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Abstract

The concept of utilizing the fatigue threshold stress intensity factor ΔK_{th} as a design criterion is discussed. This concept is of particular importance for structures sustaining very many load cycles. Determination of stress intensity factors K by means of the finite element method is treated. The experimental evaluation of ΔK_{th} is discussed and the influence of various parameters on ΔK_{th} is scrutinized.

1. Introduction

Traditionally, metal structures that must sustain a very large number of load cycles N have been designed from ordinary SN curves (Wöhler-diagrams) where it for most metals appear to exist a fatigue limit $\Delta\sigma_e$, see Fig. 1. To avoid fatigue failures of such structures they have been designed so that the applied stress range S always should be below the fatigue limit.

The experimental procedure for determining the fatigue limit of a material is very tedious and will be quite expensive if it is properly accounted for all those variables that influence the value of the fatigue limit.

This paper presents an alternative approach to the "fatigue limit" problem. This approach which is based on fracture mechanics was originally developed in the aircraft industry, but has recently come to an extensive utilization in the design also of structures such as wind turbine blades, diesel engines, steam turbines and electric generators [1].

The design concept to be discussed here is the fatigue threshold concept which simply states that for a given load there exist cracks of a certain length that will not propagate. Inversely, we can for a given crack length calculate which load magnitude that will not cause crack growth [2].

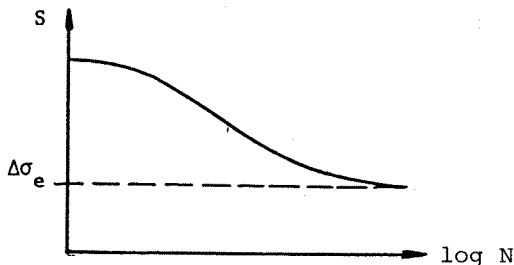


Fig. 1 SN-diagram

2. Fracture Mechanics and Fatigue

2.1 Linear Elastic Fracture Mechanics

Fracture mechanics can be defined as the applied mechanics of crack growth. Historically, early work was on brittle fracture. However, this science has been developed to a state where it now can be applied to virtually any type of crack growth. This development started with the works of Kolosov and Inglis (1909 and 1913) who showed that the elastic stress tends to infinity as a crack tip is approached. This led to the paradoxical conclusion that a cracked body cannot support any load. Griffith (1921) resolved this situation through the use of an energy balance approach which explained the fracture behaviour of glass. Griffith's approach was extended by Orowan and Irwin (1948 and 1949 respectively) to include the energy associated with plastic deformation adjacent to the new crack surfaces. Rice (1968) laid the foundation to non-linear fracture mechanics which deals with situations involving gross plasticity.

Modern linear elastic fracture mechanics (LEFM) to a great extent utilizes the concept of the stress intensity factor K . This factor, originally proposed by Irwin [3] in 1957, permits a description of the elastic stress field in the vicinity of a crack tip in form of one single parameter.

It may be shown [3, 4] that the stress field in the vicinity of a crack tip, see Fig. 2, may be written as:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) \quad (1)$$

where K is the stress intensity factor, r is the distance from the crack tip and $f_{ij}(\theta)$ is a function of the angle θ .

As seen from Eq. (1) the stresses tend to infinity as r approaches zero, i.e. the stress field has an inversed square root singularity at the crack tip. The entire stress field at the crack tip is known when the stress intensity factor K is known.

Depending on the type of load the stress intensity factor, denoted K , is assigned three different subindexes I, II or III. The three cases are shown in Fig. 3.

