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THEORETICAL AND EXPERIMENTAL FEATURES AND
METHODS OF CREEP

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THEORETICAL AND EXPERIMENTAL FEATURES AND METHODS OF CREEP

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Abstract

Problems of Aeroelasticity involving temperature effects require the knowledge of creep constitutive law. However, such law should be studied in the same conditions as the real phenomenon will occur. Experimental methods to determine dynamic damping are exposed, and creep tests conducted on wing torsion boxes are described.

1. - Introduction

At the Institute for Aerospace Technology, University of Rome, we have been concerned with creep and viscoelasticity mainly in connection with a theoretical and experimental program in aeroelasticity we started with some years ago.

A careful review of the existing literature revealed that there is a considerable scattering not only in results, but also in current ideas, so that even the formulation of problems of structural dynamics is sometimes questionable. Therefore we gradually reached the idea that experimental data referring to a given condition could not be applied to other conditions, as is done, f.i.,with elastic coefficients. In other words, creep or viscoelastic data to be used in dynamic aeroelasticity, must be obtained in experimental dynamic conditions. We developed therefore some special testing procedures that I will describe later. I simply wish to stress the point that works are still in progress (we may better say to be just at the beginning), and that our results by no means are to substitue the very important ones recently obtained in the field of general rheology they are to be viewed as an help to specific structural dynamic problems of aerospace interest.

2. - Effect of viscoelasticity on aeroelasticity

Let us consider the very simple case of a "purely viscous" beam,for which the relationship between curvature and bending moment is

$$(1) \qquad \frac{F}{EI} = \frac{\partial^2 w}{\partial x^2} + \varepsilon \frac{\partial}{\partial t} \frac{\partial^2 w}{\partial x^2}$$

very similar to the simplest model of viscoelasticity, the Voigt's model. If we insert the law (1) into the general equations of aeroelasticity for the beam, we obtain the well-known critical dynamic pressure, for the flutter of a simply supported panel of infinite with, as a function of ε (Fig. 1 represents the values of the damping of the j the mode (j=1,2) vs. dynamic pressure). I already presented this results at the ICAS meeting in Rome two year ago: it is seen that even a slight viscous damping can dangerously anticipate instability.

Seff.i.[1][2][3][4][5]

If now we go supercritical, we obtain the amplitude of the limiting cycle again as a function of ε (Fig. 2);for sake of comparison, such amplitudes are given in terms of the nondimensional overpressure referred to the respective critical value.It is an interesting result that such nondimensional overpressure gives an excellent correlation of the results even for impossibly large values of ε; this means that a good knowledge of the viscoelastic properties of the material is essential in determining critical speed, while normalized postcritical behavior is almost independent of it.

3. - Measurements on viscoelastic damping

Our first step in the experimental determination of viscoelastic damping was a measurement of the damping in vibrational motion of a cantilever bar.

A general setup of the measuring device is shown in Fig. 3. As is seen, this is a very conventional setup, but some special experimental problems were to be solved, of account of the fact that very small quantities (logarithmic decrement) were to be determined (For such details, see Ref. [6]). Figs. 4,5 provide further details of the experimental setup. The temperature effects were controlled by means of the thermostate oven,Fig. 6.

We confine ourselves to give and comment experimental results obtained with steel giving damping in terms of amplitudes (Fig. 7) and of frequency (Fig. 8). For theoretical detaits, see Appendix I; it is seen, however, that damping should be proportional to the square of the eigenfrequency, and constant with amplitude. For reasonably small amplitudes such last condition is fulfilled; whereas a large scattering occurs in the first. It seems, however, that it is possible to conclude that, for the steel considered by us in the range of frequency considered, roughly one can set in E.q. (1), $\varepsilon=3\times10^{-8}$ sec, which for a frequency of 150 Hz corresponds to a damping of 0,03 $\sec^{-1}$ ($\simeq$0,3 in db/sec). No fundamental variations occurred with temperature.

4. - Hereditary damping

A more refined, customarily accepted approach, defines viscoelastic behavior by means of the so-called "hereditary function" $H=H(\sigma)$; instead of (1), the constitutive equation is written:

$$\frac{F}{EI}=\frac{\partial^2 w}{\partial x^2}+\int_o^t H(t-\tau)\,\frac{\partial^3 w}{\partial x^2 d\tau}\,d\tau \qquad (2)$$

By using the modal approach, the governing equation for the amplitude of the n-th mode is:

$$\frac{d^2A_n}{dt^2}+\omega_n^2\left[A_n+\int_o^t H(t-\tau)\cdot\frac{dA_n(\tau)}{d\tau}\,d\tau\right]=X_n \qquad (3)$$

when X_n is the n-th component of the restoring force.

In Appendix II it is shown how the results of the tests conducted as described in Art. 1 can be used to discover some of the essential features of the function H.

