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THE STUDY OF GASEOUS JET EXHAUSTING INTO VACUUM

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Abstract

The problem of gas expanding into vacuum is formulated on the basis of the Krook's kinetic equation. The two-dimensional flow is considered as an example. The numerical scheme of solution is proposed, valid for any Knudsen's number. It consists of an iterational process combined with averaging over the lateral component of molecular velocity. After introduction of the characteristic variables the finite-difference equations substitute for differential ones. Initial equations are formulated for a distribution function but the numerical method permits to store in the grid points only macroscopic flow parameters. Numerical results showing variation of these parameters are represented in graphical form.

1. Introduction and Formulation of the problem

The problem of expansion of gaseous jets has many different aspects. Depending on the conditions of expansion itself and on the initial state of gaseous medium one could list several classes of jet flows, and everyone of them needs essentially different mathematical approach to study it. There are (1) incompressible, non-viscous jets, (2) subsonic, compressible, non-viscous jets, (3) supersonic compressible, non-viscous jets, (4) laminar, viscous, incompressible jets, (5) turbulent incompressible jets, (6) laminar, viscous, compressible jets, (7) turbulent compressible jets, (8) jets of rarefied gases.

Detailed rewievs of the methods of investigation of all but the last types of flow may be found in [1,2]. The short rewiev of methods for so called "rarefied" jets exists in [3].

Let us consider the case of jet exhausting into vacuum in steady flow conditions. In such a case inside of jet itself the regions of very low density will be inevitably created, and for the investigation it is necessary to use the methods of the kinetic theory. The case of a free molecular jet flow was considered by Knudsen [4]. Later there were found some amendments to a free molecular solution [5-8], but up to the present time the full analysis of the kinetic problem for arbitrary Knudsen number was not carried out. Such an analysis is suggested below for the case of two-dimensional motion. In principle, the present approach will do for the axisymmetrical flow, too, except of some difficulties connected with the impossibility of excluding from consideration the third component of molecular velocity.

Let the gas, having at infinity the density n_∞ and temperature T_∞, flow out of the half-space bounded by a thin wall into vacuum, through an infinitely long slit of the width 2R (Fig.1). Assuming the flow to be steady we shall use for its description the kinetic equation in the form of a Krook's model [9]:

$$\xi_x \frac{\partial f}{\partial x} + \xi_y \frac{\partial f}{\partial y} = \nu (F - f), \qquad (1)$$

$$F = n \left(\frac{h}{\pi}\right)^{3/2} \exp\left(-h(\vec{\xi} - \vec{u})^2\right),$$

$$h = m/2kT, \quad \nu = nkT/\mu.$$

Here $f(x,y,\xi_x,\xi_y,\xi_z)$ is the distribution function, ν - collision frequency, μ - viscosity coefficient, m - mass of the molecule, k - Boltzmann constant, n - numerical density, T - temperature, $\vec{u}$ - macroscopic velocity of gas.

There are standard relations to specify macroscopic parameters:

$$n=\int f\,d\vec{\xi}\,,\quad \vec{u}=\frac{1}{n}\int \vec{\xi} f\,d\vec{\xi}\,,$$
$$\frac{3}{2}kT=\frac{1}{n}\int \frac{m}{2}\left(\vec{\xi}-\vec{u}\right)^2 f\,d\vec{\xi}\,. \qquad (2)$$

In the case of maxwellian molecules, when $\mu \sim T$, the collision frequency is

$$\nu = An. \qquad (3)$$

The boundary conditions in infinity are obvious from the formulation of the problem:

$$f(\infty,y,\vec{\xi}) = \qquad (4)$$
$$= n_\infty\left(\frac{h_\infty}{\pi}\right)^{3/2}\exp\left(-h_\infty\xi^2\right) \quad \text{for } \xi_x<0,$$
$$f(-\infty,y,\vec{\xi})=0 \quad \text{for } \xi_x>0.$$

To specify the boundary conditions at the wall we assume that the wall's material is non-heat-conducting and that the molecules are reflected from the surface diffusely with the complete thermal accomodation. The distribution function of the particles emitted from the vacuum-faced side of the wall, is denoted by f^-, whereas for the particles emitted from the opposite side we shall have f^+. Then the boundary conditions at the wall are

$$f^+(y,\vec{\xi})=n_+\left(\frac{h_+}{\pi}\right)^{3/2}\exp\left(-h_+\xi^2\right)$$
$$\text{for } |y|>R,\ \xi_x>0, \qquad (5)$$

$$f^-(y,\vec{\xi})=n_-\left(\frac{h_-}{\pi}\right)^{3/2}\exp\left(-h_-\xi^2\right)$$
$$\text{for } |y|>R,\ \xi_x<0.$$

