THE ROLE OF THE SCALE PARAMETER IN SERVICE LOAD ASSESSMENT AND SIMULATION

ICAS-82-2.8.3

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Abstract

The so-called integral scale L, thought of originally only as a special turbulence concept, is in fact a universal parameter for stationary stochastic processes being complementary to the standard deviation σ. For complete description of the process, i.e. in composing the autocorrelation and the power-spectral density functions, both of them have to be used in company with one or two additional parameters peculiar to the type of the process.

Atmospheric turbulence measurement and evaluation is restricted to a finite frequency band-width. Conversion of measured to theoretical standard deviation and vice versa is feasible by use of the dimensionless parameter Ln_1. Direct calculation of time spectra from space spectra is also possible by the scale parameter. Correct and intelligent treatment of stochastic transients is also facilitated by it.

All these conveniences can be extended to road surface profile /e.g. runway roughness/ measurements, too.

Notation:

a	vertical acceleration of aircraft C.G.	m/s^2
c	wind resp. updraft mean speed	m/s
f	frequency	1/s
h	time interval between samples	s
n	wave number, reciprocal of wavelength	1/m
p( )	probability density function	
r	thermal core radius	m
t	time	s
x	road surface height above mean	m
y	system output /stress, strain, acceleration, etc./	
w	turbulent velocity component normal to the flight path	m/s
G( )	power spectral density function	
H( )	frequency response function	
L	/integral/ scale paramter	m
R( )	correlation function	
S	sample length	m
T	time scale	s
V	flight speed	m/s, km/h
α	exponent	
$\varkappa$	cutoff ratio	
σ	standard deviation	
τ	time lag	s
ξ	space coordinate parallel to the flight speed	m
ζ	space lag	m
ω	circular frequency	rad/s
Δ	relative error	
Ω	space frequency	rad/m

Subscipts:

a	for vertical acceleration of aircraft C.G.
m	measured
max	maximal
w	for turbulence velocity
x	for road surface elevation
y	for system output
o	theoretical, without frequency cutoff
1	low frequency cutoff
2	high frequency cutoff

Superscripts:

T	transpose
*	complex conjugate

1. Introduction

The unknown inventor of the wheel, while presenting mankind with one of the basic machine parts, has at the same time pioneered also in scientific abstraction. Following in his or in their steps, a very successfull edifice of deterministic physical laws has been erected by a most distinguished succession of scientific/technical experts working on it for several thousand years.

This happy state of affairs began to show the first signs of imperfections with the advent of modern thermodynamics. Further on, even some fundamental problems were raised in respect of the permissibility of thinking in terms of all-deterministic physical laws by nuclear physics respective by quantum mechanics. Roughly parallel to this, the beginning of serious research in the field of flow turbulence demanded also departure from classical deterministic concepts and a search for new methods suitable to analyse stochastic phenomena.

Needless to say, it takes a substantial time to develop stochastic concepts and methods to the level attained by deterministic line of thought in centuries. For the time being, we may be only on the first part of the learning curve.

All this began with the introduction of statistical methods and with the idea of the stationary stochastic processes. This model took its present shape by the advent of correlation technics and with the development of the spectral method.

Basic stochastic process theory is common to all brancses of science and technology but there are also a number of more specific procedures respective concepts originated by and complying with peculiar needs. Up to the present, the definition and the use of the so-called integral scale of turbulence L has been, so to say, a home affair in flow mechanics.

Nevertheless, it is actually a basic parameter common to all stationary stochastic processes in space or in time, as indicated by its mathematical definition. Recent successes in road surface roughness and rail waviness description have amply proved the soundness of this postulation. It may pay to have a closer look at this invention of turbulence specialists for it will turn out not only an ingenious research concept but also a most usefull control and development tool as well.

2. Service Load Assessment Procedures

2.1 Bulk Statistics

First in chronological order and also the most simple analysis sprocedure is straight statistics. Development of the counting accelerometer and of similar,more sophisticated counting devices opened the way to this methodology. But a review of some reports /e.g.: Steiner[18] ,Firebaugh[7], etc./ suffices to manifest the shortcommongs of the method. Without intelligent sorting out of data according to weather, flight situation, etc. several hundred flying hours are needed for arriving at statistically significant sample sizes.And not only that. Conclusions drown from the results this way are valide strictly only for the same circumstances, i.e. there is no possibility for a universal and reliable forecasting to meet future needs.