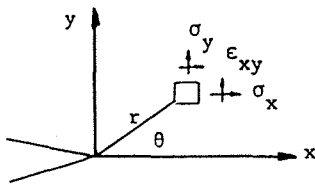


Fig. 2 Stress field in the vicinity of a crack tip

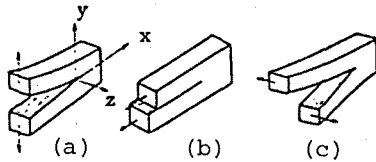


Fig. 3 Modes of crack surface displacement
(a) Opening-mode I
(b) Edge sliding-mode II
(c) Shear-mode III

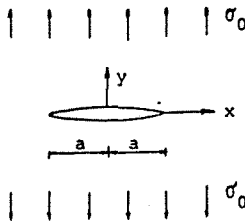


Fig. 4 Infinite plate with central crack

Of these three the first one is the by far most common one at fatigue fractures on a macroscopic scale, so attention is largely confined to mode I.

As an example an infinite plate with remote stresses σ_0 and a central crack of length $2a$ is studied, see Fig. 4.

For this simple case the stress intensity factor is given by

$$K_I = \sigma_0 \sqrt{\pi a} \quad (2)$$

For other cases the stress intensity factor is given by

$$K_I = \sigma_0 \sqrt{\pi a f} \quad (3)$$

where f is a function depending on load and geometry.

Over the past two decades a large amount of work was carried out on the evaluation of stress intensity factors for various load cases and geometries. Results from that work are put together in handbooks, see e.g. [5-7].

Fracture mechanics is further described in Refs. [8, 9].

2.2 Determination of Stress Intensity Factors

Several different techniques can be utilized for the calculation of stress intensity factors. Both analytical, numerical and experimental methods have been

used. Some of the more useful methods of evaluating stress intensity factors are scrutinized in Ref. [10].

From an engineering point of view numerical techniques are of most interest and of these the finite element method [11] is the most versatile. The treatment below will be restricted to this approach.

The stress intensity factor K may either be computed directly from the stress or displacement fields or indirectly via the relationship to other quantities such as the elastic energy. When the direct method is utilized the stress intensity factor is calculated from its relation to the stress field, Eq. (1), or from similar relations to the displacement field [3, 4]. As most finite element programs employ an assumed displacement field the stress intensity factor is best calculated from the displacement field as displacements are more accurately determined than stresses. We then utilize relations as Eq. (4), and similar relations [3, 4] for the displacements u and w .

$$v = \frac{1+\nu}{4E} \sqrt{\frac{2r}{\pi}} \left\{ K_I \left[(2\kappa+1) \sin \frac{\theta}{2} - \sin \frac{3\theta}{2} \right] - K_{II} \left[(2\kappa-3) \cos \frac{\theta}{2} + \cos \frac{3\theta}{2} \right] \right\} \quad (4)$$

By taking the displacements, calculated for elements close to the crack tip, the stress intensity factor can be determined from relations as Eq. (4). In order to improve the results the stress intensity factor is evaluated at different nodes along a ray emanating from the crack tip and then an estimate of K is obtained by extrapolating back to the crack tip [12].

The indirect methods to evaluate K usually calculate the energy release rate G [13, 14] or Rice's contour integral J [15]. A virtual crack extension δa results in a variation $\delta[K]$ in the structural stiffness matrix $[K]$ from which the energy release rate G is calculated. The stress intensity factor is then calculated from Eq. (5), where $\lambda = 1$ for plane stress and $\lambda = 1-\nu^2$ for plane strain.

$$G = \frac{\lambda}{E} (K_I^2 + K_{II}^2) + \frac{1+\nu}{E} K_{III}^2 \quad (5)$$

Alternatively the J -integral, see Eq. (6), which is path independent along any contour surrounding the crack tip may be utilized. As $J = G$ for elastic materials it follows that the stress intensity factor can be evaluated from J utilizing Eq. (5).

