ICAS-82-2.8.4

PREDICTION OF CRITICAL SPEEDS OF FLEXIBLE SUPPORT-ROTOR SYSTEMS BY SUBSYSTEM IMPEDANCE ANALYSIS

Huang Tai-Ping

Nanjing Aeronautical Institute

Nanjing, People's Republic of China

Abstract

The subsystem impedance analysis (SIA) is developed for the prediction of critical speeds of flexible support-rotor systems. The flexible support-rotor system is divided into rotor and support subsystems. By assuming impedance values of the coupling points, the critical speeds of the rotor subsystem can be determined from its frequency equation. The displacement impedance of the support subsystem is determined experimentally, or, by theoretical calculation if the mathematical model of the support subsystem can be well established. According to the conformity of the impedance at coupling points of subsystems, the critical speeds of the whole rotor system can be obtained graphically.

Examples of rotor systems with single and double flexible supports are presented. Calculations and experimental results show that SIA is not only accurate and reliable, but is also an economical method for analytical and experimental research in rotor dynamics.

I. Introduction

Flexible supports have been widely used in rotors of modern gas-turbine aeroengines. The rotor systems operate stably in a range above the first or second order critical speed and below the bending critical speed. Flexible supports are very effective in adjusting the performance of the engines.[1,2]

The effect of the flexible supports on the dynamic behaviour of rotor systems is very strong. In the engine design, some of the flexible supports, which have been studied experimentally or theoretically, may be adopted to connect with the rotor, so that the dynamic behaviour of the whole rotor system will conform to the design requirements. In the calculation of critical speeds, generally, only the static stiffness of the support is taken into account. In recent years, however, the dynamic behaviour of the support has been more and more considered.[3,4,5] In this paper, the theoretical analysis and the calculations of some single-disk rotor systems by the SIA method are presented, and the application of the mechanical impedance measurement technique to the study of dynamic behaviour of the rotor system is also introduced. The analysis and experiments show that critical speeds of whole rotor systems, determined by SIA, are in good agreement with the exact solutions and experimental results, obtained by other methods. With this approach, the design period of the rotor system will be shortened, and the calculation and test costs will also be reduced.

II. Theoretical Analysis

The dynamic behaviour of the flexible support-rotor system is quite different from that of a rigid support-rotor system. There are many methods for analytical calculation of the dynamic behaviour of rigid support-rotor systems.[6] By those methods mentioned in Ref.(6), the dynamic behaviour of the flexible support-rotor system can be calculated exactly, when a constant constraint condition is given at the supported points of the rotor. But in engine structures, because of the variable stiffness and damping of the oil film bearing, the vibrating mass and flexibility of the pedestal and the case, etc., the flexible supports of the rotor are never under a constant constraint condition. The dynamic behaviour of the flexible support varies linearly or non-linearly depending on the frequency and amplitude, so that, these methods can not be applied.

By dividing flexible support-rotor system into rotor and support subsystems at the coupling points, the dynamic behaviour of the subsystems can be investigated individually. At the coupling point, the mechanical impedance value of the rotor and support subsystem must be identical. According to this coupling condition, the dynamic behaviour of whole rotor system can be determined. This is the subsystem displacement impedance analysis (SIA).

Actually, SIA is a variant of the substructure coupling method of the dynamics of the complex structures.[7] It is an approach, in which the mechanical impedance technique is used and the theoretical calculation is thereby matched with experiments.

1. The rigid support-rotor system.

The equation of motion of any vibration system can be generally written in the force expression as:[8]

$$Z \cdot X = Q \qquad (1)$$

where

- Z -- the displacement impedance matrix of the system;
- X -- the displacement vector of the system;
- Q -- the external force vector, imposing on the system.

Equation (1) can also be written in the displacement expression as :

$$Y \cdot Q = X \qquad (2)$$

where

- Y -- the displacement admittance matrix of the system.

