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TWO-DIMENSIONAL SUBSONIC LINEARIZED THEORY OF THE UNSTEADY FLOW THROUGH A BLADE-ROW WITH SMALL STEADY PITCH AND CAMBER ANGLE

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TWO – DIMENSIONAL SUBSONIC LINEARIZED THEORY OF THE UNSTEADY FLOW THROUGH A BLADE – ROW WITH SMALL STEADY PITCH AND CAMBER ANGLE

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Abstract

The aim of this paper is to determine local unsteady pressures acting on a two-dimensional cascade made of an infinite number of straight segments, placed in a uniform, subsonic, undeflected flow, for low amplitude, harmonic vibrations. The technique aims at solving an integral equation linking the local lift with the angle of attack on a reference blade, in the configuration where all the blades vibrate harmonically in the same manner, but for a constant but undetermined outphasing from one blade to the next. The kernel is expressed as a series of exponential functions. The case is then treated where the segments are replaced by slightly cambered blades in a slightly deflected flow. To take account of these effects, on the first order, it is sufficient to modify the expression of the angle of attack in the first problem, the modification corresponding to a velocity perturbation field whose potential is defined by a convolution where the kernel is also expressed by a series of exponentials. Some computed results are presented and these will later be compared with experiments which are being carried out at Modane-Avrieux.

Résumé

On résout le problème de la détermination des pressions locales instationnaires s'exerçant sur une grille bidimensionnelle infinie de segments, placés dans un écoulement uniforme subsonique non dévié pour des petites vibrations harmoniques. La technique revient à résoudre une équation intégrale reliant la portance locale à l'angle d'attaque sur une aube de référence, dans la configuration où toutes les aubes vibrent harmoniquement de façon identique, à un déphasage près constant et quelconque d'une aube à la voisine. Le noyau est exprimé sous forme de série d'exponentielles. On traite ensuite le cas où les segments sont remplacés par des arcs faiblement cambrés dans un écoulement faiblement dévié. Pour tenir compte au premier ordre de ces effets, on montre qu'il suffit de modifier l'expression de l'angle d'attaque dans le premier problème, la modification correspondant à un champ de perturbation de vitesse dont le potentiel est défini par une convolution où le noyau s'exprime également par une série d'exponentielles. On présente des résultats numériques qui seront confrontés avec l'expérience actuellement en cours à Modane-Avrieux.

I - Introduction

Vibration problems arising in compressors are always of particular concern to engineers. These are complex because many causes are involved, and some of them, such as flow separation at high incidence, or in the transonic range, have not yet been solved theoretically. In the present paper, only the vibratory and instability phenomena in subsonic unseparated flow with a small-pressure ratio are considered (fig. 1).

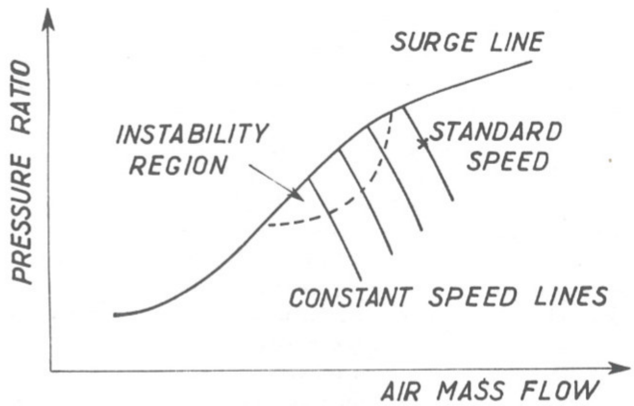

FIGURE 1

The kind of vibrations analyzed here affect the head-intake stage of jet compressors and arise at rotational speeds slightly lower than the normal running speed. These instabilities are, at present considered to be the most dangerous by operators. They generally appear in the form of a well synchronized flutter all over the rotor-stage, and the blades vibrate in either the first or second flexural mode, or in the first torsional mode. In the present theory a single rotor is considered and the interactions between different stages are neglected because it is assumed that these are not coupled but can be superimposed.