In order to find the values $n_\pm(y)$ and $T_\pm(y)$ which are not known in advance one should use the conditions of conservation of the flows of mass and energy at the wall (that is, for $x=0$, $|y|>R$):

$$-\int_{\xi_x<0}\xi_x f(0,y,\vec{\xi})\,d\vec{\xi}=\frac{n_+}{2\sqrt{\pi h_+}}\,,$$
$$-\int_{\xi_x<0}\frac{m}{2}\xi_x\xi^2 f(0,y,\vec{\xi})\,d\vec{\xi}=\frac{mn_+}{2\sqrt{\pi h_+^3}}\,, \qquad (6)$$
$$\int_{\xi_x>0}\xi_x f(0,y,\vec{\xi})\,d\vec{\xi}=\frac{n_-}{2\sqrt{\pi h_-}}\,,$$
$$\int_{\xi_x>0}\frac{m}{2}\xi_x\xi^2 f(0,y,\vec{\xi})\,d\vec{\xi}=\frac{mn_-}{2\sqrt{\pi h_-^3}}\,.$$

Thus, mathematically the problem is reduced to that of solution of the nonlinear integro-differential equation (1) with the boundary conditions (4)-(6).

2. Transformation of the Equations

The main equation (1) may be simplied fied by means of integration in the velocity space over z - component from $-\infty$ to $+\infty$. Then, after multiplying all terms of equation (1) by ξ_z^2 and making the same integration, we shall ultimately obtain two equations:

$$\xi_x\frac{\partial f_j}{\partial x}+\xi_y\frac{\partial f_j}{\partial y}=\nu\left(F_j-f_j\right) \quad /\ j=1,2\ /$$
$$F_1=n\frac{h}{\pi}\exp\left(-hc^2\right), \qquad (7)$$
$$F_2=\frac{n}{2\pi}\exp\left(-hc^2\right)$$

where $c^2=(\xi_x-u_x)^2+(\xi_y-u_y)^2$

and, by definition,

$$f_1(x,y,\xi_x,\xi_y)=\int f\,d\xi_z\,,$$
$$f_2(x,y,\xi_x,\xi_y)=\int f\xi_z^2\,d\xi_z\,. \qquad (8)$$

Macroscopic parameters are easily expressed in terms of functions f_j, namely

$$n=\int f_1\,d\xi_x\,d\xi_y\,,\quad u_x=\frac{1}{n}\int\xi_x f_1\,d\xi_x\,d\xi_y, \qquad (9)$$

$$u_y = \frac{1}{n}\int \xi_y f_1 \, d\xi_x \, d\xi_y ,$$

$$T = \frac{1}{3kn}\int (f_1 c^2 + f_2)\, d\xi_x \, d\xi_y .$$

Thus, instead of equation (1) we have now two integro-differential equations (7) for the functions f_j. These equations are simpler than the initial one, because the functions f_j depend only on two components of the molecular velocity vector, namely, on ξ_x and ξ_y. That means that from now on the said vector may be treated as two-dimensional.

Further analysis would be considerably simplified if we introduce dimensionless variables

$$\bar{x} = \frac{x}{R}\ , \quad \bar{y} = \frac{y}{R}\ , \quad \bar{\xi}_x = h_\infty^{1/2}\xi_x , \quad \bar{\xi}_y = h_\infty^{1/2}\xi_y ,$$

$$\bar{n} = \frac{n}{n_\infty}\ , \quad \bar{T} = \frac{T}{T_\infty}\ , \tag{10}$$

$$\bar{u}_x = h_\infty^{1/2} u_x\ , \quad \bar{u}_y = h_\infty^{1/2} u_y ,$$

$$\bar{f}_1 = \frac{\pi}{n_\infty h_\infty} f_1\ , \qquad \bar{f}_2 = \frac{2\pi}{n_\infty} f_2 .$$

