0. Because α-β coordinate system also has rotated 60 electrical degrees of the machine, the following relation is derived.

$$I(o)=HI(T) \tag{32}$$

where H is transfer matrix of coordinate system.

Using equations (30) (31) and (32) and transfer condition between the modes, the initial values of the system can be found.

When the machine operates at low speed, number of mode of the system is more than 2 and not definite. It is difficult to find the solution of the system directly with this method. But if we assume that certain modes doesn't appear, the number of mode is definite and the approximate initial values of the system can be obtained by this method.

In the process of finding initial values or approximate initial values of the system, it is the key to compute the values of state transition matrix e^{At} at any time t. To find e^{At} with computer, the progression solution is recommended recently, but there are many disadvantages if the solution is used here. In order to increase computation speed a new method is set forth, that is to find out the coefficient matrix and the shape parameter matrices of e^{At} with computer and the analytic expression of e^{At} can be get directly[2] . So long as t_1 replaces t at the right of the expression and the values of e^{At_1} are obtained directly without using recurrence process.

The step to solve the equation of the system operating at high speed is given in the Appendix II.

Solution for the System Equation with Salient Machine

If the machine is a salient machine the equation of the system are linear time variable coefficient homogeneous differential equation. If the initial values of the equations have been found, the digital solution of the equation can be found with Runge-Kutta method.

One method to find the initial values of the system is as follows: first regarding the salient machine as the corresponding nonsalient one, approximate initial values are found with the previous method. Second the initial values I(o) of the system with salient machine are found by Runge-Kutta method. Using this method, the computation time is long, especially when the frequency of the source voltage is high. The higher the frequency of the source voltage, the longer the computation time.

If the field current I_f is constant, unknown of the system containing a salient machine without damper windings is only one. So the initial values of the system can be found by the combining method of 0.618 optimization method and Runge-Kutta method. The new method possesses many advantages, e.g. its program is simple and computation time is shortened greatly. If the frequency of the source voltage is 400Hz, computation time may be shortened over 20 times.

The step of the method is given in Appendix II.

Two supplements on computing at low speed operation

Because α-β coordinate system steps 60 electrical degrees at the beginning of every computation period, a coordinate transformation matrix H must be introduced.

$$H = \begin{bmatrix} H_1 & & & \\ & H_2 & & \\ & & H_3 & \\ & & & H_4 \end{bmatrix} \tag{33}$$

where

$$H_i = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \qquad i=1,2,3$$

H_4 is the transformation matrix of the source.

At high speed operation regarding alternate power source as equivalent direct power source, so

$$H_4 = 1 \tag{34}$$

At low speed operation the computing is made under alternate power source. The three phase source coordinate system must be transfered into equivalent two phase coordinate system. We know that there are three forms of the transformation matrices H_4 which are demanded by the system operating in the special rotation speed.

If the beginning and the end of a commutation period of the system correspond to the same phase angle of the source voltage, the two phase source coordinate system doesn't transfer and

$$H_4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{35}$$

If the difference of two phase angles of the source voltage (the angles corresponding to the beginning and the end of a computation period of the system) is 120 electrical degrees, the two phase coordinate system should be rotated 120 electrical degrees and

$$H_4 = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \tag{36}$$

If this difference is -120 electrical degrees, the two phase coordinate system must be rotated -120 electrical degrees and

$$H_4 = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \tag{37}$$

After this the previous computation method may be used.

In addition the number of mode of the system operating at low speed is many and doesn't define beforehand. It makes to find initial values of the system difficult. Here the key is defining the number of the mode of the system with damper windings machine.

To overcome the difficulty, first regarding the machine as the corresponding nonsolient machine and assuming that mode 2, mode 4, mode 7 and mode 8 don't appear, the equation of the system becomes a constant coefficient homogeneous differential equation. In the manner the number of mode and unknowns of the system are definite. The approximate initial values and commutation time t_x may be found by previous method. If t_x is larger than 1/6 period of the source voltage, the assumption that mode 7 doesn't appear is not valid. Mode 7 should be considered and the solution of the system must be computed again. If t_x' is larger than 1/6 period of the source voltage again, mode 8 must be considered and the solution of the system must be computed again. If t_x'' is larger than 1/6 period of the source voltage still, the machine commutation break down. Thus whether the mode 7 and mode 8 could appear may be decided and approximate initial values of the system are found. Second, initial values of the system are found by Runge-Kutta method. In the process of finding initial values, whether mode 2 and mode 4 appear or not can be decided automatically. So at the end the number of mode and initial values of the system are obtained simultaneously. Then the solution of the system can be found by Runge-Kutta method but the computation time is long.