Similar problems give rise to ideal theoretical modes in which all blades vibrate harmonically at the same frequency, the same amplitude, and with an equal phase angle θ between two neighbouring blades. These we shall call fundamental modes. It is known that, for any fundamental mode (θ), an adequate criterion for stability is when the aerodynamic forces lag behind the corresponding displacements. If we substitute any multiple of $2\pi/N$ (N being the number of blades in the rotor) for all values of θ, this stability criterion also becomes necessary when the symmetry of the N^{th}-order of the rotor is perfect. The first criterion is therefore not too restricting and a structural dissymmetry offers hardly any protection against instability. This remark emphasizes the theoretical interest of fundamental modes, even if unavoidable dissymmetries exclude the possibility of flutter arising in such modes. Therefore we adopt the following equivalent stability-criterion for any blade mode : aerodynamic damping must be positive for any fundamental configuration (θ) and for any resonant mode.

We neglect the possibility of flutter involving several resonant modes because of the usually high values of mass-ratios, though such a case is hardly more difficult to treat.

Another aim in determining the aerodynamic damping for any value of θ is to permit the analysis of the forced excitation on the rotor blades produced by either an anisotropic intake flow or by the hub supporting arms, at a multiple of the rotation-frequency near a resonant mode. It is clear that the greater the damping coefficient related to the phase angle $\theta = 2\pi n/N$ (n being the order of the harmonic), the less is the amplitude of deflection.

II - Theoretical model

The blade-thickness is neglected, and at first, the camber angle and pitch are assumed negligible ; their effects will be considered later. The independence between the unsteady flows in coaxial cylindrical annuli is assumed. Steady flow as seen from the machine is supposed axial and uniform. Thus, the linearized problem is to find a two-dimensional perturbation potential, which, in any elementary annulus, is independent of the radius. The curvature of the annuli is assumed to have little effect when N is large and the radii-ratio approches unity, which is a condition already implied in the assumptions. Finally every elementary annular blade-row is replaced by an infinite straight cascade. For any fundamental vibration, a local lift-coefficient calculated in a straight cascade correspond to every blade-section.

In order to take into account the camber-angle and pitch effects, the relative steady uniform flow at infinity downstream is considered to be uniform all over the plane, and the blades are supposed in line with it. Camber angle and pitch induce a steady perturbation flow expressed by a vortex-distribution along the corresponding airfoils, which are responsible for the flow deviation and for a curl in the three-dimensional velocity-field. The only unsteady coupling of these phenomena in a fundamental mode (with $\theta \neq 0$) results from relative displacements of the attached vortices which keep their intensities during the vibration. They induce an additionnal perturbation velocity-field which only arises in the expression for the downwash, but the local lift-coefficient always remains the same functional of the downwash. Therefore, except for a modification of the expression for the downwash, the linearized problem is the same as without camber and incidence effects.

The problem of the straight two-dimensional cascade without deviation effects was first solved by **Woodston** and **Runyan** [1]. In that paper, $\tau \gg 1$; $\gamma = 0$; $\theta = 0$, where :

τ is the reduced blade-spacing refered to the semi-chord c

γ is the stagger angle.

The authors have chosen the Possio doublet-method which is also used in the present paper.

L.V. Dominas[2] has calculated the general case with neither camber nor incidence, by extending the **Haskind** method for a single airfoil[3]. The latter method is cumbersome and probably not very accurate ; moreover only a few results are given. No published work refering to this problem for compressible subsonic flow with camber and pitch effect has been found. However for incompressible flow, research carried out in Japan by **J. Shioiri**[4] and later by **Hanamura** and **Tanaka** [5] must be mentioned. The approach in the present paper is analogous to that of Shioiri.

III - The determination of the unsteady pressure field

1. Case without deviation :

We consider a fundamental mode (θ) of angular frequency ω.

We use reduced coordinates x and y, the origin of which is at the middle of the reference blade. If $\bar{x}$ and $\bar{y}$ denote dimensional coordinates, then :

$$x = \frac{\bar{x}}{c} \quad ; \quad y = \frac{\bar{y}}{c} \qquad \text{(see fig. 2)}$$