2.2 Service Load Classification

Prerequisite for an intelligent sorting of service load records is a logical classification system including primary external as well as construction parameters and facilitating the choice of a correct data processing method. Service loads are originating from gravity, inertial effects, engine and drives, environmental effects /atmospheric turbulence/,manoeuvres,etc.[17] The average respective the characteristic flying program of the airplane type may contain several different flight tasks the execution of whose leads to a number of flight manoeuvres respective situations[10]. Correct and efficient numerical procedures for data reduction and assessment have to conform to the character of the respective load time histories. A classification according to the kinematic character of the time history, a proposal amending that of Bendat and Piersol[1] is shown on Tab.1.

2.3 Spectral Method

The spectral method, principal theoretical research tool of modern vehicle dynamics, is based on linear mechanics and on the concept of an ideal stationary

Table 1: Classification of load time histories

		Deterministic: Nonperiodic	Deterministic: Periodic	Random
Process	Stationary	Constant	Sinusoidal: single-frequency multi-frequency /complex periodic/	Ergodic Non-ergodic
Process	Non-stationary	Convergence Divergence	Sinusoidal: non-stationary amplitude almost periodic	Non-stationary: mean value standard deviation power spectrum, etc.
Process	Transient		Sinusoidal transient amplitude	Stochastic transient
Event	Transient	Non-periodic transient	Damped sinusoidal	Random amplitude time function

stochastic process. The input, i.e. atmospheric turbulence, is represented in the calculations by its power-spectral density function $G_w(\Omega)$, the mechanical system of the airplane by its frequency response function $H(\omega)$ resp. $H(f)$. Dynamic response calculations are performed in the frequency domain.

Of course, there is no ideal, unlimited stationary stochastic process in nature.So the fundamental equation of the spectral method[1] /see Fig.1/

$$G_y(f) = |H_{wy}(f)|^2 \, G_w(f) \qquad /1/$$

connecting the single output y(t) of a constant parameter linear system to the input, say w(t), is to be used with some caution. For the multiple input - multiple output model, the same relation between the input gust velocity matrix $\underline{\underline{G}}_{ww}(f)$ and the output matrix $\underline{\underline{G}}_{yy}(f)$ reads[20]:

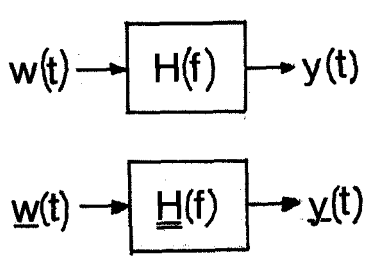

Figure 1:Linear Input - Output Relations

$$\underline{\underline{G}}_{yy}(f) = \underline{\underline{H}}^*_{wy}(f)\, \underline{\underline{G}}_{ww}(f)\, \underline{\underline{H}}^T_{wy}(f) \qquad /2/$$

$\underline{\underline{H}}_{wy}(f)$ being the complex frequency response matrix of the system. As a third step in modelling, appropriate integral relations can be developed from Equ. /2/ to account for the continuous character of atmospheric turbulence and of the airplane surfaces. This is however of minor importance, since the principal source of errors is nither discretizing nor the imperfect linearity of the airplane frequency-response function but the quasi-stationary character of atmospheric turbulence.

3.Stationary Stochasztic Turbulence Models

3.1 Discrete vs. Stochastic Gust Models

For want of a better possibility, early airplane gust load calculations have been based on isolated, deterministic gust models of the ramp- or of the one-minus--cosine type. Nobody beleived this to be a fair picture of physical reality. It has been used only to cover expectable dynamic load maxima /see e.g. Pratt and Walker[18]/.

In this respect and for sailplane performance calculations, isolated gust models have not lost their usefulness. Meanwhile some more refined updraft profiles have been worked out, e.g.[8]:

$$c = c_{max}[1-(\frac{\xi}{r})^2]\exp[-(\frac{\xi}{r})^2] \qquad /3/$$

/see Fig.2/. Even thermal groups, as reported on by Konovalov[15], can be treated in this way[9].