After changing initial variables by means of transformation (10), dropping the bar over the dimensionless variables and defining the new dimensionless parameter

$$\alpha = A n_\infty h_\infty^{1/2} R , \tag{11}$$

we shall come to a new form of equations (7)-(9):

$$\xi_x \frac{\partial f_j}{\partial x} + \frac{\partial f_j}{\partial y} = \alpha n \left(F_j - f_j\right), \quad /\!/\ j = 1,2\ /\!/$$

$$F_1 = \frac{n}{T}\exp\left(-\frac{c^2}{T}\right), \quad F_2 = n\cdot\exp\left(-\frac{c^2}{T}\right),$$

$$n = \frac{1}{\pi}\int f_1 \, d\vec{\xi}\ , \tag{12}$$

$$u_x = \frac{1}{\pi n}\int \xi_x f_1 \, d\vec{\xi}\ , \quad u_y = \frac{1}{\pi n}\int \xi_y f_1 \, d\vec{\xi}\ ,$$

$$T = \frac{1}{3\pi n}\int \left(2\xi^2 f_1 + f_2\right) d\vec{\xi} - \frac{3}{2}\left(u_x^2 + u_y^2\right),$$

where we still have $c^2 = (\xi_x - u_x)^2 + (\xi_y - u_y)^2$.

The conditions at infinity may be written in new variables in the form

$$f_j = e^{-\xi^2} \quad \text{for } x \to \infty,\ \xi_x < 0, \qquad f_j = 0 \quad \text{for } x \to -\infty,\ \xi_x > 0. \tag{13}$$

/ j=1,2 /

The boundary conditions at the wall are not difficult to obtain after transforming equations (5) and (6) first to new functions f_j and then to dimensionless variables. We have now

$$\left.\begin{aligned} f_1^{\pm} &= \frac{n_\pm}{T_\pm}\exp\left(-\frac{\xi^2}{T_\pm}\right) \\ f_2^{\pm} &= n_\pm \exp\left(-\frac{\xi^2}{T_\pm}\right) \end{aligned}\right\} \text{for } |y|>1,\ \xi_x \gtrless 0. \tag{14}$$

For abbreviation it is convenient to use the following notations

$$G_\pm = \mp\frac{2}{\pi^{1/2}}\int_{\xi_x \lessgtr 0} \xi_x f_1(0,y,\vec{\xi})\, d\vec{\xi}\ ,$$

$$E_\pm = \tag{15}$$

$$= \mp\frac{1}{2\pi^{1/2}}\int_{\xi_x \lessgtr 0} \xi_x \left[2\xi^2 f_1(0,y,\vec{\xi}) + f_2(0,y,\vec{\xi})\right] d\vec{\xi}\ .$$

Then it is possible to find the parameters $n_\pm$ and $T_\pm$ from the formulae

$$n_\pm = \frac{G_\pm^{3/2}}{E_\pm^{1/2}}\ , \quad T_\pm = \frac{E_\pm}{G_\pm}\ . \tag{16}$$

Let the mean molecular velocity be defined as

$$\langle \xi \rangle = \int \left(\xi_x^2 + \xi_y^2 + \xi_z^2\right)^{1/2} f(x,y,\xi_x,\xi_y,\xi_z)\, d\vec{\xi}.$$

Then at the large distances from the slit $\langle \xi \rangle = 2/\sqrt{\pi h_\infty}$ and the Knudsen number is

$$K = \frac{2}{\pi^{1/2}}\frac{1}{A n_\infty h_\infty^{1/2} R}\ .$$

Comparison of the last equation with equation (11) gives

$$\alpha = 2\pi^{-1/2} K^{-1} \tag{17}$$

which means that the parameter α is inversely proportional to the Knudsen number.

It is to be noted that different authors introduce the Knudsen number in somewhat different ways. All these numbers only slightly differ from each other and their radio is always close to unity. The only important thing to know is the order of the Knudsen number.

3. The Method of Solution

The system of equations (12) with the boundary conditions (13), (14) and (16) will be solved by the method of iterations. The iterational scheme is of the type

$$\xi_x \frac{\partial f_j^{(k+1)}}{\partial x} + \xi_y \frac{\partial f_j^{(k+1)}}{\partial y} = \alpha n^{(k)} \left(F_j^{(k)} - f_j^{(k+1)} \right) \quad (18)$$

In respect to the functions $f_j^{(k+1)}$ equations (18) are partial differential equations with the characteristics defined by the relation

$$(x - x_0)\xi_y = (y - y_0)\xi_x . \quad (19)$$

Along the characteristical lines we have instead of (18) an ordinary differential equation

$$\xi \frac{df_j^{(k+1)}}{dl} = \alpha n^{(k)} \left(F_j^{(k)} - f_j^{(k+1)} \right), \quad (20)$$

where the distance l is measured along the characteristics.

