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INVESTIGATIONS ON THE USE OF A KALMAN FILTERING METHOD IN TRACKING SYSTEMS FOR AIR TRAFFIC CONTROL

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Abstract

The digitized output of a surveillance radar for air traffic control consists of position information of aircraft at intermittent intervals. In track-while-scan systems real time processing of these data is performed to calculate the track of the aircraft. Two computational methods i.e. the classical α, β tracking method and a KALMAN filtering method, are analysed. The former method will be used in the new terminal area control system at Schiphol. The digital input data required for a fast-time simulation of the tracking process have been generated by means of a model of the new terminal area radar at Schiphol. The results obtained with both methods are compared and their relative merits are discussed.

I. Introduction

The recent developments in Air Traffic Control (ATC) automation are mainly based on the introduction of radar data processing by means of a digital computer and suitable interface. At the present stage it is possible to feed the computer during each radar-antenna revolution with aircraft position information. This means that the computer has the same basic information about the actual traffic situation as the controller on his radar screen. The availability of such information to the computer implies the possibility for further automation of the ATC-process. This digital radar data processing is one of the important aspects of the new ATC-system to be installed at Amsterdam Airport. In the first phase of this SARP (Signaal Automatic Radar Processing)-system, which will become operational in 1974, the digital radar data of one terminal area radar are fed into the computer system where tracking will be performed for terminal area control. This tracking process will associate radar blips (plots) with existing tracks, and will correlate these tracks with the flight plans belonging to the relevant aircraft. The first objective of this tracking process is to provide the air traffic controller with labels on his radar scope. These computer generated labels will be attached to each aircraft blip for which tracking is carried out, and can contain relevant information about these aircraft, like call sign, flight level, speed, etc.

Tracking of an aircraft is the process of smoothing and predicting the path of that aircraft, from positional input data at discrete moments; these data are updated with a time interval equal to the revolution time of the radar antenna. These input data can also contain identification information if the aircraft is equipped with a transponder. Smoothing is necessary because the aircraft plot positions as provided by the radar and its digitizing equipment contain certain errors. The result of smoothing is a so-called track, i.e. the calculated path and velocity of the aircraft. Extrapolation of the calculated track by means of prediction techniques permits the comparison of estimated and measured aircraft positions.
In this way a kind of memory is introduced into the system, without storing past plots. In the SARP-system the so-called α-β filtering process (1)(2)(3) is used, which is characterized by the smoothing or filtering parameters α and β for position and velocity respectively.

The NLR has carried out fast-time simulations of the tracking process as it will be incorporated in the SARP-system to determine the optimal parameter (α, β) values to be used for the filter. The results of these simulations have been reported in (4)(5)(6). These optimal values of the parameters (α, β) will be used in this paper, in which two filtering methods, the above α-β method and a KALMAN method (7)(8)(9)(10) are discussed. This filter can be applied to both continuous and discrete time series and it is extremely suitable for implementation on a digital computer. The method can be seen as a "popular" means of estimating the "state" of an aircraft from noisy measurements of its range and azimuth. The filter for discrete time series as developed for the track-while-scan systems is composed of a group of matrix recursion relations.
It is the concern of this paper to investigate as to how far the KALMAN filter would be superior to the α-β filter for the SARP-system.

II. Equations of motion for straight flights and problem statement.

The undisturbed target motion is a straight flight with constant velocity in a two dimensional cartesian co-ordinate system.
At discrete time intervals t_k the state vector $\bar{x}_k$ is described by a linear vector difference equation:

$$\bar{x}_k = \Phi_{k-1}\, \bar{x}_{k-1} + \bar{q}_{k-1} \qquad (2.1)$$

The 4-dimensional state vector $\bar{x}_k$ consists of the position X and Y and the velocity components $\dot{X}$ and $\dot{Y}$.

$$\bar{x}_k = \begin{bmatrix} X_k \\ Y_k \\ \dot{X}_k \\ \dot{Y}_k \end{bmatrix}$$

The transition matrix Φ_k has for straight flight with constant velocity the following simple form

$$\Phi_k = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where T is the antenna revolution time.
The vector $\bar{q}_k$ is a random 4-dimensional sequence vector which deteriorates the state at each time t_k and has known statistics. Its mean is zero and $\bar{q}_k$ is uncorrelated with $\bar{q}_j$ for $j \neq k$.