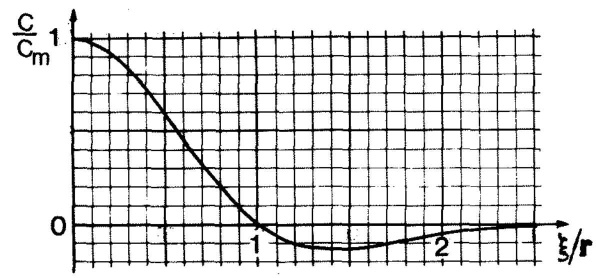

Figure 2: Isolated Thermal Model

The continuous turbulence model, too, can be used for peak design load calculations[18] but its chief usefulness is in fatigue work and in handling design and development. Inspired by the successes in these fields, some authors went even as far as to speak of turbulence modelling on a global scale. This however can be shown to be incorrect according to general stochastic process theory /see e.g. Bendat and Piersol[1,2]/ and even impractical[23]. So in the following we shall speak only of the oscillating components of the atmospheric turbulence flow field, the slowly varying mean flow being defined by appropriate deterministic equations.

3.2 Evolution and Physical interpretation of the Scale Parameter

Early theoretical work pianeered by Prandtl and Taylor relied considerably on the law of similars and on the concept of a characteristic length called then also "Mischungsweg"[21]. Taylor[22] connected in 1935 the determination of characteristic lengths with the autocorrelation function

$$R_w(\zeta) = \lim_{S\to\infty} \int_0^S w(\xi)w(\xi+\zeta)d\xi \qquad /4/$$

and defined two scale lengths for isotropic turbulence. The one of them we are concerned with, the so-called integral scale of turbulence, is/see Fig.3/:

$$L = \frac{1}{\sigma_w^2} \int_0^\infty R_w(\zeta)d\zeta \qquad /5/$$

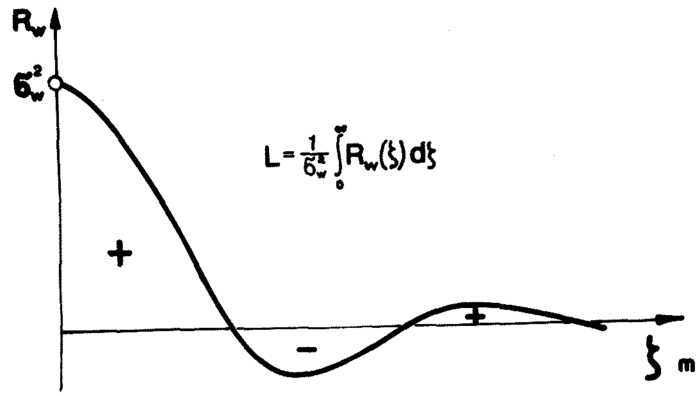

Figure 3: Determination of the Scale of Turbulence

Kármán and Howart[13] gave then a general survey of the correlation problem. Kármán put also the new parameter to quick use for eliminating the singularity of simple negative power law PSD formulae at zero wave number.

While Equ. /5/ is standard reference in every textbook, opinions on its exact physical interpretation are divided and sometimes even not unambiguous. Kovásznay, after a full and concise mathematical dedution /see Ref.[6] pp. 91-94./ does not give any physical interpretation. Duncan[5] beleives it to be a measure of average eddy size. According to Hintze[11]: "This length is to a certain extent a measure of the longest connection or correlation distance between the velocities." This latter view is the most attractive to the author, too, while guessing also some relation to the coherence length.

3.3 Current Atmospheric Turbulence PSD Formulae

There are a number of stochastic gust spectrum equations in use now. The first universally accepted one is attributed to Dryden and reads[4]:

$$G_w(\Omega) = \sigma_w^2 \frac{2L}{\pi} \frac{1 + 3\,(L\Omega)^2}{[1+(L\Omega)^2]^2} \qquad /6/$$

The value 2 for the exponent being a little on the high side, Kármán proposed in 1948[14]:

$$G_w(\Omega) = \sigma_w^2 \frac{2L}{\pi} \frac{1+\frac{8}{3}(1.339\;L\Omega)^2}{[1+(1.339\;L\Omega)^2]^{\frac{11}{6}}} \qquad /7/$$

