NEW CONCEPTS FOR DESIGN OF FULLY-OPTIMIZED CONFIGURATIONS FOR FUTURE SUPERSONIC AIRCRAFT

ICAS-80-1.5

Adriana Nastase
Lehrgebiet Aerodynamik des Fluges
Rheinisch-Westfälische Technische Hochschule
Aachen, Germany

Abstract

In this paper the author proves that the drag of a classical supersonic aircraft can be very much diminished if the conventional wing of this supersonic aircraft is replaced with a good suited one in form of a fully-optimized wing (optimum-optimorum wing).

The design of a fully-optimized wing(i.e. such a wing for which all its geometrical characteristics are optimized in order to obtain a minimum drag) leads to the solving of an enlarged variational problem for the drag-functional C_d which contains an unknown function (the surface of the wing) and an unknown boundary (the shape of the planprojection).

An original graphic-analytical method to solve this enlarged variational problem for a given set of wings is presented.

This method was used for the design of a fully-optimized delta wing model (Adela I Aachen), which was realised and tested. The experimental results are in good agreement with the theory. Further this method was used for the design of the fully-optimized wing for Concorde.

It is also possible to use this method for the design of an integrated supersonic aircraft configuration for which the fuselage and the machineries are inside the thickness of the integrated fully-optimized wing.

I. Introduction

Some previous theoretical considerations (1)-(12) in good agreement with experimental results (13) prove that the drag of the supersonic aircraft (S.A.) can be very much diminished (about 20 %) if the conventional wing of S.A. is replaced with a good suited one in form of a fully-optimized wing (called here also optimum-optimorum wing).

According to (2),(3),(4),(5),(6),(9) and (10) a fully-optimized wing is such a wing for which all its geometrical characteristics i.e. its distributions of thickness, camber and twist and also its planprojection are optimized in order to obtain a minimum pressure-drag.

As in (2),(3),(4),(5), and (10) the determination of the shape of the surface of the optimum-optimorum wing leads to an enlarged variational problem for the drag-functional C_d which consists in the simultaneous determination of the equation of the surface of the wing (which enters in the expression of the drag-functional) and of the shape of the planform (which determines its boundary) in order to obtain a minimum for C_d (at a given supersonic cruising Mach number).

In (2),(3),(4),(5) and (10) an original grafic-analytical method is presented for the approach of the solution of this variational problem for a given set of wings defined through the following properties: -their surfaces are expressed (or appro-

ached) in form of the superposition of polynomes (in two variables);
- their planprojections are polygones;
- all wings satisfy the same imposed auxiliary condition.

The optimal values of the coefficients of the polynomes and of the similarity parameters of the polygones are determined with the help of the lower-limit hypersurface (Fig.1) which is firstly introduced in[2] (for the set of delta wings) and in [5] (for the set of polygonal wings).

Each point of this hypersurface can be analytically determined through the solving of a classical variational problem as for example in [14],[15] and[16].

The position of the minimum of this hypersurface can be graphically (numerically) determined and the corresponding optimal wing for this point of the hypersurface is just the optimum-optimorum wing (of the given set of wing).

The graphic-analytical method was used in [2],[3],[4] and [10] for the effective design of an optimum-optimorum delta wing for the S.A..

For the effective design of this delta wing the results of the high conical flow theory [17] of P. Germain are used.

Further, if the hydrodynamic analogy of E. Carafoli[18],[19] is applied and the minimal singularity principle[20],[21] is taken into account then the exact solution of the boundary problem of the axial disturbance velocity u of the three-dimensional linearised supersonic flow can be written in a closed form as in [19],[1], [2] and [10] by using of singularities which are placed only along the leading edges of the wing (and, eventually, along some ridges of the wing).

The expression of the axial disturbance velocity u which satisfies the principle of the minimal singularity fulfils the asymptotic behaviour of the solution in the neighbourhood of the leading edges and also the matching condition with the boundary layer solution of the wing (in the first approximation).

This exact solution of u satisfies also the linearised boundary conditions at the infinity, of the planprojection (of the wing surface) and of the Mach cone of the apex of the wing.

The solution of u has been written in closed form in [19],[1],[2] for the flattened triangular wings at small angles of attack if the downwash w is piecewise approximated in form of suoerposition of homogeneous polynomes.

A comparison between the theoretical distribution of the pressure coefficients C_p determined with the help of the present theory and the pressure measured on several holes on the surface of two different models (The optimum- optimorum delta wing model Adela I Aachen and the wave rider *)) in two different supersonic wind tunnels shows a very good agreement (for the range of Mach numbers for which the delta wings have subsonic leading edges). This comparison between theoretical curves and experimental points has been done in [13] and [22].

