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ABSTRACT

The paper reports the results of a research performed at Pisa Institute of Aeronautics on fatigue crack propagation in stiffened panels. Two problems have been faced. -Evaluation of the Paris theory for stiffened panels, -Evaluation of the fatigue strength of overloaded stiffeners in cracked stiffened panels. The research is based on a theoretical evaluation of the stress intensity factor and of the overload coefficient in the stiffeners for cracked stiffened panels, and on fatigue tests of 2024-T 3 and 7075-T 6 riveted stiffened panels.

The theoretical approach has been based both on the classical methods of the bidimensional theory of elasticity and on the finite element methods.

The tests have been performed utilizing load apparatuses suitable to test panels in a wide range of loads and dimensions.

Correlation of the theoretical and experimental results is presented and discussed in relation to the above mentioned questions.

I. SYMBOLS

a - quantity defined by formula (3)
b - stiffener pitch
h - distance of the junction line from the y axis parallel to the sheet middle plane through the barycentre of the stringer section
l - halfcrack length
n - number of load cycles
dl/dn - fatigue crack propagation rate (also FCPR)
p - rivet pitch
s - distance of the junction line from the x axis perpendicular to the sheet middle plane through the barycentre of the stringer section
t - sheet thickness
A - stiffener section area
C - ratio of the SIF in a stiffened panel and the SIF in the same unstiffened sheet at the same nominal gross stress
C' - overload coefficient: ratio between the maximum nominal stress in a stringer in a cracked panel and the nominal stress in the same stringer in the uncracked panel under the same load
- moments and products of inertia of the stringer section
H - quantity defined in formula (2)
K - stress intensity factor (SIF)
K_F - fatigue stress concentration factor for the stiffeners
N - fatigue endurance at a given load
R - stress ratio
α_c - modified area ratio defined by formula (3)
σ_a - alternating gross stress in the panel
σ_M - mean gross stress in the panel.

I. INTRODUCTION

The problem of investigating the fatigue crack propagation rate (FCPR) in aircraft structures has received considerable attention in technical and academic literature because of the increasing interest in the design of "fail-safe" structures.

The Paris theory (1), based on the idea of Stress Intensity Factor (SIF), has proved to be succesful in the case of simple plate loaded in tension with different systems of load(2); besides some evidence exists that the said theory succeeds in the case of cylindrical vessels of constant thickness repeatedly pressurized(3).

The case of cracked stiffened panels subjected to fatigue loads has also received attention from several investigators(4), (5); however it was not studied from a theoretical and experimental point of view so deeply as the simple cracked plate, so that many important questions remain to be clarified, particularly the influence of the stringers on FCPR.

In fact the cracking mechanism in a stiffened panel may be strongly influenced by stringer action whose essential features may be so summarized:

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* *This Work was sponsored in part by United States Army through its European Research Office under Contract DAJA37-72-C-1783.*

a) when a crack grows in the sheet of a stiffened panel, load is continuously transferred from the cracked sheet to the neighbouring stringers.
Such load transfer results in a slowing down of the crack rate in the sheet of the stiffened panel with respect to the crack rate found in the same cracked unstiffened sheet.

b) Stringer overloading may initiate fatigue cracks in the stringer. These cracks generally start from a rivet hole and grow very quickly; the so cracked stringer fails after relatively few load cycles causing a sudden increase of the crack rate in the sheet.
Consequently the correct evaluation, from a fail safe point of view, of a crack propagating under fatigue load, involves answers to these questions:

1) How much of a retardation effect on the crack propagation rate do stringers have in the sheet of a stiffened panel compared with the crack rate found in the same unstiffened sheet?

2) How is the crack propagation rate in the sheet modified by the failure of one or more stringers?

3) What relationship exists between the crack length in the sheet, the fatigue strength of the most stressed stringer and the applied load?

II. RESEARCH OBJECTIVES

On the basis of the aforementioned considerations at Pisa Istitute of Aeronautics we have been carrying out a research program to face, from a theoretical and experimental point of view, the problems above stated in points 1,2,3 in the case of riveted stiffened panels, undergoing constant amplitude fatigue loads.

In planning our research we thought that the Paris theory could be a well grounded starting point for investigating the questions mentioned in points 1 and 2.

To apply the above mentioned theory to solve such problems we need, at first, theoretical methods to evaluate the SIF in the cracked stiffened panels as function of the crack pattern, the panel dimensions, and the material characteristics.

Such methods in conjunction with an adequate set of experimental data should give us a powerful device for correlating the FCPR in stiffened and unstiffened panels, and building up criteria for evaluating the efficiency of stringers in slowing down the FCPR.

To face the problem of the fatigue strength of the stiffeners (the problem stated in point 3) a well grounded starting point is the knowledge of the loads that develop in the stiffeners due to the cracking of the sheet. These loads can be adequately characterized through the overload coefficient C', namely the ratio between the maximum nominal* stress in a stringer in the cracked panel and the stress on the same stringer in the uncracked panel under the same applied load. Clearly this coefficient can be obtained through the same theoretical approach used to determine the SIF.

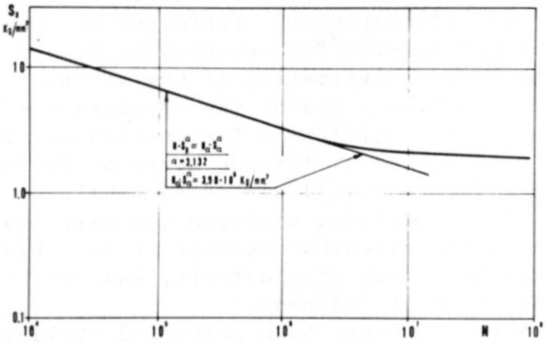

Fig. 1: 50% probability S_a-N curve for dry riveted lap joints.

The adequacy of the overload coefficient as a driving parameter in facing the problem of determining a relationship among the fatigue strength of a stiffener, the length of a crack in the sheet and the applied fatigue load, can be shown on the basis of the following considerations.