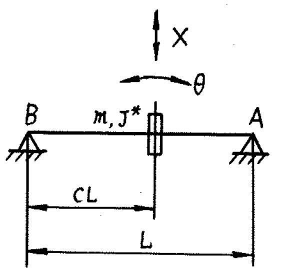


Fig. 1

For a single-disk rotor system, as shown in Fig. 1, taking into account of the mass and moment of inertia of the disk, the displacement and force expressions can be written respectively as :

the displacement expression :

$$\left.\begin{aligned} X &= Y_{11}(X+e)m\,\omega^2 + Y_{12}J^*\,\omega^2 \\ \theta &= Y_{21}(X+e)m\,\omega^2 + Y_{22}J^*\,\omega^2 \end{aligned}\right\} \qquad (3)$$

the force expression :

$$\left.\begin{aligned} \left(\frac{1}{Y_{11}} - m\,\omega^2\right)X - \frac{Y_{12}}{Y_{11}}J^*\,\omega^2\,\theta &= me\,\omega^2 \\ -\frac{Y_{21}}{Y_{22}}m\,\omega^2 X + \left(\frac{1}{Y_{22}} - J^*\omega^2\right)\theta &= \frac{Y_{21}}{Y_{22}}me\,\omega^2 \end{aligned}\right\} \qquad (4)$$

where

- x -- lateral displacement of the disk;
- θ -- angular displacement of the disk;
- e -- eccentricity of the gravity centre of the disk;
- m -- mass of the disk;

$J^* = J_d\left(\frac{\omega}{\Omega}\cdot\frac{J_p}{J_d} - 1\right)$ -- equivalent moment of inertia of the disk;

- J_d -- diametrical moment of inertia of the disk;
- J_p -- polar moment of inertia of the disk;
- ω -- rotating angular frequency of the rotor;
- Ω -- precession angular frequency of the rotor;

Y_{11}, Y_{12}, Y_{21}, Y_{22} -- influence coefficients of the flexibility. Which are :

$$Y_{11} = \frac{L^3\,C^2(1-C)^2}{3EI};$$

$$Y_{12}=Y_{21}=\frac{L^2\,C(1-C)(1-2C)}{3EI};$$

$$Y_{22} = \frac{L(1-3C+3C^2)}{3EI}.$$

- L -- span between the supports;
- C -- position of the disk;
- EI -- flexural stiffness of the shaft.

The frequency equation of the single-disk rotor system can be obtained from equation (4) as :

$$D=\begin{vmatrix} \frac{1}{Y_{11}} - m\,\omega^2 & -\frac{Y_{12}}{Y_{11}}J^*\,\omega^2 \\ -\frac{Y_{21}}{Y_{22}}m\,\omega^2 & \frac{1}{Y_{22}} - J^*\,\omega^2 \end{vmatrix} = 0 \qquad (5)$$

And the expressions of the responses are:

$$\left.\begin{aligned} X &= \frac{\begin{vmatrix} me\omega^2 & -\frac{Y_{12}}{Y_{11}}J^*\omega^2 \\ \frac{Y_{21}}{Y_{22}}me\,\omega^2 & \frac{1}{Y_{22}} - J^*\,\omega^2 \end{vmatrix}}{D}; \\ \theta &= \frac{\begin{vmatrix} \frac{1}{Y_{11}} - m\,\omega^2 & me\,\omega^2 \\ -\frac{Y_{21}}{Y_{22}}m\,\omega^2 & \frac{Y_{21}}{Y_{22}}me\,\omega^2 \end{vmatrix}}{D}. \end{aligned}\right\} \qquad (6)$$

2. The flexible support-rotor system.

Fig. 2 shows a single-disk rotor system with two supports, in which a single flexible support is used. The motion equation of this system can be written in force expression as:

$$\left.\begin{aligned}
&\left(\frac{1}{Y_{11}}-m\omega^2\right)X-\frac{Y_{12}}{Y_{11}}J^*\omega^2\,\theta-\frac{Y_{1A}}{Y_{11}}X_A = me\,\omega^2\\
&-\frac{Y_{21}}{Y_{22}}m\omega^2X+\left(\frac{1}{Y_{22}}-J^*\omega^2\right)\theta-\frac{Y_{2A}}{Y_{22}}X_A=\frac{Y_{21}}{Y_{22}}me\,\omega^2\\
&-Y_{A1}m\omega^2X-Y_{A2}J^*\omega^2\theta+\left(\frac{1}{Y_A}-m_A\omega^2\right)X_A=Y_{A1}me\omega^2
\end{aligned}\right\}\quad(7)$$

where

X_A -- displacement of the flexible support;

