REDUCTION OF THE TAKE-OFF GROUND RUN DISTANCE TO A GIVEN SET OF ATMOSPHERIC CONDITION

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Abstract

In this paper, a method has been developed which permits to determine the take-off ground run distance for any atmospheric conditions, if the value of this distance is available from direct measurement during a single take-off in known atmospheric conditions.

On the base of classical equations which defines the movement of the aircraft on the ground, are analysed the influences of the atmospheric factors (pressure and temperature) on the take-off velocity and on the thrust of the engines. Finally, it is established a formula, very easy to be applied in practice, which permits to pass from a known distance to onother one which should be obtain in other atmospheric conditions.

The method can be useful to the persons acting in the field of flight test for the establishing of the take-off ground run distance variations produces by the changes in atmospheric conditions or by the changes of the airport's hight on which the aircraft operates.

I. Introduction

The problem of determining the take-off ground run distance for a new plane (prototype) is a very important one, because this distance determines the length of the runway necessary for the practice take-off of the plane considered.

Evidently this take-off ground run distance can be determined and, it is always determined, by theoretical methods in the design period of the plane; nevertheless the calculated take-off ground run distance differs from the real distance because of the influences of many factors which can not be appreciated exactly in calculation.

This is the reason for considering the take-off ground run distance calculated by theoretical methods in the design period, only as informative for the first flights of the prototype. After we are sure that the plane has normal flying qualities and a good security in flight, we can begin the flight tests to determine the actual performances of the plane. Among the other performances which must be determined by the flight test, there are also the take-off and landing ground run distances, which must be determined during the first flight tests.

Referring only to the determination of the actual take-off ground run distance, there are two groups of difficulties.

First of all, the exact determination of this distance requires a special equipment on the ground, which makes possibile to record the trajectory of the plane during take-off. This requires a lot of foto-tape, adequate laboratory equipment, specialists for work and interpretation of the foto-tape, and a relatively long time between the test and the final results. From this point of view, and for economy of fuel, it would be advisedly to have a small numbers of flights.

Secondly, another kind of difficulties are due to the fact that the plane flight tests are made in natural atmospheric conditions, which differs from the conditions in which we should like to have the performance of the plane. Therefore, appears the necessity to change the values obtained by direct measurements in a given set of atmospheric condition, into the values which should be obtained in a wanted set of atmospheric condition. This activity is known as "reduction of parameters to a wanted set of atmospheric condition".

The present work is devoted to this last aspect for the particular case of the take-off ground run distance.

Evidently, the simplest method of reduction is to wait for a time when the real atmospheric conditions coincides with the intended conditions and to make the measurement in that moment. But this method is not practicable because we must wait too much.

Another method is to measure the take-off ground run distance in different atmospheric conditions and to build a diagram of these distances versus pressure and temperature of the air. With such a diagram we can appreciate by interpolation the take-off ground run distance for the wanted atmospheric conditions, but only in the range of pressures and temperatures in which direct measurements have been made. The method has many disadvantages that make it impossible to be practically applied. Firstly, it is necessary to have a great number of measurements (the greater is the number of measurements, the easier is the interpolation) and this implies much materials, money and time. The great number of measurements implies also another disadvantage, namely a smaller accuracy, because it is very difficult to repeat many times the same kind of piloting during the take-off ground run.

For the reasons above presented, same attempts have been made in order to find such methods which can perform the reduction on the basis of the smallest number of measurements. This is the aim of the method presented in the following sections.

II. Analytical expression used in the calculation of the take-off ground run distances

On the basis of the schema represented in Fig. 1, we can establish the equation which describes the motion of the plane :

$$\frac{W}{g}\frac{dV}{dt} = T - D - (W - L)\mu \qquad (1)$$

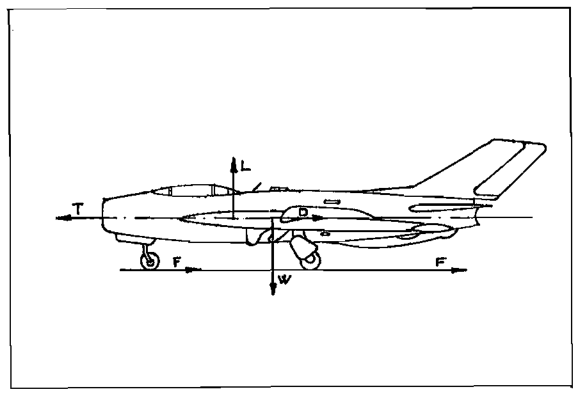

Fig.1 THE FORCES WHICH ACTS ON THE PLANE DURING TAKE-OFF GROUND RUN

where :

- W - the weight of the plane during take-off (N)
- g - gravitational acceleration (m/s^2)
- V - velocity (m/sec)
- t - time (sec)
- T - thrust of the engine (N)
- D - drag of the plane (N)
- L - lift of the plane (N)
- μ - nondimensional coefficient of friction between wheels and ground.