With reference to the most stressed stiffeners the stress significant for evaluating the fatigue strength of the stiffeners is given by:

$$S = K_F C' \sigma$$

where σ is the gross nominal stress in the panel, and K_F the fatigue stress concentration factor for the stiffener. When the nominal stress is periodically varied in the range $\sigma_M \pm \sigma_a$ the mean and the alternating stresses are given respectively by:

$$S_M = K_F C' \sigma_M \; ; \; S_a = K_F C' \sigma_a$$

Since C' increases as a crack develops in the sheet, both S_M and S_a increase with crack length.

To evaluate the fatigue strength of the

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* *To obtain the effective maximum stress one must know the stress concentration factor that depends on the factors specified in the section "Experimental Research Program".*

stiffener as function of the crack lenght in the sheet we can use Miner's rule that for the present case can be written:

$$\frac{n_o}{N_o} + \int_{n_o}^{n_1} \left(\frac{dn}{N}\right)_1 = A$$

where n_o/N_o is the fraction of the fatigue endurance of the stiffener spent before crack initiation, dn/N is the infinitesimal fraction of the fatigue endurance spent at the stress level $S_M \pm S_a$ corresponding to a crack length equal to l, n_1 is the number of cycles expected to cause the fatigue failure of the stiffener; finally, A is a suitable constant that will be specified later.

With the notations $N = F(S_M,S_a)$, SIF = K FCPR = $dl/dn=f(K_a,R)$, $R=(1-\sigma_a/\sigma_M)/(1+\sigma_a/\sigma_M)$ we obtain:

$$\frac{n_o}{N_o} + \int_{lo}^{\bar{l}} \frac{dl}{f(K_a,R)F(S_M,S_a)} = A \qquad (1)$$

where lo is the initial crack length in the sheet and $\bar{l}$ is the crack length in the sheet at which a crack starts in the stiffener.

In formula (1) f can be computed utilizing a reliable relationship among FCPR, K_a and R in conjunction with the theoretical results o SIF for stiffened panels.

The evaluation of the F function is a more complicated question. The knowledge of K_F in the case of stiffeners loaded by the applied load and rivet loads through the junction lines sheet-stiffeners, would give the opportunity to use the set of S_a-N-R curves relative to the unnotched selected material, for evaluating the function $F(S_M,S_a)$, that appears in the integral of formula (1). However, as it is difficult to asses accurately the above specified K_F, one can try to overcome the obstacle selecting a given set of S_a-N-R curves and determining through an adequate set of experimental data a nominal value of K_F. The K_F value so determined can be utilized in conjunction with the above mentioned set of curves to evaluate through formula (1) the crack length $\bar{l}$.

Among the possible sets of S-N-R curves we thought of choosing the ones relative to the dry riveted lap joint loaded in tension. Such curves represent a well grounded starting point, as the load and stress configurations in the two cases of the stiffeners and of the joints have many features in common in the zone of expected fatigue failure. As reported in (6) the fatigue strength of riveted lap joints can be considered indipendent of the mean value of the applied fatigue load. Supposing that the same is true for stiffeners in a cracked panel, the evaluation of F function can be performed only on the basis of the S_a-N curve.

With the notation of fig.1 when the stiffener fails in the range of load cycles $n \leq 3.10^6$ we can put formula (1) in the form:

$$\int_{lo}^{\bar{l}} \frac{dl}{f(K_a,R)(C')^{-\alpha}} = \left(A - \frac{n_o}{N_o}\right) N_\alpha S_\alpha^\alpha (K_F \sigma_a)^{-\alpha}$$

The above formula can be further simplified when it is $n_o/N_o \ll A$ as happen in fatigue crack propagation tests where the crack is artificially started. In such conditions we obtain:

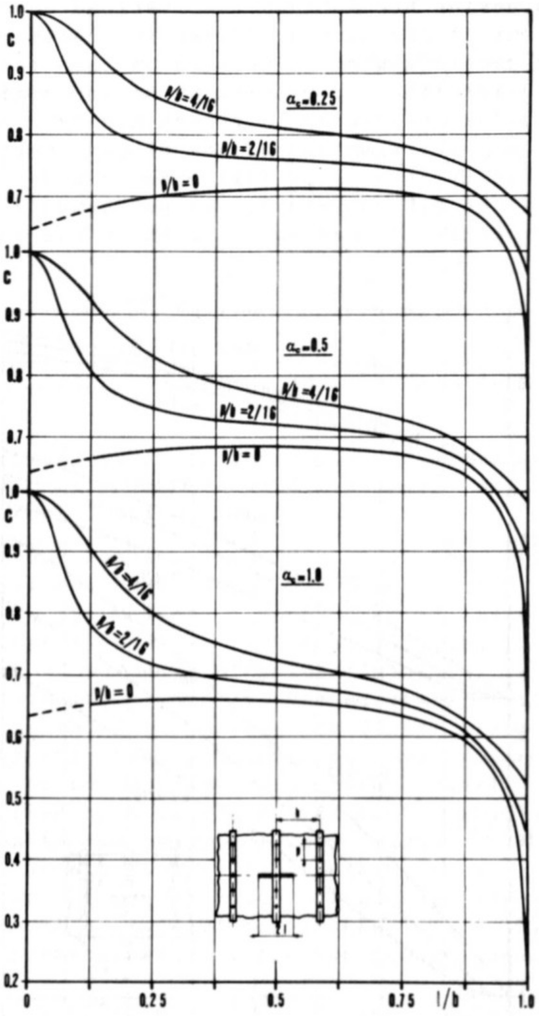


Fig.2a: C versus half crack length at different values of p/b and α_c for the crack pattern shown in figure.

$$\int_{l_o}^{\bar{l}} \frac{dl}{f(K_a,R)(C')^{-\alpha}} = AK_F^{-\alpha}\; H\; \sigma_a^{-\alpha} \qquad (2)$$

where H depends only on the chosen S_a-N curve.
Performing an adequate group of tests with appropriate loads, formula (2) gives the possibility of obtaining the quantity $AK_F^{-\alpha}$. In the absence of better information on the stress concentration in the stiffener of a cracked panel K_F can be computed on the basis of the usual position A=1.