The optimum- optimorum model Adela I Aachen for which the exact solution for u is used and which is designed with the help of the graphic-analytical method presents the following advantages: reduced drag, increased lift, stabilisation of the boundary layer along the leading and trailing edges, increased ground effect etc. For this reasons the author proposed it for the S.A. of the second generation.

---

*) The author thanks the DFG for the financial support for the experiments in Aachen and the Prof. Ganzer, U. and Dipl. Ing. Hoder, H. of TU- Berlin for their experimental results.

II. The general theory of fully-optimized (optimum-optimorum) wings

According to [1], [14], [15] and [16] a fully-optimized wing (called also optimum-optimorum wing) is such a wing for which all its geometrical characteristics i.e. its distributions of thickness, camber and twist and also its planprojection are optimized in order to obtain a minimum pressure-drag (at the cruising Mach number). The wing must also satisfy some geometrical and aerodynamical auxiliary conditions.

The design of an optimum-optimorum delta wing for the S.A. leads to an enlarged variational problem which consists in the simultaneous determination of the equation of the surface which represents the unknown function entering in the expression of the drag-functional C_d and the contour of the planform which represents its boundary, in such a way that the drag-functional C_d attains its minimum.

In order to solve this enlarged variational problem an original graphic-analytical method is proposed in [2],[3],[4],[10],[11] which allows the approach of the solution by using wings which have:

-their planforms generated by polygones in order to have a finite number of free similarity parameters $(\nu_1, \nu_2, \ldots, \nu_n)$ and

-their surface piecewise generated by finite polynomes expansions in order to have a finite number of free parameters (i.e. the coefficients of these polynomes).

This set of lifting systems is divided into classes. Two of these wings belong to the same class if:

- their planforms are polygones which can be related through an affine transformation. It follows that both planforms are defined by the same number of similarity parameters with the same significance;

-their surface-expansions contain homogeneous polynomes of the same degree.

- both wings satisfy the same auxiliary conditions.

The first step of approach consists in constructing the solution for one class. For a given set of similarity parameters $(\nu_1, \nu_2, \ldots, \nu_n)$ and an arbitrarily chosen area of the planprojection (S_0) the planform is determined and the proposed variational problem with auxiliary conditions reduces to a classical one (with given boundary for the drag-functional) which can be solved analytically in the usual way as in [1],[14],[15],[16]. The solution is the optimal drag-functional $(C_d)_{opt}$ for the prescribed planprojection.

Through systematical variation of the set of similarity parameters a, what is termed here a lower-limit hypersurface of the drag-functional i.e.

$$(C_d)_{opt} = f(\nu_1, \nu_2, \ldots, \nu_n) , \qquad (1)$$

can be analytically determined.

The "position" of the minimum of the hypersurface gives the best set of similarity parameters $(\nu_1, \nu_2, \ldots, \nu_n)$ of the planform (Fig. 1).

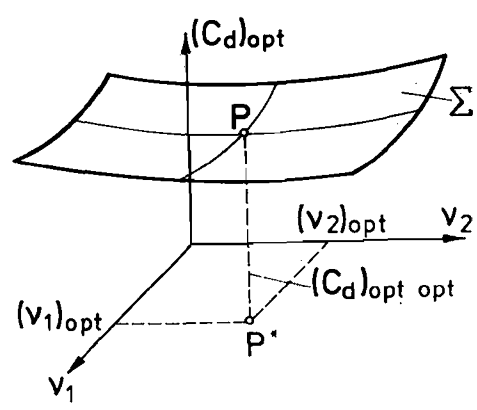

Figure 1. The lower-limit hypersurface (Σ) for two similarity parameters

This set of similarity parameters determines together with a chosen area (S_0) of the planprojection the shape of the planform of the optimum-optimorum wing of the class. The optimum-optimorum wing is exactly the optimized wing corresponding to this set of similarity parameters. The set of coefficients of the surface-expansion of this optimized wing represents in the same time the set of the optimal coefficients of the optimum-optimorum wing of the class.

The minimum value of the "ordinate" of the hypersurface represents the drag coefficient of the optimum-optimorum wing of the class.

By comparison of the minimum of the optimized drag coefficients of various classes the lowest drag coefficient of all classes can be found and the corresponding wing is the optimum-optimorum wing of all classes taken into consideration.

III. New concepts for the effective design of the optimum-optimorum delta wing

The above theory can be used for the effective design of the optimum-optimorum delta wing (of a class).