First atmospheric turbulence PSD measurements from aircraft have been made by Clementson[3] in 1950. For low altitude work Lappe proposed the formula in 1966[16]:

$$G_w(\Omega) = \sigma_w^2 \frac{L}{(1+L\Omega)^2} \qquad /8/$$

According to Firebaugh[7], this equation got the following shape at Lockheed-Georgia:

$$G_w(\Omega) = \sigma_w^2 \frac{0.8L}{(1+0.8L\Omega)^{1.8}} \qquad /9/$$

There is however a little problem concerning the constants in Eq s. /8/ and /9/. As it can be easily seen, both of them belong to the category characterized by the collective equation

$$G_w(\Omega) = \frac{G_w(0)}{(1+C\Omega)^{\alpha}} \qquad /10/$$

Now, as a special case of the Wiener--Khinchin relations, it can be shown that

$$G_w(0) = 4\int_0^{\infty} R_w(\zeta)d\zeta = \frac{2}{\pi}\,\sigma_w^2\,L \qquad /11/$$

/See e.g. Ref. [6]/. At the same time, another well-known relationship is giving for this type of formula

$$\sigma_w^2 = \int_0^{\infty} G_w(\Omega)d\Omega = G_w(0)\int_0^{\infty}\frac{d\Omega}{(1+C\Omega)^{\alpha}} = \frac{G_w(0)}{(\alpha-1)C} \qquad /12/$$

Combining Eqs. /11/ and /12/ gives:

$$C = \frac{2}{\pi(\alpha-1)}\,L \qquad /13/$$

Substituting this result in Equ. /10/ gives finally:

$$G_w(\Omega) = \frac{2}{\pi}\,\sigma_w^2 \frac{L}{(1+\frac{2}{\pi(\alpha-1)}\,L\Omega)^{\alpha}} \qquad /14/$$

According to this result, the Lappe spectrum should read:

$$G_w(\Omega) = \frac{2}{\pi}\sigma_w^2 \frac{L}{(1+\frac{2}{\pi}L\Omega)^2} = \sigma_w^2 \frac{0.63662L}{(1+0.63662L\Omega)^2} \qquad /15a/$$

or

$$G_w(n) = \sigma_w^2 \frac{4L}{(1+4Ln)^2} \qquad /15b/$$

The exponent $\alpha=1.8$ being close to the theoretically well-founded $11/6=1.8333$ value, for the Lockheed-Georgia formula we would propose:

$$G_w(\Omega) = \frac{2}{\pi}\,\sigma_w^2 \frac{L}{(1+\frac{12}{5\pi}L\Omega)^{\frac{11}{6}}} =$$

$$= \sigma_w^2 \frac{0.63662L}{(1+0.76394\;L\Omega)^{1.8333}} \qquad /16a/$$

respective

$$G_w(n) = \sigma_w^2 \frac{4L}{(1+\frac{24}{5}Ln)^{\frac{11}{6}}} =$$

$$= \sigma_w^2 \frac{4L}{(1+4.8Ln)^{1.8333}} \qquad /16b/$$

/See Fig.4/.

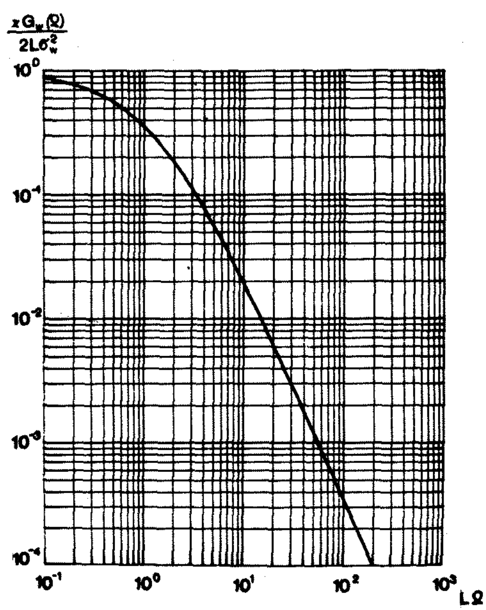

Figure 4: Modified Lockheed-Georgia Turbulence Spectrum

4. Turbulence Parameter Measurement Problems

4.1 Quasi-Stationary Atmospheric Turbulence

The uccurrence of assessment problems like the afore-mentioned one can be attributed mainly to the quasi-stationary character of atmospheric turbulence and to some peculiarities of the measurement/evaluation procedure. The nonstationarity can be easily studied e.g. on the structure of thermal convection. The flow field isn't homogeneous over greater distances. It looks much more like an ensemble of individual, different convection cells[15]. Scale of turbulence L values as published by Lappe[16] are about equal to the updraft core diameter 2r.