III. THEORETICAL RESULTS

On the basis of the above stated considerations we developed a computer program for evaluating the SIF and the overload coefficient C' for each stiffener for rectangular panels loaded in tension on the two opposite sides perpendicular to the stringer axes. The theoretical approach has been based on the matrix method of the displacements in the hypothesis of full elastic body(7), (8). The SIF computation is performed through the C coefficient (namely the ratio

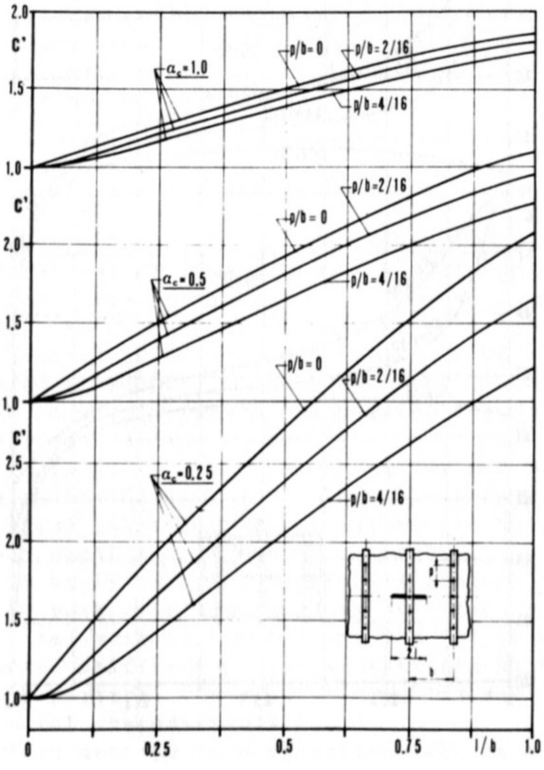


Fig. 2b: C' versus half crack length at different values of p/b and α_c for the crack pattern shown in figure.

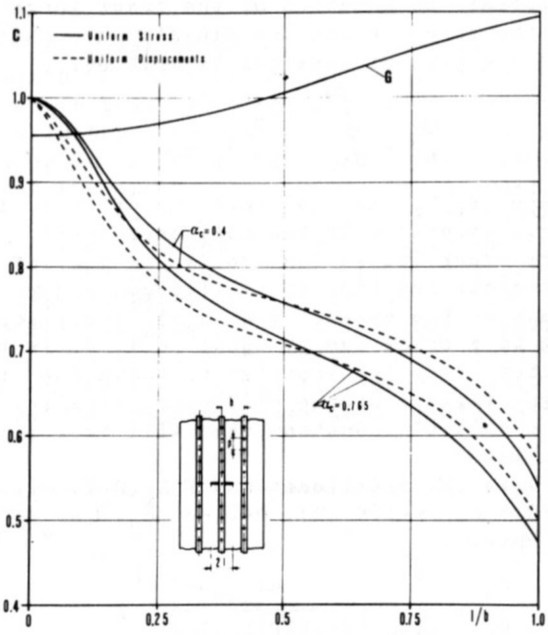


Fig. 3a: C versus halfcrack length in the two case of uniform stress and uniform displacements at the loaded edges. At the top of the figure also the coefficient G is reported; G is the ratio of SIF in a plate with uniform displacements at the loaded edge to the SIF in a plate with uniform stress at the same edge.

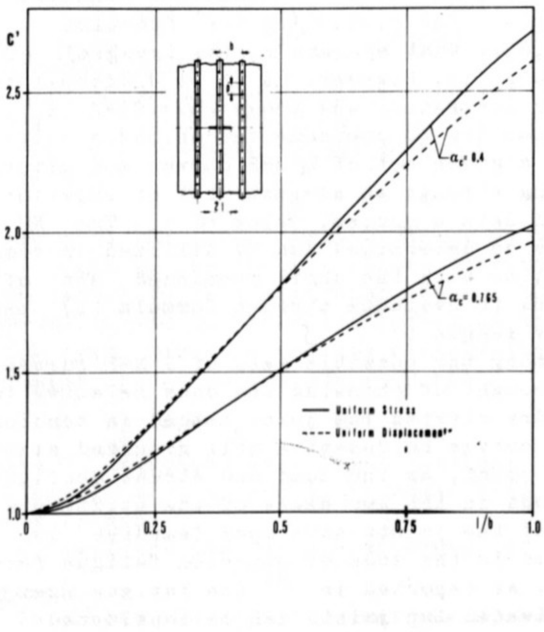


Fig. 3b: C' versus halfcrack length in the two cases of uniform stress and uniform displacement at the loaded edge.

between the SIF in the stiffened panel and the SIF in the same unstiffened panel) and the usual well known formulas for the stress intensity factor in a cracked sheet loaded in tension.

The program takes into account different panel configurations. A configuration is defined through the external dimensions (length and width) of the panel, the crack pattern in the sheet (cracks are bound to be straight and perpendicular to the stringer axes) the number of broken stringers, and the type of load distribution.

For each panel configuration the results are presented in the form:

$$C = C(l/b,\alpha_c,p/b) \qquad C' = C'(l/b,\alpha_c,p/b)$$

where l is the halfcrack length, b the stiffener pitch, p the rivet pitch, α_c the modified area ratio given by:

$$\alpha_c=\frac{bt}{A(1+a)^{-1}}\ ;\ a=\frac{h^2/I_x+s^2/I_y+2shI_{xy}/I_xI_y}{1-I_{xy}^2/I_xI_y}A \qquad (3)$$

where a is a quantity which takes into account, with sufficient accuracy, the bending deformation of the stiffeners due to the crack in the sheet.