$$E\left\{\bar{q}_k\right\} = 0; \quad E\left\{\bar{q}_k \, \bar{q}_j^T\right\} = Q_k \, \delta_{kj} \tag{2.2}$$

where δ_{kj} is the Kronecker symbol and Q_k is considered to represent speed deviations due to air turbulance. The direct perturbation of the position components is taken zero, while the distribution of the deviations of the velocity components are modeled as white noise having a Gaussian distribution. The standard deviation of $q_{3,k}$ and $q_{4,k}$ is σ_w. Hence the covariance matrix Q_k is:

$$Q_k = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_w^2 & 0 \\ 0 & 0 & 0 & \sigma_w^2 \end{bmatrix}$$

In the simulation programme the values of $q_{3,k}$ and $q_{4,k}$ at t_k have been generated with a random generator from a distribution with σ_w=1 kt . The 2-dimensional measurement vector $\bar{z}_k$ at t_k consists of range and azimuth components:

$$\bar{z}_k = \begin{bmatrix} r_k \\ \theta_k \end{bmatrix}$$

The (nonlinear) relation between the measured variables and the state variables is given by:

$$\begin{aligned} X &= r \sin \theta \\ Y &= r \cos \theta \end{aligned} \tag{2.3}$$

From (2.3) by linearization the following relation is obtained between small variations in the state and the corresponding deviations in the measured variables:

$$\Delta\bar{z}_k = H_k \, \Delta\bar{x}_k \tag{2.4}$$

The matrix H_k is the 2x4-dimensional observation matrix and has the following form

$$H_k = \begin{bmatrix} \sin\theta_k & \cos\theta_k & 0 & 0 \\ \dfrac{\cos\theta_k}{r_k} & -\dfrac{\sin\theta_k}{r_k} & 0 & 0 \end{bmatrix}$$

The error $\delta\bar{z}_k$ in the measurement vector is assumed to have the following known statistical properties:

$$E\left\{\delta\bar{z}_k\right\} = 0; \quad E\left\{\delta\bar{z}_k \, \delta\bar{z}_j^T\right\} = M_k \, \delta_{kj} \tag{2.5}$$

in which

$$M_k = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}$$

σ_r and σ_θ are the 1-σ values of the errors in the measured range and azimuth. These errors are assumed to be Gaussian distributed and to be uncorrelated.
The problem statement for the filtering process can now be formulated as follows:

Given the preceding model, determine an estimate of the state vector $\bar{x}_k$ at each t_k from the measured vector $\bar{z}_k$ and the state vector $\bar{x}_{k-1}$, in such a way that the position and velocity errors are minimized.

III. The filter processes

A The KALMAN filter.

The principle of the KALMAN filter is to minimize the sum of the diagonal elements of the covariance matrix P_k of the state vector.

$$P_k \equiv E\left\{(\bar{x}_k - E(\bar{x}_k))\,(\bar{x}_k - E(\bar{x}_k))^T\right\} = E\left\{\delta\bar{x}_k \, \delta\bar{x}_k^T\right\} \tag{3.1}$$

Mathematically this means that

$$E\left\{\delta\bar{x}_k^T \, \delta\bar{x}_k\right\} = \text{minimum} \tag{3.2}$$

In this sense the KALMAN filter is the optimal and most sophisticated smoothing process. The time-discrete KALMAN filter is composed of a group of matrix recursion relations. The simplicity of these relations makes the filter extremely suitable for implementation on a digital computer. The filter equations are derived in various papers (7)(8)(9)(10). Therefore only the algorithms for the particular process of radar tracking are briefly summarized below.
Starting at t_k a predicted state is calculated according to eq. (2.1):

$$\hat{\bar{x}}_k = \Phi_{k-1} \, \bar{x}_{k-1} \tag{3.3}$$

From equations (2.3) and (3.3) the estimated measurement vector $\hat{\bar{z}}_k$ at t_k is determined. The actual measurement vector at t_k is given by $\bar{z}_k$. The difference between $\hat{\bar{z}}_k$ and $\bar{z}_k$ is caused by measuring errors as well as by errors in the estimate of the state. The KALMAN filter now calculates on the basis of (3.2) an optimal weighting or gain matrix K_k such, that the new estimate $\bar{x}_k$ is optimal:

$$\bar{x}_k = \hat{\bar{x}}_k + K_k \left[\bar{z}_k - \hat{\bar{z}}_k\right] \tag{3.4}$$