When averaging over greater distances than say 4r then inevitably some major error is bound to occur. The same can be said if individual samples extend over different regions. In every such case the parameters calculated from the samples will be undefined averages of two or more different values.

In order to get some insight into the essence of this problem, let us review shortly the respective calculation procedures.

The standard deviation σ_w of the turbulence is by definition:

$$\sigma_w = \lim_{S\to\infty} \left[\frac{1}{S} \int_0^S w^2(\xi)d\xi\right]^{\frac{1}{2}} \qquad /17/$$

The same value can be calculated also from the power spectral density function:

$$\sigma_w = \left[\int_0^\infty G_w(n)dn\right]^{\frac{1}{2}} \qquad /18/$$

The scale parameter may be obtained from the autocorrelation function using Equ. /5/, but – choosing the most suitable from Eqs. /6/-/9/ resp. /15/-/16/ – a least squares fit procedure for smoothing of the PSD function gives also an estimation for L. When n_1 approaches zero, S approaches infinity. So the upper bound for S, agreed upon formerly, sets also a lower frequency limit n_1 to the measured power spectrum. There is also always an upper limit n_2 to the measured spectrum, imposed either by the frequency characteristics of instrumentation or by the time interval between samples h. Let us see therefore how this affects error margins in the calculation of parameter values.

4.2 Parameter Assessment by Evaluation of Finite Records

We shall choose for our investigations a PSD function according to the formula

$$G_w(n) = \sigma_o^2 \frac{4L}{(1 + \frac{4}{\alpha-1} Ln)^\alpha} \qquad /19/$$

measured from wave number n_1 to n_2 /see Fig. 5/.

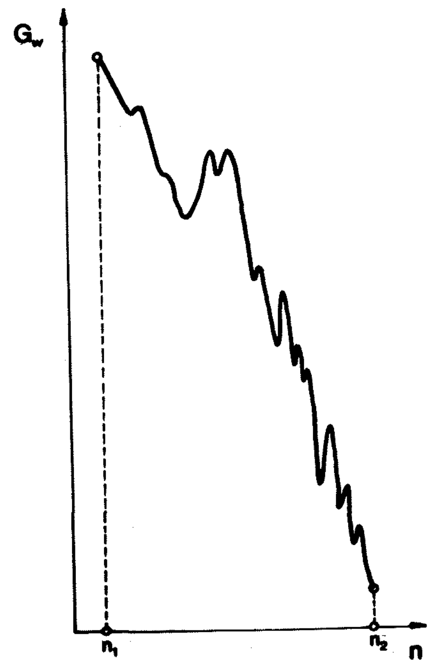

Figure 5: Truncated Turbulence Spectrum

Using a least squares fit procedure for smoothing, the infinite base theoretical standard deviation σ_o may be obtained. As against this, direct calculation of the measured standard deviation from a sample length

$$S = \frac{1}{n_1} \qquad /20/$$

i.e. from the truncated spectrum gives:

$$\sigma_m^2 = \sigma_o^2 \int_{n_1}^{n_2} \frac{4L}{(1 + \frac{4}{\alpha-1} Ln)^\alpha} dn =$$

$$= \sigma_o^2 [\frac{1}{(1+\frac{4}{\alpha-1}Ln_1)^{\alpha-1}} - \frac{1}{(1+\frac{4}{\alpha-1}Ln_2)^{\alpha-1}}] \quad /21/$$

Introducing the cutoff ratios

$$\varkappa_1 = \frac{4L}{\alpha-1} n_1 \text{ resp. } \varkappa_2 = \frac{4L}{\alpha-1} n_2$$

gives then

$$\sigma_m^2 = \sigma_o^2 [\frac{1}{(1+\varkappa_1)^{\alpha-1}} - \frac{1}{(1+\varkappa_2)^{\alpha-1}}] \quad /22/$$