$$J = \int_{\Gamma} (w dy - T^t \frac{u}{x} ds) \quad (6)$$

The initial efforts to evaluate K by means of the finite element method used conventional constant strain triangles, e.g. [12], which required hundreds or thousands of elements to secure a reasonable accuracy. To provide for better results with a reasonable number of

elements special crack tip elements have been introduced. A variety of such elements exist with the common characteristic of showing an r^{-2} singularity in the stress field. This may be accomplished either with elements, based on a Williams or Muskhelishvili stress function, embedding the crack tip, e.g. [16], or with singular elements surrounding the crack tip where the singularity is introduced in the assumed displacement field, e.g. [17]. The latter group has come to most utilization and into this category fall the so called quarter-point elements where ordinary isoparametric elements [11] are showing the correct singularity when the midside nodes are shifted to the quarter points. This may be accomplished either for triangular elements [18] or quadrilaterals [19]. The advantage of these elements is that any ordinary finite element program could be utilized for the evaluation of stress intensity factors. Extension to the three dimensional case is discussed in, for instance, [20].

2.3 Fatigue Crack Propagation

Various empirical crack-growth laws have been proposed over the past decades. Most of the early equations were of the form

$$\frac{da}{dN} = C \sigma^m a^n \quad (7)$$

However, no real success with the difficult task of correlating the crack-growth rate da/dN to applied stress, crack length etc was obtained before 1961 when Paris [21] proposed the following expression

$$\frac{da}{dN} = (\Delta K)^m \quad (8)$$

Paris came to this conclusion based on the results from experiments with aluminium. He also gave the value 4 of the exponent m for his test results. Further experiments have shown that for most metals the value of m varies between 2.3 and 6.7 [22]. The very large number of experiments that have been performed since Paris proposed his power relationship clearly indicate that Eq. (8) is not entirely valid over the whole range of ΔK . This means that a log-log plot of da/dN against ΔK is not linear but sigmoidal, see Fig. 5. In this figure we can distinguish between three distinct regions. At low ΔK the fatigue crack growth rate decreases progressively faster until the threshold, ΔK_{th} , for a non-propagating crack is reached. This region is our major concern and will be further dealt with in the following chapters. Next region is composed of a straight line and constitutes the part of the curve where Eq. (8) can be utilized. A third region exists at high ΔK where the slope of the curve increases rapidly as the maximum stress intensity approaches the fracture toughness K_c .

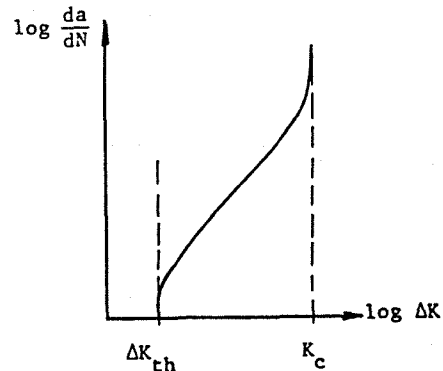


Fig. 5 Schematic shape of fatigue crack growth versus stress intensity variations

Various crack propagation laws have been proposed that include the deviations from the straight line. As example the following Eq., [23], which is assumed to be valid over the entire range of ΔK , may be studied.

$$\frac{da}{dN} = \frac{C(\Delta K^n - \Delta K_{th}^n)}{(1-R)K_c - \Delta K} \quad (9)$$

Similar expressions have been proposed by many workers. A compilation of crack propagation laws incorporating the threshold stress intensity factor is given in [2].

3. The Threshold Stress Intensity Factor ΔK_{th}

3.1 Utilization of ΔK_{th}

In the preceding chapter it was shown that the fatigue crack growth rate is a function of ΔK , the variation in stress intensity. Below a certain threshold value ΔK_{th} the fatigue crack growth rate equalizes zero as indicated in Fig. 5.

The existence of a threshold value means that specimens subjected to certain loads will not fail in fatigue if all existing flaws are smaller than a critical crack length. The reversed situation may also be of interest, when the designer knows the sizes of the cracks from measurements he will be able to calculate which stresses are allowed if fatigue should be avoided.