In the velocity space we introduce the polar coordinate system,

$$\xi_x = \xi \cos\varphi, \quad \xi_y = \xi \sin\varphi, \quad (21)$$

where φ is the angle between the characteristical line and the x-axis.

The macroscopic parameters of the (k+1)-st approximation may be calculated now from the relations substituting those which enter equations (12), namely

$$n^{(k+1)} = \frac{1}{\pi} \int_0^{2\pi} \Phi_1(x, y, \varphi)\, d\varphi,$$

$$u_x^{(k+1)} = \frac{1}{\pi n^{(k+1)}} \int_0^{2\pi} \cos\varphi\, \Phi_2(x, y, \varphi)\, d\varphi, \quad (22)$$

$$u_y^{(k+1)} = \frac{1}{\pi n^{(k+1)}} \int_0^{2\pi} \sin\varphi\, \Phi_2(x, y, \varphi)\, d\varphi,$$

$$T^{(k+1)} = \frac{1}{3\pi n^{(k+1)}} \int_0^{2\pi} \Phi_3(x, y, \varphi)\, d\varphi - \frac{2}{3}\left[(u_x^{(k+1)})^2 + (u_y^{(k+1)})^2 \right].$$

Here

$$\Phi_1 = \int_0^{\infty} \xi f_1^{(k+1)} d\xi,$$

$$\Phi_2 = \int_0^{\infty} \xi^2 f_1^{(k+1)} d\xi, \quad (23)$$

$$\Phi_3 = \int_0^{\infty} \xi \left(2\xi^2 f_1^{(k+1)} + f_2^{(k+1)} \right) d\xi.$$

Instead of equations (15) in polar coordinates we have

$$G_{\pm}^{(k+1)} = \mp \frac{2}{\pi^{1/2}} \int \Phi_4(y, \varphi) \cos\varphi\, d\varphi,$$

$$E_{\pm}^{(k+1)} = \mp \frac{1}{2\pi^{1/2}} \int \Phi_5(y, \varphi) \cos\varphi\, d\varphi, \quad (24)$$

where

$$\Phi_4 = \Phi_2(0, y, \varphi),$$

$$\Phi_5 = \int_0^{\infty} \xi^2 \left(2\xi^2 f_1^{(k+1)} + f_2^{(k+1)} \right) d\xi. \quad (25)$$

The limits of integration in equations (24) are: for the surface of the wall facing vacuum $-\pi/2 \leq \varphi \leq \pi/2$, whereas for the opposite surface $\pi/2 \leq \varphi \leq 3\pi/2$.

In spite of the fact that the flow proceeds in the unbounded space, we shall look for the macroscopic parameters inside the finite domain Q. Let us choose this domain to be of rectangular form with the center of symmetry coinciding with the origin and the bounding lines parallel to coordinate axes. The conditions (13) written for the infinite distance from the slot assume to be fulfilled at the boundary Γ of the domain Q. The part of this boundary situated at the vacuum

side will be designated by Γ_-, and the upstream part of it - by Γ_+. Subdomains divided by y-axis are designated by Q_- and Q_+, correspondingly. Of course, the domain Q should be sufficiently large to ensure the small influence of the stated assumptions on the solution of the problem.

The domain Q will be divided by a grid with equal size of cells along the both axes. The edges of the slit with coordinates $(0, \pm 1)$ must necessary coincide with any two of the grid points. It is assumed that in all the grid points the moments of the k-th approximation are known, that is, $n^{(k)}, \vec{u}^{(k)}, T^{(k)}$. Due to a symmetry in respect to x-axis we may restrict ourselves by consideration of only upper half-space, the x-axis included. It is to be noted that the wall has two surfaces, which means that at the grid points belonging to a wall one should prescribe for each of these points two values of the macroscopic parameters.

