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ON DESIGN OF DAMAGE TOLERANT BUILT-UP
AIRCRAFT STRUCTURES

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ERRATA CORRIGE

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2		54	ON THE WING	ON THE LOWER WING
4	45		DAMAGE BY	DAMAGE IN
4	56		METHODS	METHOD
6	15		6 mm.	1,27 mm.
6	23		Tau. $\le T_e$	Tau. $< T_e$
6		30	FIGS 10	FIG 10a
6		44	THE FIRST FIGURE	THE LAST FIGURE
6		48	FIG.	IN PARTICULAR FIG.
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16	FIG 10a		$(W_1-W_2)/W_1$	$(W_2-W_1)/W_1$

ON DESIGN OF DAMAGE TOLERANT BUILT-UP AIRCRAFT STRUCTURES

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Abstract

The paper deals with the problem of assessing the influence of damage tolerance requirements on the weight of built-up structures typical of commercial airplanes. After a review of main concepts underlying the design of damage tolerant structures and their relation with previously utilized design approaches, namely SAFE-LIFE and FAIL-SAFETY, the weight of a built-up structure is evaluated by means of an optimization procedure whose out put is a constrained minimum weight design. Attention has been focused on the lower wing surface structure of commercial airplanes.

The constraints selected according to Airworthiness Authority requirements and customer specifications account for static strength, durability and damage tolerance.

Special attention has been paid to the discussion of damage tolerance requirements and related problems of residual strength and crack growth evaluation.

Finally, results concerning minimum weight solutions in a wide range of design parameters (including stress level, life goal, and inspection intervals) are utilized to assess the influence of damage tolerance requirements on the design of built-up structures.

I. Introduction

The damage caused by the operational environment is one of the main sources of concern in the design of aircraft structures. Damage may impair the safety, durability and economic operation of an aircraft and greatly reduce its value.

The need to guarantee safety from catastrophic damage-induced failures has an impact on the whole aircraft industry which far exceeds that of the loss of a single aircraft and the lives of its occupants. Should in fact such a failure occur, all the aircraft of the same type would be put in a suspicious position. The suspension of the operation and, most likely, retrofit actions would be made compulsory with large economic penalties and the creation of a confidence gap affecting the whole aircraft industry.

The same considerations apply when unpredicted damage is discovered before a catastrophic failure occurs.

At the same time such a stringent need of safety must comply with the even more demanding requirements of durability and operational efficiency imposed by the market or contractual ties.

Two main lines of thought has been used with a view to reaching a well-balanced solution of this problem beset as it is by so strongly conflicting requirements.

The first line of thought identifies fatigue as the main cause of structural damage and requires designs aimed at guaranteing a fatigue crack free structures for an assigned service life.

In pursuing such an objective extensive endurance analyses and testing are performed to develop and check the design. The analyses are based on such elements as fatigue loading environment, load-stress relationship, fatigue performance data, a cumulative damage theory and a safety scatter factor. Development tests are conducted on small specimens as well as on large aircraft components, both to improve the design and to obtain the fatigue performance data to be enbodied in the durability analyses. Qualification tests are eventually performed on a production-airplane by means of a flight-by-flight fatigue test to obtain the service life of the fleet by dividing the test endurance by an appropriate scatter factor.

The other line of thought admits that some kind of damage may take place in a primary structure and requires design and substantiation tests to be carried out aimed at guaranteeing the capability of the structure to operate in the presence of damage without suffering catastrophic failure before the damage is detected in the course of a scheduled inspection.

Past experience shows that damage may stem directly from the manufacturing process or may arise during aircraft operation due to such causes as fatigue, corrosion and accidental damaging events. The damage may grow under service environment and reach such dimensions that a catastrophic failure may occur. Therefore,if safety is to be guaranteed the structure must be designed in such a way that it retains adequate strength until the damage is detected in the course of a scheduled inspection.

The designer takes this approach, through design practises based on such structural concepts as the multiple load-path, crack stopping ability and slow crack growth. He further designs the structure for inspectability to improve the possibility of damage detection and reduce the extent of the initial damage that must be allowed in the structure.

The designer carries out analyses of the damaged structure based on fracture-mechanics methodologies in order to calculate the inspection intervals allowable or to assess the crack-stopping abi lity of the stiffeners. Essential analysis factors include selection of the critical areas and the relevant damage locations and dimensions, evaluation of the crack growth time from initial dimension to successive selected dimensions, and computation of residual strength of the structure.

The analysis allows us to establish the operational time within which the structure can safely operate damaged and to deduce the inspection intervals by means of an appropriate scatter factor.

Development tests are usually performed to improve design and to obtain crack growth and residual strength data.

Full scale tests are then carried out to substantiate the results of the analysis.

From the point of view of the first line of thought, usually termed the SAFE-LIFE approach, the designer strives to provide both safety and durability at the same time by attempting to prevent cracks of significant size during service life. Further, as crack occurrence is a random event, the level of safety is strictly connected with the scatter factor which must be selected so as to ensure that no more than a certain acceptable number of aircraft develop cracks before the fleet is withdrawn, or, at least, before the most critical parts of the aircraft are replaced.

In other words, the designer strives to improve the fatigue quality of the structures as far as possible; thereafter he relies solely upon an increase in the scatter factor (and therefore in structural weight) to improve the safety level of the fleet (that is to reduce the number of aircrafts likely to develop cracks during service).

On the contrary, according to the theories of the second line of thought, usually termed damage tolerant design philosophy, the two problems of safety and durability are faced by means of different design actions.

Safety is obtained by relying on a damage-tolerant structure that is capable of accomodating flaws induced either in manufacturing or in service.

Durability is obtained by design actions, aimed at promoting structural integrity against such causes as fatigue and stress corrosion cracking, corrosion delamination, wear and other potential reasons for structural failure.

Prevention of fatigue cracking is pursued in particular in ways comparable to those used in the SAFE-LIFE approach.

However, a lower scatter factor (and therefore a lower weight at least in this respect) is required since the objective is not safety but a reduction of the in service maintenance costs and the attainment of a high level of operational readiness in terms of the whole fleet.

Both the lines of thinking have advantages and shortcomings. Without, however, going into a thorough discussion about the relative merits of the two approaches, evidence exists that the damage tolerant design is at present greatly in favour with the aircraft industry.

The commercial aircraft industry was first brought to rely on damage tolerant design as it offers a sound approach to structural safety and a rationale, through the inspection process, offers a way of attaining the long service life requested by economical operations. All the commercial aircraft produced during the last two decades have been certified from the point of view of FAIL-SAFE requirements (a structure is considered FAIL-SAFE when is capable of undergoing failure of one or more primary members without suffering catastrophic failure) which until recently represented the point of view of the Airworthiness Authorities in matter of damage tolerant design.

More recently, in military specifications too damage tolerant design is also required in order to guarantee the structural safety.

Eventually, the FAR undertook a fatigue regulatory review program which further enforced the damage tolerance design as a means of assuring structural safety.

In the present paper an attempt is made to assess modifications in structural design produced by the application of damage tolerant design rules with particular reference to changes in weight and shape. To this end attention has been focused on built-up structures typical of commercial aviation aircraft.

A method has been implemented for minimum weight design of stiffened panels representative of the lower wing surface structure. The minimum weight design is obtained by an optimization procedure whereby it is possible to comply simultaneously with the requirements of static strength, durability and damage tolerance.

The results obtained show how the design and its weight change from the conventional, buckling shaped stiffened panels to the more general panel where the stiffeners play the further role of crack stoppers and the total area is also controlled by durability and slow crack growth requirements.

II. Minimum weight design of structures complying with damage tolerance requirements

The main purpose of the present investigation is to assess the impact of damage tolerance requirements on the design of typical built-up structures of airplanes already complying with static-strength and durability constraints. To this end an optimization procedure leading to minimum weight structure has been considered meaningful. Minimum weight design can be achieved by following the main lines of Fig. 1.

Attention has been focused on the wing surface structure of commercial airplanes. This choice is felt to be significant as regards the objective of the present investigation, because of the important role played by fatigue and damage tolerance behaviour in the design of this structure. The wing structure has been idealized as a box of uniform depth and the lower surface as a skin-stringer combination with equal and uniformly spaced stringers.

The constraints the structure must comply with, have been selected according to Airworthiness Authority requirements and customer specifications which usually account for static strength, durability and damage tolerance capability.

Since bending loads predominate over other types of loadings in large parts of the wing structure, the constraints have been determined solely on the basis of bending. All the loads the structure must be capable of supporting are related to the load per unit of chordwise length, namely N_x, by means of the relevant load factors. As far as static strength is concerned both limit and ultimate load conditions have been considered with reference to positive and negative load factors.

The ultimate load for the damaged structure has been assumed to be coincident with the limit load for the integer structure.

Fig. 2 shows a sketch of the structure idealization considered in the present analysis, together with the numerical values selected for the load factors.

The MINITWIST standardized load sequence* [1] has been utilized to adequately evaluate the effects of repeated loading conditions on the durability and crack growth characteristics of the structure.

The optimization has been carried out within the framework of mathematical programming techniques [2]. The objective function, namely the weight, and the inequality constraints are combined in a unique function which is minimized regardless constraints.

Such a combination is obtained on the basis of a modified penalty function method. Following such an approach, the absolute minimum of the function

$$\overline{W}_J(X,Y) = W(X,Y) + r_J \sum_{i=1}^{n} G_i^{-1}(X,Y) \qquad (1)$$

is looked for by means of the Fletcher-Powell conjugate-direction method [3].

W is the weight of the structure, X is the vector of the design variables which are varied during the search for the optimum shape, Y stands for the vector design parameters which are kept constant during the search for the minimum weight solution. Together, design variables and parameters completely define the structure to be optimized. G_i are functions which describe the constraints and which can be expressed as follows:

$$G_i = 1 - \frac{\text{Actual } P_i \text{ value}}{\text{Allowable } P_i \text{ value}} \qquad (2)$$

* Reasons for this choice are briefly stated in discussing durability constraints.

where P_i is the i-th property of the structure (typically stress level, inspection interval, crack length and so on) which is upper-bounded by an allowable. Finally r_J is the J-th value of the penalty parameter which is progressively reduced in successive steps of optimization until stable values of the minimum are obtained.

The search for the minimum weight has been carried out by means of a computer program which gives the optimum values of the design variables for any set of design parameters.

Two main problems have been dealt with particularly carefully in preparing this computer program. The first, namely keeping a check on computer costs, has been solved by means of software suitable for minicomputers, prepared along the main lines discussed in a previous paper [4].

The second problem, more fundamental and design-oriented, concerns constraint formulation and their mathematical modelling. An effective solution to this problem is essential if realistic and meaningful results are to be obtained.

The main lines followed in pursuing this aim are briefly described in the following sections.

III. Static strength constraints

The lower wing surface structures must support limit and ultimate loads both in traction and in compression.

The statement of the constraints under tensile loading is straightforward, namely stresses in the structure under limit load are upper-bounded by yielding stress and the ultimate load must induce stresses in the structure which are not higher than the ultimate stress of the material.

Compression load conditions imply a more involved formulation of the constraints due to the presence of buckling phenomena.

Compression stresses under limit load have been constrained so that they are lower than the buckling stress of the stiffened panel.

Stresses under ultimate load must be lower than the crushing strength of the stiffened panels.

The constraint which requires no buckling below the limit load in compression is probably too restrictive. However, it ensures a safe design in all the possible shapes examined during the optimization process, without significantly influencing the optimized structure.

The buckling stress S_b for the stiffened structure has been computed by means of the following relations

$$S_b = S_L - \frac{S_L^2 \, (L'/\rho)^2}{4\pi^2 E} \qquad \frac{L'}{\rho} < \lambda_1 = \pi\sqrt{2}\,(E/S_L)^{\frac{1}{2}} \qquad (3')$$

$$S_b = \pi^2 E / (L'/\rho)^2 \qquad \frac{L'}{\rho} \geq \lambda_1 \qquad (3'')$$

$$S_L = \eta \, \frac{K\pi^2 E}{12\,(1-\nu^2)} \left(\frac{t}{b}\right)^2 \qquad (3''')$$

where η and K have been obtained by well-known methods [5].

The crushing stress S_{CR} is expressed by similar formulas with S_L substituted by the crippling stress S_c and S_b substituted by S_{CR}. S_c is the crippling stress given by well-known Gerard semi-empirical formulas [6].

IV. Durability constraints

Durability constraints are obtained by requiring that the structure can undergo the number of Life Goal Flights (LGF) without suffering significant fatigue damage.

The number of LGF must be calculated on the basis of the daily utilization of the airplane with a projected life of 20 years' use, a typical figure for commercial airplane. By combining possible usage in short, medium and long range flights, LGF numbers ranging from 15,000 to 55,000 are usually obtained.

The load spectrum encountered in each flight depends on the particular airplanes and on usage. To clear the results being considered in the present approach from being influenced by particular airplanes and kinds of usage, we shall resort to a standardized spectrum.

As already stated, the MINITWIST spectrum [1] has been selected as it offers a well balanced compromise between the representativeness of commercial airplane usage and the amount of computational effort in order for it to be implemented in the search for minimum weight. The MINITWIST comprises 4000 different flights and is based on the spectrum in Fig. 3a. Fig. 3b shows examples of the different types of flight in the MINITWIST spectrum.

As far as the amplitude of the cycles in the spectrum is concerned, all the load variations are functions of level flight stress S_{LF}.