The delta wing must satisfy some auxiliary conditions i.e. to have a given lift coefficient C_1, pitching moment coefficient C_m and relative thickess τ.

Additionally the wing must be of null-thickess along its leading and trailing edges and, at cruising Mach number, an equalization of pressure along the subsonic leading edges is required in order to avoid the birth of vortex sheets along these leading edges.

For the cancellation of vortices along the entire subsonic leading edges according to [1] - [15] it is necessary that the planprojections of these leading edges must be straight-lines, i.e. the wing must be delta (i.e. neither gothic nor ogee) and the wing shall be cambered and twisted (i.e. the mean surface of the wing does not be plane). It is also necessary that (at cruising Mach number) the parallel flow has the direction of the tangent to the skeleton of the central section at the apex of the wing (i.e. the flow must have shockless entry) and the camber and twist distribution must be correctly coupled i.e. must satisfy some supplementary auxiliary conditions, as given in [1]-[14].

Let us here consider a class of delta wings which fulfil the following properties:

- the shape of their surfaces is approximated in form of superposition of some homogeneous polynoms in two variables with undetermined coefficients;
- their planprojections are isosceles triangles (with undetermined aspect-ratio and a free chosen area S_0);
- all wings of the class fulfil the same auxiliary conditions (mentioned above).

For this class of delta wings the plan form depends only of one similarity parameter $\nu = B\ell$ (here is $B = \sqrt{M_\infty^2 - 1}$, $\ell = \ell_1/h_1$ with ℓ_1, the half span and h_1 the maximal depth of the delta wing) and the lower-limit hypersurface reduces to a curve which is here called <u>lower-limit line</u> (Fig. 2) i.e.

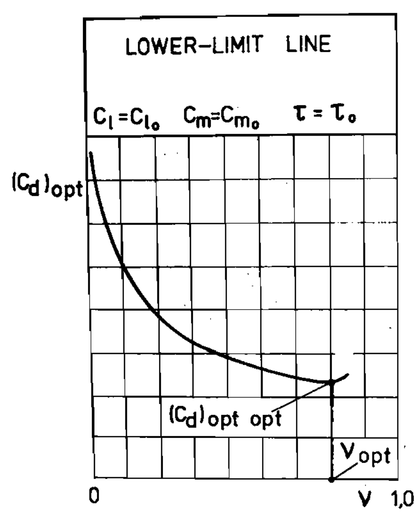

Figure 2. The lower-limit line.

$$(C_d)_{opt} = f(\nu) \qquad (2)$$

For a given ν the corresponding $(C_d)_{opt}$ which represents, the drag of the optimal wing for a given value of the similarity parameter ν, can be analytically determined as in [(1),(14),(16)] by the solving of a classical variational problem. The position $\nu = \nu_{opt}$ of the minimum is graphically (numerically) obtained. The optimum-optimorum delta wing is exactly the optimized delta wing corresponding to this value of ν.

Using this method a systematical analysis of the influence of auxiliary conditions of the variational problem (i.e. M_∞, C_ℓ, C_m and τ) on the shape and drag of about 1000 fully-optimized delta wings has been done in [(6),(7),(8)] and [(12)].

The analysis indicates that all the fully-optimized delta wings have a convex shape in the neighbourhood of the apex and take a wave shape in the neighbourhood of the trailing edge. All wings are angular along the leading edges and sharp along the trailing edge (as in Fig. 3) i.e.

The maximal thickness is located in the central section at the distance $x_1 \cong h_1/4$ from the apex.

In the vicinity of the trailing edge the transversal sections are narrower as in the vicinity of the central section and the maximal thickness in this transversal section is reached in two lateral points, which are symmetrically placed with respect to the central section.

The magnitude of the pressure drag is strongly influenced from the correct choice of:
- the angle of aperture γ (of the mean profile-section at the apex of the wing); (Fig. 4) and (Fig. 5).

