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A NEW METHOD OF CALCULATING THE NATURAL VIBRATIONS OF A FREE AEROPLANE

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Abstract

A method for calculation of the natural vibrations of an elastic aeroplane taking into account all the rigid body degrees of freedom and additional degrees of freedom, due to e.g. free controls, is presented. Contrary to commonly used methods the proposed procedure leads to an eigenvalue problem of a positive definite symmetric matrix of degree equal to the total number of coordinates used for the description of the vibration modes minus the number of rigid degrees of freedom. The basic data for calculations consist of matrices describing the rigid degrees of freedom as well as of the mass and flexibility influence coefficients matrices. For the reduction of matrices and calculations, only numerically stable methods were used.

I. Introduction

The well known, described in several papers, computational methods for determination of the natural vibration of a free - free body use the flexibility influence coefficients matrix for the structure prevented from rigid displacement by introducing additional, statically determinate constraints. In this approach, the equation of natural vibration has been obtained from the equation of vibration of the body with no rigid degrees of freedom with an additional vector which describes the displacements of the supports. This vector has been determined from the equations expressing the conditions that the momentum and moment of momentum of the body are equal to zero. The equation of natural vibration obtained in this way, determines an eigenvalue problem for an unsymmetric matrix in the case of a system with a finite number of degrees of freedom [(1)], and an eigenvalue problem for an integral operator with unsymmetric kernel in the case of a continuous system [(2)]. After some formal transformations it is possible to obtain an equivalent eigenvalue problem for a symmetric matrix too.

In the presented approach, these assumptions are preserved. However, after interpretation of one of the operators as a projection, considerable simplification and unifying of the procedure has been obtained. The procedure presented is based only on numerically stable transformations and concerns a structure with an arbitrary number of „rigid" degrees of freedom (as e.g. an aeroplane with free controls). The restriction on the free choice of the statically determinate constraints by determination of the influence matrix is also considered.

II. The computational method

Let us suppose that by means of physical considerations or mathematical simplifications, the real structure with an infinite number of degrees of freedom was replaced by an approximate one with n degrees of freedom. The position of that structure with respect to an inertial reference frame can be described by a n- dimensional vector $\{u\}$. The coordinates of this vector express usually the displacements of selected points of the structure and the rotations of elements in their neighbourhood. For example, if the lumped masses and inertia concept was used for the idealisation of a real structure, then the displacement of each lumped mass - inertia element is determined by six parameters

$$\{u^{(i)}\} = \begin{Bmatrix} u_1^{(i)} \\ u_2^{(i)} \\ u_3^{(i)} \\ \varphi_1^{(i)} \\ \varphi_2^{(i)} \\ \varphi_3^{(i)} \end{Bmatrix} \quad \text{and} \quad \{u\} = \begin{Bmatrix} u^{(1)} \\ u^{(2)} \\ \cdot \\ \cdot \\ \cdot \\ u^{(k)} \end{Bmatrix} \qquad (1)$$

where $u_1^{(i)}$, $u_2^{(i)}$, $u_3^{(i)}$ are the components of the displacement vector of the point O_i (Fig.1) and $\varphi_1^{(i)}$, $\varphi_2^{(i)}$, $\varphi_3^{(i)}$ are thouse of the rotation vector in the coordinate system $x_1^{(i)}$, $x_2^{(i)}$, $x_3^{(i)}$. The integer k denotes the number of lumped mass elements

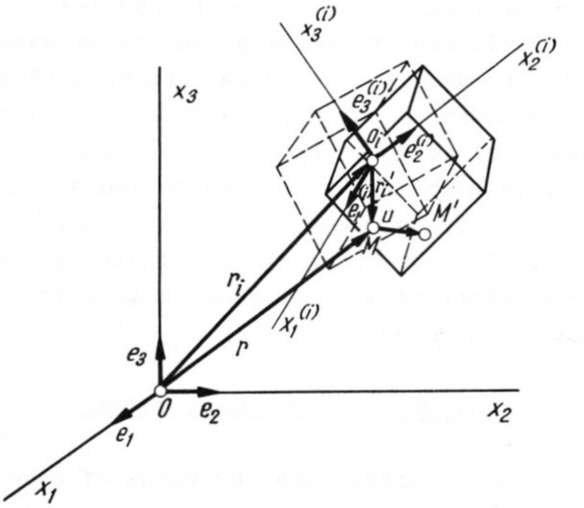

Figure 1. A lumped mass - inertia element

that constitute the structure. If some of the coordinates are dependent, then their number may be reduced and therefore we will assume that all the n coordinates of the vector $\{u\}$ are independent.