5. - Nonlinear creep

As we go to nonlinear behavior, several basic problems arise.

The most of data appearing in techinical literature refer to uni-axial state of stress.

Tests are especially conducted on cylindrical specimens subjected to constant, or timewise varying stress. For this purposes, we use in our laboratory a tensile machine, Fig. 9. Very conventional techniques allow to determine the variation of ε vs. time; some results, at a constant temperature, are given in Fig. 10, Fig. 10 a, gives the same results with a greater amplification, and subtracting the initial (elastic) strain. From the above results, the constitutive creep law, under Norton's form (Ref. [7]) could be determined. For our tests, the constitutive creep law was obtained as:

$$\varepsilon = 0{,}45\ 10^{-8}\ \sigma^3 \qquad \left| \begin{array}{l} t \rightarrow \text{hours} \\ \sigma \rightarrow \text{Kg/mm}^2 \end{array} \right|$$

6. - Torsion problems

As is well known, torsional rigidity is of paramount importance in the aeroelastic analysis of structures. For such purpose, creep effects connected with temperature exposures were considered.

If a box-beam is subjected to a constant torque, variation of the twist angle with time will be observed. If the constitutive law were known, a very simple analysis (Appendix III) would yield the time-rate variation of θ.

The idea of substituting in Norbon's law the uni-axial stress with an ideal stress has been sometimes recommended. For the case under concern, the law $\dot{\theta} = A\tau^n$ (with A,n, the same as for the case of tension) has been suggested. They appeared to be quite unadequate.

Tests were concluded on the box-beam described in Fig. 11; it can be mounted is such a way as to leave warping completely free. The torque is applied by means of a rigid arm, and transmitted to the structure by means of a rigid cage (Fig. 11). The general setup is represented in Fig. 12.

Temperature was controlled by inserting the structure between two plates brought to a given temperature. Thermocomples were monted upon the structure for the necessary checks.

Typical results are presented in Fig. 13. It should be noted that, from similar results, the constitutive creep law could be determined.

It is seen, however, Fig.14 that tests are not repetitive; tests conducted on with the same conditions at different times ($\simeq$ 1 day of intevral) provided slightly different results, due to the hereditary character of creep. In such conditions, similarity analysis are not even possible, since one should give not only the structure, but all its history, and this may become a serious problem with highly stressed, highly heated wings. However, from an experimental standpoint, a special modeling technique is beeing studied, in order to avoid excessive cost and duration of tests.

Appendix I

By means of (1), the general equation of motion of a bar becom:

$$(1) \qquad EI\left[\frac{\partial^4 w}{\partial x^4} + \varepsilon\frac{\partial}{\partial t}\frac{\partial^4 w}{\partial x^4}\right] + \mu\frac{\partial^2 w}{\partial t^2} = 0$$

If $X_n(x)$ are the normalized eigen modes of vibrations, for each modal amplitude $F_n(t)$, Eq.(1) yields:

$$(2) \qquad \frac{dF_n}{dt^2} + w_n^2\left[F_n + \varepsilon\frac{dF_n}{dt}\right] = 0$$

The general solution of (2) is a damped oscillation, with damping independent of amplitude, and proportional to the square of the eigenfrequency.

Appendix II

Vibrations of a bar with hereditary characteristies, under the action of a force $F = F_o e^{j\lambda t}$, Ref.[6]provide:

$$\frac{d^2(F_o e^{j\lambda t})}{dt^2} + \omega^2\left[T_o e^{j\lambda t} + t_o\int_o^t H(t-\tau)\cdot e^{j\lambda t}\,d\tau\right] = F_o e^{j\lambda t}$$

Letting $s=t-\tau$; the foregoing equation would yield:

$$\frac{d^2T}{dt^2} + \omega^2 T\left[1 + \int_o^t e^{-j\lambda s}H(s)ds\right] = 0$$

As t approaches infinity, the foregoing equation may yield the value of H(s) as Fourier's antitransform of the observed decay.

Appendix III

Consider a thin-walled cylinder, Fig. 15.

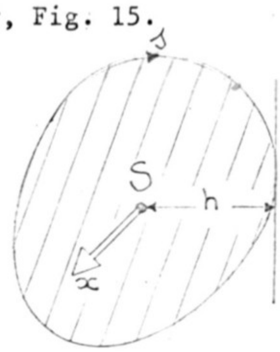

FIG. 15

subjected to a torque T, constant with time: the unit shear stress is:

$$(1) \qquad \tau = \frac{T}{2S\delta}$$

where S is the area shaded in Fig. 15, and δ is the thickeness. Shear strain γ is related to axial displacement w and angle of twist θ by:

$$(2) \qquad \gamma = \frac{\partial w}{\partial s} + h\frac{d\theta}{dx}$$

where s is the contour abscissa, x is the axial coordinate, h is the distance of a generic point of the boundary from the center of twist. By means of the constitutive creep law $\dot{\gamma} = A\tau^n$, from (1), (2), we get:

$$\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial s} + h\frac{d\theta}{dx}\right) = A\left(\frac{T}{2S\delta}\right)^n$$

Hence, integrating to all the contour:

$$\frac{\partial}{\partial t}\frac{\partial\theta}{\partial x} = \frac{AT^n}{(2S)^{n+1}}\oint\frac{ds}{\delta^n}$$

A more refined analysis should be conducted when warping is prevented, Ref. [8].