Y_{1A}-- lateral displacement of the disk caused by a unit displacement of the support ;

Y_{2A}-- angular displacement of the disk caused by a unit displacement of the support ;

Y_{A1}-- the reaction force of the support caused by a unit force, imposing on the disk. According to the principle of reciprocity, $Y_{A1}=Y_{1A}$;

Y_{A2}-- the reaction force of the support caused by a unit couple of force, imposing on the disk. Similarly, $Y_{A2}=Y_{2A}$;

$\frac{1}{Y_A}-m_A\,\omega^2 = Z_A$ -- the displacement impedance of the flexible support. In which, Y_A and m_A are the statical flexibility and the vibrating mass of the flexible support, respectively.

The equation (7) can be written in matrix form as :

$$\begin{bmatrix}
\frac{1}{Y_{11}}-m\omega^2 & -\frac{Y_{12}}{Y_{11}}J^*\omega^2 & -\frac{Y_{1A}}{Y_{11}}\\
-\frac{Y_{21}}{Y_{22}}m\omega^2 & \frac{1}{Y_{22}}-J^*\omega^2 & -\frac{Y_{2A}}{Y_{22}}\\
-Y_{A1}m\omega^2 & -Y_{A2}J^*\omega^2 & Z_A
\end{bmatrix}
\begin{Bmatrix} X\\ \theta\\ X_A\end{Bmatrix}
=
\begin{Bmatrix} me\omega^2\\ \frac{Y_{21}}{Y_{22}}me\omega^2\\ Y_{A1}me\omega^2\end{Bmatrix}\quad(8)$$

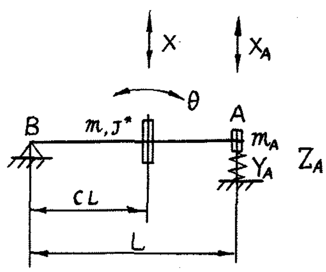


Fig. 2

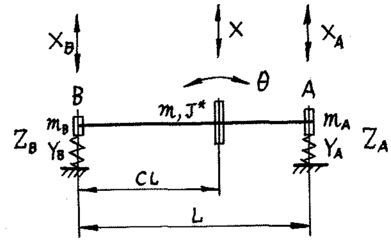


Fig. 3

The matrix form of the motion equation of a flexible support-rotor system, having two flexible supports, as shown in Fig. 3, can be written as :

$$\begin{bmatrix}
\frac{1}{Y_{11}}-m\omega^2 & -\frac{Y_{12}}{Y_{11}}J^*\omega^2 & -\frac{Y_{1A}}{Y_{11}} & -\frac{Y_{1B}}{Y_{11}}\\
-\frac{Y_{21}}{Y_{22}}m\omega^2 & \frac{1}{Y_{22}}-J^*\omega^2 & -\frac{Y_{2A}}{Y_{22}} & -\frac{Y_{2B}}{Y_{22}}\\
-Y_{A1}m\omega^2 & -Y_{A2}J^*\omega^2 & Z_A & 0\\
-Y_{B1}m\omega^2 & -Y_{B2}J^*\omega^2 & 0 & Z_B
\end{bmatrix}
\begin{Bmatrix} X\\ \theta\\ X_A\\ X_B\end{Bmatrix}
=
\begin{Bmatrix} me\omega^2\\ \frac{Y_{21}}{Y_{22}}me\omega^2\\ Y_{A1}me\omega^2\\ Y_{B1}me\omega^2\end{Bmatrix}\quad(9)$$

With equation (8) or (9), the frequency equation and the response expressions of the rotor system can be obtained. The approach mentioned above can be applied to the rotor system with any number of disks.

3. The subsystem impedance analysis (SIA).

In equations (8) and (9), the displacement impedance of the flexible supports Z_A and Z_B are contained. The equations can be solved directly when Z_A and Z_B are expressed simply as a function of the angular frequency ω . In fact, however, the displacement impedance of the flexible support cannot always be expressed as a simple function of frequency, so that the equations (8) and (9) are very difficult to solve.