The utilization of threshold values is particularly important for components which are subjected to very large numbers of cycles during service, e.g. engine parts such as crankshafts.

The threshold stress intensity factor is, however, not a genuine material constant but is dependent on some parameters

such as R-value, temperature and environment. The influence of these parameters and some others will be scrutinized in the next chapter. This dependence on other parameters must be born in mind when the experimental evaluation of ΔK_{th} is performed so that the right service conditions are imposed. Caution also to an uncritical adoption of results presented in the literature: how were these values measured, under which conditions and is the metallurgical composition exactly the same for the tested material as for the actual one?

Another application of the threshold value concept is when a comparable judgement about the fatigue qualities of several materials should be performed. This can be the case when, for instance, an old material should be replaced by a new one.

3.2 Some Numerical Values of ΔK_{th}

Some values of ΔK_{th} are given in Table 1 merely to give an indication of their magnitude and to compare various materials. Note that the threshold stress intensity factor also is meaningful for polymers. All results in Table 1 were given in [24].

| Material | Conditions | R | ΔK_{th} [MPa \sqrt{m}] |
|---------------------------|---------------|------|--------------------------------------|
| mild steel SIS 1450-01 | air | 0.1 | 12.0 |
| | | 0.5 | 7.1 |
| | | 0.9 | 4.4 |
| mild steel SIS 1450-01 | salt water | 0.1 | 7.0 |
| | | 0.5 | 3.6 |
| | | 0.9 | 2.5 |
| A 533 B | base material | 0.25 | 7.1 |
| | | 0.5 | 5.5 |
| | HAZ | 0.25 | 6.8 |
| | HAZ | 0.5 | 5.2 |
| Al 5083 | | 0.1 | 2.0 |
| Inconel | | 0.1 | 6.5 |
| | | 0.1 | 0.68 |
| PVC | | 0.5 | 0.32 |
| | | 0.75 | 0.29 |

Table 1 Some threshold values ΔK_{th} for various materials [24]

3.3 Definition of the Threshold Value

Before going into more detailed discussions in the following chapters, let us ponder about the definition of the threshold value ΔK_{th} . From Fig. 5 it follows that the existence of ΔK_{th} requires the crack growth rate to equalize zero. This would be the case when for an infinite number of cycles no crack growth occurs. However, to perform experiments with an infinite number of cycles would be not only tedious and discouraging but also

quite impractical. Therefore, it is fortunate that the threshold value is associated with a fatigue crack growth rate of around one lattice spacing per cycle. This value which corresponds to about 4×10^{-7} mm per cycle is the minimum crack growth rate possible on physical grounds, [25].

However, in a corrosive environment lower average rates could be observed, due to that the crack growth in those cases only takes place on part of the crack front during each cycle, [25].

For practical purposes, it seems appropriate to utilize a crack growth rate of the magnitude 10^{-7} - 10^{-6} mm per cycle as a definition of a valid threshold value. Recently though many workers including the present author perform tests down to a maximum crack growth of 10^{-8} mm to ensure that no further crack growth occurs.

4. Parameters Influencing ΔK_{th}

As was previously stated the threshold value ΔK_{th} is not a genuine independent material constant but is influenced by various parameters. The forthcoming chapter will deal with some of the more prominent ones.

4.1 Crack Size

Most experiments that have been performed to evaluate thresholds ΔK_{th} or fatigue crack growth data are based upon linear elastic fracture mechanics. However, in the vicinity of a very small crack the plastic zone size is not neglectable compared to the crack length and we may thus expect some peculiarities to occur if we still use the concept of stress intensity factors. Short cracks generally exhibit higher fatigue crack growth rates and lower threshold stress intensities than do longer cracks [2, 26]. There is much confusion on which crack size that should be considered short. It appears that this is dependent on the actual material as indicated in Fig. 6 [27] where ΔK_{th} is plotted versus the crack length for three different materials at $R = 0$. In the pertinent literature crack sizes below 0.5 mm are often referred to as short cracks [2].