K_k is a (4x2) dimensional matrix and is given by

$$K_k = \hat{P}_k \, H_k^T \left[H_k \, \hat{P}_k \, H_k^T + M_k\right]^{-1} \tag{3.5}$$

where H_k is the formerly mentioned observation matrix eq. (2.4), M_k is the covariance matrix of the measurements eq. (2.5), $\hat{P}_k$ is the covariance matrix of the predicted state vector $\hat{x}_k$. Hence

$$\hat{P}_k = \Phi_{k-1} P_{k-1} \Phi_{k-1}^T + Q_{k-1} \tag{3.6}$$

The covariance matrix P_k of the new estimate $\bar{x}_k$ is given by:

$$P_k = \hat{P}_k - K_k H_k \hat{P}_k \tag{3.7}$$

Equations (3.3), (3.4), (3.5), (3.6) and (3.7) constitute the KALMAN filter for the model described in section 2.
The influence of Q_k increases directly the covariance matrix $\hat{P}_k$. The gain matrix K_k is indirectly influenced by Q_k via relation (3.7) and (3.5).
If $Q_k = 0$ the matrices P_k and K_k approach to zero. Hence Q_k gives an upper limit to the state accuracy which is physically evident.
A further interesting aspect is the fact that the state as calculated by (3.4) has no direct influence on the calculation of the covariance matrix P_k and the gain matrix K_k. In practice this can mean that the calculated covariance matrix converges while the state vector does not; this can be the case when the initial estimates $\bar{x}_k$ and P_k are poor.
At k=2, i.e. the target has been observed for two consecutive antenna revolutions (r_1, θ_1 and r_2, θ_2) the filter process is initiated. The initial state at k=2 is taken as follows:

$$\bar{x}_2 = \begin{bmatrix} r_2 \sin\theta_2 \\ r_2 \cos\theta_2 \\ \frac{1}{T}(r_2 \sin\theta_2 - r_1 \sin\theta_1) \\ \frac{1}{T}(r_2 \cos\theta_2 - r_1 \cos\theta_1) \end{bmatrix} \tag{3.8}$$

From this initial state the initial covariance matrix P_2 is calculated as follows:

$$\begin{aligned} P_2 &= E\left\{\delta\bar{x}_2\, \delta\bar{x}_2^T\right\} \\ &= E\left\{B\, \delta\bar{z}\, \delta\bar{z}^T\right\} B^T \\ &= B\, E\left\{\delta\bar{z}\, \delta\bar{z}^T\right\} B^T \end{aligned} \tag{3.9}$$

in which $\left[\delta\bar{z}^T = \delta r_2 \quad \delta\theta_2 \quad \delta r_1 \quad \delta\theta_1\right]$

Since all the errors are uncorrelated one obtains,

$$E\left\{\delta\bar{z}\quad \delta\bar{z}^T\right\} = \begin{bmatrix} \sigma_r^2 & 0 & 0 & 0 \\ 0 & \sigma_\theta^2 & 0 & 0 \\ 0 & 0 & \sigma_r^2 & 0 \\ 0 & 0 & 0 & \sigma_\theta^2 \end{bmatrix} \tag{3.10}$$

The relation matrix B can be derived from (3.8):

$$B = \begin{bmatrix} \sin\theta_2 & r_2\cos\theta_2 & 0 & 0 \\ \cos\theta_2 & r_2 \sin\theta_2 & 0 & 0 \\ \frac{1}{T}\sin\theta_2 & \frac{1}{T}r_2\cos\theta_2 & -\frac{1}{T}\sin\theta_1 & -\frac{1}{T}r_1\cos\theta_1 \\ \frac{1}{T}\cos\theta_2 & -\frac{1}{T}r_2\sin\theta_2 & -\frac{1}{T}\cos\theta_1 & \frac{1}{T}r_1\sin\theta_1 \end{bmatrix}$$

The equations (3.8), (3.9) and (3.10) yield the elements of the initial covariance matrix P_2:

$$P(1.1) = \sigma_r^2 \sin^2\theta_2 + r_2^2\sigma_\theta^2 \cos^2\theta_2$$

$$P(2.2) = \sigma_r^2 \cos^2\theta_2 + r_2^2\sigma_\theta^2 \sin^2\theta_2$$

$$P(1.2) = P(2.1) = \sin\theta_2 \cos\theta_2 (\sigma_r^2 - r_2^2\sigma_\theta^2)$$

$$P(1.3) = P(3.1) = \frac{1}{T}\ P(1.1)$$

$$P(2.3) = P(3.2) = \frac{1}{T}\ P(1.2)$$

$$P(2.4) = P(4.2) = \frac{1}{T}\ P(2.2)$$

$$P(3.3) = \frac{1}{T^2}(\sigma_r^2 \sin^2\theta_2 + \sigma_r^2 \sin^2\theta_1 +$$