The next step in durability assessment is to establish a relationship between S_{LF} and the number of flights which can be flown safely, that is without any significant fatigue damage by a given structure.

Since no systematic endurance data of built-up structures undergoing the MINITWIST type of spectrum is available, the usual approach based on S-N curves and on Miner's rule has been adopted.

The main advantages of such an approach, in relation to the objectives of the present investigation, are easiness of implementation and abundance of data on built-up structures under constant amplitude loadings. The well-known shortcomings of the methods, and already discussed in several papers, are the potential lack of accuracy due to load sequence effects (retardation and acceleration phenomena) and the damaging effects of small cycles whose amplitude is below the constant amplitude endurance limit. To counteract such shortcomings, following the suggestion of Smith and Goranson [7], S-N curves of the type shown in Fig. 4 may be utilized.

These curves are straight lines on a log-log plot with a reduced slope, in order to take the prevailing retardation effects of flight by flight loads into account, at least on average. A linear variation of fatigue strength at a given number of cycles with the mean stress of the cyclic load complete the model of S-N curves.

This type of S-N curves, together with reliable test data in $(S_a^*)_{REF}$, has been used for establishing definitive procedures for effective design against fatigue [8].

The main problem encountered in the present investigation was to obtain adequate information on $(S_a^*)_{REF}$ for all the structural shapes which it is necessary to examine in an optimization procedure. The $(S_a^*)_{REF}$ factor depends not only on the type of material and its status, on fastener systems and fastener loads, but also on the geometric ratios between the fastener diameter and the thickness of the sheets fastened.

This problem has been solved by processing a large amount of data on the fatigue endurance of stiffened panels obtained during tests lasting several year at the Institute of Aeronautics of the University of Pisa and by utilizing data from published and unpublished reports and handbooks. The final result was that it was possible to specify $(S_a^*)_{REF}$ as a 95% confidence and a 95% reliability value for the survival of the structural component without the appearence of any significant damage.

The values of $(S_a^*)_{REF}$ were established for a set of structural shapes suitable for the purposes of the present investigation.

Fig. 5 shows, as a typical example, the S-N curves for a 2024-T3 alluminium alloy stiffened panel with different types of fasteners. Fig. 6 shows the allowable value of S_{LF} as a function of the number of MINITWIST-LGF, for different fastener systems.

Using these results, the durability constraint for any given structural component can be established on the basis of the following inequality:

$$N_x/\bar{t} \leq \bar{S}_{LF} \qquad (4)$$

where $\bar{S}_{LF}$ is the 1g stress which must not be exceeded in order for the aircraft to survive a given number of LGF, with the value of $(S_a^*)_{REF}$ relevant to the given component.

V. Damage tolerance constraints

A built-up stiffened structure can be designed to be damage tolerant in different ways. Consequently, the formulation of the constraints follows different patterns in relation to the structural features which ensure damage tolerant behaviour.

Irrespective of these features, one or both the following two main questions must be faced in the implementation of constraints, namely:

- the evaluation of the critical stress-critical crack length relationship;
- the evaluation of the time a crack takes to grow from its initial length to a final size under a flight-by-flight spectrum and, in particular, a MINITWIST spectrum.

Both the problems are typical of any application of fracture mechanics. Nevertheless, they deserve particular attention in relation to the type of structure and the numerical problems being considered in the present investigation.

The solution of the first problem consists here in utilizing the basic relationship of linear fracture mechanic, $K = K_C$.

One problem encountered in the solution of such an equation stems from the difficulty of obtaining accurate solutions of the stress intensity factor K. This difficulty can be ascribed to uncertainty in the idealization of fastener behaviour due to such influences as fastener flexibility in the elastic and plastic range and friction between faying surfaces.

A second problem is of a numerical nature and is related to the need to reduce the computer time necessary to evaluate K as far as possible owing to the large amount of repetition in the K calculation encountered in the optimization procedure.

The method utilized in the present investigation is a modification of the original Poe method[9], often called the Displacement Compatibility Method. The modified method allows for fastener flexibility both in the elastic and in the plastic range, for finite dimensions via corrective formula developed ad hoc and for the actual shape of stiffeners.

Appendix A presents, for the sake of completeness, a more detailed discussion on these modifications and their relevant background.

The Poe method, so modified, provides a high degree of accuracy with a reasonably reduced computer time.

The method also gives the stringer overload coefficient L, which plays an important role in ensuring the crack-stopping ability of the stringers.

The second problem concerns the calculation of the crack growth time. Interaction effects, namely retardation and acceleration due to load sequence, play an important part and must be taken into account when utilizing a cycle-by-cycle calculation procedure.

However, a procedure of this type is prohibitively time-consuming if implemented in an optimization procedure and it is, therefore, necessary to find simplified methods providing comparable accuracy with substantial reduction of computer time.

A satisfactory solution to this problem was found in the course of an investigation carried out at the Institute of Aeronautics of the University of Pisa on the simplified methods for predicting the crack growth rate under flight-by-flight spectrum loading[10,11].

The results of this investigation, which are summarized in Appendix B, show it is possible to obtain the crack growth time by simple integration of the following relationship:

$$\frac{da}{dF} = A^{*}\ \overline{\Delta K}^{n^{*}} \quad ; \quad \overline{\Delta K} = \bar{S}\ \sqrt{\pi a}\cdot C \qquad (5)$$

which relates the crack growth per flight $\frac{da}{dF}$ to an average value $\overline{\Delta K}$ in the flight. $\bar{S}$ is a reference stress assumed, in the present case, as the RMS of MINITWIST.

A^{*} is a constant which depends on the type of spectrum and material, but is independent from the geometry of the structure; n^{*} depends only on the material and is virtually coincident with the exponent in the Paris law.

The main achievement of this investigation is the conclusion that A^{*} is independent from the geometry of the structure so that A^{*} can be obtained by tests on simple sheets.

Both the methods, previously summarized, have been used to quantify the constraints obtained in the two following types of damage tolerant structures, namely:

- fail-safe crack-arrest structure
- slow-crack growth structure.

V.1 - Fail-safe crack-arrest constraints

The degree of damage to be tolerated without catastrophic failure in the structure of the wing lower surface must be consistent with the requirements of the Airworthines Authorities.

Typical requirements state that a fail-safe structure must be capable of operating safely after failure or obvious partial failure of a simple principal structural element when subjected to the fail-safe load conditions. In the case of the skin stringer structure, a rational interpretation of such a statement may be a two bays skin crack with the central stringer broken. This kind of damage may be ascribed to undetected damage growing in the skin and in the underlying stringer, to such an extent that the stringer breaks completely and the skin crack grows to an unstable length, propagates and then stops near the following stringers.

To establish the constraints for this case, reference is made to Fig. 7 which shows the residual strength and the stringer allowable as functions of the crack length in the skin.

If the stress in the structure under fail-safe load conditions S_{FS} lies between S_{MIN} and $\beta\ S_{MAX}$ and the overstress in the stringer remains lower than the ultimate stress, compliance with tolerance of a two bay length damage is ensured.

The β factor ensures that this damage can be sustained for a certain number of flights without catastrophic failure.

As a consequence of the above, the constraints can be stated as follows, namely:

$$S_{MIN} \leq S_{FS} \leq \beta\ S_{MAX} \qquad (6')$$

$$L\ (a_2) S_{FS} \leq (1 - M \cdot S) \sigma_u \qquad (6'')$$

where σ_u is the ultimate strength of the material.

A margin of safety (MS) of 10% has been assumed. Furthermore $\beta = 0.99$ was found to guarantee at least 10 flights without catastrophic failure.

V.2 - Slow crack growth constraints

In this case it is necessary to specify the extent of the initial damage, a_o, and the limit inspection interval, T_1, that is, the number of flights possible before damage reaches catastrophic dimensions. The dimension of the initial damage, a_o, has been identified with a broken fully stringer and a 6 mm crack issuing from a fastener hole.

Crack length equal to a_1 in Fig. 7 has been considered catastrophic, not allowing for the potential arrest capability of the integer stringers.

The constraint is therefore obtained from the following inequality for the allowable inspection interval T_{all}, namely

$$T_{all} \leq T_1 \qquad (7)$$

V.3 - Fail-safety with a specified inspection interval

A third case considered in the present investigation is a two bay length damage fail-safe structure with a prescribed operational time without damage greater than a_1.

In this case, both the constraints specified in the previous sections apply.

VI. Analysis of results

The optimization procedure described in the previous sections has been applied in the search for the minimum weight design of stringer-skin combinations of the type shown in Fig. 8.

Design variables, design parameters and constraint functions are also shown in the same figure, for the sake of completeness.

The results obtained have been presented in terms of the minimum value of the average skin-stringer thickness w which is proportional to the structural weight.

The influence of several design parameters on weight variations has been examined.

The parameters which varied in successive optimization runs are the following:

- load per unit length in level flight conditions, ($0 < N_x \leq 180$ kg/mm)
- material (2024 and 7075 aluminium alloys)
- number of LGF
- fastener fatigue quality (three different types of behaviour)
- inspection intervals.

To obtain a better appreciation of the influence the different constraints have on the structure of minimum weight, successive optimization runs have been carried out with different combinations of constraints, namely:

- Static Strength Constraints (SSC),
- Static Strength and Durability Constraints (SSC + DC),
- Static Strength, Durability, and Slow Crack Growth Constraints (SSC + DC + SCGC),
- Static Strength, Durability, Slow Crack Growth and Crack Arrest Constraints (SSC + DC + SCGC + + CAD).

Table I shows, for each combination of constraints, the design parameters which have been modified in successive runs of optimization.

Fig. 9 shows the influence of the static strength constraints for two aluminium alloys, namely 2024 and 7075.

Buckling and crippling phenomena have major effects on the weight only in the $N_x < (N_x)_b$ range. For higher values of N_x, the ultimate strength in traction controls the average thickness which is simply given by $w_1 = 4.05\ N_x/\sigma_u$. Buckling and crippling strengths only influence the share of material between skin and stringers.

Figs. 10 concerns weight variation due to the addition of durability constraints.

The increase in weight for a given number of LGF, namely $(w_2 - w_1)$, is semply proportional to w_1 for $N_x > (N_x)_b$ and therefore the ratio $\frac{w_2 - w_1}{w_1}$ is independent from N_x, at least in the range of N_x and the number of LGF investigated in the course of the present investigation.

Three different scales of the number of LGF are shown on the axis of abscissas. The $90 \cdot 10^3$ scale is relevant to the number of LGF obtained by applying the methods outlined in the section on "Durability Constraints". The other two scales have been obtained by division by life reduction factors 1.5 and 4 respectively. The first figure is likely to be applied in the case of Safe-Life design to comply with the safety requirements of the Airworthiness Authorities.

Fig. 10a concerns a skin-stringer combination made of 2024 aluminium alloy, and shows the influence of differing fastener fatigue quality on the minimum weight.

Curve M is representative of skin-stringer joints obtained by means of a standard riveting of countersunk head rivets with a shank diameter appropriate to the stringer flange-skin thickness and a driven head diameter of a high standard. The L curve is representative of low fatigue quality fasteners like rivets with ecxessive countersunk depth, and too a value of the shank and driven head diameters. This curve can be also considered to be representative of standard riveting and poor surface finishing such as is obtained by chemical milling.

The H curve is typical of the highest fatigue quality fasteners like hi-squeeze rivets for hi--lok installed in high standard cold-worked holes. The weight variation due to these extreme variations in fastener quality is noteworthy; use of high quality fasteners can reduce weight with respect to standard riveting by up to 20% of static strength weigth w_1.

Fig. 10b shows a comparison between two aluminium alloys, that is 2024 and 7075.

The increase in weight with alloy 7075 is such as to cancel out the initial advantage of this alloy due to its higher static strength. This result is in accordance with the usual design practice which resorts to alloy 2024 in lower wing structure.

Fig. 11 shows the weight increase due to the addition of crack arrest constraints. The average thickness w_{3a} is plotted versus the number of LGF for different values of N_x. The reduction factor 1.5 has been utilized to obtain the number of LGF.

Values of w_2 are also shown for the sake of comparison.

The data are relevant to aluminium alloy 2024 and a standard riveting joint.

Two values of the rivet diameter (3/16 inch for $N_x < 90$ kg/mm and 3/8 inch for a higher value of N_x) have been selected to calculate stress intensity factors and overload coefficients on the basis of the elasto-plastic idealization of the fastener utilized in constraint implementation. Further, the critical crack lenght has been obtained by allowing for K_c variation with skin thickness.

The data shown in Fig. 11 allows us to estimate the weight increase necessary to guarantee crack arrest ability.

Some comments must be made on these results:

The optimization process strives to reduce the thickness of the skin and the stringer spacing in order to reduce the load transfer from the cracked skin to the stringers and to minimize K_c reductions. At the same time, the area of the stringer is increased to exploit their crack ability to the utmost. In this process, the geometric constraints on t/b, b/b_w and b_f/b embodied in the computer program to limit the ranges of the design variables, play a important role in defining lower limits for the stringer spacing and upper limits for the stringer area. The shape of the minimum weight skin stringer combination, particularly in the high N_x range, is characterized by low-gage skin, stiffened by close high area stringers.

Structural configurations of this type are probably open to criticism as they present problems from the point of view of cost due to production and maintenance difficulties. In fact, they imply an excessive number of fasteners, a great difference between the thickness to be fastened and a considerable extension of faying surfaces.