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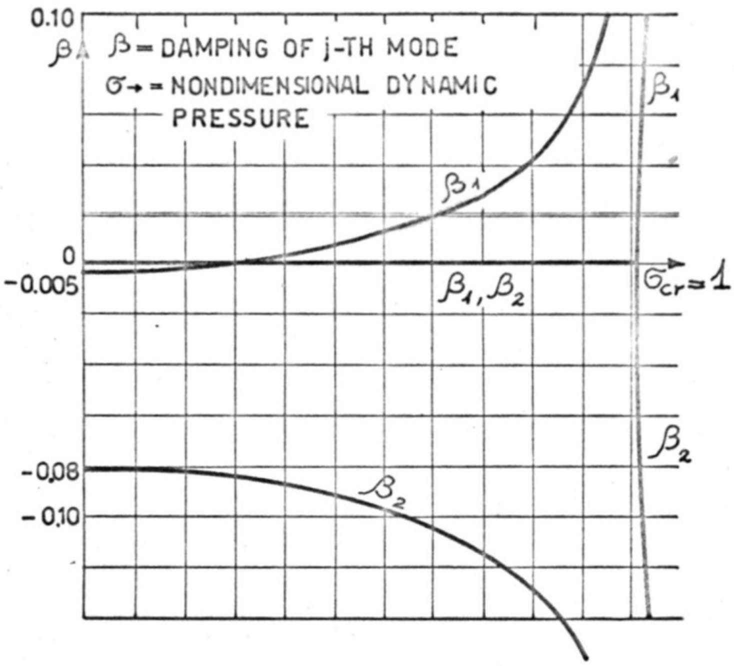
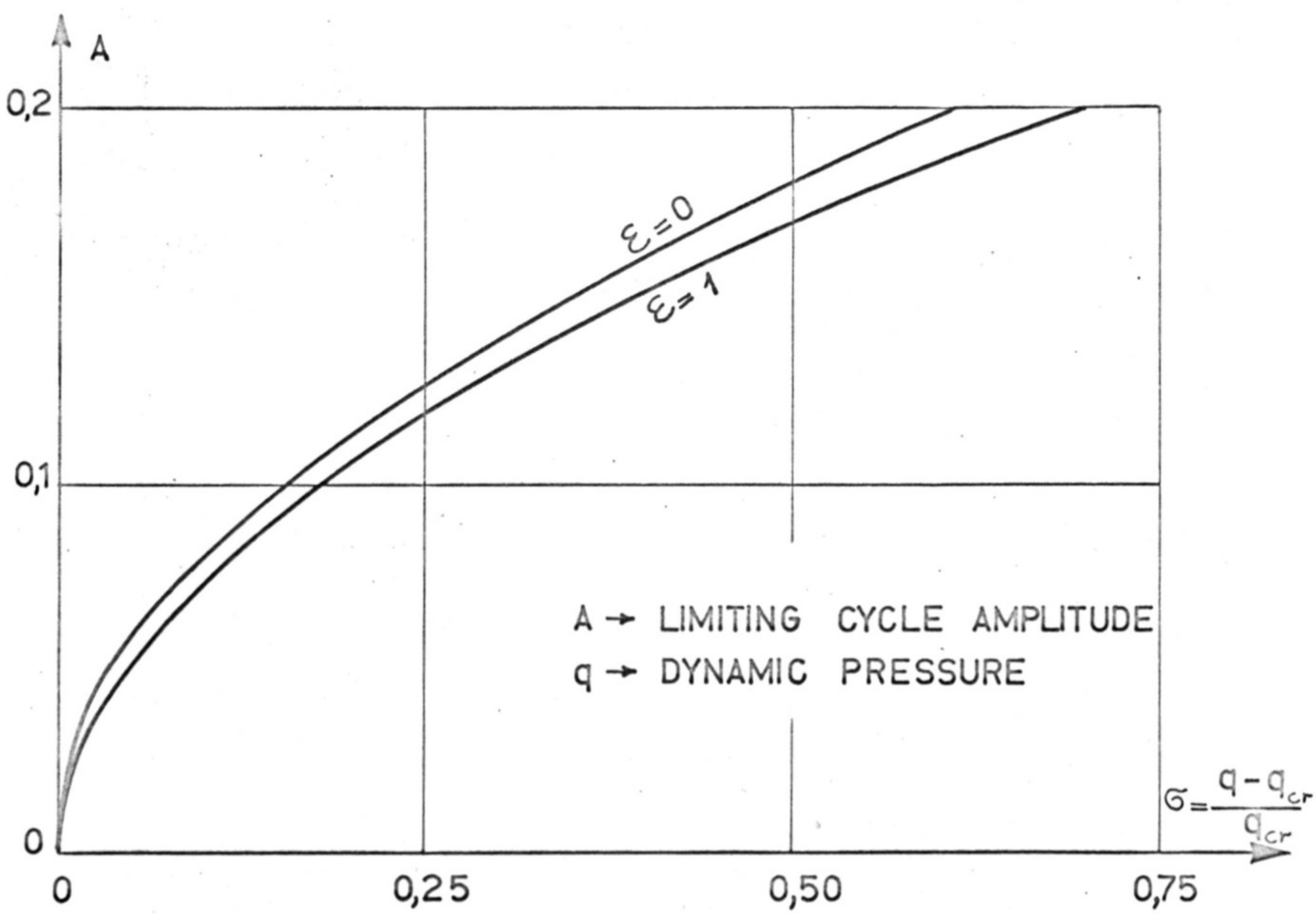