The important assumption is that all the macroscopic variables are changing between the neighbouring grid points according to a linear law. It is easy to see that this assumption is not very good in the vicinity of the edges, where the flow parameters are changing most quickly. Moreover, for the large Knudsen numbers these parameters lose their continuity at the points $(0, \pm 1)$ (see, for example, analytic formulae for the free molecular flow, (28) and (29)). Therefore, at these grid points three values of the flow parameters must be prescribed: two for both sides of the wall and one for the limit from inside of the slit along the y-axis.

The essential features of the computational process are the following. For the chosen angle φ the characteristic lines are drawn, covering the whole domain Q. Along each of these lines the equation (20) is solved with the appropriate initial conditions. Then, after integration over ξ one gets the values of the integrals (23) and (25) along the characteristics; the values at the grid points are found by means of linear interpolation between the neighbouring characteristics. Making such a calculations for various values of φ between zero and 2π and adding together all the Φ-functions multiplied by the coefficients, which depend on the type of the integration formula, one obtains the integral sums necessary for getting at the grid points the moments of the (k+1)-th approximation. Repeating the process we shall obtain the (k+2)-th approximation and so on, until the iterations converge.

By conducting the calculations it is necessary to make a distinction for several families of characteristics depending on their direction and inclination. These factors have an essential influence of the choice of the initial conditions, as well as on the some details of the interpolation process. The full description of all the possible variants would take too much place, we have but to draw attention to the following fact. As it was already mentioned, the flow parameters are discontinuous along the y-axis, and thus the edge of the slit is the singular point. By the calculations, the macroscopic parameters at this point were taken to be equal to the half-sums of their limit values near the positive and negative sides of the wall. It is assumed that such a simplification will not essentially influence the solution as a whole, if only the cell size h is less than the mean free path of the molecules.

The integrals over the variables were calculated by the Simpson's formulae in which the upper limit was determined by the condition $\xi < u + 3\sqrt{T}$. The step of integration was chosen to be 0.2. By the way, the calculations showed that the application of Gaussian formulae with the Legendre's or Tchebysheff-Hermite's polynomials is leading to diminishing of the accuracy, especially in the region Q_-.

The integration over φ was made by the Simpson's method, too. Depending

on the variant of calculations, the step of integration was taken between 2.5^0 and 1.25^0.

To integrate the equation (20) along the characteristic line we have applied the implicit finite difference scheme of the type

$$\xi \frac{f_{ji}^{(k+1)} - f_{j,(i-1)}^{(k+1)}}{\Delta \ell} = \alpha n_i^{(k)} \left(F_{j,i}^{(k)} - f_{j,i}^{(k+1)} \right), \quad (26)$$

where indics i and $i-1$ correspond to values at the two successive points of the characteristic line, ℓ_i and ℓ_{i-1}. The scheme (26) is stable for any values of ξ and α.

When solving the equations (20) it is possible to write them down in integral form (see [3]) and to calculate the results using the quadrature relations. Such an approach might to be of higher accuracy but demands larger computer time due to a complex structure of integrals a and integrands. As to the present method, it permits to save a great amount of the computer time just because the calculations along the characteristics are conducted simultaneously for the whole domain Q. In spite of the fact that the calculations are made on the level of distribution function, it is possible to restrict oneself by storing in the grid points only the moments of distribution function of the k-th approximation and the integral sums for the (k+1)-th approximation.

4. The Numerical Results

Any iterational process leading to a solution of the problem for the fixed and finite Knudsen number should take as an initial step some solution known. It is natural that for the large Knudsen numbers such an initial solution would be the free molecular flow. Taking into account that the free molecular distribution function is conserved along the rectilinear trajectories of the particles, one could easily obtain the analytic expressions for the macroscopic parameters in the upper half-space. In the high-pressure region $(x \geq 0)$ we have

$$n = n_\infty \left(1 - \frac{\vartheta_2 - \vartheta_1}{2\pi}\right),$$

$$\vartheta_2 = \begin{cases} \frac{\pi}{2} - \operatorname{arctg} \frac{y-R}{x} & \text{for } x \neq 0, \\ \frac{\pi}{2} - \frac{\pi}{2} \operatorname{sign}(y-R) & \text{for } x = 0, \end{cases} \quad (27)$$

$$\vartheta_1 = \operatorname{arctg} \frac{x}{y+R},$$

$$u_x = -\frac{n_\infty}{2n} \left(\frac{kT_\infty}{2\pi m}\right)^{1/2} (\cos\vartheta_1 - \cos\vartheta_2),$$