Writing in full the drag and the lift, equation (1) becomes :

$$\frac{W}{g}\frac{dV}{dt} = T - \frac{\rho}{2} SV^2C_D - (W - \frac{\rho}{2}SV^2C_L)\mu \qquad (2)$$

where :

– S represents the reference surface (the wing surface) (m^2)

- C_D represents the coefficient of the drag
- C_L represents the coefficient of the lift
- ρ represents the density of the air (kg/m^3)

Because we can write $\frac{dV}{dt} = \frac{1}{2}\frac{dV^2}{Vdt}$ and because V.dt represents the distance run on the ground during the time dt, therefore V.dt = dL, we obtain :

$$\frac{dV^2}{dL} = 2g\left(\frac{T}{W} - \mu\right) - \frac{\rho}{2} SV^2 (C_D - \mu C_L)\frac{2g}{W} \quad (3)$$

The further work with this equation depends on the fact whether we consider or not the influence of the velocity on the engine's thrust. In the case in which the considered plane is equipped with jet engines, we can practically consider that the thrust is independent of the velocity in the range of take-off ground run velocities.

In this case, we can write

$$2g\left(\frac{T}{W} - \mu\right) = \text{const.} = A \quad (4)$$

and

$$\frac{2g}{W}\frac{\rho}{2} S(C_D - \mu C_1) = \text{const.} = B \quad (5)$$

Using the notations (4) and (5), equation (3) becomes:

$$dL = \frac{dV^2}{A - BV^2} \quad (6)$$

By integration, we obtain the expression of the take-off ground run distance :

$$L_r = \frac{1}{B} \ln \frac{A}{A - BV^2_{t.off}} \quad (7)$$

where $V_{t,off}$ represents the velocity at which the plane becomes airborne.

III. Reduction of the take-off ground run distance in the case in which we do not consider the influence of the atmospheric conditions on the thrust of the engine

In equation (7) found before, we can see the take-off velocity $V_{t.off}$. This velocity is determined by the formula :

$$V_{t.off} = \sqrt{\frac{2W}{\rho S C_{Lt.off}}} \quad (8)$$

If it is established a method of piloting which is repeated from one take-off to another, then $C_{L_{t.off}}$ remains unchanged; the result is that the take-off velocity will depend only of atmospheric conditions through ρ , evidently if the take-off weight remains also unchanged.

Using index t for the values obtained by direct measurements in known atmospheric conditions, and index 1 for the values corresponding to other atmospheric conditions, we can write :

$$V_{t.off_t} = \sqrt{\frac{2W}{\rho_t S C_{L_{t.off}}}} \quad \text{and}$$

$$V_{t.off_1} = \sqrt{\frac{2W}{\rho_1 S C_{L_{t.off}}}} .$$

By division, we obtain :

$$W_{t.off_1} = V_{t.off_t}\sqrt{\frac{\rho_t}{\rho_1}} \quad (9)$$

Similarly we can obtain from (5) :

$$B_1 = B_t \frac{\rho_1}{\rho_t} \quad (10)$$

Writing equation (7) for the atmospheric conditions existing during the test (using index t), and for other atmospheric conditions (using index 1), we obtain :

$$L_{r_t} = \frac{1}{B_t} \ln \frac{A}{A - B_t V^2_{t.off_t}}$$

and respectively :

$$L_{r_1} = \frac{1}{B_1} \ln \frac{A}{A - B_1 V^2_{t.off_1}}$$

Dividing this two expressions and using relations (9) and (10), we obtain :

$$L_{r_1} = L_{r_t} \frac{\rho_t}{\rho_1} \tag{11}$$

In this case, when the thrust of the engine is considered to be independent of atmospheric conditions, it is sufficient to determine by direct measurements the take-off ground run distance L_{r_t} in known atmospheric conditions (respectively p_t - the pressure of the air during the test and t_t - the temperature of the air, because with them we can calculate immediately the density $\rho_t = \frac{p_t}{gRT_t}$), and formula (11) permits the calculation of the take-off ground run distance in all other atmospheric conditions.