0.10
β
β = DAMPING OF j-TH MODE
σ → = NONDIMENSIONAL DYNAMIC PRESSURE
β1
β1
0
-0.005
β1, β2
σcr = 1
β2
-0,08
-0,10
β2

FIG. 1



A
0,2
0,1
0
ε=0
ε=1
A → LIMITING CYCLE AMPLITUDE
q → DYNAMIC PRESSURE
σ = (q - qcr)/qcr
0
0,25
0,50
0,75

FIG. 2

① MODEL ④ OSCILLATOR

② EXCITATOR ⑤ AMPLIFIER

③ SENSOR ⑥ RECORDER

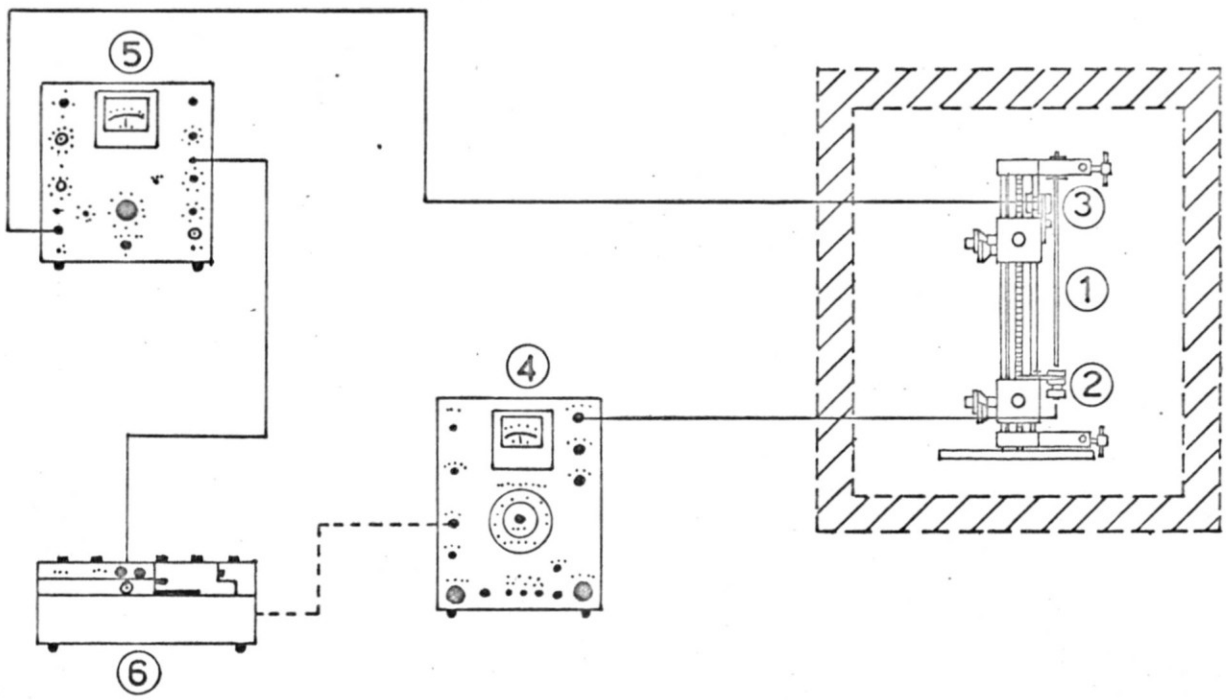