Figs 2a and 2b give typical values of

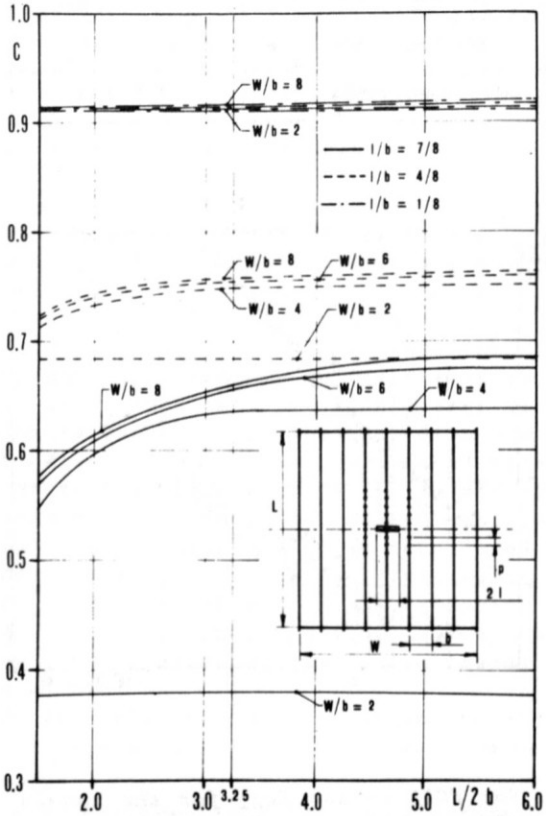


Fig.4a: C versus reduced panel height L/2b for different reduced widths w/b and crack lengths.

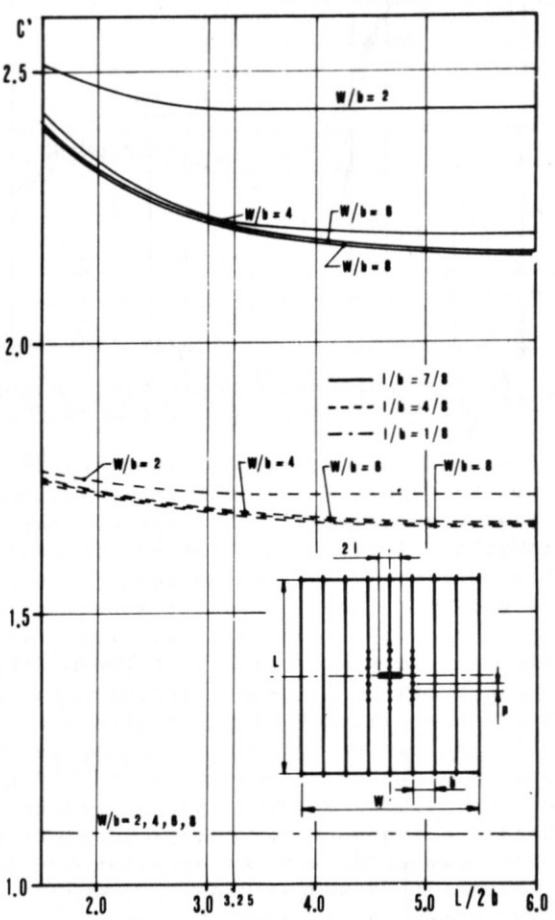


Fig.4b: C' versus reduced panel height L/2b for different reduced widths w/b and crack lengths.

the above mentioned coefficients. The influence of the load distribution (uniform stress, and uniform displacement at the loaded edges) is shown in fig.3; finally, the effect of the general panel dimensions are shown in fig.4a and 4b. The accuracy of the results obtained through the finite element method was checked through the available results on an infinite wide cracked panel, stiffened by a single stiffener(9) relative errors in the range ±2% were found (10).

To obtain further information about the accuracy of the above described approach we developed a new computer program utilizing an approach based on the continuum mechanics.

The coefficients C and C' were evaluated through a resolution of the well known equations of the plane elasticity based on the conformal mapping. The approach is the same as used in (9), modified to take into account the presence of several stringers. The results obtained, which can only be applied to infinite panels, are very useful

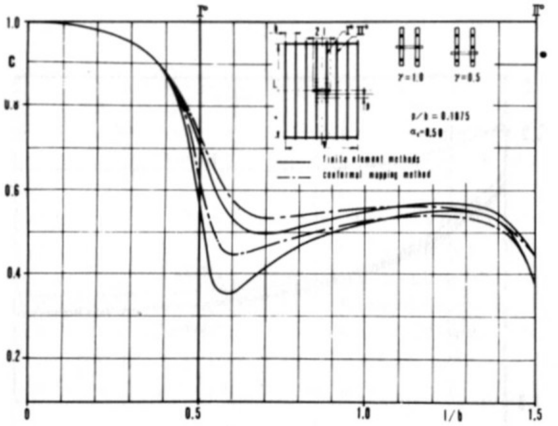


Fig. 5: C as obtained through the two methods quoted at the top right of the figure.

to controll the accuracy of the results obtained through the finite element methods in a wide and significant range of panel dimensions. A comparison between typical results obtained with the two methods is shown in fig.5. The finite elements method generally gives quite good results, except in a small zone in the proximity of the minimum. Further research is being carried out to improve the accuracy of the finite element methods utilizing an approach based on the use of a singular cracked element $^{(11)}$. However the results so far obtained can be considered adequate for the purpose of the present research, also in relation to the difficulty to assess accurately C and C' in the aforementioned zones through a theoretical approach.

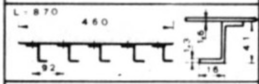
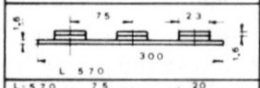
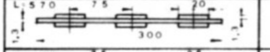
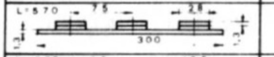
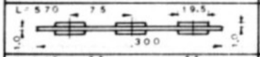
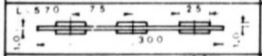
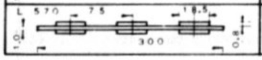
IV. EXPERIMENTAL RESEARCH PROGRAM

The theoretical approach is undoubtedly a powerful device for facing the problem on a sound basis, but only a convenient set of experimental data, interpreted and generalized on the basis of the theoretical ones, can give a correct answer, to the design problem previously stated in points 1,2,3 of the Introduction.

In fact it is necessary not only to verify the reliability of the Paris theory for stiffened panels, with cracked or uncracked stringers, but also to investigate the fatigue behaviour of overloaded stringers in cracked stiffened panels; this investigation should evaluate the reliability of the coefficient C' as fundamental parameter in correlating the fatigue strength of stringers in cracked panels on the basis of the ideas discussed in section II.