An approach to account for the behaviour of short cracks was proposed by El Haddad [28] where a constant length λ_0 , see Eq. (10) is added to the physical crack size. The term λ_0 which is assumed to be a material constant can be determined from the ordinary threshold stress intensity factor ΔK_{th} and fatigue limit $\Delta \sigma_e$ as

$$\lambda_0 = \frac{1}{\pi} \left(\frac{\Delta K_{th}}{\Delta \sigma_e} \right)^2 \quad (10)$$

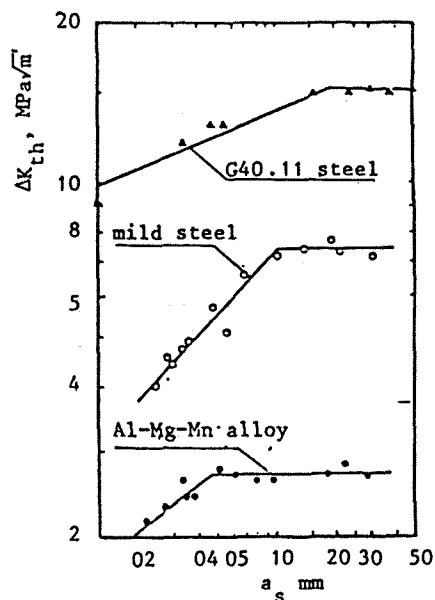


Fig. 6 Dependence of ΔK_{th} on crack length [27]

In the following we will restrict ourselves to situations with "normal" crack sizes when discussing how different parameters influence the threshold stress intensity factor.

4.2 Loading modes

Almost all performed experimental evaluations of ΔK_{th} have been restricted to pure mode I loading, Fig. 3. Recently, however, interest has been shown in the establishment of threshold stress intensity factors also for other modes, particularly mode II and mixed-modes [29,30]. It was found that the mode II and the mixed mode I and II thresholds for mild steel can be lower than the pure mode I threshold [29, 30]. Experimental results from [29] are shown in Fig. 7 together with a lower bound derived in [30]. It was proposed in [29] that the reason for the very low values shown in Fig. 7 might partially be due to that crack initiation rather than crack extension was monitored. In the following only pure mode I thresholds are to be discussed. For more information on other loading modes it is referred to [2, 29 and 30].

4.3 R-dependence

It is nowadays well established that the stress ratio $R = \sigma_{min}/\sigma_{max}$ greatly influences both the fatigue crack growth and the threshold stress intensity factor. The general conclusion from studies on the effect of stress ratio R on the threshold value ΔK_{th} is that an increasing R tends to decrease the threshold value. This behaviour is confirmed for various materials by a great number of authors, e.g. [2, 31].

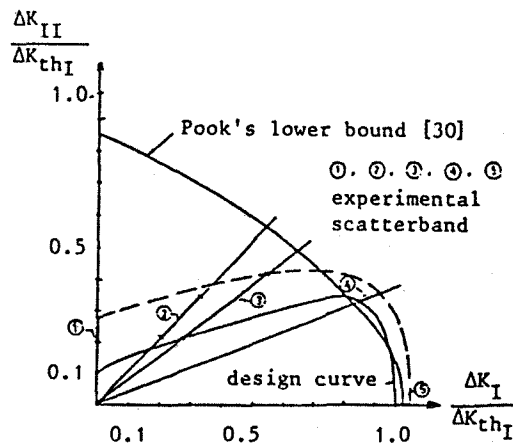


Fig. 7 Mixed mode thresholds for $R=0.1$ [29]

There is however, some confusion about the varying influence of R on different materials and also how environmental conditions influence the R -dependence. One might ponder about why the stress ratio influences the rate of fatigue crack growth. At very high propagation rates when K_{IC} , the fracture toughness, is approached, the influence is attributed to different fracture mechanisms such as intergranular cracking, cleavage and void coalescence, [32]. In the intermediate region relatively little effect is observed. This results in a schematic curve as in Fig. 8 when the influence of R is considered. The effect of R on region I in Fig. 8 can, however, not be clarified by means of different fracture mechanisms as accounted for above for the case of very high crack propagation rates.