FIG. 3 - GENERAL SETUP OF MEASURING DEVICE

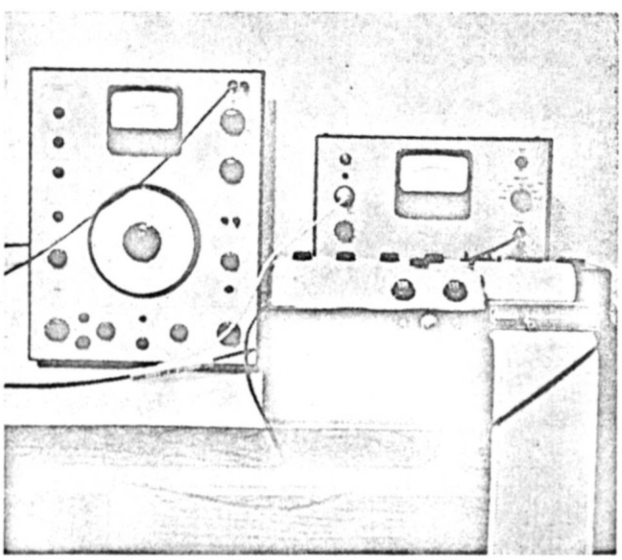


FIG. 4 - OSCILLATOR, AMPLIFIER, RECORDER

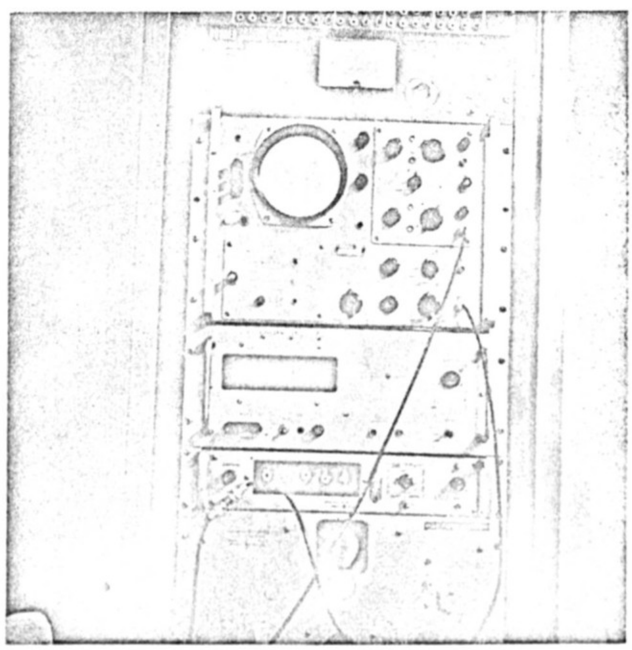


FIG. 5 - OSCILLOSCOPE, FREQUENCYMETER

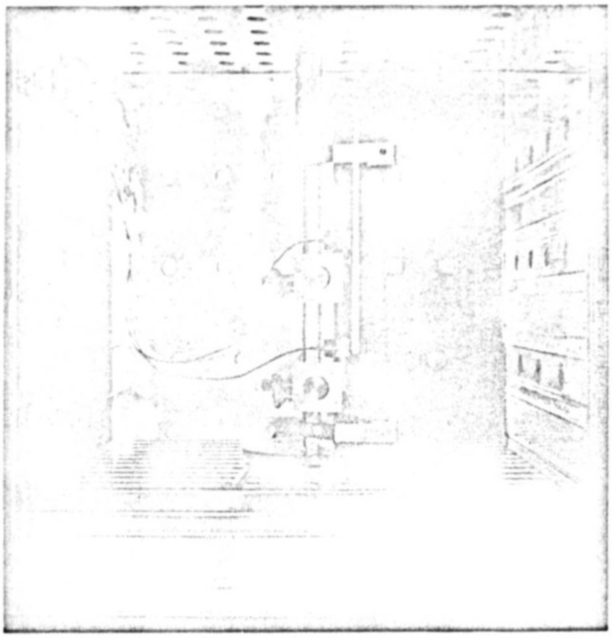


FIG. 6 - THERMOSTATIC OVEN

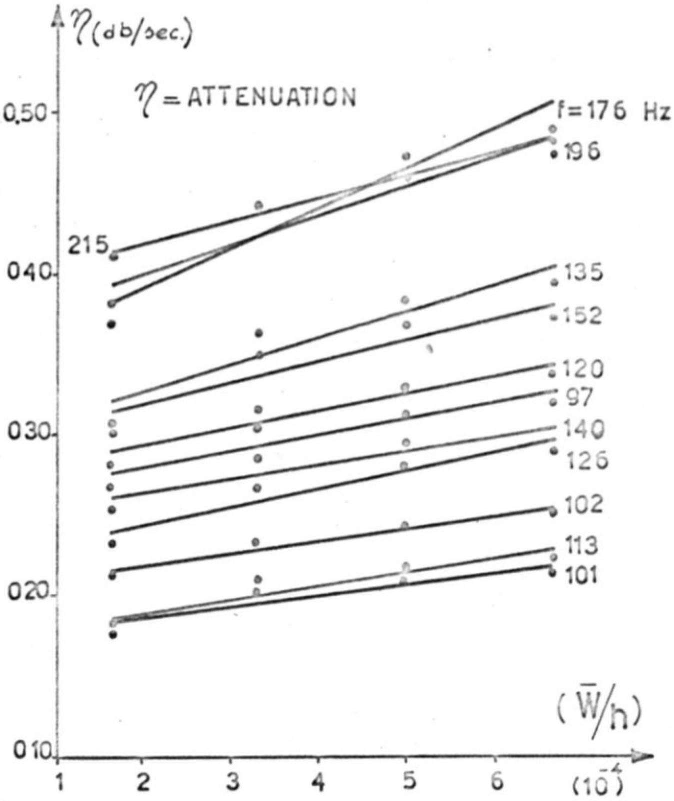


FIG. 7 - VARIATION OF DAMPING WITH AMPLITUDE

η = ATTENUATION (db/sec.)