With the degree of approximation considered, the inertia properties of the structure may be described by a mass matrix $[M]$ of degree n . In the case when the lumped mass concept was used, the mass matrix is a direct sum of the inertia matrices of all elements

$$[M] = [M_1] \dotplus [M_2] \dotplus \cdot \cdot \cdot \cdot \dotplus [M_k], \qquad (2)$$

where (in the general case)

$$[M_i] = \begin{bmatrix} M^{(i)} & 0 & 0 & 0 & S_3^{(i)} & -S_2^{(i)} \\ 0 & M^{(i)} & 0 & -S_3^{(i)} & 0 & S_1^{(i)} \\ 0 & 0 & M^{(i)} & S_2^{(i)} & -S_1^{(i)} & 0 \\ 0 & -S_3^{(i)} & S_2^{(i)} & I_1^{(i)} & -I_{12}^{(i)} & -I_{13}^{(i)} \\ S_3^{(i)} & 0 & -S_1^{(i)} & -I_{21}^{(i)} & I_2^{(i)} & -I_{23}^{(i)} \\ -S_2^{(i)} & S_1^{(i)} & 0 & -I_{31}^{(i)} & -I_{32}^{(i)} & I_3^{(i)} \end{bmatrix}.$$

$M^{(i)}$ is the mass of the i-th element, $I_1^{(i)}$, $I_2^{(i)}$, $I_3^{(i)}$ are its moments of inertia with respect to the axes $x_1^{(i)}$, $x_2^{(i)}$, $x_3^{(i)}$; $I_{12}^{(i)}$, $I_{23}^{(i)}$, $I_{31}^{(i)}$ - the deviation moments and $S_j^{(i)} = \int_{M^{(i)}} x_j^{(i)} dm$ $(j = 1, 2, 3)$ - the static moments. If for the description of displacements of the i-th element less than six coordinates were used, then the degree of the matrix $[M_i]$ is subjected to an appropriate reduction. As a consequence of the independence of coordinates, the mass matrix is always positive definite

The elastic properties of the structure considered, may be described by a flexibility influence coefficients matrix $[C]$ of degree n for the same structure in which rigid degrees of freedom have been eliminated by means of additional statically determinate constraints. The element c_{ij} of the matrix $[C]$ express the change of the i-th coordinate of the vector $\{u\}$ produced by the generalized force appropriate to the j-th coordinate. If the structure consists of several elastc parts that can undergo relative displacements without deformation then the influence matrix $[C]$ must be determined after suppression of the rigid motion by introducing additional, arbitrary statically determinate constraints.

In order to make the problem of natural vibration unique, we must have a number of additional relations between the coordinates equal to the number of rigid degrees of freedom. It is convenient to suppose, that the generalized momenta for the rigid degrees of freedom are equal to zero. Let us suppose that the structure has N degrees of freedom for the rigid displacements and, if all their elastic parts become rigid, its configuration can be described by means of generalized coordinates $q_1, q_2, \ldots, q_N$. Now, if we define the „rigid modes" of the structure by the relations

$$\{\psi_j\} = \frac{\partial}{\partial q_j}\{u\},$$

for $j = 1, 2, \ldots, N$, then the condition of nullity of the generalized momenta may be represented in the form

$$\{\psi_j\}^T [M] \{\phi\} = 0, \text{ for } j = 1,2,..,N, \qquad (3)$$

where $(^T)$ denotes the transposed matrix and $\{\phi\}$ is an arbitrary vibration mode.