$$u_y = -\frac{n_\infty}{2n} \left(\frac{kT_\infty}{2\pi m}\right)^{1/2} (\sin\vartheta_2 - \sin\vartheta_1),$$

$$T = T_\infty - \frac{m}{3k}(u_x^2 + u_y^2).$$

By the similar way, in low-pressure region $(x \leq 0)$ we obtain

$$n = n_\infty \frac{\vartheta_2 - \vartheta_1}{2\pi},$$

$$\vartheta_2 = \begin{cases} \frac{\pi}{2} + \operatorname{arctg} \frac{y-R}{x} & \text{for } x \neq 0, \\ \frac{\pi}{2} - \frac{\pi}{2} \operatorname{sign}(y-R) & \text{for } x = 0. \end{cases} \quad (28)$$

$$\vartheta_1 = -\operatorname{arctg} \frac{x}{y+R},$$

$$u_x = -\frac{n_\infty}{2n} \left(\frac{kT_\infty}{2\pi m}\right)^{1/2} (\cos\vartheta_1 - \cos\vartheta_2),$$

$$u_y = \frac{n_\infty}{2n} \left(\frac{kT_\infty}{2\pi m}\right)^{1/2} (\sin\vartheta_2 - \sin\vartheta_1),$$

$$T = T_\infty - \frac{m}{3k}(u_x^2 + u_y^2).$$

In addition to the usual flow parameters it is important to know the mass flow rate of the gas through a slit. The local mass flow rate is

$$\dot{m}(y) = \int \xi_x f(0, y, \vec{\xi}) \, d\vec{\xi} = u_x \cdot n,$$

whereas the total mass flow rate is equal to

$$M = \int_{-R}^{R} \dot{m}(y) \, dy.$$

The corresponding expressions for the free molecular flow are

$$\dot{m}_0 = \frac{n_\infty}{2\sqrt{\pi h_\infty}} = const, \quad M_0 = \frac{n_\infty R}{\sqrt{\pi h_\infty}} = const.$$

For the Knudsen numbers $K \geq 2$ an initial approximation was taken in the free molecular form defined by equations (27) and (28). The solution obtained for $K=2$ was considered as an initial approximation for the case $K=1$, whereas the last solution was, in its turn, initial for $K=0.5$. The establishment of the profiles of macroscopic parameters was considered as a criterion for convergency of iterations. For the case $K=5$ such a convergency (with the accuracy of the order of 0.2%) was achieved after 5 iterations, whereas for other values of K it was necessary to make from 10 to 12 iterations.

To chose the linear size of the cell the condition was set that the mean change of the macroscopic parameters after dividing that size by two should not exceed 1%. The value of h satisfying such a condition was 0.25 (for $K \geq 2$) or 0.125 (for $K < 2$).

Rather serious difficulties were encountered by the choice of the outer boundary of the domain Q. As a matter of fact, the boundary conditions for the particles entering this domain (see equations (13)) ought to be satisfied at the infinity, whereas we should set the same conditions at the finite distance from the slit. It might seem that the farther is the boundary Γ, the better accuracy will be achieved in satisfying real boundary conditions. But if the boundary Γ is moved too far away, then it will certainly increase both the computer time and the memory required. Moreover, accuracy of the results becomes even worse due to the fact that some of the flow parameters are very small at the large distances, so that the relative error is increased. In the opposite case of the boundary Γ situated too close to the slit, the layer where the condition of conservation of mass is not fulfilled, might influence the solution in the immediate vicinity of the slit, at any rate from the high-pressure side.

Happily enough, of the most interest for us is the low-pressure region which is the least subjected to the influence of the boundary's replacement. Nevertheless, by the calculations the mass flow rates through several control surfaces inside the domain Q were computed, and by means of a trial-and-error method such an outer boundary Γ was determined, for which the accuracy of the mass conservation law fulfillment is not less than 3%.

Figures 2-4 show the changes of the density, velocity and temperature along the x-axis for various Knudsen numbers. It is evident that the influence of the Knudsen number on the density and velocity is rather small in the high-pressure region ($x>0$). The temperature in this region tends to its limit value the faster, the less is K. In the low-pressure region ($x<0$) the pattern of density differs very little from the free molecular solution, too. However, by decreasing of K there is a notable increase of the velocity and decrease of the temperature. It is interesting to note that the same properties of the expansion into vacuum are observed by the study of source-type flows [10-13].