Further, the low-gage skin is, in many cases, inadequate to comply with aeroelastic requirements of torsional stiffness.

As a consequence, the results of Fig. 11 must be considered merely as the bottom reference values of the lower wing surface weight, which could seldom be achieved in actual design practise.

Cost and aeroelastic constraints previously indicated might be allowed for, in carrying out a well-defined design task. In this case one might obtain different minimum weight solutions for different skin thicknesses and stringer spacing constraints and then trade-off to obtain a safe minimum-cost design.

Fig. 12 concerns the influence of slow crack growth constraint on the weight of the lower wing surface. In this case, the structure is required to operate with growing damage for a selected fraction of the life goal. In the present approach, an inspection interval equal to 1/3 of the life goal has been chosen; further, to allow for scatter in crack growth evaluation, a safety factor of 2 has been assumed so that the interval calculated amounts to 2/3 of the life goal. The type and extension of initial damage assumed in interval calculations are also shown in Fig. 12. A stringer which is completely broken, plus a crack in the skin of the dimension shown, are felt to be representative of the maximum damage which is likely to be missed during inspection checks.

The results are presented in terms of average thicknesses w_{3s}, w_{3a} and w_2 versus the number of LGF. Comparison of both the thicknesses w_{3s} and w_{3a} indicates that the slow crack growth design can be a weight saving solution only for relatively low values of N_x and life goal.

Calculations have been also carried out, according to which the structure must comply simultaneously with slow-crack growth and crack arrest constraints. The results of these calculations, show that only low values of N_x (say $N_x < 80$ kg/mm) and of life goal (say 10÷15 thousand flights) give a minor increase in weight with respect to w_{3a} and w_{3s}.

VII. Conclusions

The paper deals with the problem of evaluating the influence of the main design constraints on the weight of the lower wing structures of commercial airplanes. The problem has been faced within the framework of an optimization approach which minimizes the weight of a structure while it complies with a set of prescribed design constraints.

Both Safe-Life and Damage Tolerance design concepts have been considered.

Constraints deriving from static strength, durability and damage tolerance requirements have been implemented in mathematical models to be used in a optimization computer program.

Further, several geometrical constraints have been introduced to obtain meaningful solutions.

The results have been presented in terms of the minimum average thicknesses of the lower wing structure for significant ranges of important design parameters such as the load per unit chordwise length in level flight, the life goal, the fastener fatigue quality and the inspection interval.

Results are available for different combinations of the constraints and for two typical aluminium alloys, namely 2024 and 7075.

The results obtained can be considered to be significant particularly for comparison purposes between different design approaches or different selection of design parameters.

The computer program, if implemented with specific constraints dictated by specific design needs, is felt to be suitable to select realistic structural configurations of minimum weight.

SYMBOLS

a : Half-crack length, mm.
a_o : Initial value of half-crack, mm.
A_r : Rivet area, mm^2.
A : Constant of Paris law in semplified model of crack growth.
b : Stiffener spacing, mm.
b_f : Stiffener flange dimension, mm.
b_w : Stiffener web dimension, mm.
d : Rivet diameter, mm.
E : Young's modulus, kg/mm^2.
E_c : Sheet Young's modulus, kg/mm^2.
E_r : Rivet Young's modulus, kg/mm^2.
E_s : Stiffener Young's modulus, kg/mm^2.
FFQ : Fastener fatigue quality.
G_i : Constraint function.
h : Box depth, mm.
K : Stress intensity factor, $kg/mm^{3/2}$.
K_C : Critical value of stress intensity factor, $kg/mm^{3/2}$.
l : Chord length of the box, mm.
L : Stiffener overload in the cracked panel.
L' : Effective length of the stiffened panel, mm.
L_b : Rib spacing of the box, mm.
LGF : Life goal expressed via MINITWIST number of flights.
MS : Margin of safety.
n : Constant of Paris law in semplified model of crack growth.
N : Number of load cycles.
N^* : Reference number of load cycles.
N_x : Load per unit of chord wise length in level flight, kg/mm.
p : Rivet pitch, mm.
r_j : Numerical parameter of minimization process.
S : Stress, kg/mm^2.
$(S_a^*)_{ref}$: Reference value of fatigue strength of structure component, kg/mm^2.
S_b : Buckling stress of stiffened panel, kg/mm^2.
S_c : Crippling stress of stiffened panel, kg/mm^2.
S_{CR} : Crusching stress of stiffened panel, kg/mm^2.
S_{FS} : Stress relevant to fail-safe load, kg/mm^2.
S_{LF} : Stress in level flight, kg/mm^2.
$\underline{S}_{LF}$: Allowable value of stress in level flight for a fixed value of LGF, kg/mm^2.
S_m : Mean value of stress, kg/mm^2.
t : Sheet thickness, mm.
$\bar{t}$: Mean value of stiffened panel thickness, mm.
t_s : Stiffener thickness, mm.
T_{all} : Inspection interval allowable.
T_I : Inspection interval of panel.
w : Thickness per unit of chordwise length, mm.
$\bar{W}$: Objective function in minimization process.
w_1, w_2, w_{3a}, w_{3s} : Minimum values of the average thickness, defined in Tab. I.
X : Vector of design variables.
Y : Vector of design parameters.
β : Safety factor.
ν : Poisson's modulus.
ξ : Flexibility parameter.
ρ : Radius of giration in skin-stiffener combination, mm.
σ_u : Ultimate stress, kg/mm^2.
σ_y : Yield stress, kg/mm^2.

REFERENCES

1 - Lowak H., De Jonge J.B., Franz J., Schütz W., "MINITWIST, a Shortened Version of TWIST", NLR MP 79018 U, 1979, ICAF Doc.1147.

2 - Fox L.R., "Unconstrained Minimization Approaches to Constrained Problem", in Structural Design Applications of Mathematical Programming Techniques, Agardograph 149, 1971, pp. 55-78.

3 - Fletcher R., Powell D.J., "A Rapidly Convergent Descent Method for Minimization", The Computer Journal, Vol. 6, 1963, pp. 163-168.

4 - Cavallini G., "An Approach to Minimum Weight Design of Lower Wing Surface", To be published.

5 - E.S.D.U. - Structure Sub-Series-, Vol. 8, "Local Bucking", Data Sheets 71014, 1976.

6 - Gerard G., "The Crippling Strenght of Compression Elements", J. of the Aeronautica Science, Vol. 25, 1958, pp. 37-52.

7 - Smith H.W., Goranson U.G., "Spectrum Loading in Relation to Aircraft Design", Proceedings of the Eight ICAF Symposium, Lausanne, 1975, ICAF Doc. 801.

8 - Craig L.E., Goranson U.G., "Airworthiness Assessment of Boeing Jet Transport Structures", Proceeding of Tenth ICAF Symposium, Brussels, 1979.

9 - Poe C.C., "Stress Intensity Factor for a Cracked Sheet with Riveted and Uniformly Spaced Stringers", NASA TR-358, 1971.

10 - Cavallini G., Maltoni C., "Metodi Semplificati per la Determinazione dell'Accrescimento di Difetti sotto Carichi Affaticanti ad Ampiezza Variabile", Doc. Istituto di Aeronautica, Università di Pisa, DIA 79-8, 1979, To be published in 'L'Aerotecnica - Missili e Spazio'.

11 - Cavallini G., "Fatigue Life and Structural Integrity Monitoring in Operative Aircrafts", 2°Convegno di Studi sulla Fatica nelle Strutture Aerospaziali, Napoli, 1980.

12 - Salvetti A., Cavallini G., Lazzeri L., "The Fatigue Crack Growth Under Variable Amplitude Loading in Built-Up Structures", DA-ERO 78-G-107, 1st Annual Technical Report, 1979, ICAF Doc. 1139.

13 - Giavotto V., Merlini F., "Su alcuni problemi relativi alla determinazione del fattore di intensità degli sforzi in difetti tridimensionali", 2 Convegno di studi sulla fatica nelle strutture aerospaziali, Napoli, 1980.

14 - Chang J.B., "Improved Methods for Predicting Spectrum Loading Effects", Rep. NA-78-491, Rockwell Intern., Los Angeles, 1978.

APPENDIX A: Stress intensity factor evaluation in built-up structures.

The computation of the stress intensity factor in a built-up structure is in principle straightforward. First a solution must be found to the problem of load transfer from the cracked elements to those integers by means of the fastener system and then the stress intensity factor by superposition must be computed by utilizing a relation of the type:

$$K = K_s + \sum_i (K_f)_i \, f_i = K_s \, C \qquad (1\text{-}A)$$

as in the Displacement Compatibility Method (DCM) developed by Poe [9] or directly by a finite element method. In this last case, K may be obtained either by an approximate evaluation of the intensity of the stress singularity at the crack tip or by computing the rate of change of the strain energy with respect to the crack area.

Irrespective of the method of computation used, the K values obtained depend significantly on the idealization selected for the fastener behaviour. Two points deserve particular attention in this respect, namely the fastener flexibility both in elastic and in plastic ranges and the friction between faying surfaces.

The types of influence they can have on the values of the stress intensity factor have been discussed in some detail in a previous report [12]. Figs. A1 show typical results obtained with different values of flexibility and friction forces.

The main question stemming from such results is how to obtain appropriate values of this quantities, namely fastener flexibility and friction. Both these quantities, being influenced, for a given type of fastener, by hole preparation, installation techniques and faying surface status, are likely to be scattered over a certain range due to the random nature of such influences.

Consequently one should rely only on plenty of test data which is satisfactorily representative of the specific fastener behaviour suitable for treatment by statistical analysis.

As an example of such an approach, Fig. A2 shows the cumulative probability distribution of the adimensional rivet flexibility in the case of hand driven countersunk rivets. Such results, obtained by test data on the basis of the basis of the approach outlined in ref. 12, are typical of the behaviour of the rivets mentioned above, and give a good measure of the scatter which affects the flexibility of such rivets in built-up structures. Further data on the combined effects of flexibility and friction are discussed in ref. 12, already quoted.

Consequently, a rational selection of the values of flexibility and friction ought to be given in percentages (e.g. 50% probability of occurrence) obtained from a probability distribution deduced from a coherent set of test data.

Obviously K values so obtained mean percentiles in the K distribution.

A second aspect of evaluating K concerns the availability of methods that are easy to apply and require comparatively short computer running time.

The Displacement Compatibility Method (DCM) is, in this respect, more effective than finite element methods. The DCM, however, has limitations as it provides exact results only for infinite panels. Other available solutions treat the stiffeners as simple straps in tension.

It is therefore useful to have simple methods in order to correct the DCM results for the actual shape of the stiffeners and finite panel dimensions.

As far as the shape of the stiffeners is concerned, a reduced-area concept has been developed. Following this approach, a stiffener of a given cross-section with area A is substituted by a strap with an equivalent reduced area A*. Equivalency means equal displacements at fastener locations, evaluated within the frame work of the beam theory.

The reduced area is then given by:

$$\frac{1}{A^*} = \frac{1}{A} + \frac{x_1^2 I_x + y_1^2 I_y - 2x_1 y_1 I_{xy}}{I_x I_y - I_{xy}^2} \qquad (2\text{-}A)$$

where I_x, I_y and I_{xy} are the moments and products of inertia of the actual cross section, x_1 and y_1 are the rectangular coordinates of the fastener line with respect to the centraid cross section, axis y being perpendicular to the middle-plane skin.

This relationship has been carefully checked by a FEM approach[13]. Fig. A3 shows typical results which demonstrate the substantial accuracy of the reduced area approach. In more detail, the FEM predicts K values which are a little higher than the predictions obtained with the reduced area. This difference can be ascribed to the fact that FEM idealization of the stringer allows for plate deformation in the stringer flange, whereas the reduced area concept only takes into account bending deformation. However, these differences are not significant, since flexibility relationship in question[12] allows for local deformation in the stringer flange due to localized action of the fastener forces.

As far as the finite dimension effect is concerned, the FEM approach has been utilized to check formulas developed for simple plate.

The work on this topic is still in progress. Preliminar results are reported in Ref. 13.

APPENDIX B: Crack growth prediction with simplified methods.

All the cycle-by-cycle crack growth prediction methods have one fundamental shortcoming, namely the long computer running time, necessary to predict crack growth, from the initial size to its final critical dimension. This is an important problem particularly in the preliminary design phase when many different solutions have to be compared.

It is therefore worthwhile to develop simplified methods to predict crack growth which significantly reduce computer time without substantially impairing prediction accuracy.

Several methods have been considered[10,14], but the most promising method is the one based on a Paris-like model, namely:

$$da/dF = A^{*}\ \overline{\Delta K}^{\,n^{*}} = A^{*}(\overline{\Delta S}\cdot f(g,a))^{n^{*}} \qquad (1\text{-}B)$$

where F is the number of flights. A^{*} and n^{*} can be deduced from a regression analysis of data obtained by a cycle-by-cycle method. $\overline{\Delta S}$ is a characteristic value of the loading spectrum.

The root mean square value of the stress spectrum has been found to be a meaningful choice in Ref. 8 where results obtained with four different spectra, namely FALSTAFF, TWIST, MINITWIST, Random Gaussian, have been used to substantiate this choice.

Should the values of parameters A^{*} and n^{*} be determined by means of a cycle-by-cycle method, no improvement would be achieved with the use of a simplified method.