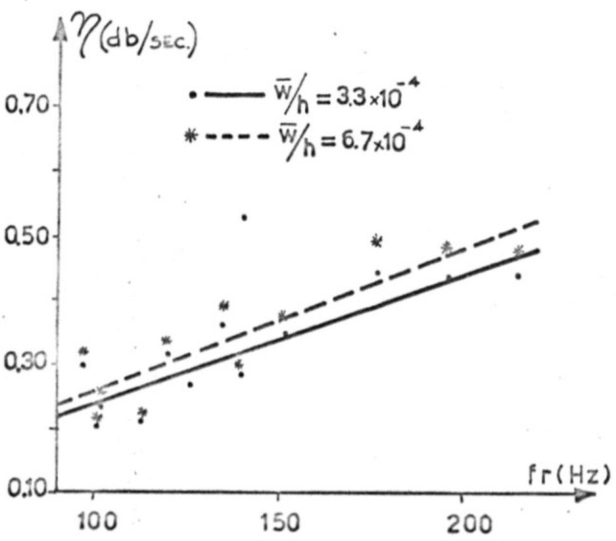


FIG. 8 - VARIATION OF DAMPING WITH FREQUENCY

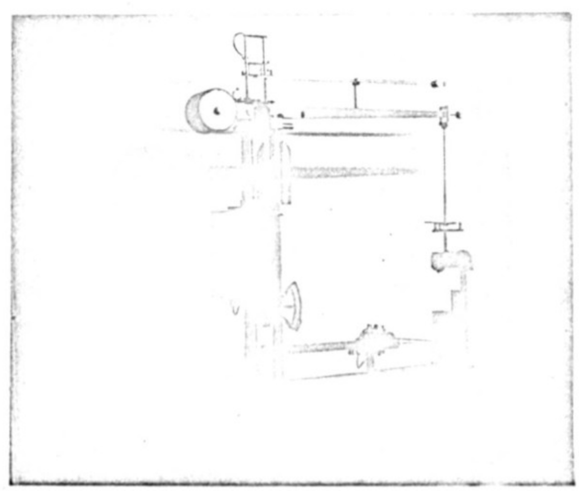


FIG. 9

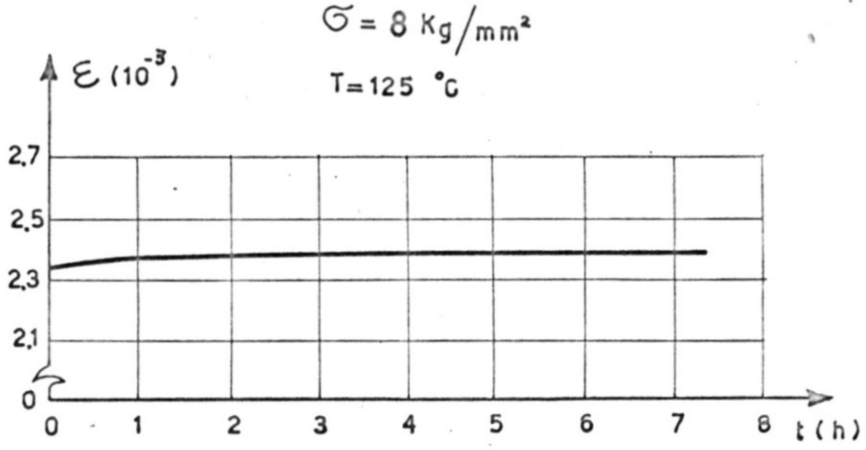


FIG. 10 - CREEP TEST UNDER CONSTANT STRESS

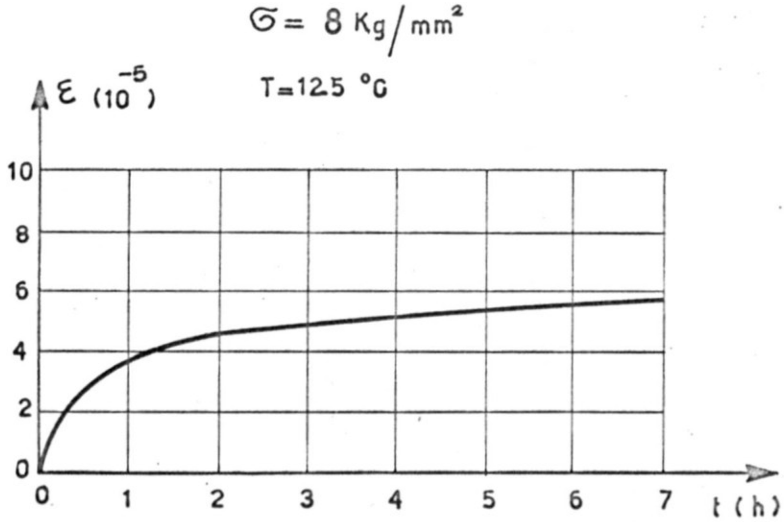


FIG. 10 a - CREEP TEST UNDER CONSTANT STRESS.