$$+ r_2^2\sigma_\theta^2 \cos^2\theta_2 + r_1^2\sigma_\theta^2\cos^2\theta_1) \approx \frac{2}{T^2} P(1.1)$$

$$P(4.4) \approx \frac{2}{T^2}\ P(2.2)$$

$$P(1.4) = P(4.1) = \frac{1}{T} P(1.2)$$

$$P(3.4) = P(4.3) \approx \frac{2}{T^2} P(1.2)$$

An alternative form of factoring the covariance matrix will be discussed in section 6.

B The α-β filter

The performance of the KALMAN filter will be compared with that of an α-β filter method (1) (2) (3).
The α-β method considered here is the same as will be implemented in the SARP-system at the Amsterdam ATC-centre. The tracking equations will be briefly summarized below.
Again the predicted state $\hat{x}_k$ at t_k is obtained from eq. (2.1):

$$\hat{x}_k = \Phi\bar{x}_{k-1} \tag{3.11}$$

From the measurement vector $\bar{z}_k$ at each time t_k the position vector $\bar{y}_k$ is obtained using eq. (2.3)

$$\begin{aligned} y(1) &= z(1)\sin z(2) \\ y(2) &= z(1)\cos z(2) \end{aligned} \tag{3.12}$$

The new estimated state at t_k is now formed by

$$x(1)_k = \hat{x}(1)_k + \alpha_k(y(1)_k - \hat{x}(1)_k)$$
$$x(2)_k = \hat{x}(2)_k + \alpha_k(y(2)_k - \hat{x}(2)_k)$$
$$x(3)_k = \hat{x}(3)_k + \frac{1}{T}\beta_k(y(1)_k - \hat{x}(1)_k) \qquad (3.13)$$
$$x(4)_k = \hat{x}(4)_k + \frac{1}{T}\beta_k(y(2)_k - \hat{x}(2)_k)$$

These equations can be rewritten as

$$\bar{x}_k = \hat{\bar{x}}_k + A_k \; \Delta\bar{x}_k \qquad (3.14)$$

where A_k is a (4x2) dimensional weighting matrix

$$A_k = \begin{bmatrix} \alpha_k & 0 \\ 0 & \alpha_k \\ \frac{1}{T}\beta_k & 0 \\ 0 & \frac{1}{T}\beta_k \end{bmatrix} \quad \text{and} \quad \Delta\bar{x}_k = \begin{bmatrix} y(1)_k - \hat{x}(1)_k \\ y(2)_k - \hat{x}(2)_k \end{bmatrix}$$

The method used here is quasi-adaptive during the first 6 scans after iniation i.e. 6 values of the matrix have been stored. For the following scans the final values of α, β are used. Optimal values of α_k and β_k have been determined through simulations reported in ref. (4) (5) and (6).

The six sets of α and β used during the first six steps of the tracking process are:

$$\alpha_k = 0.76;\ 0.59;\ 0.48;\ 0.40;\ 0.34;\ 0.34 \qquad (3.15)$$
$$\beta_k = 0.47;\ 0.25;\ 0.16;\ 0.11;\ 0.06;\ 0.045$$

The equations (3.11), (3.12) and (3.13) charaterize a second order linear prediction filter, wherein damping is introduced by means of α_k and β_k to improve the performance of the system in the presence of noise. This demping, however, degrades the transient response. This means that always a compromise has to be made between good smoothing of noise errors and a good transient response.
In deriving the optimal sets of α_k, β_k most emphasis was laid upon minimizing the r.m.s. errors in the magnitude of the velocity vector and in the distance between the true and filtered positions.

The α-β filter is a very simple filter and requires little memory space and computer time. It is an essential property of the filter that the values of the matrix A_k i.e. α_k and β_k are not calculated during the filter process itself as is the case with the gain matrix K_k in the KALMAN filter.
After two consecutive antenna revolutions the tracking is initiated. The initial state vector $\bar{x}_2$ at k=2 is the same as in the KALMAN process, see eq. (3.8) which means that at initiation α= 1 and β=1 are selected.