But the extensive calculations carried out[10,11], showed two very significant results, namely, the value of n^{*} is a typical property of the material and is independent from the spectrum type and is therefore the same value as the Paris law exponent, and the value of A^{*} depends on the material, the spectrum type and the cycle-by-cycle method used, but A^{*} is essentially independent from the geometry of cracked structure, taken into account by means of function $f(g,a)$.

These two results have been proved to be accurate for several types of spectrum loading (TWIST, MINITWIST, FALSTAFF, Random Gaussian), for several kinds of geometry (simple panel, stiffened panel with central stiffener broken), for several cycle-by-cycle methods (non interactive, Wheeler, Willenborg and Bell-Eidinoff) and for several types of material.

Then the number of flights during which the crack grows from a_1 to a_2, can be easily obtained in the form:

$$F = (1/A^{*}\ \overline{\Delta S}^{\,n^{*}})\int_{a_1}^{a_2}(1/f(g,a))^{n^{*}}\,da \qquad (2\text{-}B)$$

where A^{*} can be determined once and for all for a given spectrum by applying a cycle-by-cycle method to a simple geometry.

Figs. B1,2 show the application of the relation (2-B) to the case of stiffened panels loaded by standardized spectra.

These results, together with many others reported in Ref. 10, are likely to subtantiate the proposed approach.

Another valuable outcome of the relation (2-B) is the possibility of drawing normalized curves which are a significant tool in designing and especially in maintaining aircraft structures. Indeed the relation (2-B) can be written as

$$F_1(a_o\to a_1)/F_2(a_o\to a_2)=\int_{a_o}^{a_1}(1/f(g,a))^{n^{*}}\,da\Big/\int_{a_o}^{a_2}(1/f(g,a))^{n^{*}}\,da \qquad (3\text{-}B)$$

where F represents the number of flights necessary for the crack to grow from initial damage a_o to a_1 or a_2. If a_2 represents the final dimension of the crack which cannot be exceeded, the ratio of life elapsed can be evaluated merely by measuring crack dimension a_1.

It is worthnoting that the ratios of (3-B) are independent from A^{*} and therefore from spectrum loading and from the particular cycle-by-cycle prediction method which represents the crack growth phenomenon.

In particular, the normalized curve can be prepared very quickly by using constant amplitude loads and then utilized for other spectrum loads.

Fig. B3 shows a typical application of this concept which has been extensively discussed and substantiated by detailed results reported in Ref. 11.

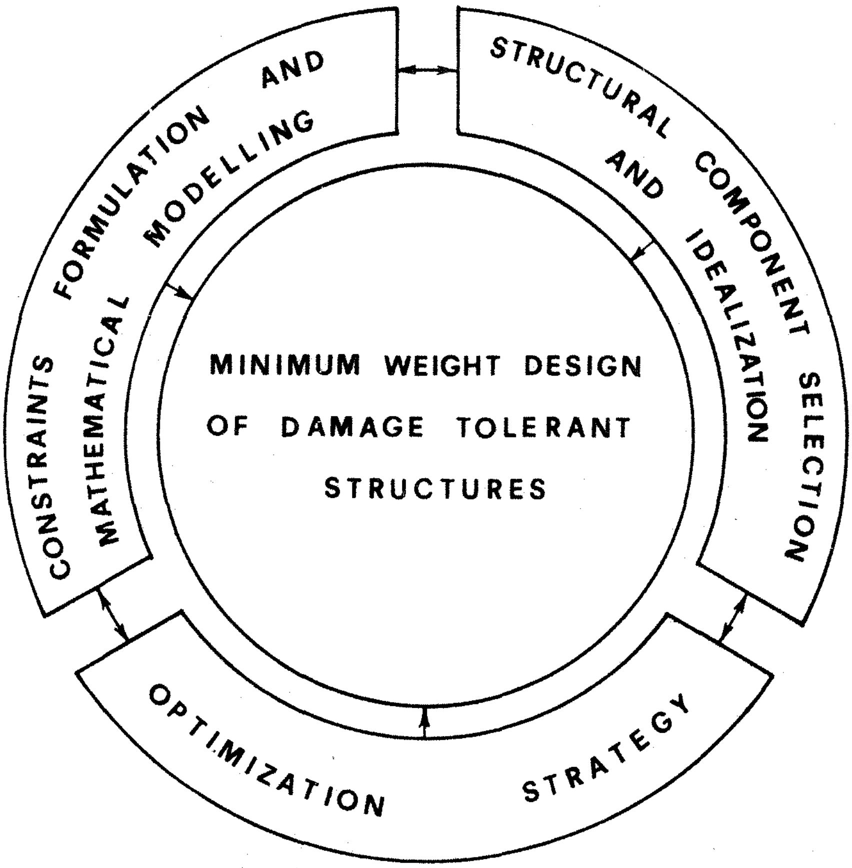
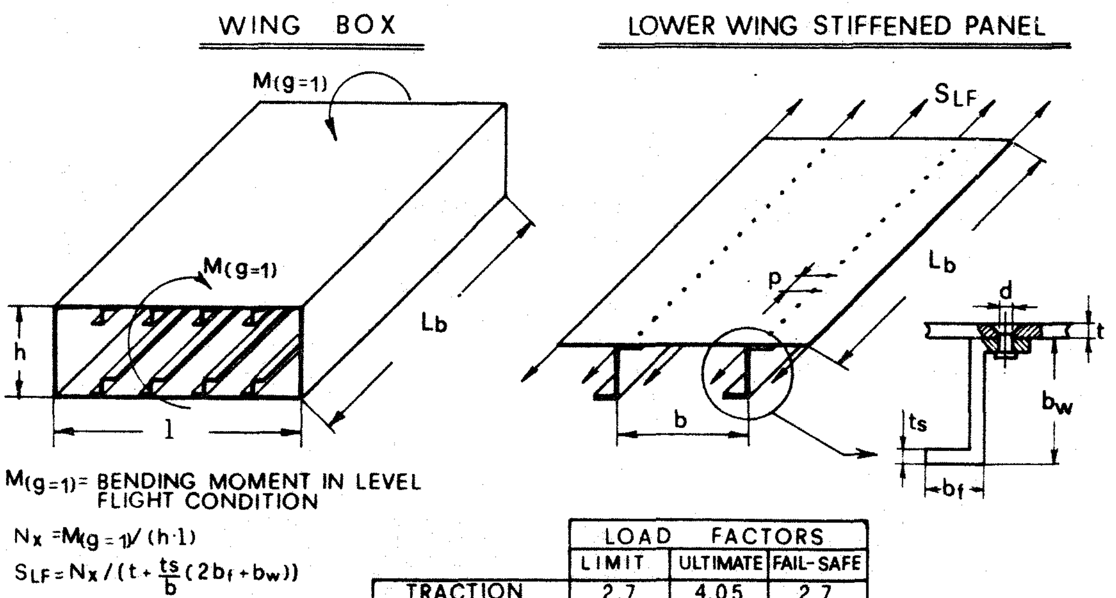


Fig. 1 - Rationale for minimum weight design of damage tolerant structures.



	LOAD FACTORS		
	LIMIT	ULTIMATE	FAIL-SAFE
TRACTION	2.7	4.05	2.7
COMPRESSION	-1.0	-1.5	—

Fig. 2 - Sketch of the structure idealization table and values of load factor values.

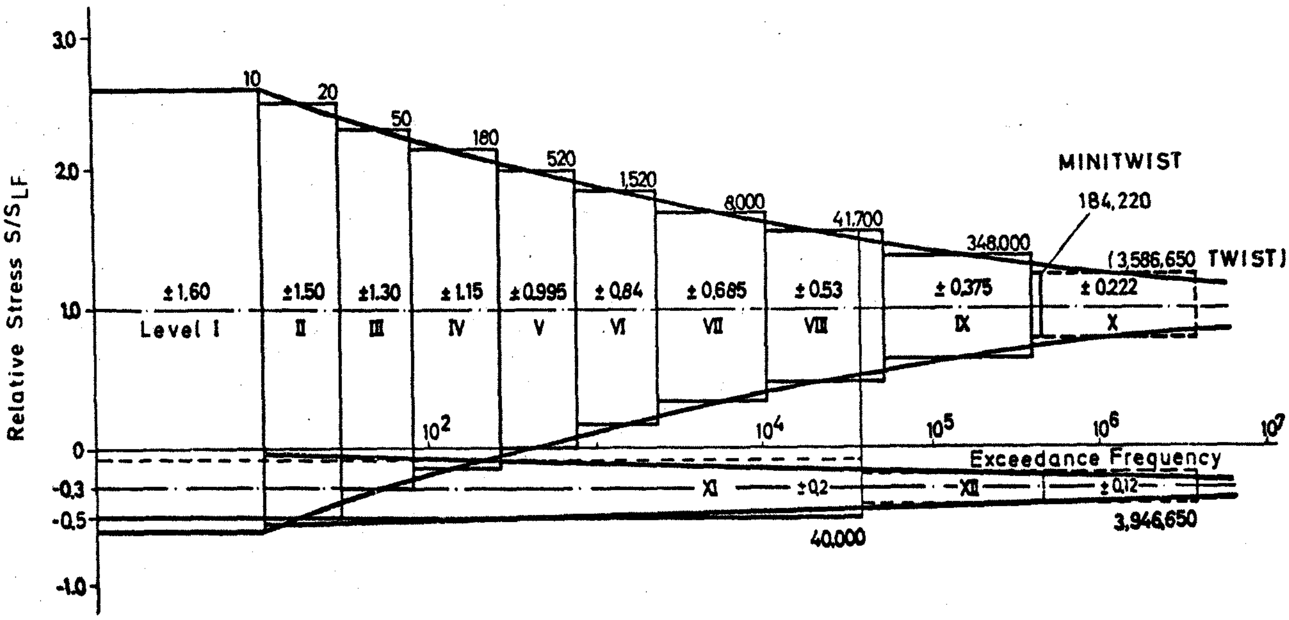


Fig. 3a - MINITWIST load spectrum, Ref.|1|.

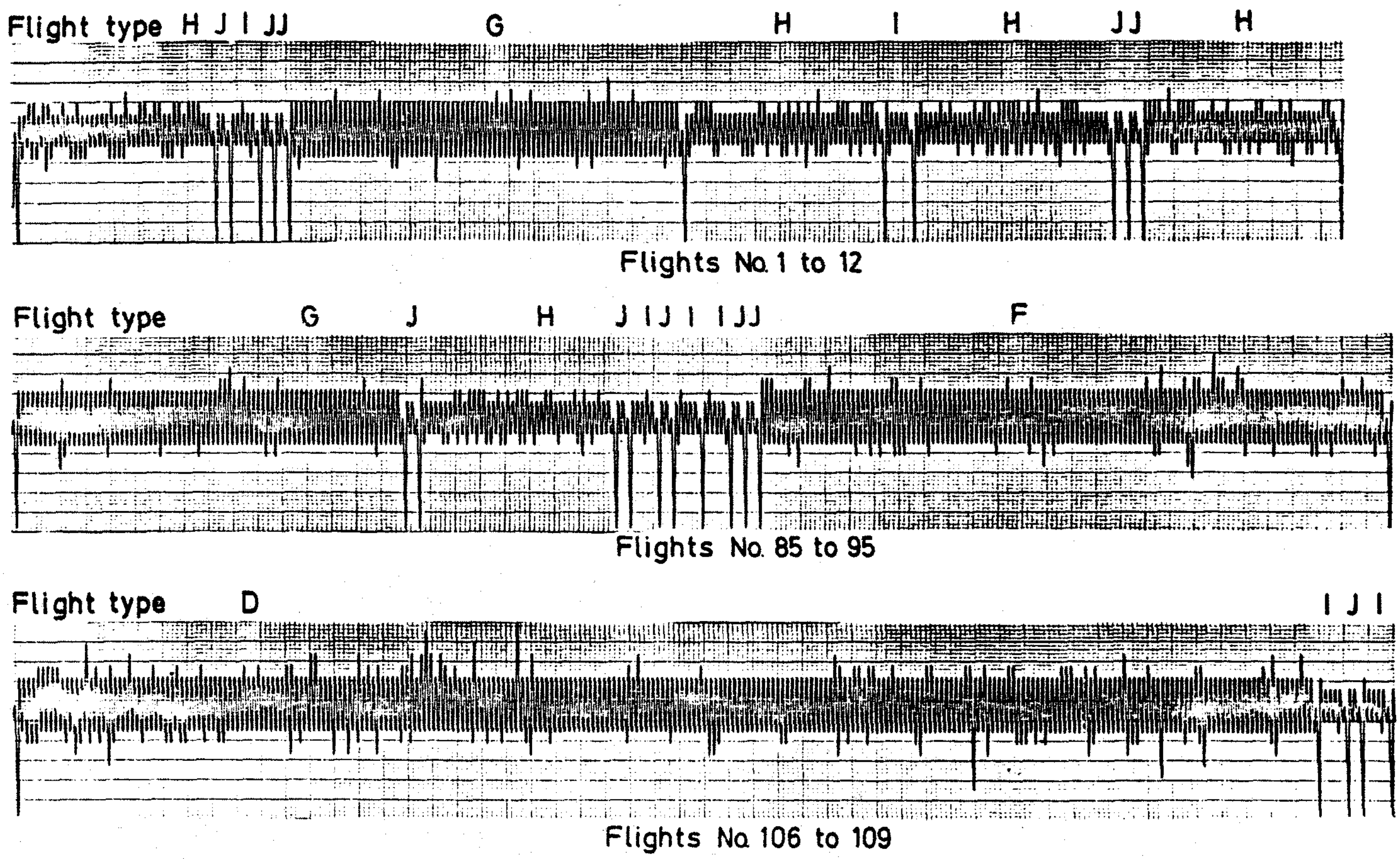


Fig. 3b - Examples of different flight types of MINITWIST, Ref.|1|.