Both the questions, moreover, must be investigated taking also into account the effects of simplifications introduced into the theoretical analysis; the following ones are particularly significant in relation to the topics under investigation:

a) the sheet buckling in the area surrounding the crack has been neglected;
b) the effect on the stress state due to the geometrical discontinuities of the rivet holes has been disregarded;
c) the effects on the stress state in the zones surrounding holes, due to the forces acting through the rivets, have been treated in a simplified manner; in fact these forces have been replaced by concetrated ones applied at nodal points;
d) Yielding due to local rivet bearing stresses has been neglected

TYPE OF PANELS	SYMBOLS	TYPE OF STRESSES		K-Values at which the first stiffener failed.
		σ_{MAX}	R	
	▼	9	0.6	52.3
	▽	8.5	0.6	No K
	○	9	0.6	No K
	■	8.5	0.6	No K
	✱	12	0.6	61.0
	+	7	0.6	No K
	▲	11	0.6	56.0
	●	12	0.6	61.6
	△	11	0.6	56.4

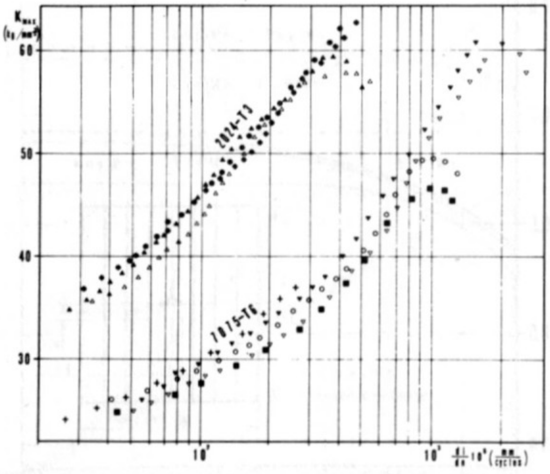


Fig. 6a: FCPR versus K_{MAX} for the panels shown at the top of the figure for two different materials, 2024-T3, 7075-T6. Stresses quoted in Fig. 6a as well as in Fig. 6b are given in Kg/mm^2.

TEST APPARATUS	MAX. PANEL DIMENSIONS LENGTH x WIDTH	TYPE OF STRINGERS	STATIC TESTS LOADS RANGE	DYNAMICS TESTS	
				LOADS RANGE	FREQUENCY (CYCLES/MIN.)
1	600x400	Only strip type	0 + 20	0+10	200 + 1000
2	950x600	Every type	-60 ÷ 40	-40+40	30+70,200+600
3	1300x900	Every type	0 + 60 -60 + 0	-0+60 -40+0	30+70,200+600

Tab. I - Principal characteristics of the three load apparatuses working at Fatigue Laboratory of Pisa Institute of Aeronautics. Loads, reported in metric tons, are given by hydraulic jacks; the pressure may be controlled by two different machines: a SBE low frequency (30+70 c/m) high deformation machine or a 300 cc. pulsator controlled by a programmed load system.

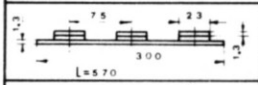
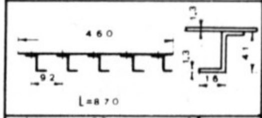
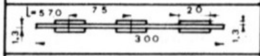
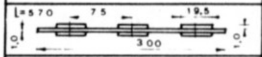
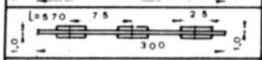
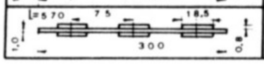
All the simplifications may be important.

The buckle pattern in a cracked stiffened sheet is influenced not only by the dimensions and the pitch of the stiffeners but also by the position of the crack with respect to the stiffeners. Two typical configurations can be considered for exemplification purposes: a crack starting from a junction line, and one starting from a line between two stringers, both the cracks propagating simmetrically with respect to the starting line.

In the first case the buckling surface is antisymmetrical with respect to the starting line; in the second case the buckling surface is symmetrical with respect to the starting line *

The simplifications stated at the points c,d,e clarify principally, the reason why coefficient C' cannot be the only parameter for assessing the fatigue strength of an overloaded stringer in a cracked panel; we need in fact to utilize K_F to take into account those effects due to the presence of holes and rivets that are significant in relation to the fatigue behaviour of the stiffener. Also the SIF values calculated with the above described methods can be affected by some error owing to the stated simplifications, particularly when we considered cracks whose tips approach a rivet hole or small cracks starting from a rivet hole or large cracks for which yelding due to rivet bearing stresses has been exceeded in some points.

To gain all the information on the above discussed topics we had, above all, to test

TYPE OF PANELS	SYMBOLS	TYPE OF STRESSES		K-Values at which the first stiffener failed.
		σ_{MAX}	R	
	▼	9	0.4	46.1
	■	8	0.4	40.0
	▽	9	0.4	48.6
	+	8	0.4	42.0
	○	7.5	0.4	52.5
	✱	12	0.6	61.0
	▲	11	0.6	56.0
	●	12	0.6	61.6
	△	11	0.6	56.4

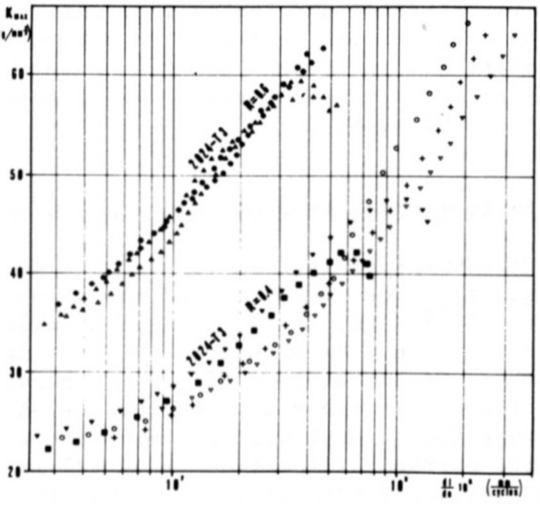


Fig. 6b: FCPR versus K_{MAX} for the panels shown at the top of the figure for two different values of the stress ratio R.

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* *Such considerations have been substantiated by several tests performed at Pisa Institute of Aeronautics as resported in (12)*

stiffened panels.

To do this we designed and built three different load apparatuses$^{(7)}$ whose principal performances are reported in Tab.I.

At the same time we set up an experimental program to investigate the fatigue crack-propagation in aluminium alloy rectangular panels stiffened by riveted stringers.