In the figures 5 and 6 are shown the curves of the constant density and constant temperature for the free molecular flow ($K=\infty$) and for the case $K=0.5$. The velocity fields for $K=\infty$, $K=5$ and $K=0.5$ are shown in the figures 7-9, where the arrows indicate the direction of velocity vector. By the letter S the sonic line is designated.

Finally, in the fig.10 there is shown the change of the local mass flow rate inside the slit, measured in the units of the free molecular value $\dot{m}_0$. It seems that the calculated mass flow rates are somewhat too low, since in all the cases

enlarging of the domain Q or diminishing of the cell size h leads to increasing of the flow rates. The dotted lines in fig.10 correspond to the data given in (5,6). As should be expected, the difference between these data and the present results is the more, the less is Knudsen number.

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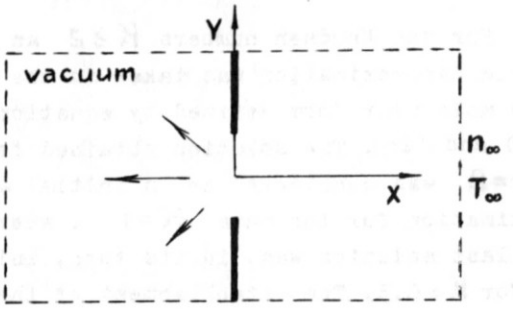


Fig. 1.

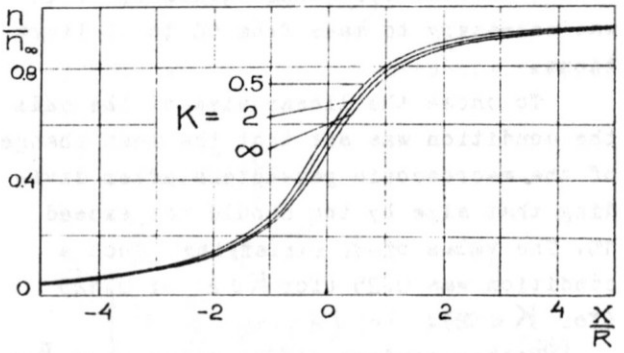


Fig. 2.

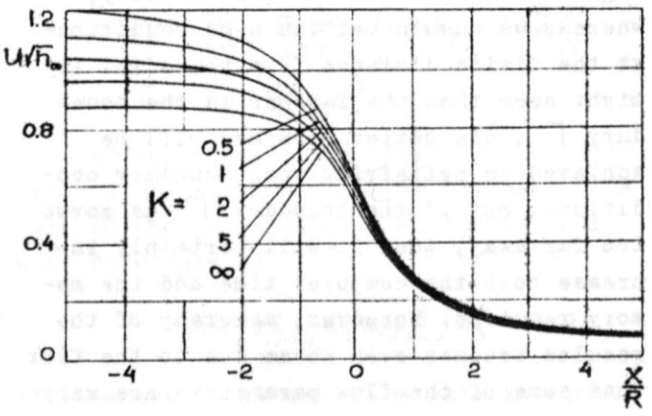


Fig. 3.

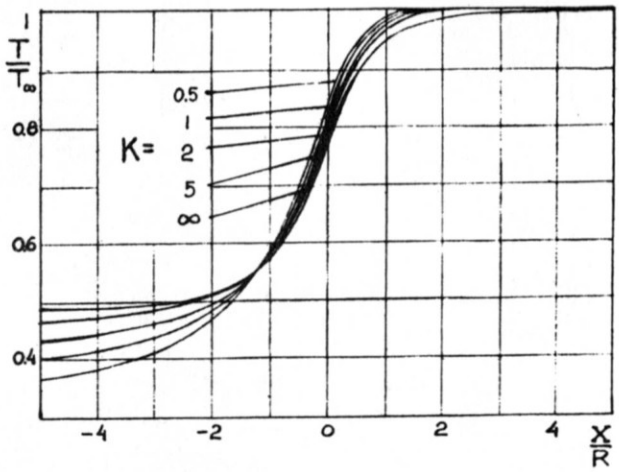


Fig. 4.