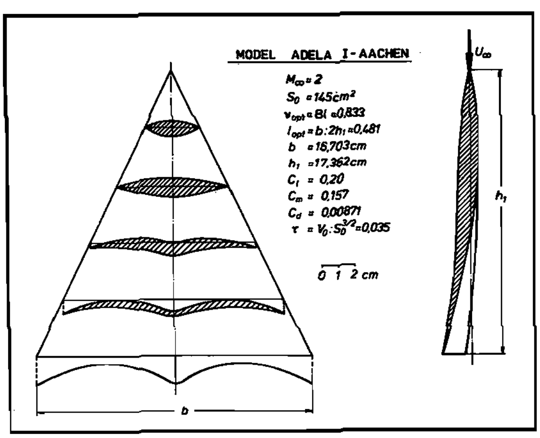

Figure 3. The fully-optimized delta wing Model Adela I Aachen

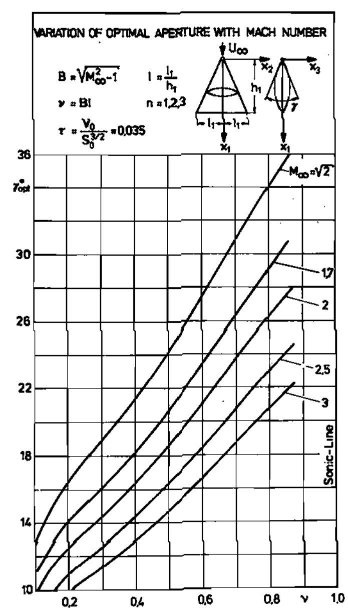

Figure 4. The dependence of γ_{opt} versus M_∞ (by τ = 0,035)

-the aspect-ratio ℓ (Fig.6).

In the (Fig. 4) and (Fig. 5) the dependence of the optimal aperture (γ_{opt}) versus the Mach number M_∞ (by τ =0,035) and the relative thickness (by M_∞ = 2) is presented.

The optimal aperture increases when the cruising Mach number M_∞ decreases and the thickness τ increases (for given $\nu = \nu_{opt}$) thickness is neglected).It results the following range for ν_{opt} :

$$0,75 < \nu_{opt} < 0,85 \qquad (3)$$

The pressure drag decreases when the distance k of the pressure center (with respect of the apex of the wing) increases.

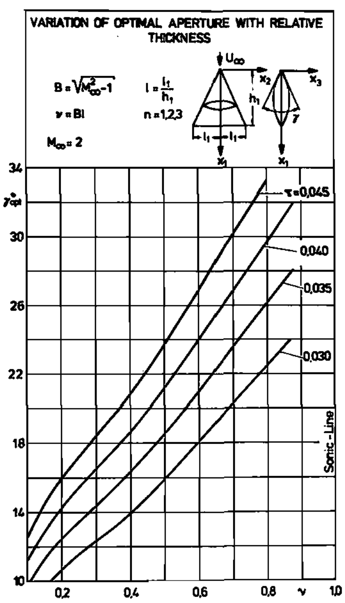

Figure 5. The dependence of γ_{opt} versus τ (by M_∞ = 2)

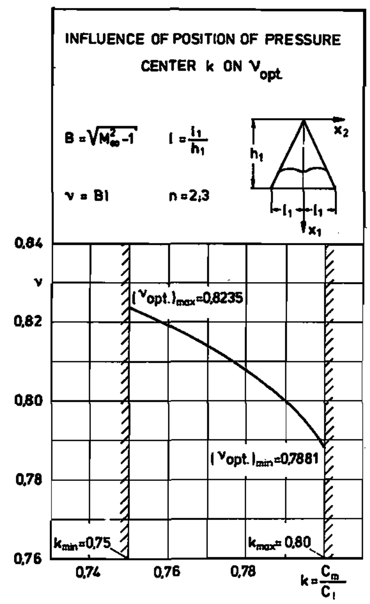

Figure 6. Influence of pressure center k on ν_{opt} .

In the (Fig. 6)is given the dependence of the optimal value ν_{opt} of the similarity parameter versus the pressure center $k = C_m / C_\ell$. (Hereby the influence of

The extensive analysis of the influence of the cruising Mach number M_∞ , the relative thickness τ the lift coefficient C_ℓ and the pitching moment coefficient C_m on the drag coefficient C_d shows the importance of the correct choice of the angle $\gamma°$, the position k of pressure center and of the similarity parameter ν of the plan-projection.

IV. Design of a fully-optimized Model Adela I Aachen

The present theory was used in (5),(6) (9) and (10) for the design of a fully-optimized delta wing model given above (Fig. 3), called Adela I Aachen. This wing was fully-optimized for the cruising Mach number $M_\infty = 2$.

For this wing:

$$S_0 = 145 \text{ cm}^2 \tag{3}$$

$$\tau = 0,035 \tag{4}$$

$$C_\ell = 0,2 \quad (\text{by } M_\infty = 2 \text{ and } \alpha = 0°) \tag{5}$$

$$C_m = 0,157 \quad (\text{" } \quad \text{" } = \text{" } \quad \text{" } \quad \text{" }) \tag{6}$$

Additionally the delta wing must be of null-thickness along the leading and trailing edges and the axial disturbance velocity u (of the thin component of the thick-lifting delta wing) must be cancelled along the subsonic leading edges of the wing (at cruising Mach number).