The stop of this method is specified in Appendix II.

VI. Conclusion

The mathematics models of the VSCF power system at high speed motor state and low speed motor state have been confirmed by experiments. The results of experiment and computation show that the mathematics models can describe accurately the characteristics of the system.

Using the mathematics models, the influence of the parameters of reactor and machine is analysed and some useful results have been obtained.

Appendix I

Equation of Synchronous Machine in α-β Coordinate System

Under the condition that the field current I_f is constant the equation of synchronous machine is as follows.

$$V=ZI=(Z_1+Z_2p)I$$

where

$$V=col\left[V_{1\alpha}\ V_{1\beta}\ V_{2\alpha}\ V_{2\beta}\ V_{3\alpha}\ V_{3\beta}\right]$$

$$I=col\left[i_{1\alpha}\ i_{1\beta}\ i_{2\alpha}\ i_{2\beta}\ i_{3\alpha}\ i_{3\beta}\right]$$

Z is the impedance matrix of machine. $V_{1\alpha}$, $V_{1\beta}$, $i_{1\alpha}$ and $i_{1\beta}$ are voltages and currents of the machine in α-β system respectively. $i_{2\alpha}$ and $i_{2\beta}$ are field currents, $V_{3\alpha}$, $V_{3\beta}$, $i_{3\alpha}$ and $i_{3\beta}$ are voltages and currents of the damper windings respectively.

The matrices Z_1 and Z_2 are as follows.

$$Z_1=\begin{bmatrix} r-2L_{1s}\dot\theta\sin2\theta & 2L_{13}\dot\theta\cos2\theta & 0 & 0 & -2M_{13s}\dot\theta\sin2\theta & 2M_{13s}\theta\cos2\theta \\ 2L_{1s}\dot\theta\cos\theta & r+2L_{1s}\dot\theta\sin2\theta & 0 & 0 & 2M_{13s}\dot\theta\cos2\theta & 2M_{13s}\dot\theta\sin2\theta \\ 0 & 0 & 0 & \dot\theta & 0 & 0 \\ 0 & 0 & -\dot\theta & 0 & 0 & 0 \\ -M_{13s}\dot\theta\sin2\theta & \dot\theta\,(M_{13}+M_{13s}\cos2\theta) & 0 & 0 & R_3+R_{3s}\cos2\theta-L_{3s}\dot\theta\sin2\theta & R_{3s}\sin2\theta+\dot\theta\,(L_3+L_{3s}\cos2\theta) \\ -\dot\theta\,(M_{12}-M_{13s}\cos\theta) & M_{13s}\dot\theta\sin2\theta & 0 & 0 & R_{3s}\sin2\theta-\dot\theta\,(L_3-L_{3s}\cos2\theta) & R_3-R_{3s}\cos2\theta+\dot\theta L_{3s}\sin2\theta \end{bmatrix}$$

$$Z_2=\begin{bmatrix} L_1+L_{1s}\cos2\theta & L_{1s}\sin2\theta & M_{12} & 0 & M_{13}+M_{13s}\cos2\theta & M_{13s}\sin2\theta \\ L_{1s}\sin2\theta & L_1-L_{1s}\cos2\theta & 0 & M_{12} & M_{13s}\sin2\theta & M_{13}-M_{13s}\cos2\theta \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ M_{13}+M_{13s}\cos2\theta & M_{13s}\sin2\theta & 0 & 0 & L_3+L_{3s}\cos2\theta & L_{3s}\sin2\theta \\ M_{13s}\sin2\theta & M_{13}-M_{13s}\cos2\theta & 0 & 0 & L_{3s}\sin2\theta & L_3-L_{3s}\cos2\theta \end{bmatrix}$$

where

$L_1=\frac{L_d+L_q}{2}$; $\quad L_{1s}=\frac{L_d-L_q}{2}$;

$L_3=\frac{L_{rrd}+L_{rrq}}{2}$; $\quad L_{3s}=\frac{L_{rrd}-L_{rrq}}{2}$;

$R_3=\frac{R_{rrd}+R_{rrq}}{2}$; $\quad R_{3s}=\frac{R_{rrd}-R_{rrq}}{2}$;

$M_{13}=\sqrt{\frac{3}{2}}\frac{L_{ard}+L_{arq}}{2}$; $\quad M_{13s}=\sqrt{\frac{3}{2}}\frac{L_{ard}-L_{arq}}{2}$;

$M_{12}=\sqrt{\frac{3}{2}}L_{afd}$.