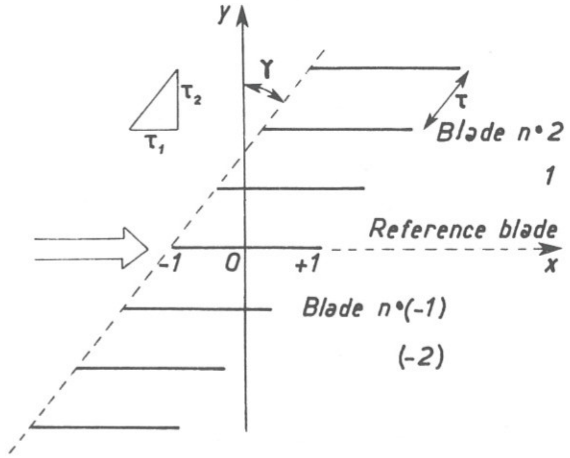

FIGURE 2

U is the speed of the undisturbed flow and we define the reduced frequency as : $k = \dfrac{\omega . c}{U}$

Let $U c \varphi(x,y)\, e^{j\omega t}$ be the perturbation velocity-potential. The acceleration potential ψ , or the pressure, is determined by :

$$\psi(x,y) = \varphi_x + jk\varphi \tag{1}$$

The local lift is thus expressed by :

$$C_p(x) = \frac{p(x,-0) - p(x,+0)}{\rho U^2} = \psi(x,+0) - \psi(x,-0)$$

ρ being the density of air.

If $\hat{\psi}(x-\xi, y-\eta)$ denotes the potential of an unit harmonic source placed at (ξ, η), we have, according to Green's theorem :

$$\psi(x,y) = \int_{-1}^{+1} C_p(u) \sum_{n=-\infty}^{+\infty} e^{jn\theta}\, \hat{\psi}_y\left[x-(u+n\tau_1), y-n\tau_2\right] du \tag{2}$$

where, to satisfy the Sommerfeld condition :

$$\hat{\psi}(x-\xi, y-\eta) = \frac{j}{4\beta}\, e^{j\frac{kM^2}{\beta^2}(x-\xi)}\, H_0^{(2)}\left(\frac{kM}{\beta^2}\sqrt{(x-\xi)^2+\beta^2(y-\eta)^2}\right) \tag{3}$$

M is the Mach number, and $\beta^2 = 1 - M^2$

Let $h(x)\, e^{j\omega t}$ be the deflection function of the reference-blade. Then the boundary condition is expressed by :

$$\alpha(x) = \left(\frac{\partial}{\partial x} + jk\right) h(x) = \left(\frac{\partial}{\partial y}\right)_{y=0} \int^{x} e^{-jk(x-\xi)}\, \psi(\xi, y)\, d\xi \tag{4}$$

After substituting (2) in (4), we eliminate $\hat{\psi}_{yy}$ by means of the equation defining $\hat{\psi}$;

$$\left(\beta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - 2jkM^2\frac{\partial}{\partial x} + k^2M^2\right)\hat{\psi} = \delta(x-\xi)\,\delta(y) \tag{5}$$

where we recognize the form of the linear potential equation in the left handside. Thus, we find :

$$K(x) * C_p(x) = \alpha(x) \qquad (6)$$

where :

$$K(x_0) = \frac{j}{4\beta}\left\{ e^{j\frac{kM^2}{\beta^2}x_0}\left(jk - \beta^2\frac{\partial}{\partial x_0}\right)\sum_{n=-\infty}^{+\infty} e^{jn\theta_1} H_0^{(2)}\left(\mu\sqrt{(x_0 - n\tau_1)^2 + (\beta n\tau_2)^2}\right) \ldots \right. \qquad (7)$$

$$\left. + k^2 e^{-jkx_0}\sum_{n=-\infty}^{+\infty} e^{jn\theta_1}\int^{x_0} e^{j\frac{ku}{\beta^2}} H_0^{(2)}\left(\mu\sqrt{(u - n\tau_1)^2 + (\beta n\tau_2)^2}\right) du \right\}$$

with : $\mu = \frac{kM}{\beta^2}$; $\theta_1 = \theta - \frac{kM^2}{\beta^2}\tau_1$

In order to respect the Sommerfeld condition, the indefinite integrals in (4) and (7) must be chosen such that φ_y does not contain any periodic terms in x or x_0 common at infinity downsteam and upstream. It will be seen that expressions (4) and (7) can be other than zero at infinity upstream. Making φ, φ_y or ψ zero at infinity upstream, in this case, would mean that we have superposed a periodic solution such as $A\ e^{-jkx_0}$ for any range of x, which is irrelevant because of the Sommerfeld condition. The form (7) of the Kernel function indicates the singularity which is the same as in the single airfoil problem. Numerical computation makes use of this point which also shows that the form of equation (6) is that of a Cauchy integral. However, expression (7) cannot be used for numerical computation because the series in the right handside converges very slowly.