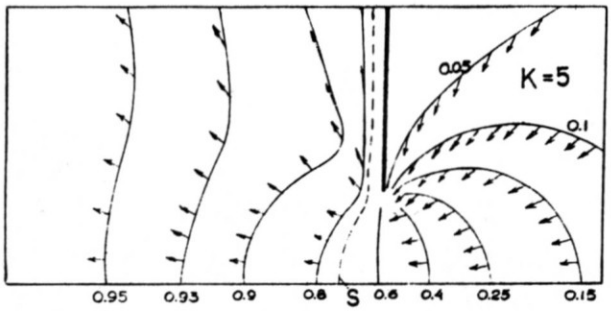


Fig. 8.

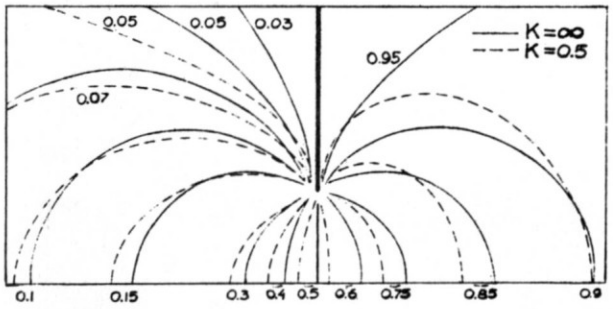
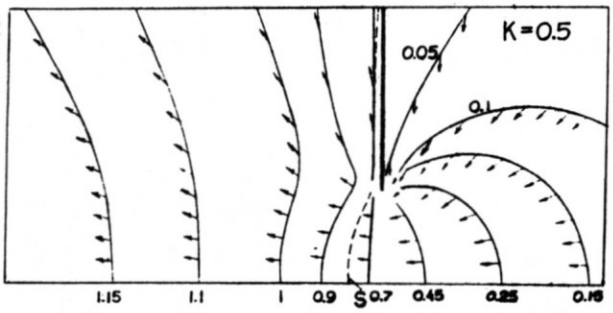


Fig. 5.



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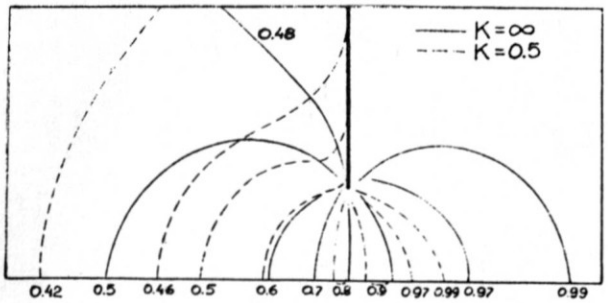

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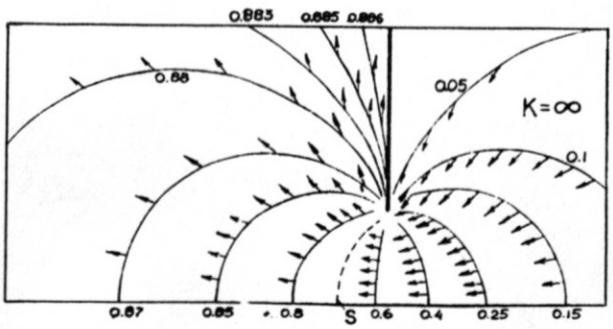

Fig. 7.

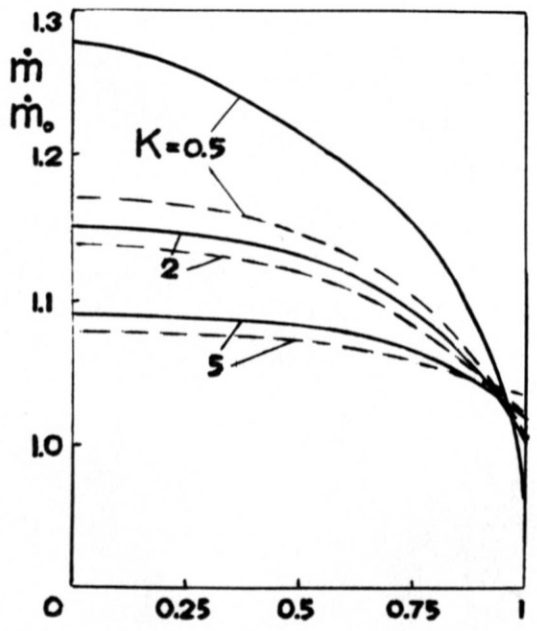

Fig. 10.