The panels are loaded in tension on two opposite sides and stiffened by stringers whose axes are parallel to the applied load direction.

Different types of stiffened panels have been considered for test: panels stiffened by strips and panels stiffened by Z section stringers, both in 2024-T3 and 7075-T6 aluminium alloy.

Panels stiffened by strips have two or three stiffeners, panels stiffened by Z section stringers have five, seven or eight stiffeners.

In all the tested panels cracks were started through a stress raiser; all the cracks propagate approximately perpendicularly to the applied load; for the stiffened panels different points of crack initiation were considered, namely:

- crack starting from a rivet hole
- crack starting from a point between two rivets
- crack starting from the middle line between two stiffeners and propagating on a line joining two rivets
- crack starting from the middle line between two stiffeners and propagating on a line intersecting junction lines between two rivets.

We are planning to test 60 panels. A lot of 50 panels have been tested so far; all the experimental and theoretical results pertaining to each test will be reported in a report now in preparation$^{(13)}$. Some of the more significant results will be discussed in the next section.

V. FATIGUE CRACK PROPAGATION DATA EVALUATION

Figs.6 report typical values of FCPR versus K_{max} for two values of the stress ratio R and two different type of materials. Data on FCPR were obtained testing at constant amplitude load the panels shown on the top of the figures; K was evaluated through the theoretical approach discussed in the section "Theoretical Results". In each test the crack started from a rivet hole on the central stringer and propagated in both directions. The tests were interrupted when crack stopped in a rivet hole on a stringer near the central one. The results of figs 6 shows that stiffened panels with different geometrical dimensions have approximately the same FCPR for the same theoretical K value.

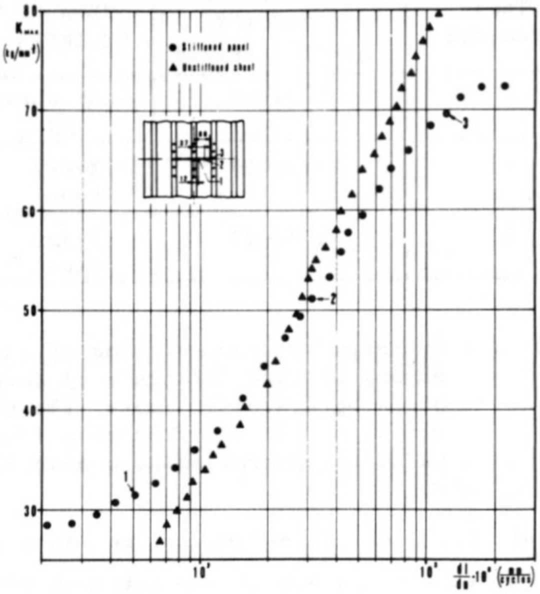


Fig.7a: FCPR versus K_{MAX} for a simple sheet and for a stiffened panel. The stiffened panel curve shows in the low and high crack length range different trends from the one of the unstiffened sheet.

Further the same results indicate that the curves of FCPR versus K_{MAX} conform generally to the ones found in the case of unstiffened panels at least in the middle crack length range; different trends are found on the contrary in the low and high crack length ranges.

To emphasize these differences in fig.7a we reported K and l as function of FCPR for a stiffened panel for which these trends were particularly noticeable, together with the data for an unstiffened sheet. For a given K value the low crack length part of the stiffened panel curve generally exhibits lower FCPR than the unstiffened panel curve. On the contrary in the high crack length range the stiffened panel curve shows an opposite trend, particularly beyond the point of maximum.

The trend in the low crack length range depends on the theoretical approach that overestimates the SIF. In fact, owing to the symmetry of the rivet forces with respect to the crack line, the rivets on the crack line are unloaded; consequently the theoretical approach, owing to the simplifications discussed in points b and c of section IV, gives the same K values both when the rivets

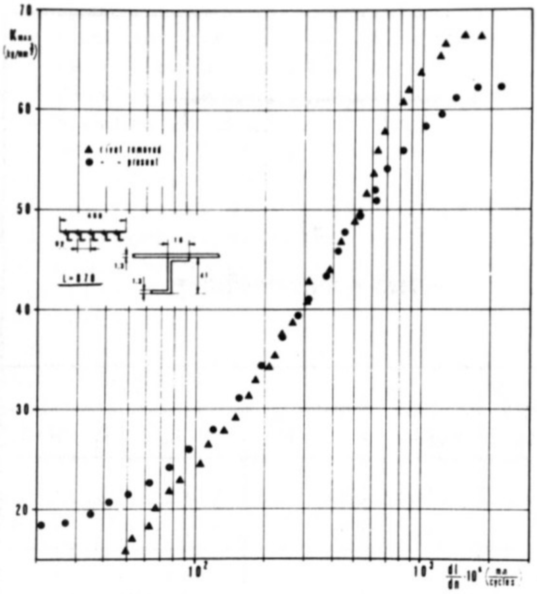


Fig. 7b: FCPR versus K_{MAX} for two stiffened panels. The cracks start from a rivet hole in both the case. In one panel the rivet was removed from the hole before starting the test.

on the crack line are present and when the same rivets are non existent. However this two situations are not really equivalent.

The riveting clamps the sheet to the stiffener so that friction forces are introduced in the sheet by the stiffeners. For small cracks, these friction forces are, probably, of the same order of the ones which develop in the case of continuous bonding. In such conditions riveting acts approximately as a continuous bonding. Increasing the crack, friction forces become negligibly smaller and smaller with respect to the rivet forces so that the theoretical approach gives correct results.

This point of view is confirmed by the results reported in fig. 7b which shows FCPR versus K in a stiffened panel where the rivet was removed from the crack starting hole before the test; the curve in the low l range conforms to the one of a simple plate.

Further if one conforms experimental data, obtained in the low crack length range to the expected trend of the FCPR versus K curve, values of coefficient C are found in good accordance with the theoretical ones for continuous bonding at least for small cracks as it is shown in fig. 7c.