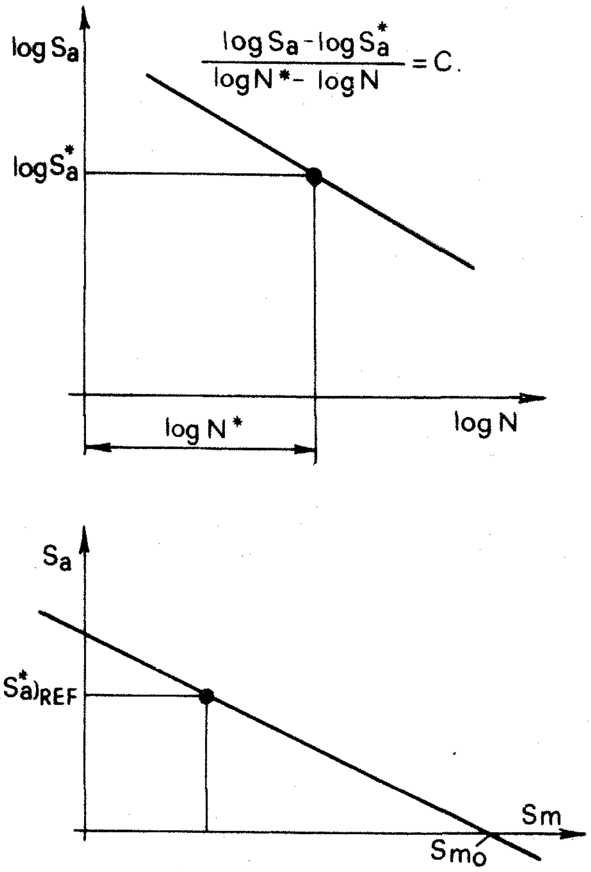


Fig.4 - Mathematical model for S-N curves.

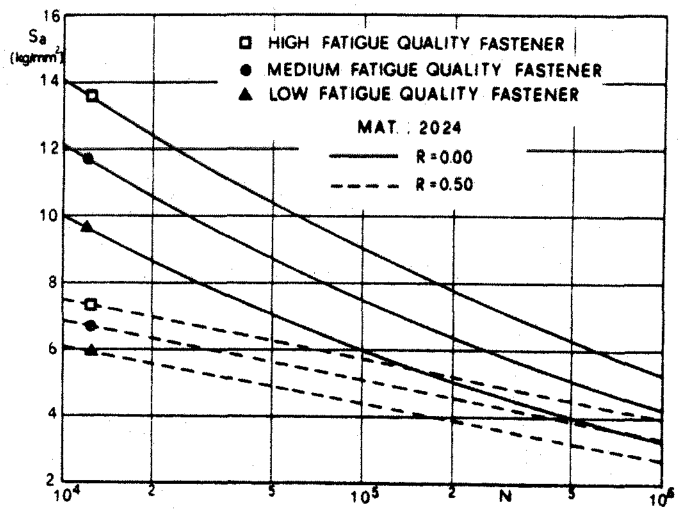


Fig.5 - S-N curves representative of the behaviour of 2024 aluminium alloy stiffened panel built-up with fastener of different three fatigue performance.

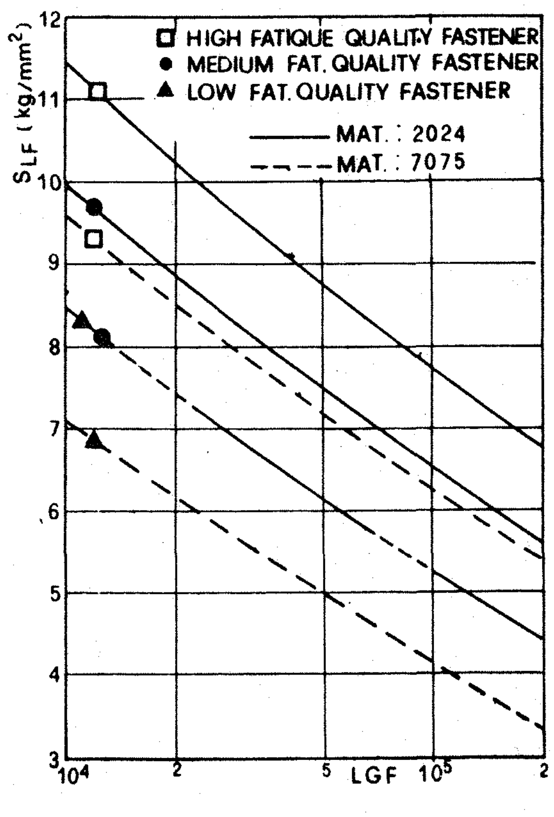


Fig.6 - Allowable value of the level flight stress versus the number of life goal flights for MINITWIST spectrum.

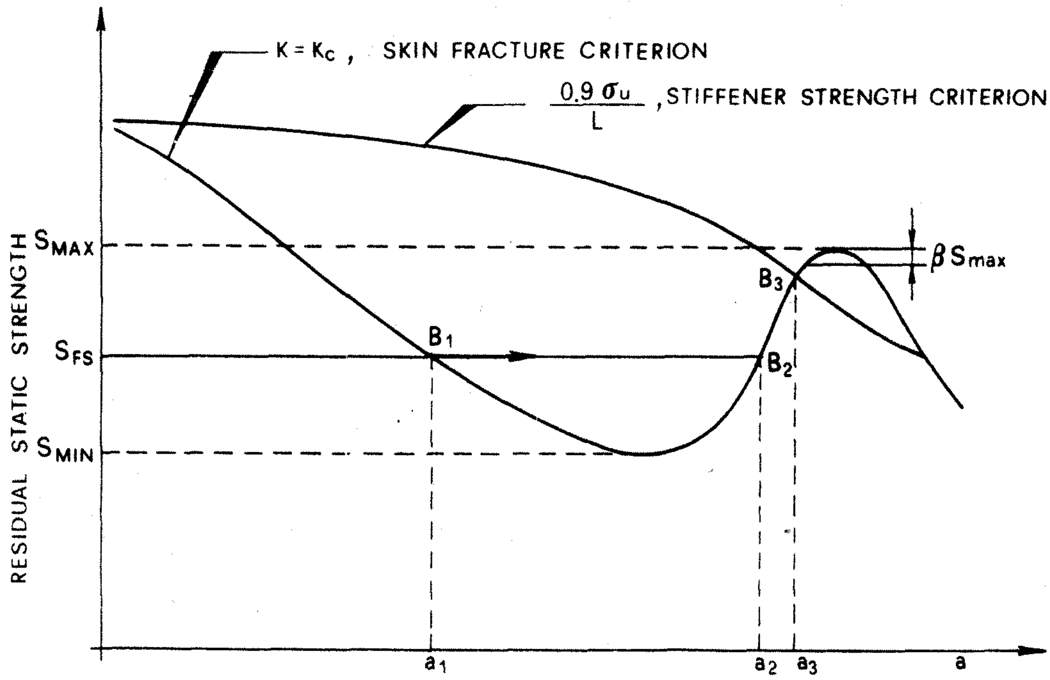
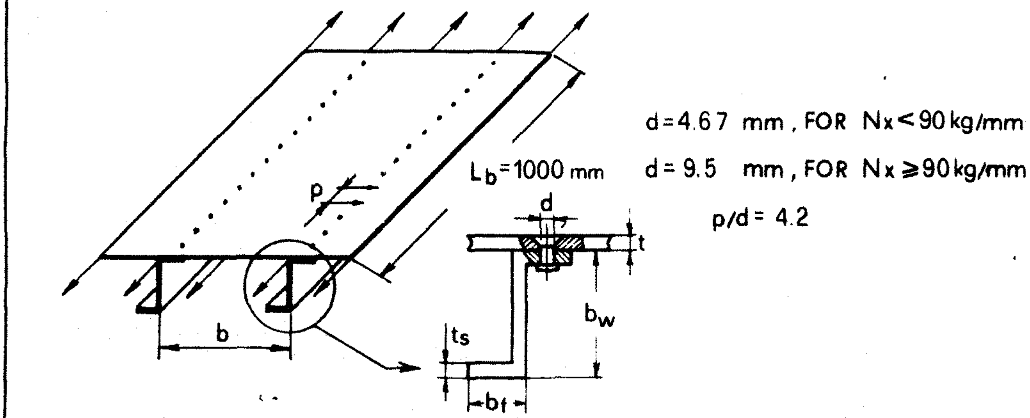


Fig.7 - Residual static strength curve of stiffened panel.



DESIGN VARIABLES	DESIGN PARAMETERS	CONSTRAINT FUNCTIONS
t	N_x	$G_1 = 1. - 2.7\ S_{LF}/\sigma_y$
b	MATERIAL	$G_2 = 1. - 4.05\ S_{LF}/\sigma_u$
t_s	L.G.F.	$G_3 = 1. - 2.7\ S_{LF}/S_b$
b_w	FASTENER FATIGUE QUALITY	$G_4 = 1. - 4.05\ S_{LF}/S_{CR}$
b_f	INSPECTION INTERVAL	$G_5 = 1. - S_{LF}/\bar{S}_{LF}$
		$G_6 = 1. - S_{MIN}/S_{FS}$
		$G_7 = 1. - S_{FS}/(\beta S_{MAX})$
		$G_8 = 1. - L\ S_{FS}/(0.9\sigma_u)$
		$G_9 = 1. - T_{all}/T_1$

Fig.8 - Summary of constraints,design parameters and design variables considered in the present analysis.