Let us denote by L the n - dimensional vector space containing all vectors $\{u\}$ (and $\{\phi\}$) . The vectors $\{\psi_j\}$ $(j = 1,2,..,N)$ constitute a basis of a N - dimensional subspace R of the space L. If we suppose that the metric in the space L is defined by the mass matrix $[M]$, then the relations (3) indicate that each vibration mode is orthogonal to the subspace of rigid displacements, that is $\{\phi\} \perp R$. A vector $\{u\} \in L$ can be, in a unique manner represented as a sum of two components $\{u\} = \{u_R\} + \{u_E\}$ where $\{u_R\} \in R$ and $\{u_E\} \perp R$. This expression determines a decomposition of L into a direct sum

$$L = R \oplus E \quad (\text{where } E \perp R).$$

Let us denote by $[P_R]$ and $[P_E] \equiv [P]$ the projection matrices on R and E respectivelly in the space L. Now, the orthogonality conditions (3) may be expressed in equivallent forms

$$[P_R]\{\phi\} = \{0\} \quad \text{or} \quad [P]\{\phi\} = \{\phi\} \qquad (4)$$

where $\{0\}$ is a matrix, the elements of which are all zero. Determination of the projection matrices $[P_R]$ and $[P]$ is not difficult.

Let us introduce a $n \times N$ matrix $[R]$, the columns of which are the rigid modes, that is

$$[R] = [\psi_1 \,\vdots\, \psi_2 \,\vdots\, \ldots \,\vdots\, \psi_N] .$$

The projection matrix on R in L has the form

$$[P_R] = [R][A]^{-1}[R]^T [M]$$

where

$$[A] = [R]^T [M] [R].$$

The projection matrix on E in L can be obtained as

$$[P] = [I] - [P_R], \qquad (5)$$

where $[I]$ denote the unity matrix. However, for numerical purposes, a direct determination of the matrix $[P]$ is more covenient. Let us consider a $n \times (n-N)$ matrix defined by the condition that its columns constitute an orthonormal basis of the $n-N$ - dimensional subspace E . We have then

$$[E]^T [M][E] = [I]$$

and therefore

$$[P] = [E] [E]^T [M] .$$

The equation of natural vibration may be expressed in the form

$$\{\phi_R\} + \{\phi\} = \omega^2 [C] [M] \{\phi\} \qquad (6)$$

where $\{\phi_R\} \in R$. The vector $\{\phi_R\}$ depends on the manner in which the structure was supported by determination $[C]$, and has different forms for different vibration modes. It can be determined by the orthogonality relations (3). This is the usual way of deduction of the vibration equations However the projection of (6) on the subspace E leads to the same result

$$\{\phi\} = \omega^2\ [P]\ [C]\ [M]\ \{\phi\} \qquad (7)$$

The commonly used equation of vibration is equal to (7) with the form (5) of the projection matrix. Using the second relation of (4) we may derive from (7) also an equation with a symmetric (in the space L) operator:

$$\{\phi\} = \omega^2\ [D]\{\phi\} \qquad (8)$$

where

$$[D] = [P][C][M][P].$$

To obtain, on the basis of (8), an equation with a symmetric matrix we can perform the Banachiewicz - Cholesky decomposition of the positive definite matrix $[M]$

$$[M] = [L]\,[L]^T$$

where $[L]$ is a lower triangular matrix. Bearing in mind the particular structure (2) of the matrix $[M]$, this operation can be carried out separately for each matrix $[M_i]$ of degree six at most. Let us now introduce the notations

$$\{\bar{\phi}\} = [L]^T\{\phi\},$$
$$[\bar{R}] = [L]^T\ [R] \quad \text{and} \quad [\bar{E}] = [L]^T\ [E] \qquad (9a)$$

and also

$$[\bar{C}] = [L]^T[C]\ [L],$$
$$[\bar{P}] = [L]^T\ [P]\ [L]^{-1\ T} = [\bar{E}]\,[\bar{E}]^T. \qquad (9b)$$

On multiplying (8) by the matrix $[L]^T$ we obtain the following eigenvalue problem for a symmetric matrix

$$\{\bar{\phi}\} = \omega^2\ [\bar{D}]\{\bar{\phi}\} \qquad (10)$$

where

$$[\bar{D}] = [\bar{P}]\,[\bar{C}]\,[\bar{P}].$$

The transformations (9) determine a space $\bar{L}$ isomorphic with L , the metric being defined by a unity matrix. At the same time the N - dimensional subspace R corresponds to the subspace $\bar{R}$ generated by $[\bar{R}]$. Similarly, the subspace E corresponds to the subspace $\bar{E}$ generated by $[\bar{E}]$. $[\bar{P}]$ is a projection matrix on $\bar{E}$ in the space $\bar{L}$.