With reference to the high crack length range the theoretical approach underestimates K owing to the approximations, discussed in points b, c at the section IV, which can produce significant effects above all when the crack tip approaches a rivet line. The rivet hole, particularly, causes an increase of the SIF that is completely disregarded in the theoretical approach. Fig. 7c

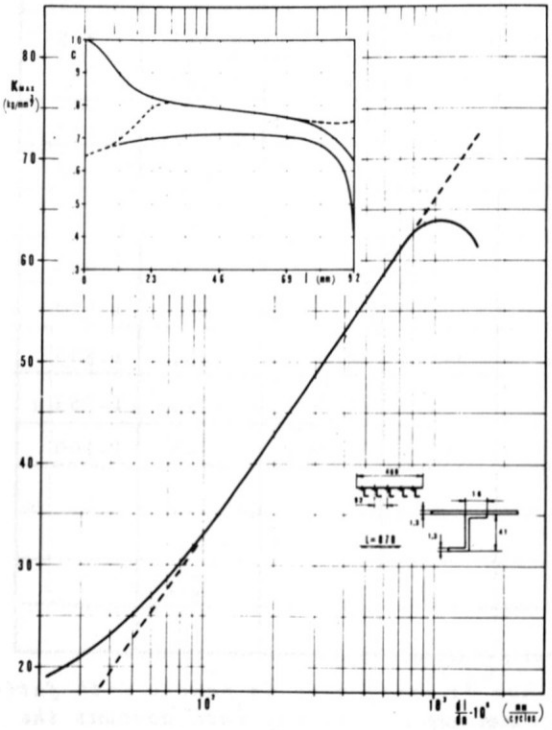


Fig. 7c: Coefficient C versus halfcrack length as suggested by the expexted trend of the FCPR versus K curve.

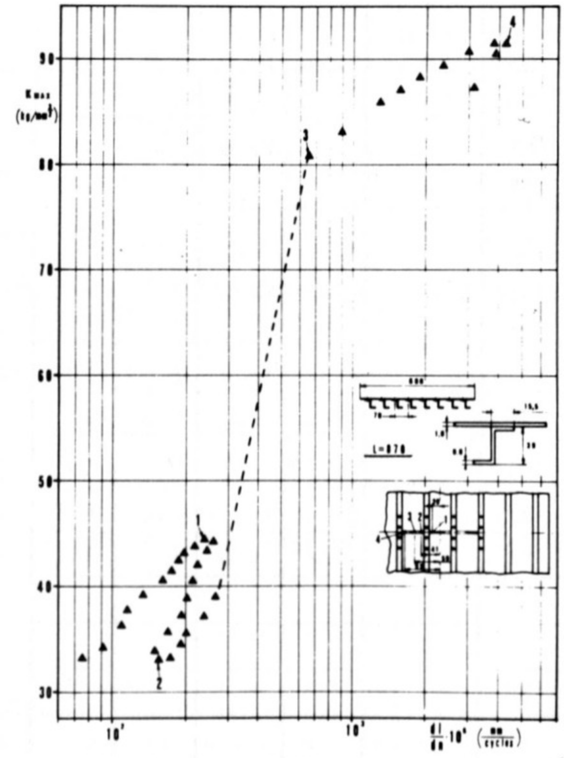
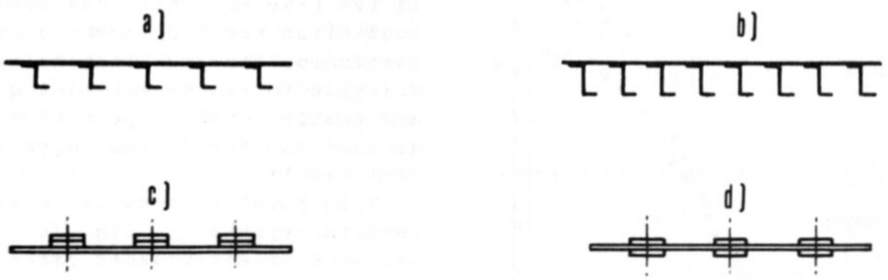


Fig. 8: FCPR versus K_{MAX} for the stiffened panel shown in figure. The dashed curve refers to a part of crack propagation test during which cracks developed in the first two stiffeners.



SPECIMEN N°	Type of panel	α_c	σ_{MAX} kg/mm²	σ_m	l_0 -mm-	$\bar{l}$ -mm-	$n_0 \cdot 10^{-3}$ cycles	$\bar{n} \cdot 10^{-3}$ cycles	$\alpha = 3.132$		
									$H \cdot 10^{-6}$	$A \cdot K_F^{-\alpha}$	K_F *(A=1)
41	a	0.765	8.5	6.8	8	75	285	1580	19	0.477	1.265
45	a	0.614	9.0	6.3	7	70	120.	419.5	24	0.600	1.175
46	a	0.614	8.0	5.6	7	75	80	468	21.25	0.534	1.220
47	a	0.25	9.0	6.3	7	45	32	169.7	9.92	0.249	1.560
48	a	0.25	8.0	5.6	8	40	80	258.5	12.96	0.325	1.430
49	a	0.25	7.5	5.25	8	60	98	351	18.54	0.466	1.277
51	d	0.53	9.0	7.2	11	41	120	940	12.83	0.322	1.440
52	d	0.53	13.0	10.4	12	35	46	263	11.18	0.281	1.500
53	d	0.53	12.0	9.6	11	35	50	352	10.6	0.266	1.528
54	d	0.53	11.0	8.8	11	37	45	402.2	9.47	0.238	1.580
55	d	0.53	10.0	8.0	11	39	54.5	102.8	11.6	0.292	1.485
56	d	0.53	11.0	8.8	12	32	108	540.3	11.71	0.294	1.480
57	d	0.53	10.0	8.0	11	41	178	779.8	16.23	0.408	1.345
58	d	0.53	12.0	9.6	11	51	39	372.6	15.58	0.392	1.350
59	d	0.53	11.0	8.8	12	54	57.5	447.	13.65	0.343	1.410
60	d	0.53	11.0	8.8	11	47	50	373.4	10.49	0.264	1.528
71	a	0.25	10.0	6.5	13	39	251	930	61	1.530	0.875
76	d	0.52	11.0	8.8	12	75	103	627.9	30	0.755	1.095
78	d	0.40	11.0	8.8	13	43.5	50	343	12.34	0.310	1.450
79	d	0.667	12.0	9.6	15	57.5	46.1	341.6	15.8	0.397	1.345
82	b	0.26	11.0	8.8	8.5	50.5	129.8	630.5	19.7	0.495	1.253
87	b	0.26	9.0	6.3	11	56.5	30.2	320	31.9	0.926	1.100

Tab. II - Data from 22 tests on the fatigue endurance of stiffeners in panels with fatigue cracks propagating in the sheet. K_F has been computed taking into account the ratio n_0/N_0.

gives also an idea of the differences in the high crack length range between the the theoretical value of coefficient C,and the one evaluated on the basis of the expected trend of the FCPR versus K curve.