IV. Comparison of the filter accuracies for straight flights.

In order to evaluate the two filter processes, simular simulations have been performed with both. The generation of the errors in the simulated radar data were based on a Gaussian distribution in range as well as in azimuth. Flights of many different orientations relative to the radar site were analysed. The results of one representative flight will be discussed in this paper since the results appeared hardly to depend on the orientation of the flight. The characteristics of this flight are summarized in the following table:

Table 1

Initial true state:	X_o = 28 NM ; Y_o = 7 NM $\dot{X}_o$ = 75.5 kts; $\dot{Y}_o$ = 238.8 kts
Antenna revolution time:	T = 4 sec.
Standard deviation:	Range σ_r = 50 m Azimuth σ_θ = 0.08 degrees

To determine which number of runs is sufficient (Monte Carlo trials), the results based on 25,50 and 100 trials were compared. The difference in the results from 25 runs and 50 runs was more than 20 %. The difference between 50 and 100 runs was less than 5 %. A number of 100 runs was therefore considered to be sufficient.
The covariance matrix P_k of the state vector in the KALMAN filter has been calculated automatically each scan. The diagonal elements of this matrix are the variances of the errors in the state components; these variances have also been calculated from the 100 runs.
It appeared that the standard deviations from the covariance matrices were some 5 % larger than the actual r.m.s. values, which may be expected. The parameters on the basis of which the performance of both filters will be discussed are:
- the difference Δs between true position and smoothed track position; $\Delta s = |\bar{r}_{true} - \bar{r}_{track}|$
- the difference Δv between true ground speed and the smoothed track speed; $\Delta v = |\bar{v}_{true}| - |\bar{v}_{track}|$
- the difference ΔH between true and smoothed heading; $\Delta H = H_{true} - H_{track}$.

At each scan the value of Δs, Δv and ΔH has been calculated. The mean and the r.m.s. error at each scan have been calculated from the 100 runs.
Although the mean values of ΔX and ΔY appeared to be zero, it is clear that the mean of Δs, which is always positive, cannot be zero.
Figure 1 shows the mean and the standard deviation of the error Δs at successive scans.
At initiation the values for "α-β" and "KALMAN" are exactly the same, which is evident since the same initial state vector has been used.
It can be seen from the figure, that for the following scans, although the KALMAN filter gives superior results, the difference between the two filtering processes is so small (some 5 %) that the α-β filter is quite acceptable.

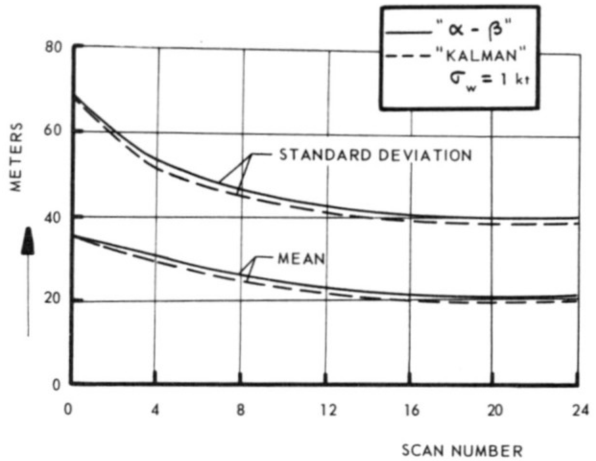

Figure 1 Mean and standard deviation of the position error Δs.

If the error distribution of Δs would be Gaussian, then for about 17 % of the cases the error Δs will be larger than the mean plus 1σ. Then from figure 1 it follows that after about 8 scans (32 secs after initiation) the error Δs will - for about 83 % of the cases - be smaller than some 70 m.
Deviation in the order of this magnitude will not be perceptable on the display screen of the air traffic controller.
Figure 2 and 3 show the picture of the standard deviation of the velocity error and the heading error respectively.

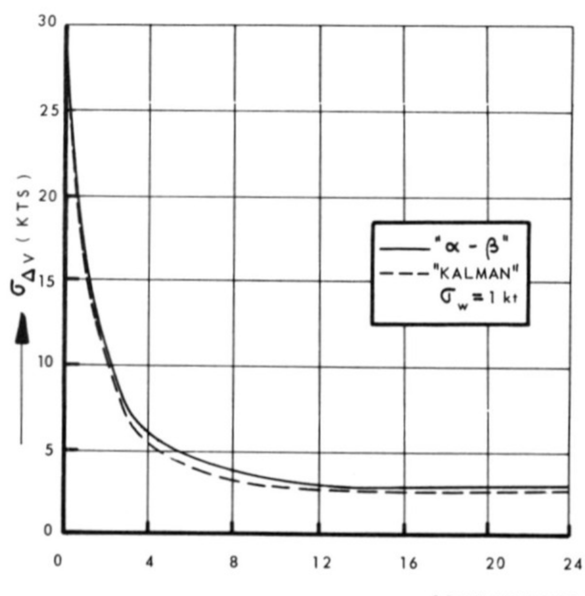

Figure 2 Velocity standard deviation.