Some different theories have been proposed to account for R effects on the threshold value ΔK_{th} .

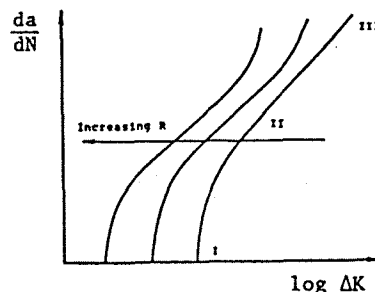


Fig. 8 R -dependence on fatigue crack-growth rate
 I Threshold region
 II Intermediate region
 III Fracture toughness region

A crack closure concept originally proposed by Elber is often utilized. Elber [33] stated that, as a result of permanent tensile plastic deformation left in the wake of a propagating fatigue crack, the cracks are partially closed, even though loading may be tension-tension. Crack extension takes place only during those portions of the cycle where the crack is open. It follows that for a given maximum load, the crack will remain open longer during each cycle for higher values of the mean load, i.e. higher minimum loads and thus higher R-values. This will explain the trend shown in Fig. 8.

Crack closure can, however, not entirely explain the effect of R on the threshold behaviour. It is shown, e.g. in [34], that the threshold value in vacuum is independent of the stress ratio R. This independence of ΔK_{th} on R should according to [35] indicate that there is no crack closure in vacuum. Recent work, however, closely shows that crack closure indeed occurs in vacuum. In [36] it is stated that crack closure is more pronounced in vacuum than in air. Thus, it is obvious that crack closure alone cannot adequately be used to explain the effects of stress ratio R.

Several researchers, e.g. [34, 37 and 38], have proposed that the R-dependence could be explained by environmental effects. In [37] it is stated that environmental effects due to hydrogen embrittlement explains the stress-ratio effect while crack closure is assumed to be of only secondary importance. In [34] a two-component mechanism is introduced. This two-fold mechanism is proposed to take account for environmental actions such as intergranular corrosion. One of the two components involves the creation of damage ahead of the crack tip in the form of intergranular facets and the other component can be viewed as a linking of these facets to the main crack by a tensile tearing process. Ritchie, [38], agrees with Beevers et al, [34] about the R-dependence being due primarily to environmental effects. However, he claims that the above mentioned mechanism does not correctly describe the affection by environment.

The most recent opinions [1] seem to be that both crack closure and environmental effects should be accounted for, e.g. [39, 40], when the R-dependence on ΔK_{th} is discussed.

Discussions as above on the very physical nature of the influence of the stress ratio are basically important as a source to better understanding of the general topic of fatigue. For engineering purposes though, it is more important to find out to what extent the stress ratio influences

the threshold stress intensity factor at different conditions and for various materials.

The following empirical relation was originally proposed by Klesnil and Lukás, [41].

$$\Delta K_{th} = \Delta K_0 (1-R)^\gamma \quad (11)$$

where ΔK_0 is the threshold stress intensity factor at $R = 0$ and γ a parameter characteristic of the material.

Several similar relations, discussed in [2] have been proposed including a simplification of Eq. (11) with $\gamma = 1$ and also more complicated expressions. No such relations seems to be more successful than Eq. (11) why they are not further discussed here. The value of γ in Eq. (11) has been found to vary between 0.53 and 1.0 for different materials [2].

4.4 Thickness

The threshold intensity factor ΔK_{th} is generally not supposed to be affected by the thickness of the test specimen. Radon [42] has, however, recently found an influence of thickness at low stress ratios. It was found that ΔK_{th} decreased with thickness for a low alloy steel. It remains to be seen whether this will be shown also for other materials.