Fig.8 refers to a different crack pattern; the results were obtained testing a eight stringer panel where the crack crossed rivet lines between two rivets; the crack was started from the middle line of the panel between two stringers. In this case the decrease of the rivet crack rate experimentally found when the crack tip crosses the first rivet line conforms better to the decrease in SIF given by theoretical approach, in accordance with the fact that the rivet hole can have only minor influence on the SIF, with this crack configuration.

However the decrease in FCPR is lower than the predicted through the SIF theoretical diminution; this fact may mean that also the other approximations may be portant. The part of the curve shown at the top right of the figure refers to a crack approaching the second rivet lines; the crack rates experimentally found are higher than expected in relation to the theoretical SIF values. This fact can be probably ascribed to the overcoming of the yelding bearing stress around holes lodging highly stressed rivets; such an occurence gives rise to a redistribution of the rivet forces that probably lowers the efficiency of the stiffeners in reducing the SIF of the cracked panel.

Fig.9 finally compares the results obtained testing two sheet one free to buckle, one with antibucking guides, and a stiffened panel free to buckle with crack starting from a rivet hole.. These results show that buckling has a minor effect on the FCPR versus K trend and that the stiffened panel has a behaviour similar to the unbuckled, unstiffened sheet in accordance with the fact that stiffeners restrain the buckling of the sheet. Caution must be used however to generalize these data because buckling is a non linear phenomenum with respect to the applied load and is strongly dependent on the thickness of the sheet; however,even if the data on this topic are very limited, it seems that in the range of fatigue stresses and crack length that are likely to be found in aircraft structures buckling has a minor effect on fatigue crack propagation rate.

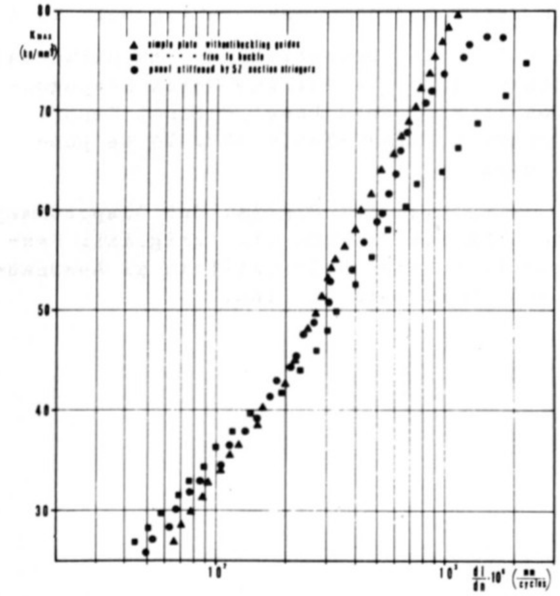


Fig. 9: Effect of buckling on FCPR versus K_{MAX}.

VI. FATIGUE STRENGTH OF THE STIFFENERS

Tab.II reports the results of 22 tests on stiffened panels where stiffeners failed during the propagation of fatigue cracks in the sheet. The panel dimensions and the load conditions, particularly the mean stress,were varied in a range adequate to the research objective. The data have been treated on the basis of the method outlined in section II. The K_F values were obtained with the usual position A=1. Log K_F conforms to an extreme-value distribution with a characteristic value $(K_F)_m = 1,4$ and a dispersion parameter $d = 10,45$ These results suggest that the fatigue strength of a stiffener in a panel with cracked sheet can be evaluated on the basis of the proposed method assuming A=1. The K_F percentile can be chosen, on the basis of the aforementioned statistical properties of the K_F population,in relation to the allowed reliability of stringer endurance estimate.

The method is strictly valid only for constant amplitude loads; for other types of fatigue loads further research need to be carried out to define the modifications to be introduced in the aforementioned method.

VII. CONCLUSIONS

A research on the behaviour of cracked stiffened panels under constant amplitude loads has been performed at Pisa Institute of Aeronautics.

The research has faced the following two main problems:

- evaluation of the reliability of Paris theory for cracked stiffened panels

- evaluation the fatigue endurance,of stiffeners overloaded owing to the cracking of the sheet,as function of crack length in the sheet.

At first a theoretical method has been developed to evaluate the stress intensity factor in the sheet and the overload coefficient in stiffeners of a given cracked stiffened panel. The fundamental idea about the theoretical approach have been discussed in section II.

The theoretical results, obtained through a computer program, have been used to correlate the data on the fatigue crack propagation rate in the sheet and on the fatigue endurance of stiffeners obtained testing, under constant amplitude loads, riveted stiffened panels of different geometric dimensions with several crack patterns. Particularly the data on the fatigue endurance of the stiffeners have been treated on the basis of an original approach based on the Miner's rule discussed in section II.

Some of the results obtained with the outlined procedure have been discussed in section V and VI. Some significant conclusions can be drawn from such results.

At first the results so far obtained seems to validate the Paris theory for stiffened panels; in fact different stiffened panels, fig.6a and 6b, have approximately the same crack rate for the same value of K computed with the theoretical methods discussed in section III.

However when the FCPR versus K curves found for stiffened panels are compared with the ones found for unstiffened sheet, some remarkable differences exist;above all in the crack length range discussed in section V. These differences seem to depend on the inadequacy of the theoretical approach to evaluate correctly the SIF in the aforementioned crack length ranges. Some possible explanations of such inadequacy have been suggested; from which matter for further research can be drawn.

Finally the experimental results reported in section VI indicate that the evaluation of the fatigue endurance of stiffeners undergoing fatigue loads through the cracked sheet can be performed in a quite reliable way through the methods proposed in section II.

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