By assuming the displacement impedance of the supports Z_A and Z_B as constants, the equations (8) and (9) can be solved simply. These are the critical speeds of the rotor subsystem, with assumed displacement impedance values at the supported points of the rotor. Assuming a series of various values of displacement impedance Z_A and Z_B , the critical speeds of the rotor subsystem depending on the values of Z_A and Z_B can be found. Generally, the displacement impedance properties of the support subsystems are determined experimentally. It can also be determined by calculation if the mathematical model of support can be well established. The displacement impedance value at the coupling point of the rotor must be identical with that of the support subsystem. Thus, according to this coupling condition, the critical speeds of the whole rotor system can be readily determined.

Fig. 4 shows a way to determine critical

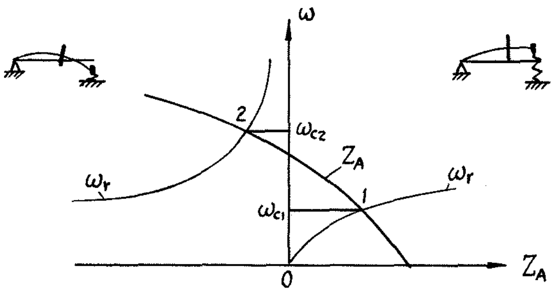


Fig. 4

speeds of a single-disk rotor system, having a flexible support and a rigid support, by means of SIA. The curves ω_r are the critical speeds of the rotor subsystem with various assumed displacement impedance values at the supported point. The curve Z_A is the displacement impedance of the flexible support. The intersections of the curves ω_r and Z_A are points 1 and 2. The frequency ω_{c1} and ω_{c2}, corresponding to the intersection 1 and 2, is the first and second critical speed of the rotor system, respectively. The vibration mode of the rotor system can be determined from the impedance values of the support.

When the rotor system has two flexible supports, the critical speeds can be determined by a simple way as shown in Fig. 5. At first, a frequency ω' will be guessed as near the critical speed as possible. At frequency ω', the value of displacement impedance of support B is Z_B', represented by point 1 on curve Z_B. A curve corresponding Z_B' can be found up from the curves of the critical speeds of rotor subsystem, which were drawn for a series of assumed displacement impedance values at supported point. The curve intersects with the displacement impedance curve Z_A of the support A at point 2, and gives the corresponding frequency ω''. If $\omega''=\omega'$, it is the critical speed of the rotor system. And if $\omega''\neq\omega'$, repeat the same procedure as above until the values of the last two ω are identical. In fact, the repeated procedures are not necessary. A simple way, mentioned as follow, can be used to determine the critical speed. The value of the displacement impedance of support B at frequency ω'' is Z_B'', represented by point 3 on curve Z_B.

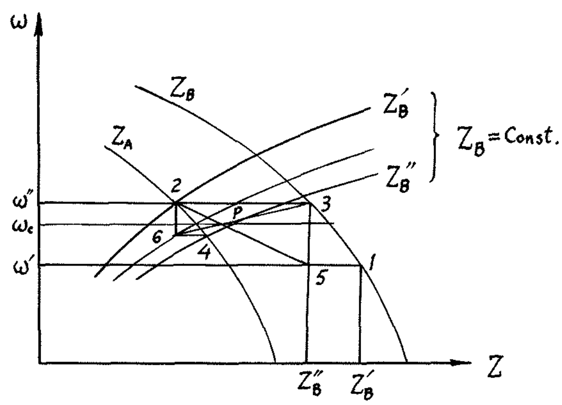


Fig. 5

Another curve corresponding Z_B'' can be found up from the curves of the critical speeds of rotor subsystem. It intersects with curve Z_A at point 4. Draw a vertical line through point 3 and a horizontal line through point 1. They cross at point 5. Again, draw a vertical line through point 2 and a horizontal line through point 4. They cross at point 6. Connect points 2,5 and 3,6 respectively. The intersection of the lines 2-5 and 3-6 is point p. Thus, the frequency ω_c, represented by point p, is the critical speed of the rotor system with these flexible supports Z_A and Z_B. The higher order critical speeds may be determined in the same way.

The comparison between the critical speeds, that were predicted by SIA and the exact solutions of several rotor systems are presented in Table. I. It can be seen that the errors are less than 0.2%.

Figs. 6-8 are graphs for determining critical speeds of flexible support-rotor systems by SIA.