In (Fig. 7) a perspective view of the optimum-optimorum delta wing model Adela I-Aachen is given.

For flattened delta wing, at small angle of attack, the downwash w is related to the geometry of the wing in the following form

$$w_a = \frac{w}{U_\infty} = \frac{\partial Z}{\partial x_1} \tag{7}$$

Here is w_a the dimensionless down wash, U_∞ is the velocity of the undisturbed flow and Z is the equation of the

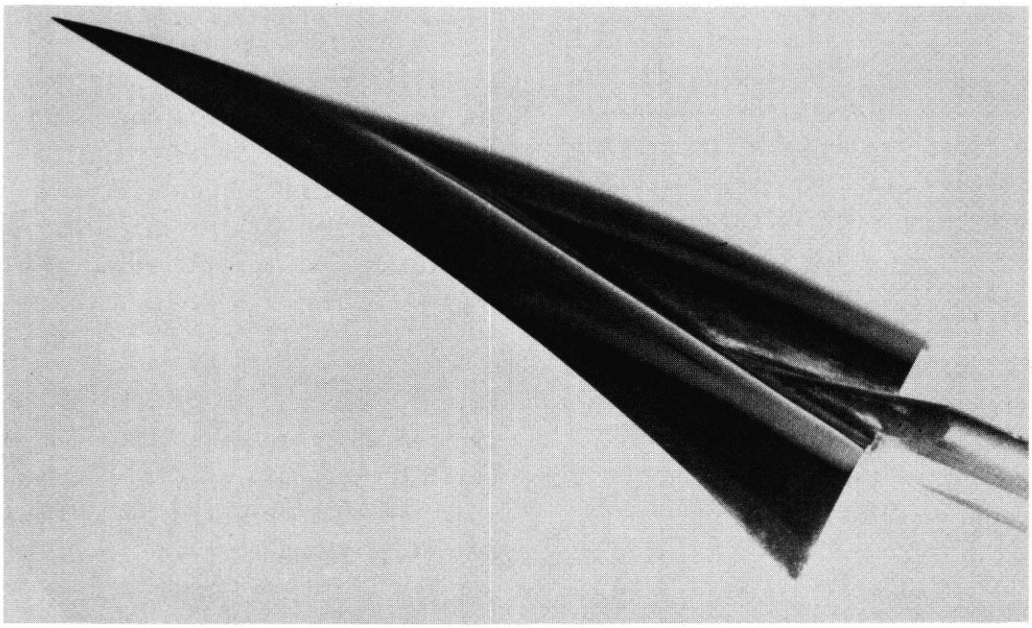

Figure 7. Perspective view of the optimum-optimorum delta wing model Adela I- Aachen

wing surface.

The pressure coefficient C_p is proportional to the axial disturbance velocity u

$$C_p = - \frac{2u}{U_\infty} \tag{8}$$

The thin and the thick-symmetrical wing components of the thick lifting delta wing are considered separately as in (1)- (16) and (21).

For the thin component the dimensionless downwash w_a is expressed under the form of superposition of homogeneous polynomes of order n = 2; 3 i.e.

$$w_a = \frac{w}{U_\infty} = x_1 (w_{10} + w_{01} |y|) + x_1^2 (w_{20} + w_{11} |y| + w_{02} y^2) \tag{9}$$

and for the thick-symmetrical delta wing component the corresponding dimensionless downwash $w_a{}^*$ is represented under the form of superposition of homogeneous polynomes of order n = 1; 2; 3 i.e.

$$w_a{}^* = \frac{w^*}{U_\infty} = w^*_{00} + x_1 (w^*_{10} + w^*_{01} |y|) + x_1^2 (w^*_{20} + w^*_{11} |y| + w^*_{02} y^2) \tag{10}$$

The design of this optimum-optimorum delta wing model leads to the determination of the unknown similarity parameter ν and of the unknown coefficients w_{ij} and w^*_{ij} in order to obtain a minimum drag. The wing must satisfy in addition the auxiliary conditions (4), (5) and (6). The wing must also be of null-thickness at the leading and trailing edges and the axial disturbance velocity u (due to the thin component) must be cancelled along the leading edges (at $M_\infty = 2$).

In the (Fig. 8) the lower-limit lines are plotted separately for the thin, the thick-symmetrical delta wing (component wings of the thick lifting delta wing model) and for the thick lifting delta wing. The lower-limit line of the thick lifting delta wing is to be obtained through the addition of both lower-limit lines of the component wings.