In applying the Poisson-formula to (7), we fortunately obtain a series of exponentials in x_0 which is very convergent for any value of x_0 whose modulus is not too small, and for values of τ not too large. An additional term such as $A\ e^{-jkx_0}$ arises when $x_0 > 0$. It must be noted that this term, periodic in x, is absent upstream. The coefficient A is a simple series of a rational function of the index.

For example, apply Poisson-formula to the serie :

$$S(x_0, y, \theta_1) = \sum_{n=-\infty}^{+\infty} e^{jn\theta_1} H_0^{(2)}\left(\mu\sqrt{(x_0 - n\tau_1)^2 + \beta^2(y - n\tau_2)^2}\right) \qquad (8)$$

We find :

$$S(x_0, 0, \theta_1) = \sum_{m=-\infty}^{+\infty} \frac{2j}{\Delta_m}\exp\left(j\Theta_{1,m}\frac{\tau_1 x_0}{\tau'^2} - \beta\frac{\tau_2}{\tau'^2}\Delta_m |x_0|\right) \qquad (8')$$

where :

$$\tau'^2 = \tau_1^2 + \beta^2\tau_2^2$$

$$\Theta_{1,m} = 2\pi m + \theta_1$$

$$\Delta_m = \sqrt{\Theta_{1,m}^2 - (\mu\tau')^2}$$

Expression (7) is, in fact, a similar series, only a little more complicated.

When : $\Theta_{1,m}^2 - (\mu\tau')^2 < 0$, Δ_m must be chosen imaginary with argument equal to $(+\pi/2)$. Expression (8') shows that $S(x_0, \theta_1)$ becomes infinite when θ and m are such that $\Delta_m = 0$. It is always possible to find a range of θ values for which there exists at least one value of m such that Δ_m is imaginary. Thus, $S(x_0, \theta_1)$ is not zero far upstream. These are also properties of $K(x_0, \theta)$ expressed in equation (7).

If a is the speed of sound, θ_1' and θ_2', the two critical values for which $K(x_0, \theta)$ is infinite, then, when $\omega\tau' c/a < \pi\beta^2$, $K(x_0, \theta)$ is equal to zero far upstrean only for : $0 < \theta_1' < \theta < \theta_2' < 2\pi$

Outside these conditions, the acceleration potential ψ , or perturbation pressure, tends to two different plane wave potentials far upstream and downstream (see fig. 3).

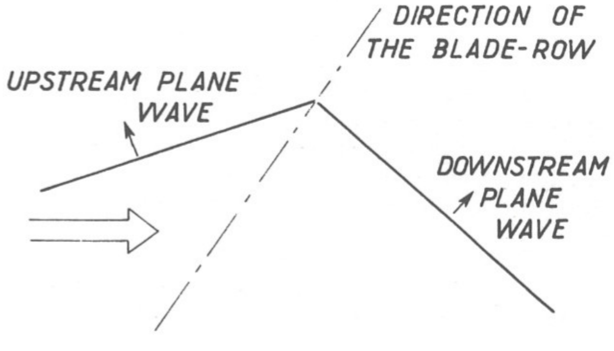

FIGURE 3

These waves move upstream in front of the blade-row and downstream behind it. The propagation direction is the same only when θ is critical. For these critical values, the solution of (6) shows that the contribution of the finite terms in $K(x_0, \theta)$ is of the same order of magnitude as that of the single term. This implies that $C_p(x)$ is not identically zero for critical values of θ. However, when θ takes on a critical value, a sudden step appears in the variation of the aerodynamic coefficients. For flexural modes, the coefficients become nearly zero at critical values of θ, but for torsional modes, it may happen that the imaginary part of the moment at the rotational axis becomes positive, and then there is an instability near a critical value of θ . It must be noted that this very sharp instability disappears in the strip-theory applied in the spanwise direction.