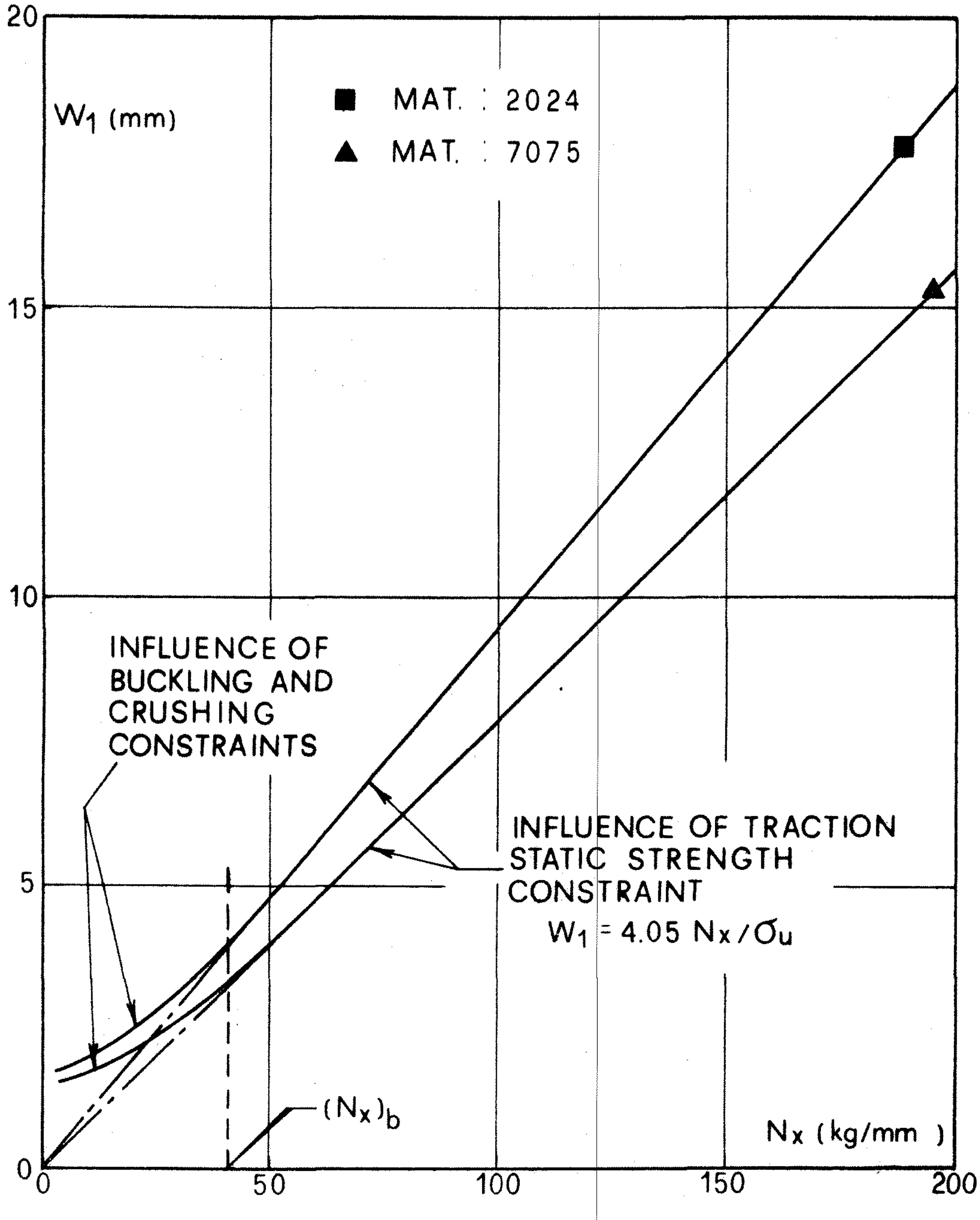
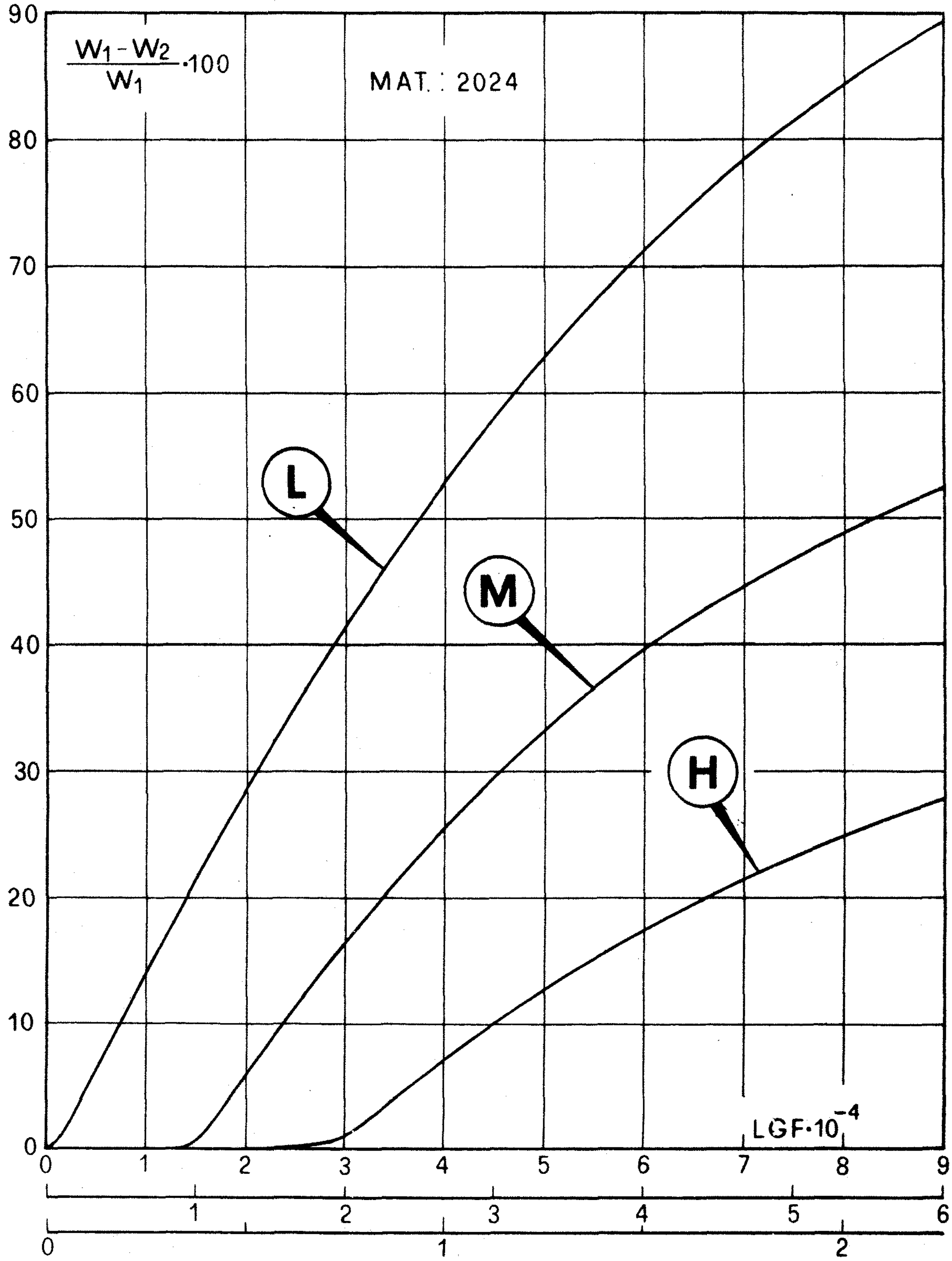


Fig.9 - Minimum weight stiffened panel complying merely with static strength constraints.



,Fig.10a- Influence of fastener fatigue quality on minimum weight stiffened panels complying with static strength and durability constraints.

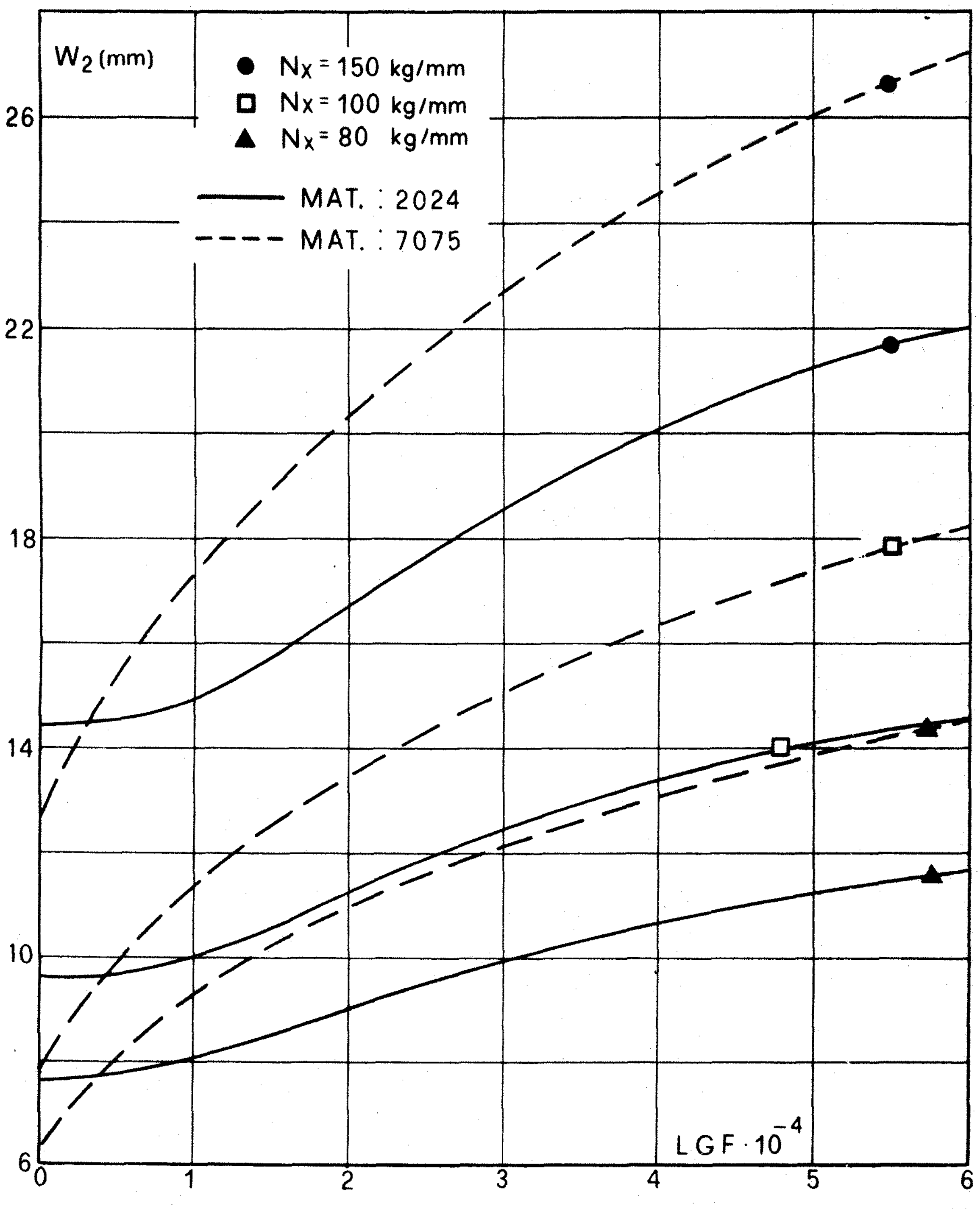


Fig.10b - Comparison between minimum weight values for stiffened panel made of 2024 and 7075 aluminium alloys. Static strength and durability constraints.

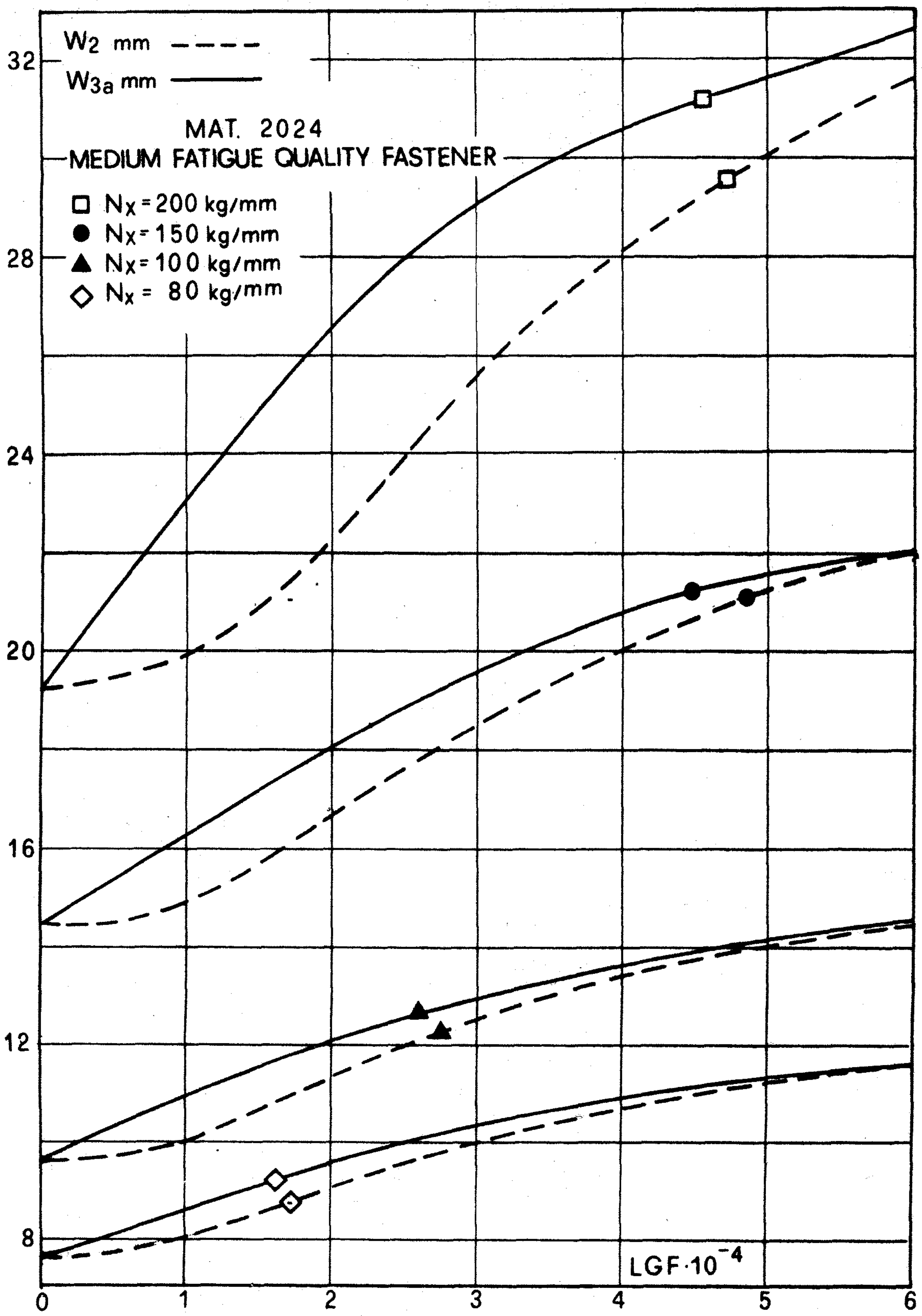


Fig.11 - Minimum weight values for stiffened panels complying with static strength, durability and Fail-Safe crack arrest constraints. Material 2024 aluminium alloy.