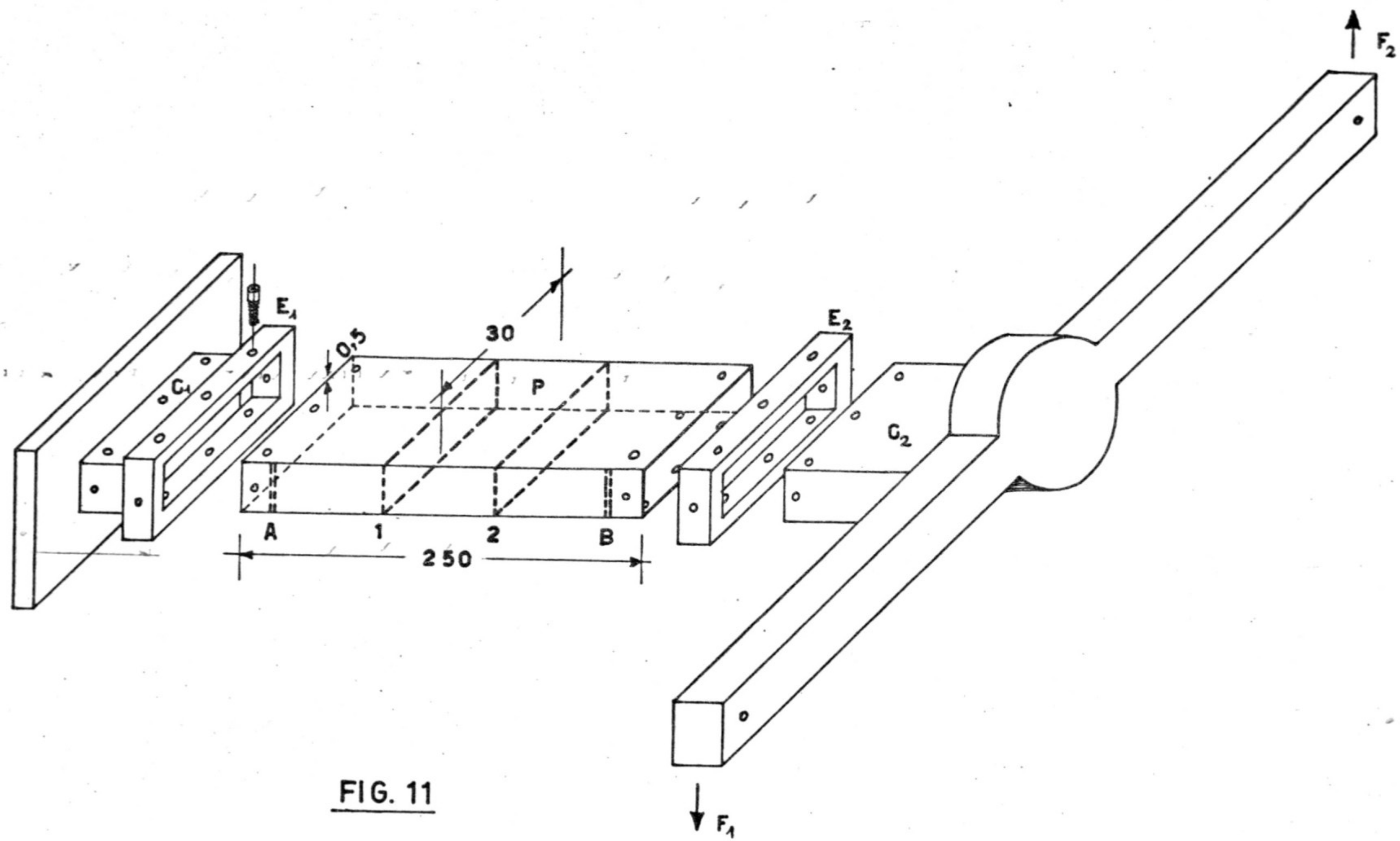


FIG. 11

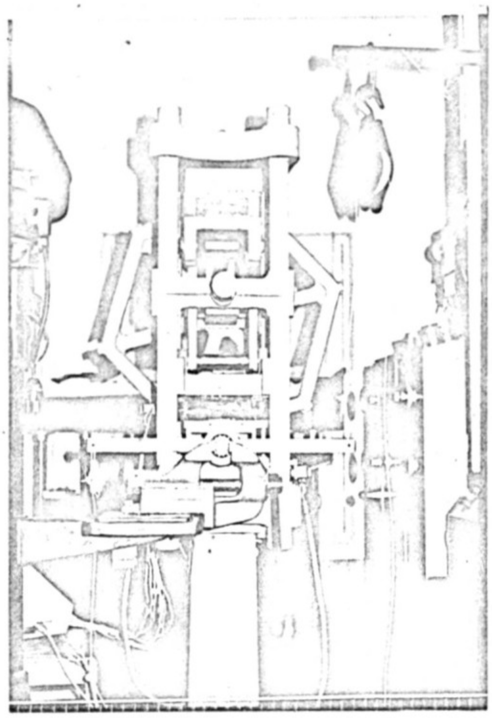


FIG. 12