4.5 Frequency

At threshold conditions it seems that the influence of frequency is almost negligible [2]. Slight variations in the threshold value due to influence of the frequency can, however, be found in the literature. In [35] a lowering of the threshold value with increasing frequency from 342 to 1000 Hz was found. It was suggested that this behaviour might be due to a localized heating of the crack tip. In [43] the opposite behaviour was observed, i.e. an increased threshold value with increasing frequency from 30 to 1000 Hz. It therefore seems that no general statement can be done about the influence on ΔK_{th} of the frequency. When exposure to various detrimental environments occurs the situation is entirely different. Increasing frequencies then generally yield higher threshold values due to less time for environmental interaction. This is especially noticeable at very low frequencies.

4.6 Temperature and environment

Results on the effect of temperature and environment on ΔK_{th} are unfortunately somewhat contradictory why it is difficult to draw any general conclusions. Recent work by Ritchie [40] and Stewart [44] indicate that a concept of oxide-induced crack closure satisfactorily explains the influence of environment on ΔK_{th} . It is found that ΔK_{th} often is lower in dry environment, hydrogen or air, than in moist air and water at low stress ratios. This is explained [40, 44] with corrosion/

/fretting debris building up at the crack tip and thus reducing the effective cyclic stress intensity range.

The effect of temperature on ΔK_{th} is quite unclear and apparently varies for different materials. As example it was found in [45] that the threshold value for a mild steel plate was lower and higher, respectively, at 200°C and 360°C than at 20°C. For weld metal [45] ΔK_{th} was similarly shown to be lower between 200°C and 200°C, but higher at 360°C, than at 20°C.

It is concluded that all experimental evaluations of ΔK_{th} should necessarily be performed in an environment and at temperatures closely simulating the operational conditions.

4.7 Microstructure

Mostly it appears that the threshold value increases with increasing grain size [39, 46-48]. This has been found for many metals such as aluminium, various steels and titanium. Some materials though, for instance copper [49], do not show any variation in ΔK_{th} with grain size changes. Metallographic observations in [46] showed that at threshold the crack often arrests at grain boundaries and inclusions which may clarify the relatively large scatter in test results for materials with coarse grain sizes and inclusions. The structure sensitivity of ΔK_{th} is primarily considered to be associated with the closure process [39, 47]. The closure level is highest in the threshold region due to changes in the transgranular crack mode from the opening mode at higher ΔK -values to a combination of shear mode and opening mode.

It has been found that the threshold stress intensity factor is a function of yield strength and Young's modulus [39, 47]. A nearly linear plot of ΔK_{th} versus the square root of $E\sigma_y$ for several metals is considered [39] to be indicative of ΔK_{th} being related to a critical crack tip opening displacement.

4.8 Amplitude

All discussions so far have been presented assuming a constant load amplitude. It is, however, well known that both single overloads and random loading greatly affect the threshold value. When a specimen is subjected to overloads a plastic zone forms at the crack tip. This plastic zone can be of sufficient magnitude to prevent further crack growth which results in that a too high threshold value is registered.

To account for the load history, Klesnil and Lukás [50] proposed the following relationship

$$\Delta K_{th} = \Delta K_{th,0} \left(\frac{\Delta K_{Ip}}{\Delta K_{th,0}} \right)^\alpha \quad (12)$$

where $\Delta K_{th,0}$ is the threshold value without prior loading, ΔK_{Ip} is the just preceding stress intensity factor and α is an exponent strongly dependent on the tensile strength of the steel.

In 51 the fatigue strength of cracked mild steel plate specimens was studied. It was found that a narrow-band random loading produced significantly greater fatigue strength than did estimates based on linear summation of constant amplitude fatigue crack growth data. More accurate results were obtained when the effect of prior loading on the threshold value, utilizing Eq. (12), was accounted for.