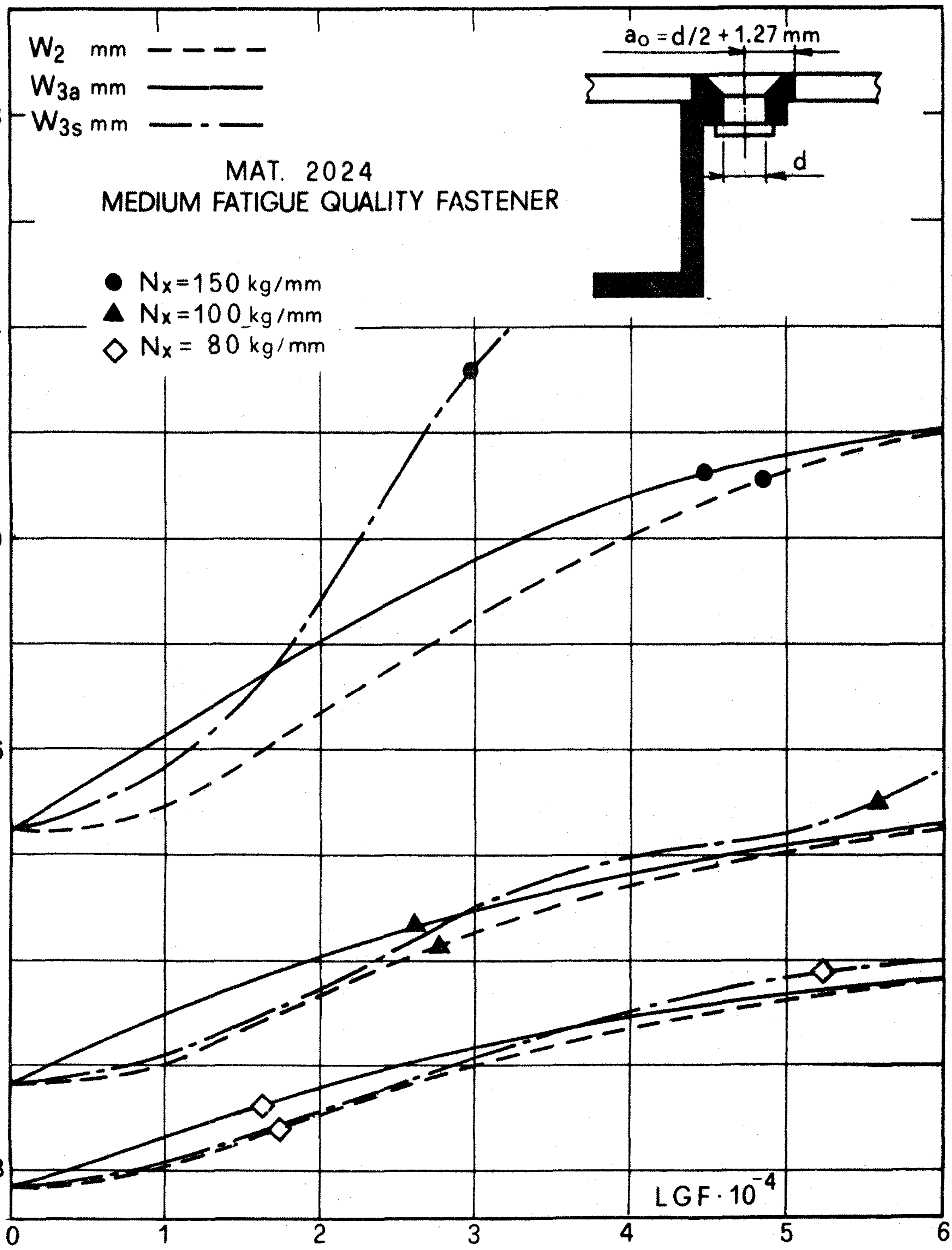
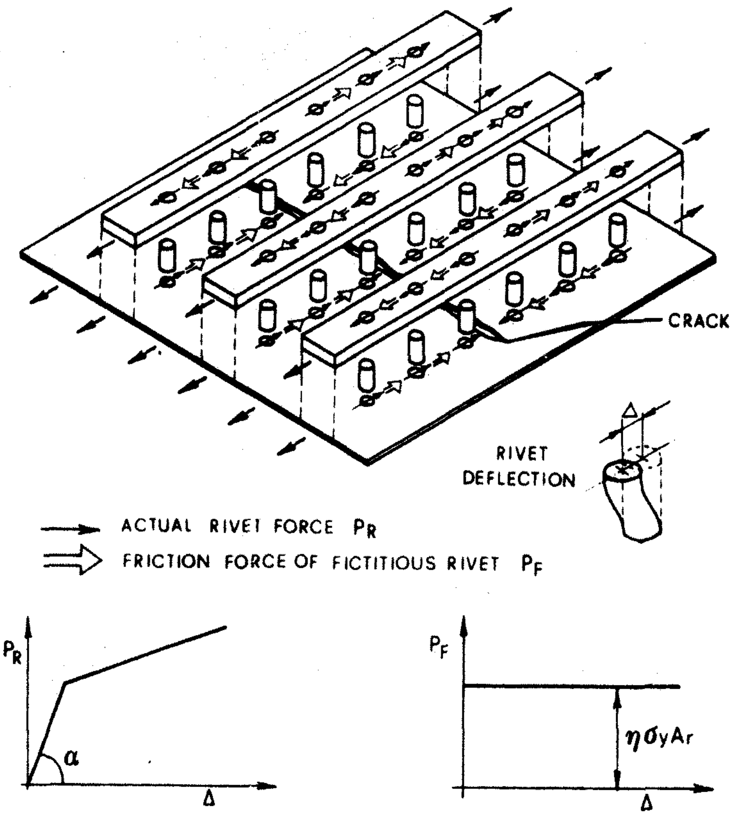


Fig.12 - Minimum weight values for stiffened panels complying with static strength, durability and slow-crack growth constraints.



$$\xi = \frac{E_r\, d}{tg\,\alpha\; F} \quad ; \quad F = 5. + 0.8\, \frac{E_r}{E_s} \cdot \frac{d}{t_s} + 0.8\, \frac{E_r}{E_c} \cdot \frac{d}{t}$$

Fig.A-1a - Idealization of skin-stiffener riveted junction.

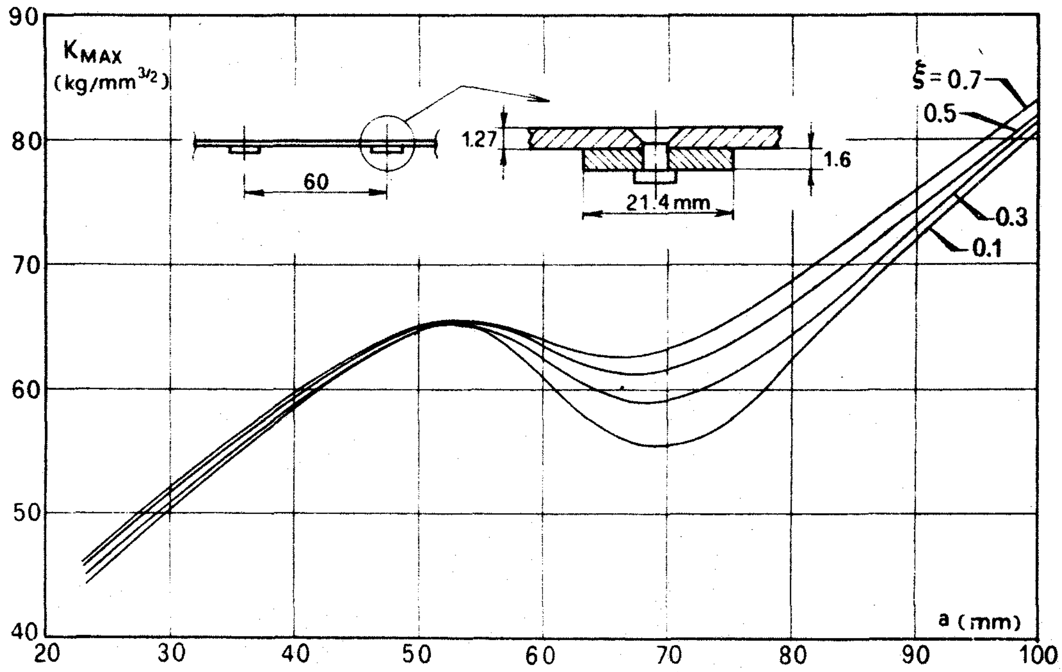


Fig.A-1b - Influence of the rivet flexibility on the stress intensity factor.

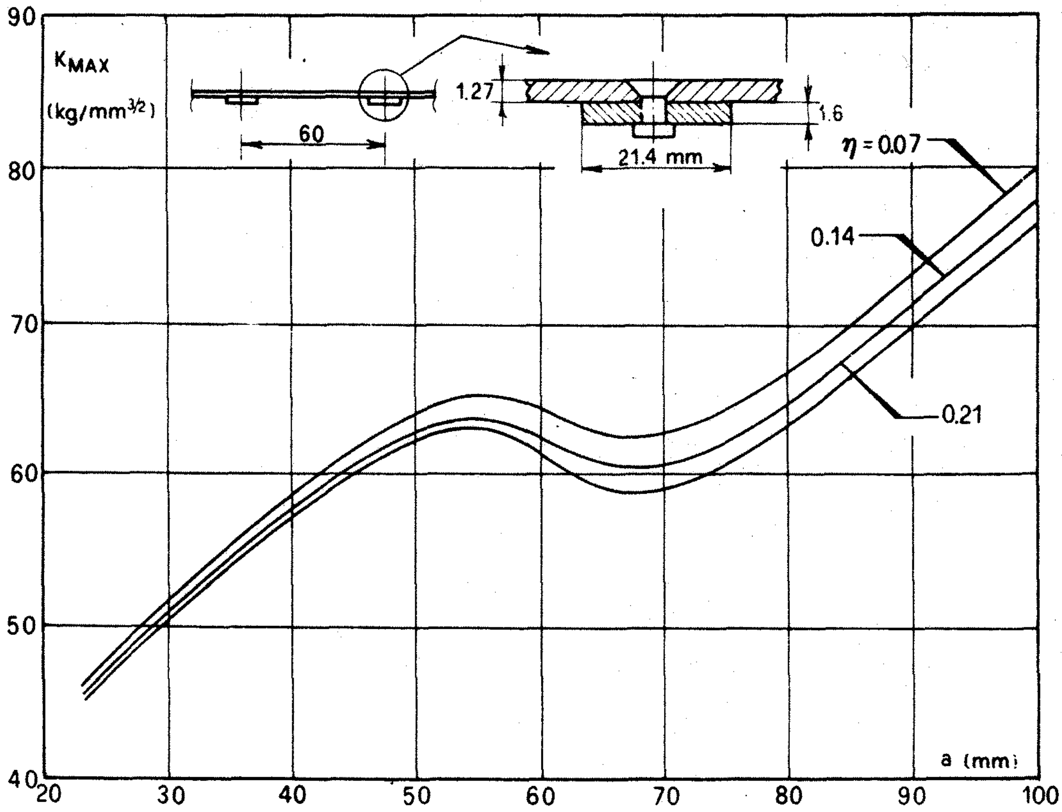


Fig.A-1c - Influence of the friction force on the stress intensity factor.

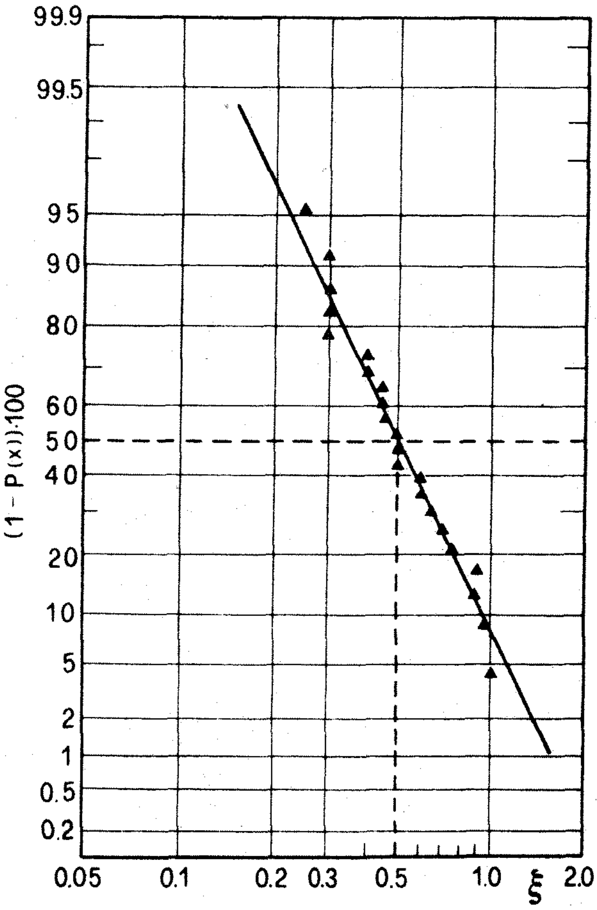


Fig.A-2 - Cumulative probability distribution of rivet flexibility parameter (countersunk rivets hand driven).