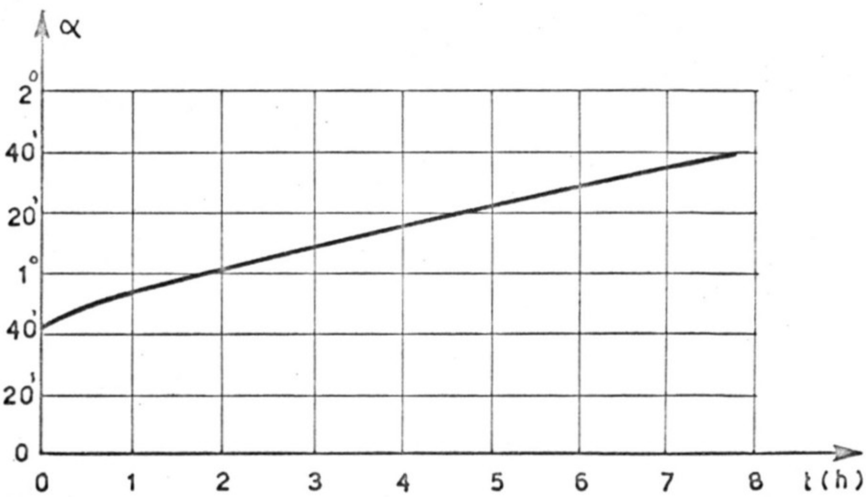


FIG. 13

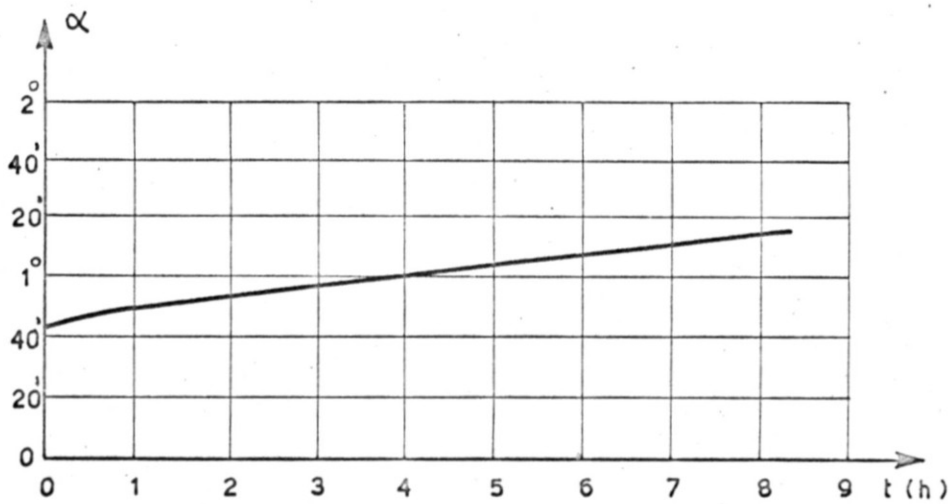


FIG. 14