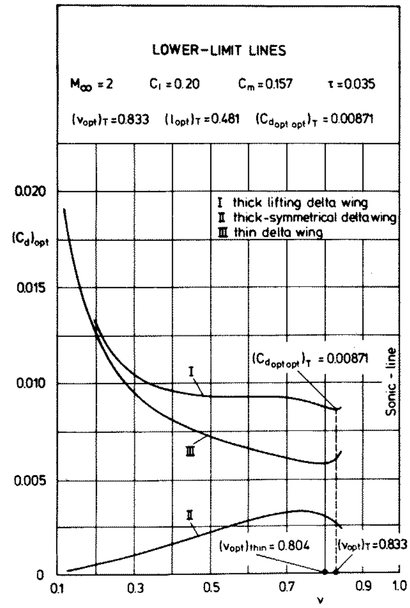

Figure 8. The lower-limit lines of the optimum-optimorum delta wing model Adela I Aachen and for its component wings.

The position of the minimum of the lower-limit line of the thick lifting wing determines the optimal value of the similarity parameter $\nu = \nu_{opt}$ i.e.

$$\nu_{opt} = 0,833 \tag{11}$$

and the optimal thick lifting delta wing which corresponds to this value of ν_{opt} is just the optimum-optimorum delta wing model Adela I Aachen.

The theory predicted that the here proposed fully-optimized delta wing has half frictionless drag as an equivalent planar delta wing which is flying at the same cruising speed and has the same planprojection and the same lift.

That is the reason for which I proposed it for the S.A. of second generation.

V. The agreement between theory and experimental results

The model Adela I Aachen was tested in two supersonic wind tunnels i.e.
- the supersonic wind tunnel of the Aerodynamisches Institut of RWTH-Aachen *) (test section 40 x 40 cm^2);
- the supersonic wind tunnel of DFVLR-Institut für Experimentelle Strömungsmechanik (Test section 60 x 60 cm^2).

This model has 17 holes which allowed the measurement of the pressure distribution on its surface.

The pressure measurements have been done in the supersonic wind tunnel of the Aerodynamisches Institut of RWTH-Aachen.

A very good agreement between the theoretical distribution of the pressure coefficients C_p determined with the help of present theory and the pressure coefficients measured on the surface of the model (see Fig. 9) is obtained over the entire surface of the wing and for whole range of Mach numbers for which the delta wing has subsonic leading edges, i.e. in this case the Mach number of the supersonic flow doesn't exceed the following upper limit

$$M_\infty < 2,307 \qquad (12)$$

*) Experiments done with the amiability of Prof. Egon Krause, Ph.D., Director of this Institute, together with Ing. A. Scheich and Techn. L.Simons and a group of students (H.Reimerdes, G.Bozinis and P. Bürger).

**) Experiments done with the amiability of Prof. Dr.-Ing. A.Heyser, and Dr.-Ing. G.Maurer, together with Dipl.-Ing. Esch, Dipl.-Ing. H.Riedels and Ing. H.Emunds.

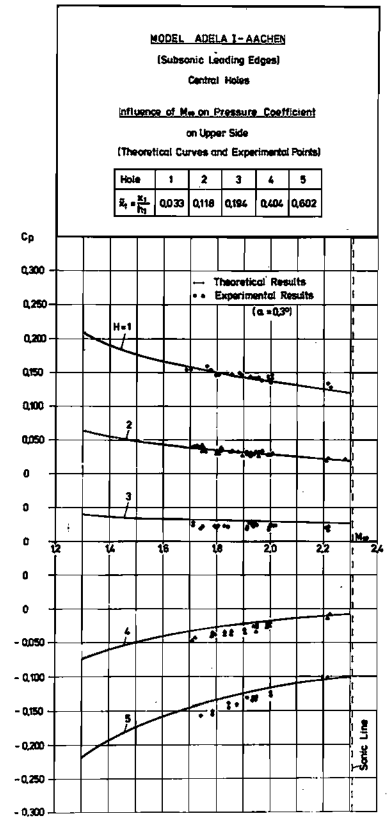

Figure 9 . Variation of C_p versus M_∞ for the central holes

The experimental results concerning the variation of lift coefficient and pitching moment coefficient with Mach number and incidence have been done in supersonic wind tunnel of the Institut für Experimentelle Strömungsmechanik.

The comparison between the theoretical aerodynamic characteristics (i.e. lift- and pitching moment coefficient) determined with the help of present theory and the aerodynamic characteristics measured on the model (with the strain gauge balance) are also in good agreement with each other.

The hypothesis, that there are no flow detachments along the trailing and leading edges of the fully-optimized delta wing model (at cruising Mach number M_∞ = 2 and for the range of the angle of attack α

($-2 < \alpha < 2$), is also experimentally confirmed with the help of oil pictures.