The square of the relative error

$$\Delta_m^2 = \frac{\sigma_m^2 - \sigma_o^2}{\sigma_o^2}$$

is therefore

$$\Delta_m^2 = [\frac{1}{(1+\varkappa_1)^{\alpha-1}} - 1] - \frac{1}{(1+\varkappa_2)^{\alpha-1}} = \Delta_1^2 + \Delta_2^2 \quad /23/$$

Both terms are negative indicating that the measured value of the variance is always less than the theoretical one. The absolute value of the error due to low-frequency cutoff is:

$$|\Delta_1| = \sqrt{1 - \frac{1}{(1+\varkappa_1)^{\alpha-1}}} \quad /24/$$

For the high-frequency cutoff:

$$|\Delta_2| = \sqrt{\frac{1}{(1+\varkappa_2)^{\alpha-1}}} \quad /25/$$

Let us discuss first the problem of high-frequency cutoff /Fig.6/. A little calculation shows that it isn't very serious. So e.g. for a flight speed of V = = 720 km/h and L = 200 m /656 ft/ and h = = 0.002 s f_2 will be 125 1/s giving $\varkappa_2$= = 600. For α = 1.8333 the high-frequency cutoff error is therefore 4.8 %. Even error margins under 1 % are not unachievable.

The low-frequency cutoff error is much greater /see Fig.7/. Even without reference to instrumentation problems, the fact is that record lenths of over say 4L are no more stationary and therefore inevaluable. This gives $\varkappa_1$ a lower limit of

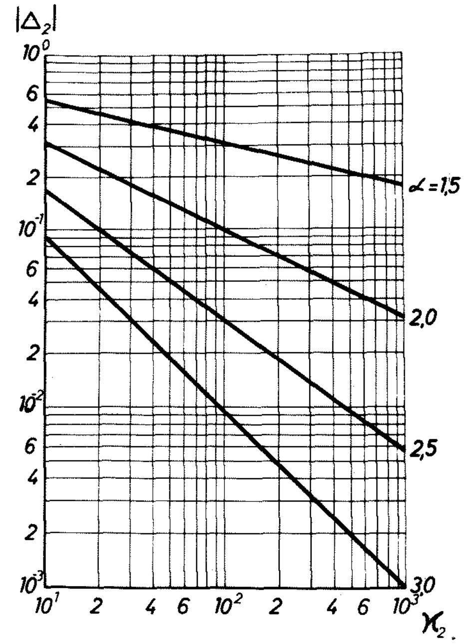

Figure 6: High-Frequency Cutoff Error

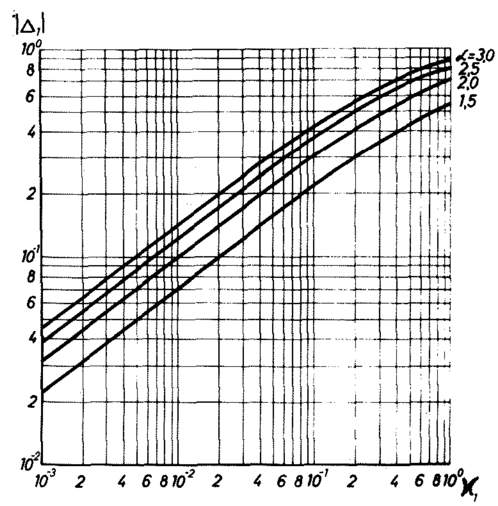

Figure 7: Low-Frequency Cutoff Error

about 1.25. For α=1.8333 the low-frequency error works out to $|\Delta_1|$=63 %. Fortunately this difference is no error in the proper sense but only the difference between infinite frequency range theoretical σ_o and bounded frequency bandwidth measured σ_m values. For a control on the accuracy of measurement and evaluation they have to be compared using Equ./22/. Anyway, the frequency response characteristic of the plane, too, has a very strong filtering effect on these very low frequencies.

Similar relations apply to the Kármán turbulence model.