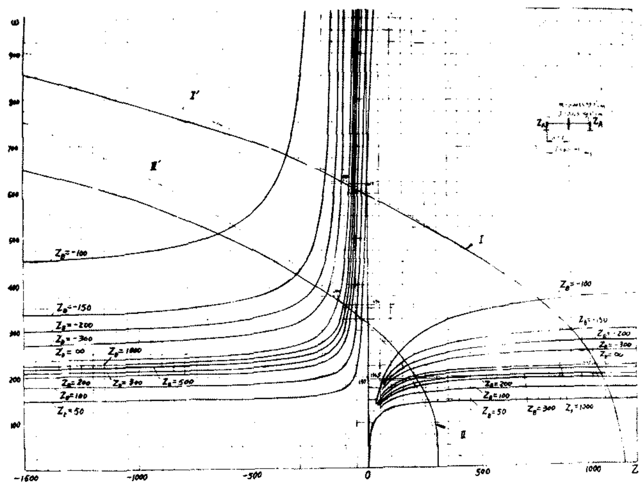


Fig. 6

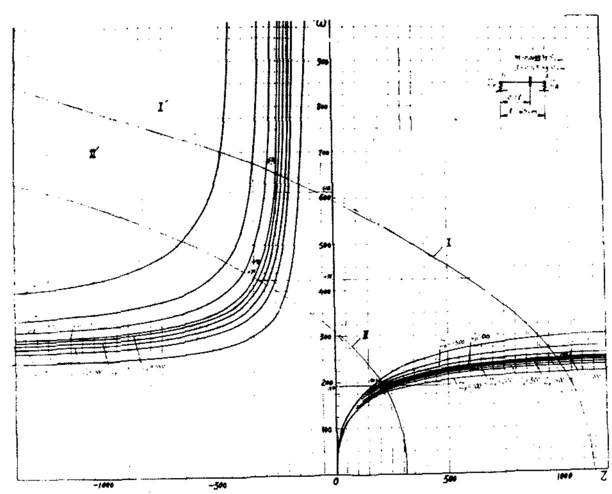


Fig. 7

Table I Comparison between the critical speeds determined by SIA and exact solutions.

ω rad/sec supports A	B	rotor* order	No. 1 exact	No. 1 SIA	No. 1 err. %	No. 2 exact	No. 2 SIA	No. 2 err. %	No. 3 exact	No. 3 SIA	No. 3 err. %
I′	rigid	1st	220.98	221	.009	248.58	248.5	.032	269.20	269	.074
		2nd	619.86	620	.023	674.40	674	.06	606.80	607	.033
II′	rigid	1st	194.49	194.5	.005	190.38	190.5	.063	239.90	240	.042
		2nd	366.06	366	.016	458.36	458	.08	353.60	354	.113
II′	I′	1st	190.20	190	.105	189.28	189	.148	226.30	226	.132
		2nd	356.70	356	.196	437.65	438	.08	338.00	338	.000
		3rd	617.80	618	.03	613.10	614	.147	664.90	664	.135

*No.1, No.2 and No.3 rotor is shown in Fig. 6, Fig. 7 and Fig. 8 respectively.

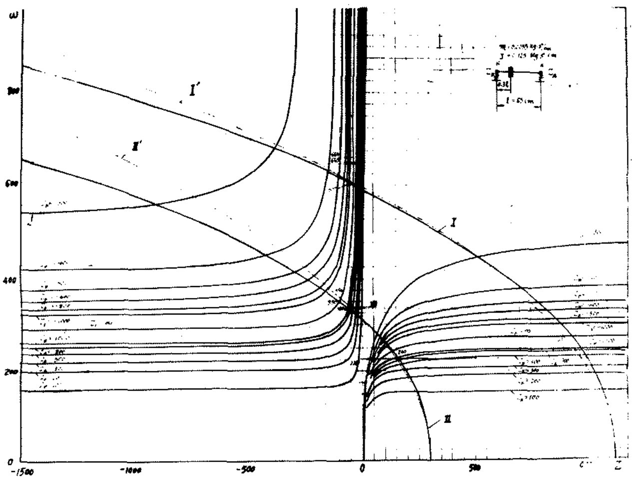
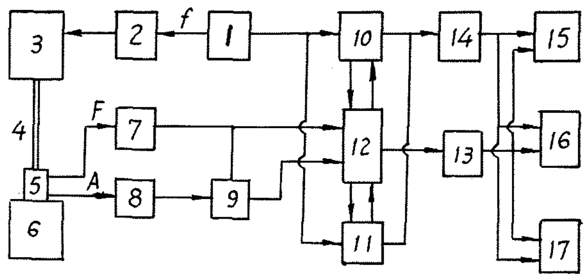


Fig. 8



1-generator, 2-amplifier, 3-vibrator, 4-flexible bar, 5-impedance head, 6-tested support, 7,8-amplifier, 9-mass canceller, 10,11-following filter, 12-A/D converter, 13-computer, 14-phase meter, 15- X-Y recorder, 16-teletype, 17-puncher.