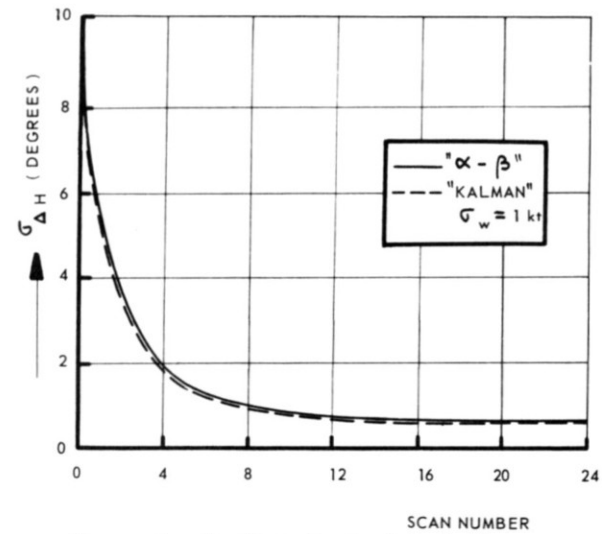

Figure 3 Heading standard deviation.

The mean values of the errors appeared to be zero in general, however, at initiation a bias can arise due to the limited number of 100 runs.
At initiation the velocity standard deviation is for both filters 30 kts.
From figure 2 it can be seen that during the scans nr. 4-10 "KALMAN" is slightly superior to "α-β". The difference,however,does not exceed 10 %.
The 8th scan e.g. shows a velocity standard deviation of 3.4 kts for "α-β" and 3.1 kts for "KALMAN".
The difference is due to the fact that the α, β values of the "α-β" filter during the 4th, 5th and 6th scans after initiation are not optimal from the point of view of the standard deviation of the velocity; when these values would be taken slightly smaller (5-10 %) no significant difference would remain.
The picture of the heading error is about the same. The accuracy of both filters is very good. The 1σ value of the heading error is within 8 scans (32 secs) reduced to about 1 degree from 10 degrees at initiation.
For both filters the velocity standard deviation approaches to about 2.5 kts. The magnitude of this limit for the α-β filter depends on the selected α, β values and the standard deviation σ_w representing the air turbulance effects.
For $\sigma_w = 0$ the error would converge further to zero in case of the KALMAN filter. To illustrate this effect, figure 4 shows the velocity standard deviation for the same flight but with $\sigma_w = 0$.

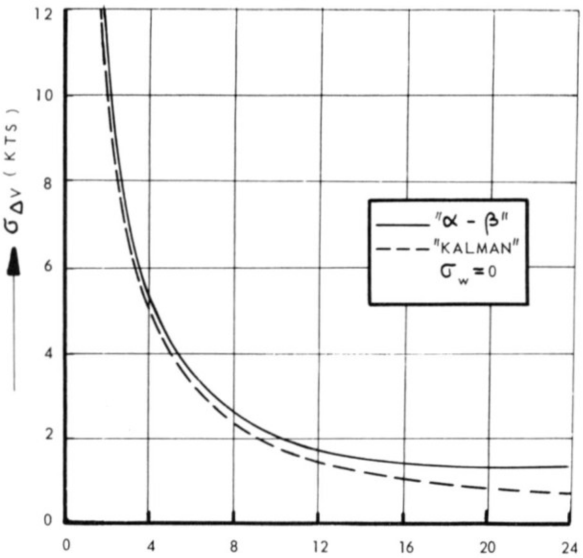

Figure 4 Velocity standard deviation without air turbulance.

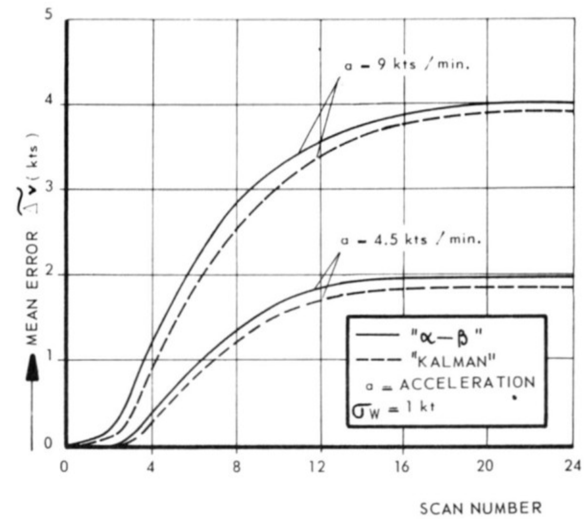

Figure 5 Velocity bias (mean error) for straight accelerated flights.