In conclusion, there is a good agreement between the theoretical results obtained by using the high conical flow theory and the experimental results for the whole range of Mach numbers for which the delta wing has subsonic leading edges.

VI. New concepts for the design of second generation of fully-optimized supersonic aircraft (S.A.)

The principal conclusion of this paper is that the rentability of a S.A. can be achieved if the classical gothic wing of the S.A. is replaced with a good suited wing i.e. with a fully-optimized delta wing. Such a replacement has been done in (Fig. 10) for the S.A. of type Concorde.

The Concorde with fully-optimized delta wing has the same fuselage and the same wing area as the classical Concorde.

The wing was fully-optimized for the same cruising Mach number.

A geometrical comparison between the both wings provides the following differences

- it is delta shaped (i.e. the projections of the leading edges on the planform are straight lines) and not gothic;
- it has angular leading edges and not rounded leading edges;
- the mean surface is not plane but cambered and twisted;
- all geometrical parameters of the new wing (camber, twist, thickness, planform) are optimized in order to obtain minimum drag (at the same cruising Mach number).

The Concorde with fully-optimized delta wing presents the following aerodynamical advantages:

- its pressure drag is reduced because the wing is fully-optimized (at cruising speed); it results a reduced fuel-consumption

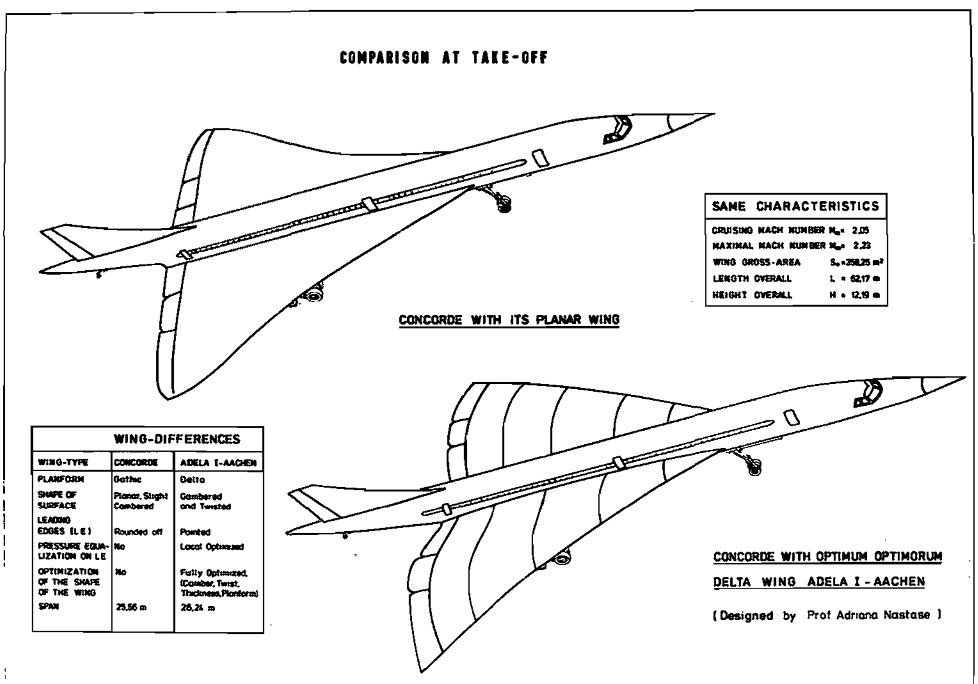

Figure 10. Comparison between a classical Concorde and the Concorde with fully-optimized delta wing, by start

(for the same trajectory,by the same cruising speed);
- its lift is increased (at the same cruising speed and for the same wing area) because of the condition of pressure equalisation along the leading edges (at cruising speed); it results in an increased possibility of transport of payload;
- its lift is also increased at all regimes of flight because of its twist, therefore this S.A. needs shorter run-ways for take-off and landing;
- its induced drag is cancelled (at cruising Mach number) and is reduced at all other regimes of flight because of the twist of the wing.
- its twist produces a lateral stream which transports additional central energy in the central part of the back region of the upper side of the wing and therefore a stabilization of the boundary layer in this region is achieved; the detachment at the trailing edge is retarded. It results in an increased lift and a supplementary reduction of the drag.

A better concept for the design of S.A. of second generation is to optimized the whole configuration of the aircraft i.e. wing-fuselage-machineries and to design an integrated fully-optimized wing which contains inside its thickness the fuselage and the machineries.