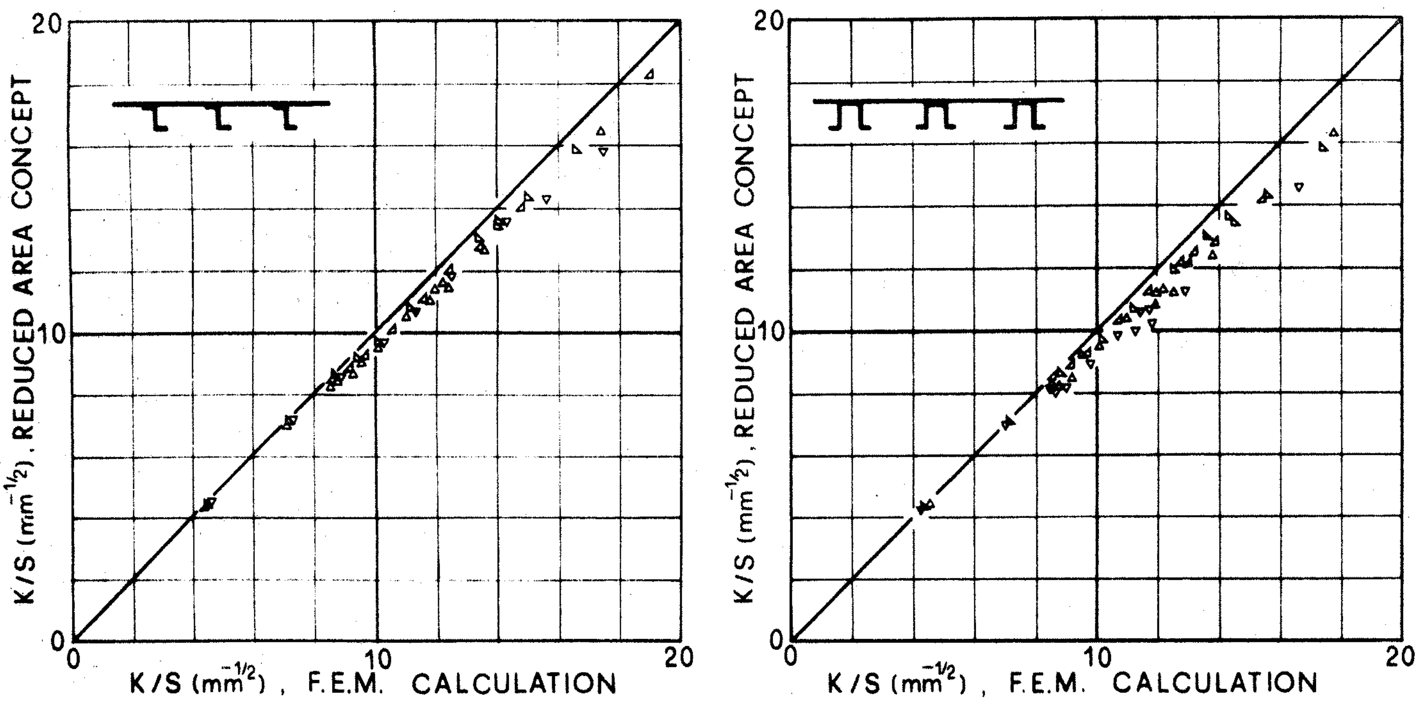


Fig.A-3 - Comparison between the values of the stress intensity factors obtained with a finite Element Method and the reduced area concept.

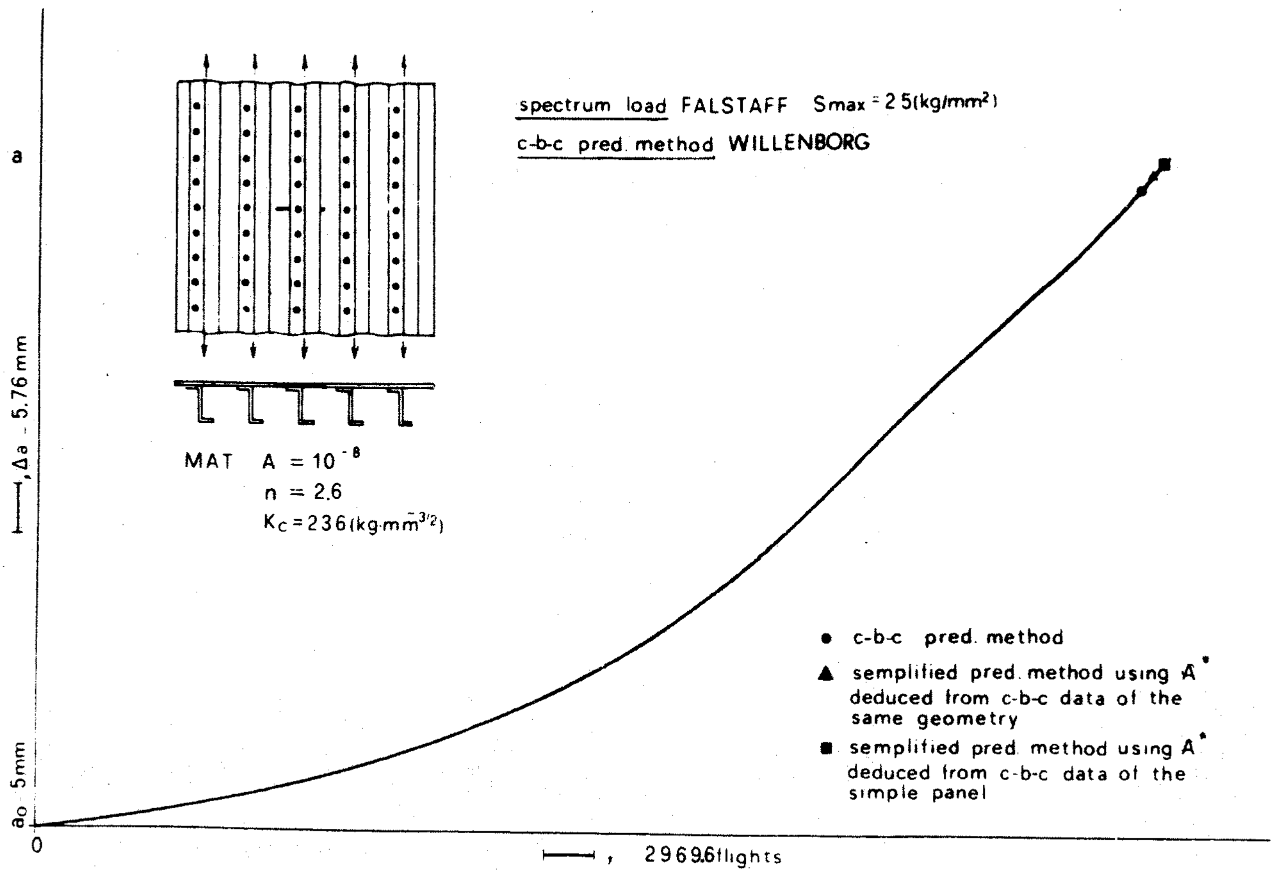


Fig.B-1 - Results of application of the semplified method in crack growth evaluation. FALSTAFF load spectrum.

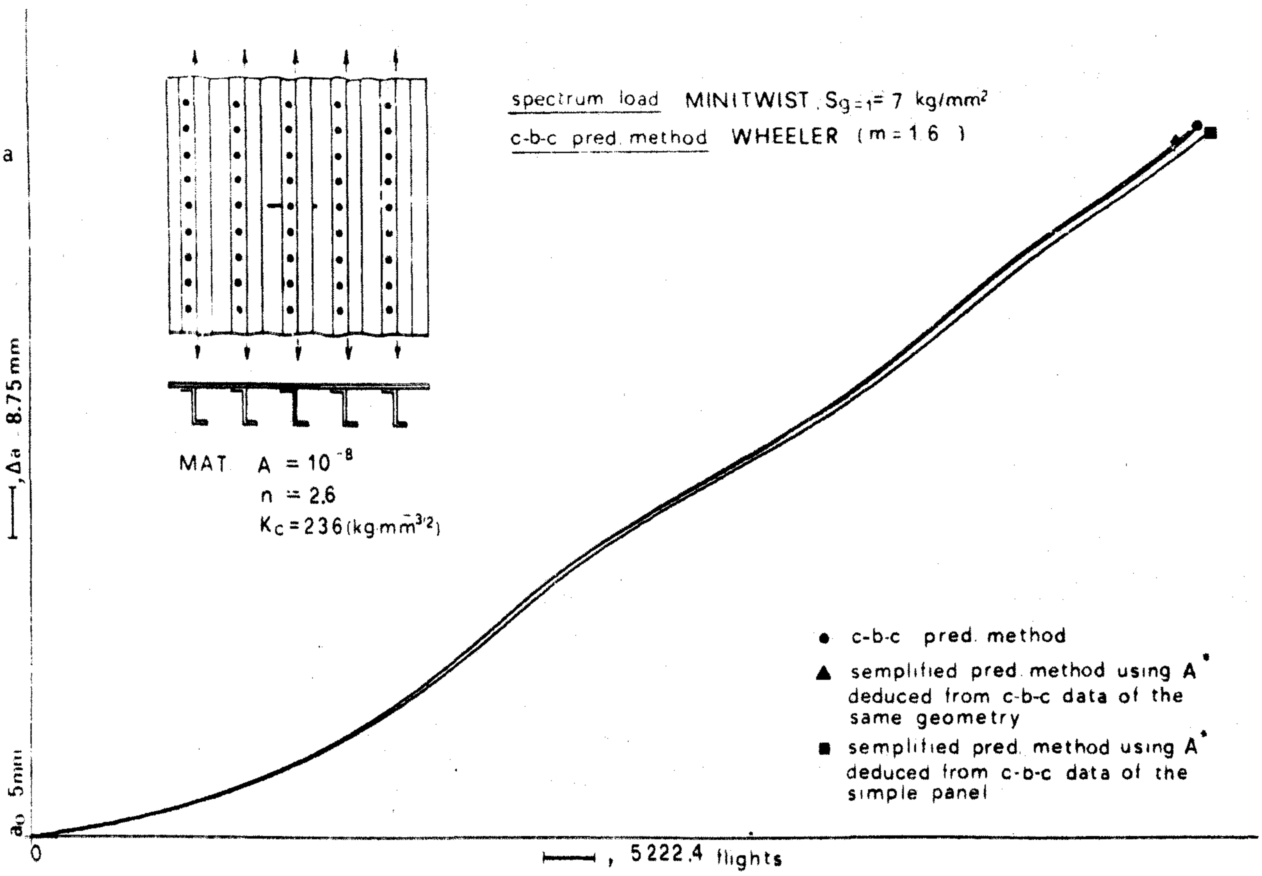


Fig.B-2 - Results of application of the semplified method in crack growth evaluation. MINITWIST load spectrum.

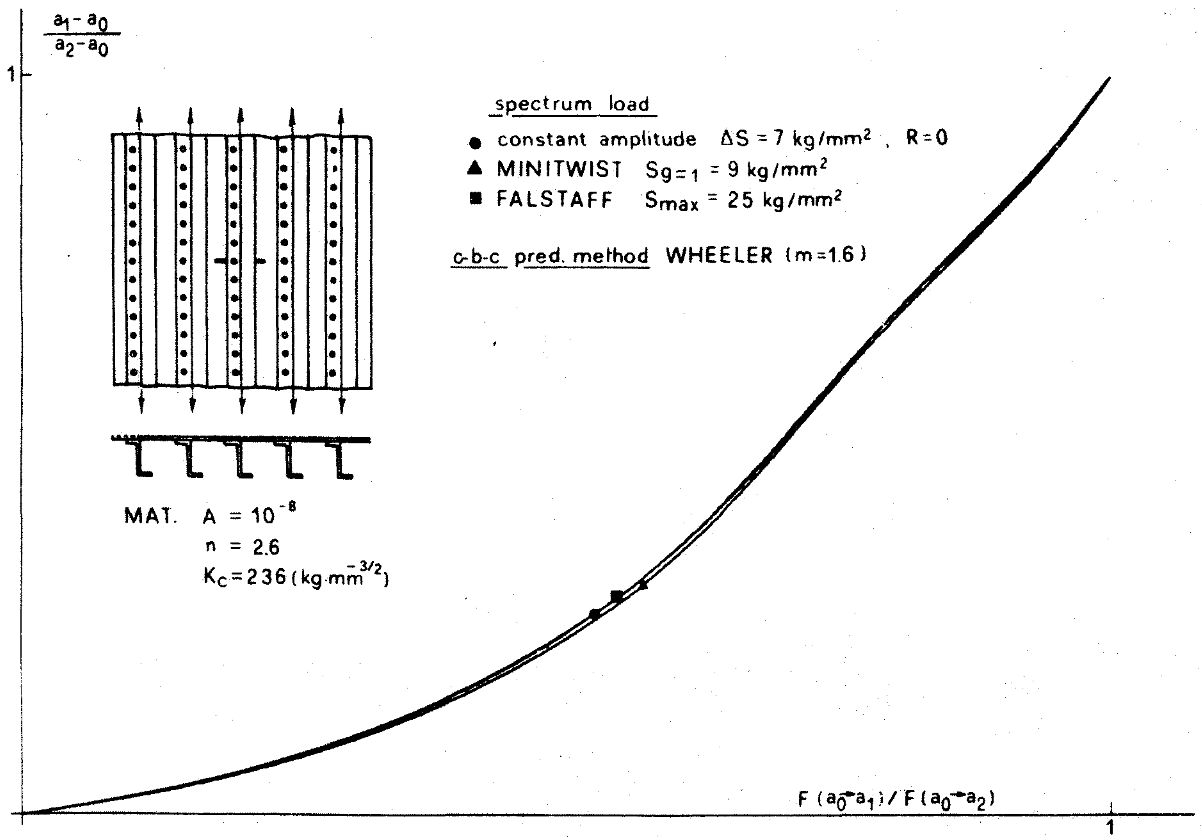


Fig.B-3 - Example of application of normalized curve concept for crack growth evaluation.

CONSTRAINTS	CONSTRAINT FUNCTIONS	DESIGN PARAMETERS	MINIMUM VALUE OF THE AVERAGE THICKNESS
SSC	G_1, G_2, G_3, G_4	N_x, MATERIAL	w_1
SSC + DC	G_1, G_2, G_3, G_4, G_5	N_x, L.G.F., F.F.Q. MATERIAL	w_2
SSC + DC + CAC	G_1,G_2,G_3,G_4,G_5,G_6,G_7,G_8	N_x, L.G.F.	w_{3a}
SSC + DC + SGC	G_1, G_2, G_3, G_4, G_5, G_9	N_x, L.G.F. INSPECTION INTERVAL	w_{3s}

Tab.I - Constraints, Constraint Functions and Design Parameters combination utilized in successive optimization runs.