In this case it can be seen that the KALMAN filter further approaches to zero whereas the α-β filter approaches to about 1.7 kts which value depends on the selected values of α and β. However, if the quasi-adaptive character of the α-β filter would be extended to scan numbers above 6 better results would be obtained.

Though the radar errors used in these simulations are based on the radar specifications, it may turn out that the actual errors are higher. Further it should be kept in mind that the blip to scan ratio has been assumed to be 1 in the simulations, while in practice some plots will be missed.

V. Some aspects of tracking manoeuvring flights.

Although the tracking process for manoeuvring flights will not extensively be discussed in this paper, some attention will be paid to slowly accelerating straight flights. To test the performance of both filters, two flights have been simulated in which the aircraft accelerated at a rate of 4.5 kts/min and 9 kts/min respectively. The initial conditions were the same as given in table 1, while for the α-β filter as well as for the KALMAN filter the model used is the same as described in section III.
To illustrate the results of this simulation figure 5 shows the mean of the velocity error.

The performance for both filters is within 10 % the same. Within a some 10 scans the mean approaches to a constant value of about 1.9 kts for an acceleration of 4.5 kts/min (3.8 kts for 9 kts/min). The results indicate that both filters have the same capability in responding to the navigational corrections of the pilot and/or inadvertant changes in aircraft speed. However, when the acceleration becomes larger or turns are executed, an unacceptable mean will result in the velocity and track position errors if the filters are unaltered.
To track such flights the α-β filter has to be extended with a manoeuvre detection logic. The KALMAN filter can be extended by adding acceleration terms to the dynamical equations (3.1). This aspect has been investigated in a number of papers (11) (12). The problem, however, is that the type of the manoeuvre is not known, i.e. the acceleration in the cartesian co-ordinates is not a constant for a turn. The problem can be attacked by developing special models for the manoeuvre equations in such a way that at each scan it is assumed with a certain probability that a manoeuvre is executed. It depends on the actual consecutive measurements which manoeuvre is detected by this model. The author has some feeling that by such models results are obtained which are not significantly better than for an α-β filter with a manoeuvre detection logic.
In the SARP α-β system the discrepancy between plot and predicted position at each scan is used as a means to decide whether the target is executing a manoeuvre. If such a manoeuvre is detected then the α, β values are adapted.

VI. Computational aspects.

An important aspect of the KALMAN filter is the calculation of the gain matrix K_k at each scan. The gain matrix is merely dependent upon the statistics described by P_k, M_k and Q_k. At each scan

the state covariance matrix P_k and the measurement covariance matrix M_k must be determined. To initiate the proces initial values of P_o and M_o must be derived. The matrix M_o is easily calculated from eq. (2.5). The matrix P_o, as derived in section 3, however, requires an extensive set of calculations. The performance of the KALMAN filter process depends on the initial covariance matrix. To illustrate this influence of P_o some runs have been made with $P_o = N\ I$ where I is the identity matrix and N is a large number. Actually this means that almost no statistical information on the initial state is available.
For the calculation of the gain matrix and the covariance matrix at the first scan the following relations must be used now (9)

$$P_1^{-1} = P_o^{-1} + (H_1^T M_1^{-1} H_1) \; ; \; K_1 = P_1 H_1^T M_1^{-1} \quad (6.1)$$

Figure 6 illustrates the results of the velocity standard deviation for this initiation process.

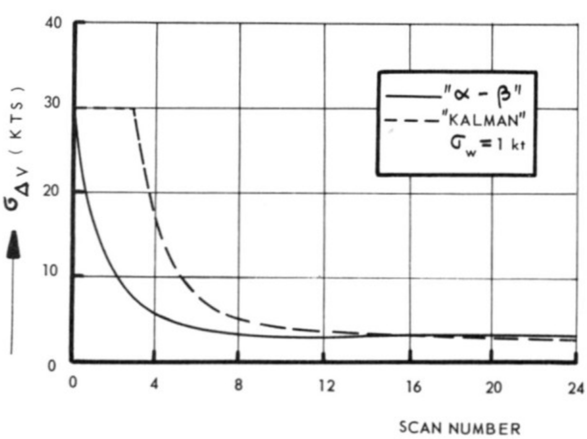

Figure 6 Velocity standard deviation for a a large initial covariance matrix P.