4.3 Space-Time Conversion

The mainstay of modern aircraft fatigue development is simulation testing reproducing as far as possible actual service loads in the laboratory. An advanced method for this purpose in the so--called remote parameter control of servo-hydraulic testing machines. Drive signals for the actuators can be generated directly from analog tapes registered on test flights. It is however much more reliable to start from the PSD functions of the input signals. After a systematic elimination of random noise and other detectable errors[1,2], a correct equivalent of average gust loads can be calculated via the inverse Fourier-transformation of the input spectral matrix[20].

Power spectra containing the essentials of atmospheric turbulence are written as function of the wave number n respective of the space frequency Ω. For the generation of input time functions w(t) it is inadmissible to transform only the frequency coordinate of the PSD function $G_w(n)$ resp. $G_w(\Omega)$ using the relation

$$f = nV \qquad /26/$$

or

$$\omega = \Omega V \qquad /27/$$

The correct classical elementary process for this purpose is to transform the autocorrelation function $R_w(\zeta)$ into $R_w(\tau)$ by the relation

$$\zeta = \tau V \qquad /28/$$

From $R_w(\tau)$ $G_w(n)$ can be generated by Fourier-transformation.

This laborious process can be evaded the following way. Displacement of ζ in Equ. (5) by τV as given by Equ.(20)proves the time scale of turbulence

$$T = \frac{1}{\sigma^2} \int_0^\infty R_w(\tau)\, d\tau \qquad /29/$$

to be equal to

$$T = \frac{L}{V} \qquad /30/$$

With the time scale T the time-equivalent of say the modified Lockheed-Georgia spectrum Equ. /16b/ for an airplane with a true airspeed V is:

$$G_w(f) = \sigma_w^2 \frac{4T}{(1+4.8\ Tf)^{1.8333}} \qquad /31/$$

A numerical example for this at three different airspeeds is to be seen on Fig. 8.

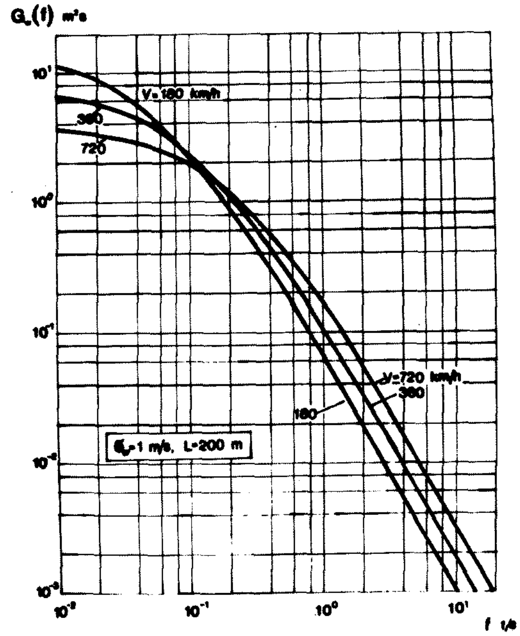

Figure 8: Space-Time Conversion for Different Airspeeds

A glance at the graphs reveals also the cause of the so-called velocity paradoxon. This semi-technical term relates to the peculiarity that when thinking only in terms of the negative power function section of the PSD curve or when transforming the PSD function only according to Equ. /26/ resp. /27/ an apparent increase of the variance with the flight speed is bound to occur.

Integration of Equ. /31/ [see Eqs./12/, /13/] proves without any doubt this opinion to be wrong, the variance of time spectra turning out independent of the vehicle speed V and identical with the original value σ_w^2 .

In spite of this, direct calculation of standard deviation from time records registered on moving vehicles may show a trend of input intensity increasing with speed. A comparison of Figs. 5 and 8 reveals the cause of it. Spectrum truncation at the frequency boundaries of instrumentation resp. evaluation as well as the frequency transfer characteristics of the vehicle may easily cut out a zone of the spectrum increasing in area with speed. So the measured or the effective variance of environmental load input may increase with speed while the theoretical, unlimited frequency width variance remains, of course, constant.