It is interesting to note that the possible excitations of the n^{th} harmonic of the rotor frequency have a phase angle between two neighbouring blades determined by : $\theta = 2\pi n/N$ which may easily coïncide with a critical value for a particular elementary annulus. Thus, it is sometimes possible to increase the damping by a slight modification of the critical values of θ which are expressed by :

$$\theta = \frac{\omega\tau c}{a\beta^2}\left(M\sin\gamma \pm \sqrt{1 - M^2\cos^2\gamma}\right), \mathrm{mod}.2\pi \qquad (9)$$

Solution

The solution for $C_p(x)$ is required in the form :

$$C_p(x) = \sqrt{\frac{1-x}{1+x}}\sum_{i=1}^{I} d_i\, Q_i(x) \qquad (10)$$

which takes into account the Kutta condition. $Q_i(x)$ denotes the sequence of orthonormal polynomials on (-1, +1) with the weighting function $\sqrt{\frac{1-x}{1+x}}$. Equation (6) is discretized by projecting the downwash $\alpha(x)$ on the polynomial sequence $Q_j(-x)$ with the weighting function $\sqrt{\frac{1+x}{1-x}}$. Therefore, with the dumb index convention,

$$\alpha_j = \int_{-1}^{+1}\sqrt{\frac{1+x}{1-x}}\, Q_j(-x)\,\alpha(x)\,dx$$
$$= \left\langle \sqrt{\frac{1+x}{1-x}}\, Q_j(-x),\ K(x) * \sqrt{\frac{1-x}{1+x}}\, Q_i(x) \right\rangle d_i \qquad (11)$$

which is of the form :

$$\alpha_j = M_{ij} \, d_i \tag{11'}$$

The matrix M_{ij} reduces to a diagonal in the corresponding problem where the cascade is replaced by the single reference airfoil, and also when the reduced frequency tends to zero. In the present case, it is the relative importance of the diagonal terms that permits to restrict the rank of M to a value only slightly higher than the degree of the polynomial representation of α (x). The generalized Gauss method is used for the two integrations expressed in (11). If the same number of points R is chosen for the two sets of integration-points, these become interwoven and respectively symmetrical to the origin. With this choice, Cauchy's singularity is automatically accounted for [6].

For R = 10, the logarithmic singularity and terms like $x_0 \log x_0$ can be taken out and these are easily integrated analytically. The smallest value of x_0 considered, does not generate really large numbers so that there is no loss of precision in differencing large numbers when integrating numerically.

2. Case with pitch and camber :

The linearized steady flow problem must first be solved.

By using the Prandtl-Glauert transform, the potential equation becomes the Laplace equation, let :

$$\alpha(x) = \int_{-1}^{+1} C_{p,o}(\xi) \, G_y(x-\xi, 0) \, d\xi \tag{12}$$

where $C_{p,o}(\xi)$ represents the steady local lift coefficient at the point (ξ) of the abscissa (ξ). G (x-ξ,y) denotes the potential, before the Prandtl-Glauert transform, of a train of vortices of unit intensity having the reduced spacing τ and moving in the blade-row direction with one vortex on the reference blade at (ξ, 0). In order to maintain a zero velocity at infinity upstream, a uniform perturbation velocity field must be added to this. Thus, the equation (12) becomes :

$$\alpha(x) = \gamma(x) - \bar{\alpha} \tag{12'}$$

$$= \beta \oint_{-1}^{+1} C_{p,o}(\xi) \left[-\frac{1}{2\pi} \Re q' \coth q'(x-\xi) - \frac{\beta \cos\bar{\gamma}}{2\tau(\sin^2\bar{\gamma} + \beta^2\cos^2\bar{\gamma})} \right] d\xi$$

where γ (x) denotes the angle between the x-axis and the normal to the blade-row (see fig. 4).