To compare the results the same results for "α-β" from fig. 2 are plotted as well. The "KALMAN" results show an error of about 30 kts during the first 3 antenna scans. During the later scans the performance of the filter improves as rapidly as in the case of the optimal initiation matrix. Such a non-optimal initiation apparently causes a delay of 12 secs (3 scans).
The KALMAN filter requires more memory space since - besides the flight plan information, the vectors $\bar{x}_k$ and $\hat{\bar{x}}_k$ - also the covariance matrix P_k has to be stored for each aircraft each antenna revolution. Compared to the α-β method in the SARP-system it is estimated that the KALMAN will require about 25 % more memory space. The total computation time seems also to be considerably higher than for the α-β filter.
Although this time depends on the type of the computer used, a rough indication is given by comparing required computer time for both filters on the NLR CDC 3300 computer. It appeared that in general the "KALMAN" takes about 200 % more computer time than the "α-β" method.

VII. Conclusions

On the basis of a model of the characteristics of the new terminal surveillance radar at Amsterdam airport simulations have been carried out in order to evaluate the accuracy aspects of two filtering methods ("Kalman" and "α-β") for use in the process of aircraft tracking.

It appeared that for straight flights the Kalman filter gives somewhat superior results (5-10 % smaller errors). The order of magnitude of the r.m.s.-value of the error in the calculated velocity is about 6 kts (after 4 radar scans), which decreases to 3 kts after some 8 scans; at this moment an accuracy in the calculated heading of about 1^o has been reached.

Although the performance of the filters was not extensively analyzed for the case of manoeuvring flights the results obtained indicate that the difference in performance of the two filters is of the same order of magnitude (some 10 %).

The tracking process as carried out by means of the Kalman filter, however, requires about 25 % more computer memory and - at least for the NLR computer used for the investigation - about three times as much computer time as in the case of the much simpler "α-β" filter.

It is therefore concluded that for the terminal system at Schiphol the "α-β" filter is quite acceptable for aircraft tracking purposes.

List of symbols

a	acceleration in kts/min.
$\bar{q}_k$	random sequence vector deteriorating the dynamical system at t_k
r_k	measured range at t_k
$\bar{r}$	radius vector in polar co-ordinates
t_k	time at k^{th} antenna revolution (k = 0, 1, 2 ......)
$\bar{x}_k$	the linear estimate of the state $\bar{x}_k$ at t_k using the data $\bar{z}_k$
$\hat{\bar{x}}_k$	the predicted linear estimate of the state $\bar{x}_k$ at t_k before $\bar{z}_k$ is used
$\bar{y}_k$	position vector directly calculated from $\bar{z}_k$
$\bar{z}_k$	measurement vector at t_k
$\hat{\bar{z}}_k$	the predicted estimate of $\bar{z}_k$ from $\hat{\bar{x}}_k$ at t_k
H	difference between true and smoothed heading
Δs	difference between true position and smoothed track position
Δv	difference between true ground speed and smoothed track speed
α	position smoothing parameter in the "α-β" filtering method

β velocity smoothing parameter in the "α-β" filtering method

θ_k measured azimuth at t_k

σ standard deviation

σ_w standard deviation of the speed perturbations due to air turbulance

σ_r standard deviation of the error in the measured range

σ_θ standard deviation of the error in the measured azimuth

A_k weighting matrix in the "α-β" filtering method.

B relation matrix between errors in the initial state and errors in the initial measurements

H heading

H_k observation matrix at t_k

I identity matrix

K_k optimal gain matrix for the KALMAN filter at t_k

M_k covariance matrix of the measurement data $\bar{z}_k$

P_k covariance matrix of the state vector $\bar{x}_k$ at t_k

$\hat{P}_k$ covariance matrix of the predicted state vector $\hat{\bar{x}}_k$

Q_k covariance matrix of $\bar{q}_k$

T antenna revolution time (4 secs)

Φ_k the state transition matrix at t_k

A^T the matrix transpose of A

A^{-1} the matrix inverse of A

$E\{\ \}$ the expected value of $\{\ \}$

$(\ )_k$ the quantity ( ) at the time t_k

δ_{kj} the Kronecker symbol

$$\delta_{kj} = \begin{cases} 1 & \text{if } k = j \\ 0 & \text{if } k \neq j \end{cases}$$

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