The mean surface of the integrated wing looks like the fully-optimized isolated wing, but its thickness-distribution must be modified in order to allow the introduction of the fuselage and the machineries inside its thickness.

Literature

(1) NASTASE, A.: Forme aerodinamice optime prin metoda variationala (Optimum Aerodynamic Shapes by Means of Variational Method). Ed. Acad. R.S. Romania Bucarest 1969, Chap. IV.

(2) NASTASE, A.: Utilizarea calculatoarelor in optimizarea formelor aerodinamice (Use of Computers in the Optimization of Aerodynamic Shapes). Ed. Acad. R.S. Romania Bucarest 1973, Chap. I-III, VI.

(3) NASTASE, A.: Eine graphisch-analytische Methode zur Bestimmung der Optimum-optimorum Form dünner Deltaflügel in Überschallströmung. RRST-SMA+) 1, 19 (1974) (Bucarest).

(4) NASTASE, A.: Eine graphisch-analytische Methode zur Bestimmung der Optimum-optimorum Form symmetrisch-dicker Deltaflügel in Überschallströmung. RRST-SMA 2, 19 (1974) (Bucarest).

(5) NASTASE, A.: Die Theorie des Optimum-optimorum Tragflügels im Überschall. ZAMM 57 (1977).

(6) NASTASE, A.: Einfluß der Anfangswerte der Varitationsaufgabe auf die Geometrie und den Widerstandsbeiwert des Optimum-optimorum Deltaflügels in Überschallströmung. ZAMM 56 (1976).

(7) NASTASE, A.: Einfluß der Anfangswerte auf die Wölbung, Verwindung, Grundrißform und Wellenwiderstandsbeiwert des Optimum-optimorum dünnen Deltaflügels in Überschallströmung. Abh. Aerodynamisches Institut Aachen, Heft 22(1975).

(8) NASTASE, A.: Einfluß der Anfangswerte auf die Flügelform und Grenzkurve symmetrisch dicker Deltaflügel in Überschallströmung. Abh. Aerodynamisches Institut Aachen, Heft 21 (1974).

(9) NASTASE, A.: Modern Concepts for Design of Delta Wings for Supersonic Aircraft of Second Generation ZAMM 59 (1979).

(10) NASTASE, A.: Optimierte Tragflügelformen in Überschallströmung. Braun Verlag Karlsruhe 1981. Chap. V, VI.

(11) NASTASE, A.: Contribution a l'etude des formes aerodynamiques optimales Faculté des Sciences de Paris en Sorbonne, 1970, 150 S.

(12) NASTASE, A.: Analytische und numerische Untersuchungen für Optimum-opti-

morum Modelle von Deltaflügeln in Überschallströmung DFVLR-Braunschweig. Int. Bericht 74/15, 1974.

(13) NASTASE, A.: Agreement of High Conical Flow Theory with Experimental Results for Supersonic Flow. ZAMM 60 (1980).

(14) CARAFOLI, E. and NASTASE, A.: Minimum Drag Thin Delta Wing with Finite Velocity of the Leading Edges in Supersonic Flow. RRST-SMA Nr. 5, Vol. 12, (1967).

(15) CARAFOLI, E. and NASTASE, A.: Minimum Drag Thin Triangular Wing in Supersonic Flow. RMA Nr. 5, Vol. 5 , (1960).

(16) CARAFOLI, E. and NASTASE, A. : On the Minimum Drag Problem of the Triangular Wing of Symmetrical-Thickness in Supersonic Flow. RMA, Nr. 2, Vol. 7, (1962).

(17) GERMAIN, P.: La théorie des mouvements homogenes et son application au calcul de certaines ailes delta en régime supersonique,Rech. Aero., 7, 3-16 (1949).

(18) CARAFOLI, E.: Asupra caracterului hidrodinamic al solutiilor in miscarile conice aplicate la teoria aripilor poligonale. Com. Acad. RPR, II, 9-10 (1952).

(19) CARAFOLI, E., MATEESCU, D. and NASTASE, A.: Wing Theory in Supersonic Flow. Pergamon Press, London, 1979, Chap. 4.

(20) VAN DYKE, M.:Perturbation Methods in Fluid Mechanics. New York,Academic Press, 1964.

(21) NASTASE, A.: L'étude du comportement asymptotique des vitesses axiales de perturbation aux voisinage des singularités. RRST-SMA, Nr. 4 , Vol. 17, (1972)

(22) NASTASE, A., STAHL, H.: Use of High Conical Flow Theory for the Determination of the Pressure Distribution on the Wave Rider and its Agreement with Experimental Results,for Supersonic Flow. ZAMM 61 (1981).