But the hypothesis considerd is well far from the reality and it can introduce great errors.

IV. Reduction of the take-off ground run distance when we consider the influence of the atmospheric conditions on the thrust of the engine

Remaining in the field of turbo-jet engines, the incfluence of the atmospheric conditions on the thrust can be expressed by the formula :

$$T = T_o \left(\frac{\rho}{\rho_o}\right)^n \tag{12}$$

where

- T is the thrust when the density of the air is ρ
- T_o is the thrust in standard atmospheric conditions (p_o = 760 mm Hg column and $t_o = 15^oC$ from which results ρ_o = 1.225 kg/m^3)
- n is an exponent, which in the first approximation can be considered 0.7.

Returning to the equation (4), we can write:

$$A_t = 2g\left(\frac{T_t}{W} - \mu\right) = 2g\left[\frac{T_o\left(\frac{\rho_t}{\rho_o}\right)^n}{W} - \mu\right] \tag{13}$$

and

$$A_1 = 2g\left(\frac{T_1}{W} - \mu\right) = 2g\left[\frac{T_o\left(\frac{\rho_1}{\rho_o}\right)^n}{W} - \mu\right] \tag{14}$$

Dividing the last two expressions, we obtain :

$$\frac{A_1}{A_t} = \frac{\frac{T_o}{W}\left(\frac{\rho_1}{\rho_o}\right)^n - \mu}{\frac{T_o}{W}\left(\frac{\rho_t}{\rho_o}\right)^n - \mu} \tag{15}$$

Because the value of the term $\frac{T_o}{W}$ is practically at least 0.3 and the value of μ is approximately 0.03, we can admit :

$$\frac{A_1}{A_t} \approx \frac{\frac{T_o}{W}\left(\frac{\rho_1}{\rho_o}\right)^n}{\frac{T_o}{W}\left(\frac{\rho_t}{\rho_o}\right)^n} = \left(\frac{\rho_1}{\rho_t}\right)^n \tag{16}$$

One can easily see that the equations (9) and (10) remain valid also in this case. Conside-ring equations (9), (10) and (16), we can write :

$$L_{r_t} = \frac{1}{B_t} \ln \frac{A_t}{A_t - B_t V^2_{t,off_t}} \tag{17}$$

and

$$L_{r_1} = \frac{1}{B_1} \ln \frac{A_1}{A_1 - B_1 V^2_{t.off_1}} = \frac{\rho_t}{\rho_1}\frac{1}{B_t} \ln \frac{A_t\left(\frac{\rho_1}{\rho_t}\right)^n}{A_t\left(\frac{\rho_1}{\rho_t}\right)^n - B_t V^2_{t.off_t}} \tag{18}$$

Dividing the last two relations, we obtain :

$$\frac{L_{r_1}}{L_{r_t}} = \frac{\rho_t}{\rho_1} \frac{\ln \frac{A_t}{A_t - B_t V^2_{t.off_t}\left(\frac{\rho_t}{\rho_1}\right)^n}}{\ln \frac{A_t}{A_t - B_t V^2_{t.off_t}}} = K \frac{\rho_t}{\rho_1} \tag{19}$$

where

$$K = \frac{\ln \dfrac{A_t}{A_t - B_t V^2_{t.\,off_t} \left(\dfrac{\rho_t}{\rho_1}\right)^n}}{\ln \dfrac{A_t}{A_t - B_t V^2_{t.\,off_t}}} \qquad (20)$$

Formula (19) permits the reduction of the take-off ground run distance to every atmospheric conditions. Comparing the formulae (19) and (11) we can see that the influence of thrust variation with the atmospheric conditions is determined by a corrective coefficient K. For calculating the value of this coefficient it has to be known the values of the two constants A_t and B_t and also the value of the take-off velocity $V_{t.\,off_t}$.

To determine this values we can apply the following procedure.

We do a take-off during which the plane is piloted in a way which will be used normally. During the ground run, we record or determine by cineteodolites the variation of the velocity as a function of time, starting from the moment when the plane begins to move and untill the plane is airborne. The diagram has generally the forme of the one represented in Fig.2.