Fig. 9

III. Experiments and Results

1. Impedance of the flexible supports.

The displacement impedance is an important data to determine the critical speeds of the rotor system by SIA. In recent years, the measurement technique of impedance property of the support has been introduced in many papers. (9,10,11)

Fig. 9 shows a scheme of impedance measurement of a "squirrel-cage" flexible support. The sweeping signal is amplified and then drives a electro-magnetic vibrator to impose an exciting force on the flexible support. The impedance head transmits the signals of the exciting force and the acceleration response of the support. These signals are treated by the electric charge amplifier, mass canceller, following-filter, A/D converter, computer and are sent out by a X-Y recorder or teletype. They can also be connected to a data processing unit when necessary.

In the mechanical impedance measurement, the following points must be attended to:

(1) The measurement position.

The position of excition and measurement must be identical with the coupling point of the rotor and support.

(2) The exciting force.

The exciting force must be imposed exactly along the normal direction at the exciting point.

(3) The fit situation.

The fit situation of the tested support must conform to the actual operation situation.

(4) The mass cancelling.

The additional masses of the jig and other parts attaching on the tested support must be cancelled by means of mass canceller or other ways.

(5) Numerical calibration of the measurement system.

The numerical calibration of the measurement system is an important key to the experiments. Generally, the calibration can be done by means of the impedance measurement of a "free" mass.

(6) The data treatment.
When necessary, the data may be transformed into the velocity impedance or admittance and displacement impedance or admittance.

Fig. 10 and Fig. 11 show the experimental results of the displacement impedance of two "squirrel-cage" flexible supports.

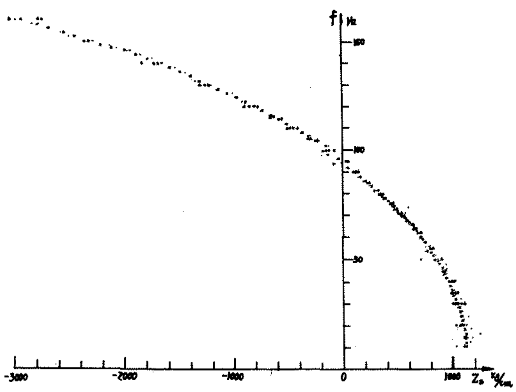


Fig. 10

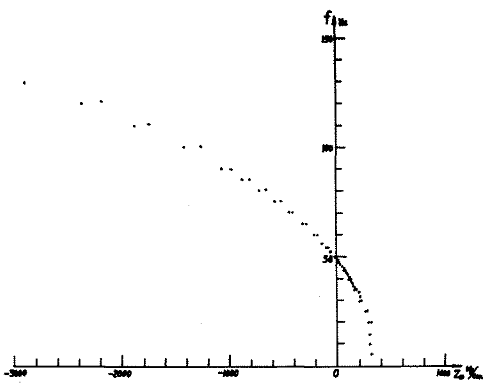
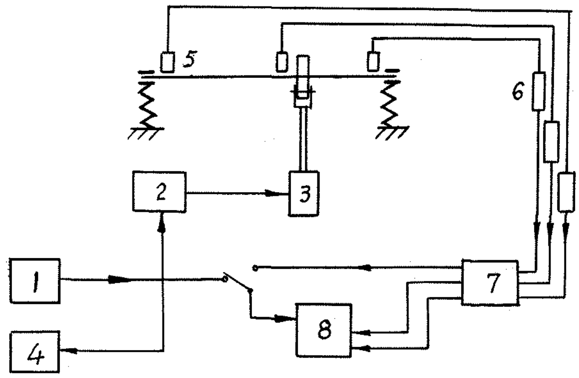


Fig. 11

2. Lateral vibration test of rotor system.
Fig. 12 shows a scheme of lateral vibration test of rotor system. The sinusoidal signal is amplified and then drives the electro-magnetic vibrator. The vibrator is connected to the disk by a flexible bar and excites the rotor system. The eigenfrequencies and modes are measured by means of three eddy-current transducers, located near the disk and supports, respectively.