5. Accelerating and Decelerating Flight

A shorthand calculation of atmospheric turbulence-induced air loads on an accelerating or decelerating airplane can also be accomplished, at least in first order linear approximation. The proposed numerical method is based on the condition of the load frequencies significant from the fatigue damage viewpoint being associated with turbulence wavelengths well below the scale of turbulence. This justifies the approximation of the turbulence spectrum by a simple power law giving the familiar descending straight line on log-log plots. The calculation may be done strictly by using weight functions or approximately by adapting quasi-stationary procedures. Lack of space prevents us from giving particulars of this possibility and of its restrictions.

6. Runway Surface Profile Spectra

Every flight starts and closes on the runway, so we are interested also in the correct description of its surface roughness resp. undulations. Indeed, probably the first road surface spectrum has been measured by Walls, Houbolt and Press in 1954 on a runway.

Runway roughness spectra are following the familiar negative power trend for higher wave numbers. Most authors preferred therefore to use simply a negative power function for their analytical description. The singularity of these functions at zero frequency has been circumvented by truncating or by various arbitrary postulations. Recently it has been shown by the author that the concept of the scale parameter can and has to be extended to the road surface description, too[17].

A deduction analogous to Eqs./10/-/14/ gives for the general PSD formula /see Fig. 9/:

$$G_x(n) = \sigma_x^2 \frac{4L}{(1+\frac{4}{\alpha-1}Ln)^\alpha} \qquad /32/$$

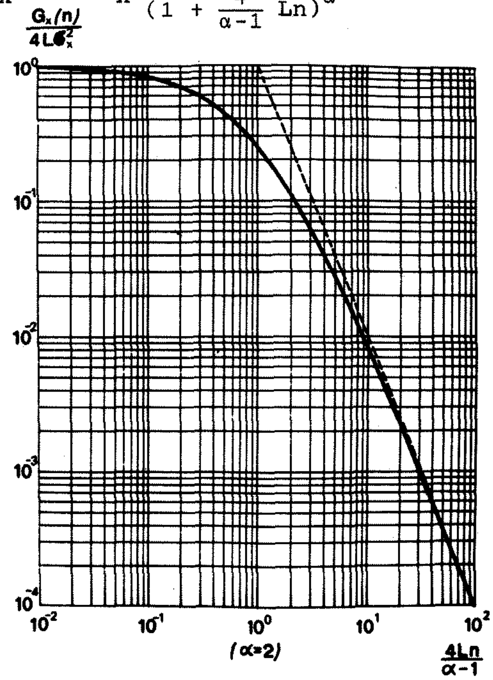

Figure 9: General PSD Diagram for Road Surface Raughness

Evaluation of road profile measurements is giving about L=18-140 m and α=1.6-2.7 according to the type and condition of the surface.

Everything said about parameter assessment, time-space conversion, etc. of the atmospheric turbulence can be adopted logically to the road surface problem,too, except the cellular character.

Some road surface texts may require a more complex PSD formula, e.g.:

$$G_x(n) = \sigma_1^2 \frac{4L_1}{(1+\frac{4}{\alpha_1-1}L_1 n)^{\alpha_1}} + \sigma_2^2 \frac{4L_2}{(1+\frac{4}{\alpha_2-1}L_2 n)^{\alpha_2}} \qquad /33/$$

The theoretical standard deviation is:

$$\sigma_o = \sqrt{\sigma_1^2+\sigma_2^2} \qquad /34/$$

The scale parameter as calculated from the autocorrelation function being:

$$L = \frac{L_1}{1 + (\frac{\sigma_2}{\sigma_1})^2} + \frac{L_2}{1 + (\frac{\sigma_1}{\sigma_2})^2} \qquad /35/$$

7. Conclusions

The integral scale L is a universal parameter for stationary stochastic processes and complementary to the standard deviation σ. Its efficient use in atmospheric turbulence research includes determination of correct constants for PSD formulae, parameter assessment from finite frequency-bandwidth records, direct space--time conversion for fatigue test load programing, etc.

The same concept applies also to runway roughness description.

Acknowledgement

The studies reported on in this paper were part of the research work done on the Department for Mechanics of the Faculty for Transport Engineering under the auspieces of Professor P. Michelberger. The author wishes to express his gratitude for his interest and furthering of the work.

Acknowledgement is due to Mr.F.Galambosi for computer graphs on Figs. 4 and 9 and to Mrs. H. Nagy for the neat typing.

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