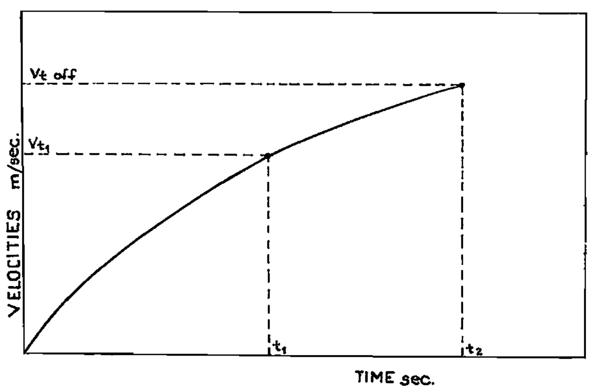

Fig.2 DIAGRAM OF THE VELOCITI VRS. TIME DURING TAKE-OFF GRUND RUN

In this diagram, we choose a point, for instance t_1 to which corresponds a velocity V_{t_1}; the point t_2 represents the moment when the plane is airborne and therefore the velocity is $V_{t.\,off}$.

Farther, we determine the distances which correspond to the moments t_1 and t_2 respectively; let these distances be $L_{r_{t_1}}$ and $L_{r_{t_2}}$.

Also during the take-off ground run we can measure the pressure and the temperature of the air and then we can calculate the density ρ_t.

Returning to the equation (7), we can write it for the moments t_1 and t_2.

$$L_{r_{t_1}} = \frac{1}{B_t} \ln \frac{A_t}{A_t - B_t V^2_{t_1}} \qquad (21)$$

$$L_{r_{t_2}} = \frac{1}{B_t} \ln \frac{A_t}{A_t - B_t V^2_{t.\,off}} \qquad (22)$$

These expressions represent a system of two equations with two unknown values, A_t and B_t respectively. In order to solve this system we can put it in the following form :

$$A_t(1 - e^{L_{rt_1} B_t}) = - B_t V^2_{t_1} e^{L_{rt_1} B_t} \qquad (23)$$

and

$$A_t(1 - e^{L_{rt_2} B_t}) = - B_t V^2_{t.\,off}\, e^{L_{rt_2} B_t} \qquad (24)$$

Dividing, we can eliminate one of the unknown values and we shall obtain :

$$\frac{V^2_{t.\,off}}{V^2_{t_1}} = \frac{e^{L_{rt_1} B_t} (1 - e^{L_{rt_2} B_t})}{e^{L_{rt_2} B_t} (1 - e^{L_{rt_1} B_t})} \qquad (25)$$

Equation (25) contains only one unknown value, B_t, which can be determined by successive approximations or by graphic representation.

Having the value of B_t we can determine also the value of A_t from equation (23) or (24)

$$A_t = \frac{B_t V_{t_1}^2 e^{L_{rt_1} B_t}}{1 - e^{L_{rt_1} B_t}} = - \frac{B_t V_{t.off_t}^2 e^{L_{rt_2} B_t}}{1 - e^{L_{rt_2} B_t}} \qquad (26)$$

Having the values A_t and B_t we can determine the coefficient K and thereby we can use the formula (19), which permits the calculation of the take-off ground run distances in any other atmospheric conditions. We underline that the formula (19) can reduce the take-off ground run distance for a given take-off weight, for a runway of the same nature and for the same technique of piloting as those existing during the test.

When we wish a greater accuracy of the calculus, we can also determine the value of the exponent "n" from formula (12), but for this we must have two measurements of the take-off ground run in different atmospheric conditions.

From equation (19) we can have :

$$\ln \frac{A_t}{A_t - B_t V_{t.off_t}^2 \left(\frac{\rho_t}{\rho_1}\right)^n} = \frac{L_{r_1}}{L_{r_t}} \frac{\rho_1}{\rho_t} \ln \frac{A_t}{A_t - B_t V_{t.off_t}^2} \qquad (27)$$

or

$$\frac{A_t}{A_t - B_t V_{t.off_t}^2 \left(\frac{\rho_t}{\rho_1}\right)^n} = \left(\frac{A_t}{A_t - B_t V_{t.off_t}^2}\right)^{\frac{L_{r_1}}{L_{r_t}} \frac{\rho_1}{\rho_t}} \qquad (28)$$

from which we can obtain :

$$A_t - B_t V_{t.off_t}^2 \left(\frac{\rho_t}{\rho_1}\right)^n = \frac{A_t}{\left(\frac{A_t}{A_t - B_t V_{t.off_t}^2}\right)^{\frac{L_{r_1}}{L_{r_t}} \frac{\rho_1}{\rho_t}}} \qquad (29)$$

One can easily see that equation (29) contains only one unknown value and this is the exponent "n" which can be thus determined.

We also underline that for the calculation of the value of exponent "n", during the second test we must determine only the take-off ground run distance (L_{r_1}) and the density of the air ρ_1 (p_1 and t_1 respectively).

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