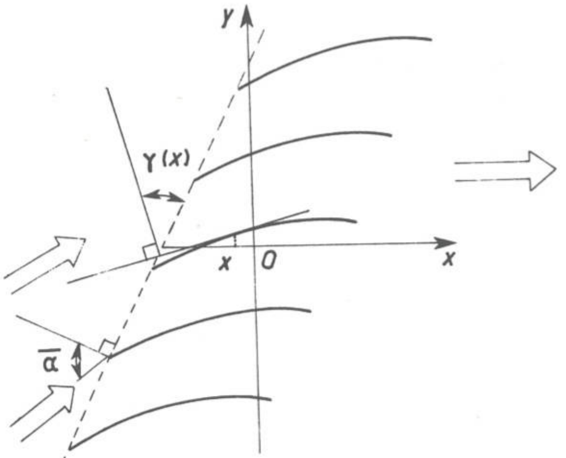

FIGURE 4

$\bar{\gamma}$ is a computed term representing the stagger angle of the equivalent blade-row without camber (it is a kind of mean value of γ(x) ; $\bar{\alpha}$ denotes the pitch angle with respect to the normal to the blade-row ; $q' = \frac{i\pi}{\tau_1 + i\beta\tau_2}$, where : $\tau_1 = \tau \sin\bar{\gamma}$, $\tau_2 = \tau \cos\bar{\gamma}$

The fluctuating flow is then linearized with respect to the downstream steady flow. The basic problem is to define the y-component along the reference-blade of the fluctuating velocity induced by an infinite train of equal vortices vibrating as if each one is attached to its respective moving blade, and this in any fundamental mode. In the same way as the contribution of a bound vortex on its blade is of the second order, an error of the same order is introduced when these vortices are replaced by near by vortices still having their true vibrational amplitudes, but now bound to straight segments near their corresponding blades and parallel to the downstream flow (see fig. 5).

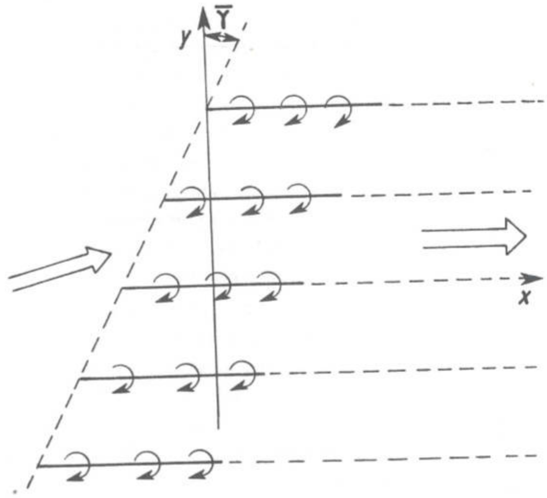

FIGURE 5

The linearized fluctuating field is the same as that induced by an infinite train of harmonic vortex-doublets normal to the downstream flow, whose intensities are proportional to the product of the steady vortex intensity and the amplitude of deflection. To determine such a potential, we differentiate the multiform potential induced by a harmonic vortex with respect to (y). A harmonic vortex is defined here as a vortex whose intensity is a circular function of time. It is not usual to consider the perturbation associated with such a singularity because no uniform pressure corresponds to it ; but when differentiating with respect to (y), uniformity returns. Let φ be the potential of a unit harmonic vortex placed at the origin. φ is antisymmetric in (y) and increases by a unit every time the origin is circumscribed.

Therefore, according to Green's formula, and respecting the Sommerfeld condition, we have :

$$\varphi(x,y) = -\frac{j}{4\beta} \int_0^\infty e^{j\frac{kM^2}{\beta^2}u} \frac{\partial}{\partial y} H_0^{(2)}\left(\mu\sqrt{(x-u)^2+\beta^2 y^2}\right) du \tag{13}$$

We can then deduce the expression $f(x-\xi, y)$ for the potential of an infinite train of vortex-doublets parallel to the blade-row containing the point (ξ, 0), having equal phase angle (θ) between two neighbouring doublets. For y = 0, putting :

$$x_0 = x - \xi$$

we obtain by differentiating :

$$f(x_0,y) = \sum_{n=-\infty}^{+\infty} e^{jn\theta} \varphi_{yy}(x_0 - n\tau_1, y - n\tau_2)_{y=0}$$

$$= -\frac{j\beta}{4}\left\{ e^{j\frac{kM^2}{\beta^2}x_0}\left(\frac{\partial}{\partial x} - j\frac{kM^2}{\beta^2}\right)\left[\frac{\partial}{\partial y}\right]_{y=0} S(x_0, y, \theta_1) + \ldots \right. \tag{14}$$

$$\left. \frac{kM^2}{\beta^2}\left[\frac{\partial}{\partial y}\right]_{y=0} \int^{x_0} e^{j\frac{kM^2}{\beta^2}u}\left[S(u,y,\theta_1) - H_0^{(2)}\left(\mu\sqrt{u^2+\beta^2 y^2}\right)\right] du \right\}$$