Table II is a comparison between the eigenfrequencies, determined by SIA, and the experimental results. It can be seen that the errors are less than 2%.



1-signal generator, 2-amplifier, 3-vibrator, 4-frequency meter, 5-displacement transducer, 6-transformer, 7-displacement & amplitude meter, 8-occilloscope.

Fig. 12

Table II Comparison between the eigenfrequencies, determined by SIA, and the experimental results of the flexural vibration of the rotor systems.

No	Support condi.	fre. Hz	1st	2nd	3rd
1	a flex. + a rigid	SIA	29.50	101.00	-----
		exp.	29.20	102.10	-----
		err. %	1.03	1.08	-----
2	two flexible supports	SIA	25.00	55.7	97.0
		exp.	24.80	56.15	96.95
		err. %	0.80	0.80	0.05
3	two flexible supports	SIA	30.4	63.0	102.3
		exp.	30.95	63.80	101.80
		err. %	1.77	1.25	0.49
4	two flexible supports	SIA	28.70	63.88	96.23
		exp.	28.20	64.36	96.50
		err. %	1.78	0.75	0.27

3. The critical speed measurement of the rotor system.

Fig. 13 shows a test rig of a flexible support-single-disk rotor system. A D.C. motor drives the rotor by means of a flexible coupling. The shaft is 20 mm dia. and 800 mm long. The span between two supports is 650 mm. The disk is 200 mm dia. and 20 mm width, and can be set at the midspan or any other position.

(1) The critical speed measurement method.
There are two methods of the measurement. One of them is to measure the maximum of the amplitude response, the other is to measure the change of phase angle (the turn of the eccentricity of the gravity centre). The measurement

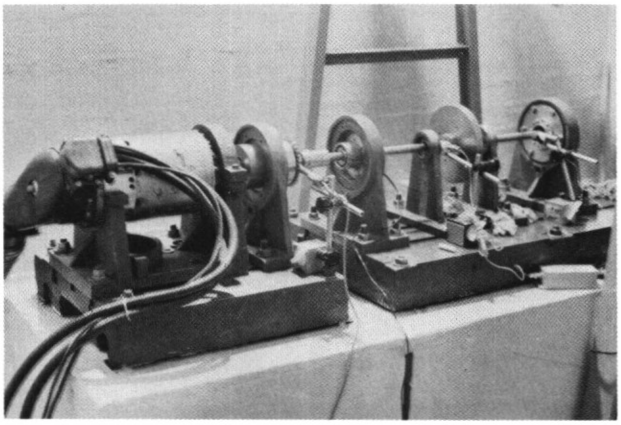


Fig. 13

data of these two methods are in good agreement with each other when the test is in steady state.

(2) The vibration mode.

The response-frequency behaviour of several points can be plotted by means of the association of the measurement of multi-points amplitude-frequency curves and the phase-changes. Then the vibration mode can be obtained. Fig.14 shows the procedure to determine the vibration mode from amplitude-frequency curves and phase changes.

(3) Comparison between the data by calculation and by test.

Table III is a comparison between the critical speeds, determined by SIA (See Figs. 7 and 8), and the experimental results of several rotor systems. It can be seen that the results of the calculation are in good agreement with those of the measurement. The errors are only about 1%.

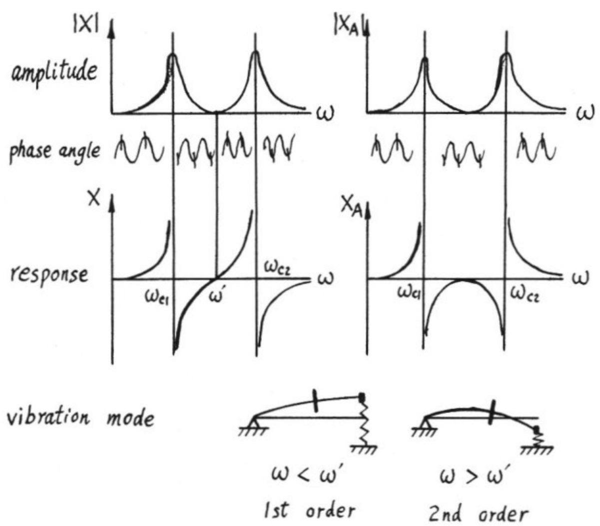


Fig. 14

Table III Comparison between the critical speeds, determined by SIA, and measurement results of several single-disk rotor systems.