- r — resistance of the armature winding per phase in Ω.
- R_{rrd}, R_{rrq} — D-axis and Q-axis resistance of damper windings respectively in Ω.
- L_d, L_q — D-axis and Q-axis inductance of the armature windings respectively in H.
- L_{afd} — maximum value of mutual inductance between armature winding and field winding.
- L_{ard}, L_{arq} — maximum value of mutual inductance between armature winding and damper winding in D-axis or in Q-axis respectively in H.
- L_{rrd}, L_{rrq} — inductance of damper winding in D-axis and Q-axis respectively in H.
- θ — electrical angle between the axis of phase winding A and D-axis of the rotor along rotor rotating direction. The relation between θ and γ is $\theta=\omega t-\pi/2-\gamma$.

The subscripts a, f and r express phase winding A, field winding and damper winding respectively.

Appendix II

Step for non-salient machine

The step to solve the equation of the system with a non-salient machine operating at high speed is as follows.

1. Based on the equations of the mode of the system find out the matrix A for every mode.
2. With the method for e^{At} find analytic solution of the following equation.

$$I(0)=H\,e^{A_2(\pi/3\omega-t_1)}\,e^{A_1t_1}\,I(0)$$

and obtain a nonlinear equation set.

3. Add the following equation to preceding nonlinear equations

$$i_{1\alpha}(t_1)=\sqrt{3}\, i_{1\beta}(t_1)$$

and solve this equation set. Thus find out the initial values I(o) and the transfer time t_1 .

4. With following equations

$$I(t)=e^{A_1 t} I(0) \qquad 0 \le t \le t_1$$

and

$$I(t)=e^{A_2(t-t_1)} I(t_1) \qquad t_1 \le t \le T$$

to find I(t).

For salient machine with damper windings at high speed and at low speed, approximate initial values of the system all must be found out at first. The step is similar to preceding except term 2 has a little difference. The form of nonlinear equation set is different with various modes of the system.

Step for Salient machine without Damper Windings

1. Choose roughly the limits (l_1 , l_2) of initial value, where $l_1 < l_2$.
2. Optimize first initial value within the limits.

$$i_{1\alpha}(0)=l_1+0.618(l_2-l_1)$$

3. Compare $i_{1\alpha}(o)$ with l_1 , if $i_{1\alpha}(o)$ is better than l_1 , $i_{1\alpha}(o)$ replaces l_1 , else $i_{1\alpha}(o)$ replaces l_2 and form a new shortened range (l_1 , l_2).
4. Continue to optimize within the new limits (l_1 , l_2) and repeat terms 3 and 4 until find out the initial value.
5. Solve system equation by Runge-Kutta method.

This step is suitable for the system operating at high speed and at low speed except the transformation matrix H_4 has a little difference.

Step for Damper Winding Machine at low speed

1. Define the transformation matrix H_4 by $\omega, \omega_e, \psi, \alpha$.
2. Judge whether ψ is larger than α . Thus deside if mode 1 and mode 2 appear or not.
3. Assume that mode 2, mode 4, mode 7 and mode 8 don't appear and machine is corresponding to a nonsalient machine. Find the approximate initial values of the system by the method for nonsalient machine system. In progress define progressively whether mode 7 and 8 appear or not.
4. Find the closed initial values and solve the system equation by Runge-Kutta method.

Reference

1. Fumio Harashima "Dynamic Performance Self Controlled Synchronous Motors Fed by Current-Source Inverters" IEEE IA-15 1979. No.1

2. Hu Yuwen "The Method for finding the Analytic Solution of the State transition Matrix e^{At} with Computer" Journal of Nanjing Aeronautical Institute 1981. No.3