The preceding remark concerning the integrals $\int^{x} \ldots du$ applies here. It must be noted that

$$\left[\frac{\partial}{\partial y}\right]_{y=0} \int^{x_0} e^{j\frac{kM^2}{\beta^2}u} H_0^{(2)}\left(\mu\sqrt{u^2+\beta^2 y^2}\right) du$$

is zero if $x_0 < 0$, and discontinuous and equal to $\mp 2j\beta$ if $x_0 > 0$ and $y = \pm 0$. This discontinuity is just equal and opposed to that of $\left[\frac{\partial}{\partial y}\right]_{y=0} \int^{x_0} e^{j\frac{kM^2}{\beta^2}u} S(u,y,\theta_1)\, du$. By transforming (14) with the Poisson-formula, we obtain a new exponential series in x_0. When $x_0 > 0$, an extra term which is independent of x_0 and is expressed by a series of rational fractions is added to it. Let $\alpha_s(x)$ be the additional perturbation downwash relating to the camber and pitch effects. This term is equal and opposed to the y-component of the induced velocity, and thus, is expressed by :

$$\alpha_s(x) = -\int_{+1}^{-1} \varepsilon_0(\xi)\, h(\xi)\, f(x-\xi, y)\, d\xi \tag{15}$$

where : $\varepsilon_0(\xi) = -C_{p,0}(\xi)$

The integrals are computed by the Gauss method with respect to the weighting function $\sqrt{\frac{1-\xi}{1+\xi}}$, since, in order to satisfy the Kutta condition, $C_{p_0}(\xi)$ is found from (12') in the form :

$$C_{p,0}(\xi) = \sqrt{\frac{1-\xi}{1+\xi}}\left(\delta_0 + \delta_1\xi + \delta_2\xi^2 + \ldots\right) \tag{16}$$

Finally, when they are moderate, camber and pitch effects can be taken into account by modifying only the second part of (6) ; thus :

$$K(x) * C_p(x) = \alpha(x) + \alpha_s(x) \tag{6'}$$

Only Gauss values of x need to be considered in the integral of (15). Here, we can repeat a preceding remark concerning accuracy and that is that the smallest considered value of $(x-\xi)$ is sufficiently large not to become a special case in series calculations.

IV - Theoretical results

In order to allow an easy comparison to be made, we have treated the vibratory configuration where only the reference blade vibrates, the other blades being fixed. For such a mode, the potential is obtained by integrating the fundamental potentials with respect to (θ), giving the mean potential over the range $(0, 2\pi)$, the phase origin being taken at the reference-blade.

Some precaution must be taken in integration because of the existence of the two critical values of (θ). The integration is carried out by the Gauss method in three ranges : $(0, \theta_1')$; (θ_1', θ_2') ; $(\theta_2', 2\pi)$.

From the experimental point of view, the principal interest of the chosen vibratory configuration is that the requirements are much simpler, especially in the excitation procedure. As for the measurements, the amplitude of vibration is imposed by a forced excitation and measured with a velocity transducer. Local unsteady pressures on every blade are obtained by means of a sufficiently large number of miniaturized pressure transducers. The effects of an unavoidable limitation of N,(†) and the presence of the walls are expected to be small for the chosen vibratory mode. Experiments are carried out in a blow-down wind-tunnel especially built at Modane-Avrieux for pressure fluctuation measurements on blade-rows. The blow-down wind-tunnel and measuring equipment and techniques will be described, and a comparison of the theory with the experiment will be discussed in a future publication in 'La Recherche Aérospatiale'.

In order to illustrate the method which has just been described, we shall only give three examples of graphs showing typical results. In these figures, the mode considered is always rotation about the point : X = 0,042. Blades are supposed straight and the blade-row is defined by :

$\gamma = 0{,}785$ rad. $\quad \tau = 1{,}964$

Mach number is M = 0,512, and the reduced frequency is :

k = 0,215

1. Results for the case where only the central blade is moving

In figure 6, the central blade pressure coefficient at the upper and lower surfaces is represented against the position along the chord.