Supports A	Supports B	Order	Critical speed $n_c$rpm meas.	SIA	errors %
rigid	II	1st	1820	1819	0.05
		2nd	4200	4154	1.10
II	rigid	1st	2257	2291	1.54
		2nd	3314	3342	0.84
II	I	1st	2182	2158	1.09
		2nd	3274	3223	1.56
		3rd	6164	6159	0.08

IV. Conclusions

1. The flexible supports have a significant influence on the dynamic behaviour of the rotor system. When a single flexible support is used, an additional resonance of the rotor system will occur at lower frequency, and the bending critical speed will be higher than the first critical speed of the rigid support-rotor system. When two flexible supports are used, two additional resonances will occur at lower frequencies, and the bending critical speed is also higher than the first critical speed of the rigid support-rotor system. Generally, the vibrations of flexible support-rotor system at lower frequencies are like that of a rigid body. They are mainly caused by the flexible supports, and can be damped easily. Thus, the stable operation range of the rotor system is increased.

2. The effect of the flexible support on the rotor system can be readily determined by means of SIA. The dynamic behaviour of the flexible supports, by which the design requirements to the rotor system are fulfilled, can be graphically determined from the curves of the critical speeds of rotor subsystem.

3. The design and experimental research of the flexible support must be done systematically. So, a flexible support suitable for new engine design can be found that will satisfy the performance requirements. Thus, the time and cost of rotor dynamic design will be considerably reduced.

V. Acknowledgement

The author is indebted to the Laboratory Engineer Tian Shu-Fang and Chou Di-Lian, Mr. Chian Chung-Lung and Mr. New Chu-Wen for their fine co-operation in carrying out the experiments.

References

(1) M. Balda, Dynamic Properties of Turboset Rotors. DYNAMICS OF ROTORS, IUTAM SYMPOSIUM, 1974.

(2) K.M. Kullem, J. Magge, A New Concept for Critical Speed Control. SAE 670347 1967.

(3) Huang Tai-Ping, Predetermine the Critical Speeds of the Rotor-Flexible Supports System By the Subsystem Impedance Analysis. Journal of Nanjing Aeronautical Institute, No.2, 1980.

(4) E. Krämer, Computation of Vibrations of the Coupled System Machine-Foundation. The Second International Conference Vibrations in Rotating Machinery. 2-4 Sep. 1980

(5) M. Jacker, Vibration Analysis of Large Rotor-Bearing-Foundation-System Using a Modal Condensation for the Reduction of Unknowns. The Second International Conference Vibrations in Rotating Machinery. 2-4 Sep. 1980

(6) L. Eshleman, Flexible Rotor Bearing System Dynamics, I--Critical Speeds and Response of Flexible Rotor System. ASME 1972

(7) Roy R. Craig Jr. & Ching-Jone Chang, A Review of Substructure Coupling Methods for Dynamics Analysis. NASA CP-2001 1976

(8) Huang Tai-Ping, The Application of Mechanical Impedance in the Vibration Analysis of Rotor System. Report in Science and Technology Nanjing Aeronautical Institute, No. 284, 1978.

(9) Huang Tai-Ping, The Subsystem Impedance Analysis and Measurement of the Vibration of a Flexible Support-Rotor System. Report in Science and Technology Nanjing Aeronautical Institute, No. 284, 1980.

(10) Toshio Miyachi, Oil Squeeze Film Dampers for Reducing Vibration of Aircraft Gas Turbine Engine. ASME 79-GT-133.

(11) David P. Fleming, William J. Theyer and Robert E. Cunmingham, Experimental Dynamic Stiffness and Damping of Externally Pressurized Gas-Lubricated Journal Bearings. NASA TN D-8270 1976.