Experiments performed on a commercial nickel-base and titanium-base alloy, [43], disclosed various interesting effects of single and multiple cycle overloads on the threshold value. It was found that the overload modified threshold value, ΔK_{th}^* , increased exponentially with the magnitude of the prior overload. Further, it was shown that the effect of relative overload on relative threshold value is independent of the stress ratio R , but that overloads produce much larger absolute magnitude increases in the threshold value at low R than at high R . Also, it was shown that the number of overloads, overload rate, cycle shape, temperature and frequency can affect the threshold value but that these variables are much less important than the magnitude of the overload.

5. Experimental Methods

It is obvious from the preceding chapter that all experimental evaluations of ΔK_{th} should account for such parameters as temperature, environment, stress ratio etc. simulating the operational conditions as closely as possible. Also, in order to avoid load history interactions, the load must be decreased very carefully.

Concerning test equipment and test specimens most workers use servo-hydraulic testing machines and standardized fracture mechanics specimens such as the Center-Cracked-Tension and the Compact Tension specimen. It has recently been proposed that a modified wider compact Tension specimen would be more suitable [24].

The conventional test method is the load shedding technique where the load is manually decreased at selected crack length intervals. This method is thoroughly investigated by Bucci and his co-workers, [52, 53], who have developed a tentative ASTM standard, [54], based on their results. Stringent requirements [52 - 54] are proposed for the load shed-

ding procedure to avoid load history effects. It is stated that the reduction in P_{max} for adjacent load steps must be lower than 10 per cent of the previous P_{max} and further must the crack extension within each adjustment of the maximum load be at least 0.5 mm. This method has, however, some drawbacks, such as being rather time consuming with respect both to test equipment and personell. This is due to that the crack length has to be monitored during the test and that the load must be reduced very slowly in order to eliminate any load history effects from prior overloads. Further there will be cusps on the ΔK versus time curve which are not very appealing from a physical point of view.

These disadvantages are substantially reduced when the load shedding is performed in a continuous manner utilizing a computer controlled test system [55, 56].

In Ref. [55] a stress intensity gradient technique is used and in Ref. [56] a technique is described which utilizes crack opening displacement as controlling parameter. The main objection against these methods is the relatively high cost of computerized test systems and even if such systems are available they ought to be saved for more sophisticated tests such as spectrum or random fatigue testing.

A simple method to measure ΔK_{th} was proposed by Jerram and Priddle, [57]. Their method which operates automatically seems to be rather cost effective as computer control is avoided. One serious disadvantage is, however, the inability of the method to keep the load ratio R , constant throughout the test. As it is well established that R significantly influences the threshold stress intensity ΔK_{th} , the usefulness of the method seems rather limited.

If it would be possible to combine the advantage with a control unit as in [57], with the possibility of keeping R constant throughout the test, as in [55, 56], then a quite efficient method would be available. How this is accomplished is described in [51].

6. Applications

It is evident from many recent papers that the threshold stress intensity factor

ΔK_{th} is a parameter of great technical importance. Applications where ΔK_{th} has been used include different structural components such as: boiler feed pump shafts [58], low pressure turbine rotor shafts [58] turbo generator rotor [58], nuclear reactor gas circuitry [59], pipes [60], cylinder cover of diesel engines [61], engine shafts [61], connecting rods [61] and turbine blades [62].

7. Conclusions

The fatigue threshold concept as a design criterion has been discussed. This concept which simply states that the applied stress intensity range ΔK should be below the threshold stress intensity factor ΔK_{th} is of particular importance for structures sustaining very many load cycles, i.e. where no crack growth can be allowed.

It has been discussed how stress intensity factors K are determined by means of the finite element method and how the threshold stress intensity factor is experimentally evaluated. The influence of various parameters was scrutinized and it is concluded that all experiments should necessarily simulate the operational conditions as closely as possible.

8. References

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