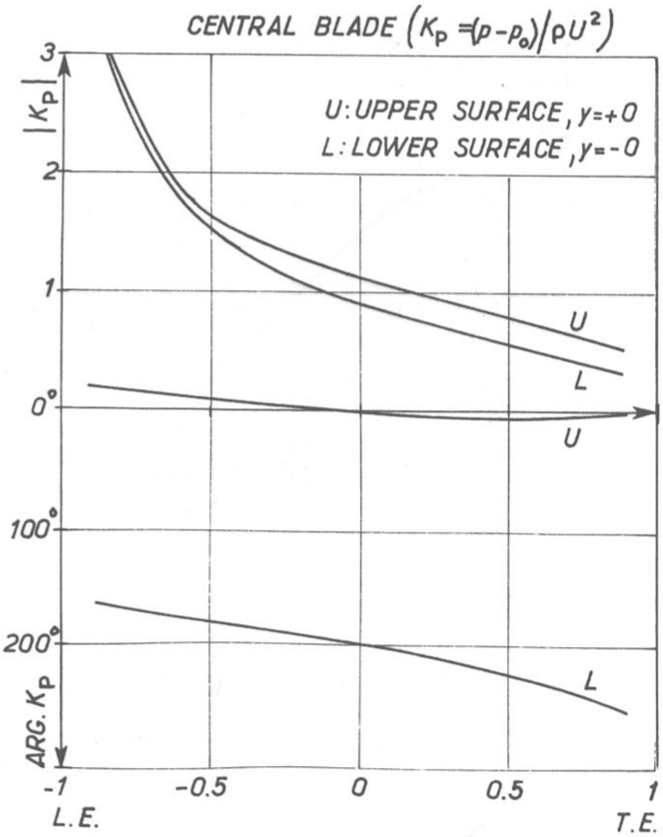

FIGURE 6

(†) N is the total number of blades in the tested cascade

Figure 7 represents the lift coefficient of blade N° (-1).

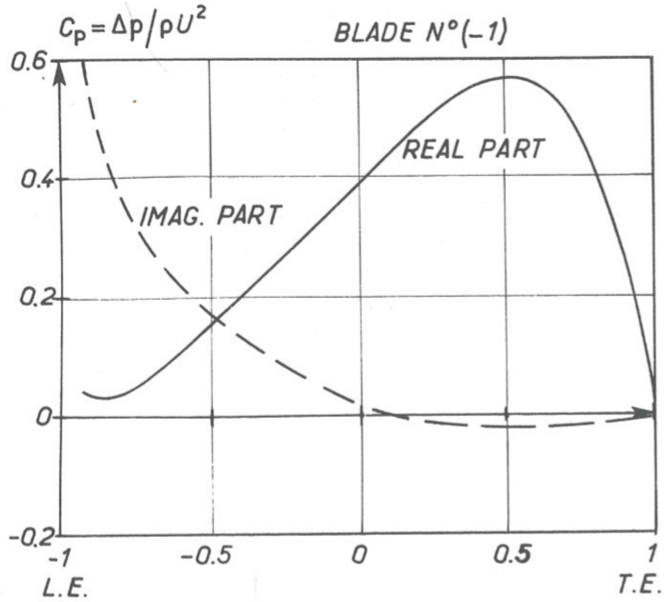

FIGURE 7

2. Results for the fundamental modes with and without incidence

Let : M = M' + jk M''' be the moment coefficient of the aerodynamic forces about the axis. Figure 8 represents M''' with respect to the phase angle (θ). The incidence considered is +5° (pressure ratio > 1). Instability region corresponds to positive values of M'''.

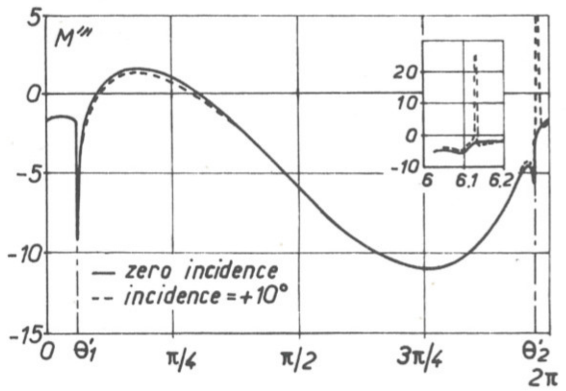

FIGURE 8

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