Let us introduce the notation

$$\{\phi_D\} = [\bar{E}]^T\{\bar{\phi}\} \quad \text{and} \quad [D_D] = [\bar{E}]^T\,[\bar{C}]\,[\bar{E}]. \qquad (11)$$

On multiplying (10) by $[\bar{E}]^T$ and bearing in mind the relations (11) it is seen that we obtain an equation of vibration with a positive definite matrix of degree $n-N$

$$\{\phi_D\} = \omega^2\ [D_D]\{\phi_D\}. \qquad (12)$$

The $n-N$ dimensional vectors $\{\phi_D\}$ contain all information on the vibration modes, since those belong to the $n-N$ dimensional subspace E. From (11) we obtain

$$[\bar{E}]\{\phi_D\} = [\bar{E}]^T\ [\bar{E}]\,\{\bar{\phi}\} = [\bar{P}]\ \{\bar{\phi}\} = \{\bar{\phi}\}.$$

Finally, the determination of vibration frequencies and modes may be divided into the following steps:

1. Choice of a set of coordinates to describe the displacements of a given structure.
2. Determination of the mass matrix $[M]$ and the flexibility coefficients matrix $[C]$.
3. Definition of the possibly rigid displacements of the structure and thus determination of the matrix $[R]$.
4. Performance of the Banachiewicz - Cholesky decomposition of the mass matrix

$$[M] = [L]\,[L]^T$$

5. Computation of the products

$$[\bar{R}] = [L]^T [R] \quad \text{and} \quad [\bar{C}] = [L]^T [C] [L].$$

6. Determination of any orthonormal basis of the n - N dimensional subspace $\bar{E}$.
7. Computation of the eigenvalues ω^{-2} and eigenvectors $\{\phi_D\}$ of the symmetric, positive definite matrix (of degree n - N):

$$[D_D] = [\bar{E}]^T [\bar{C}] [\bar{E}].$$

8. Inversion of the triangular matrix [L].
9. Calculation of the vibration modes from the relations:

$$\{\phi\} = ([L]^{-1})^T \; [\bar{E}]\{\phi_D\}.$$

The data of the particular structure considered, are involved in the steps 1 - 3 only. All the remaining steps can be performed by universal , stable numerical methods. The computation method consisted of reducing the matrix $[D_D]$ to the tri - diagonal form by the Householder method with subsequent determination of eigenvalues by the bisection method and the determination of eigenvectors by the Wielandt inverse iteration method (3) which is particulary convenient to solve the eigenvalue problem for the matrix $[D_D]$.

The natural vibration frequencies and modes are determined by the matrix [D] . The influence coefficients matrix [C] includes some additional information on the way the structure is constrained which is of no use in the analysis of the natural vibration and eliminated finally by the projection matrix [P] . In practical applications we are often concerned with structures composed of several parts connected by statically determinate constraints. In such cases we may introduce further simplifications for the determination of the matrix [D] by replacing the matrix [C] with a sum of influence coefficients matrices, constructed in a manner analogous to [C] , but for the structure in which all their parts except one are rigid, and the constraints for fixing each of these may be different. As an example of such a procedure, let us consider an aeroplane, whose wings and tail planes are connected to the fuselage by statically determinate constraints.

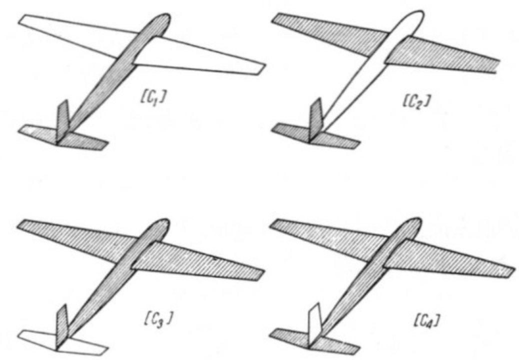

Figure 2. Determination of the influence coefficients matrix

In this case we may determine the matrices $[C_1]$ to $[C_4]$ independently, assuming the shaded (in Fig.2) parts of the aeroplane as rigid and obtain the matrix [C] as a sum

$$[C] = [C_1] + [C_2] + [C_3] + [C_4]$$

III. Determination of the matrices $[\bar{E}]$ and $[D_D]$

From the above consideration it follows, that the central problem in the estabilishment of the equation of natural vibration is that of setting up the matrix $[\bar{E}]$. Bearing in mind that the matrix $[\bar{E}]$ is arbitrary to a considerable degree, we can choose for its determination methods particulary convenient from the point of view of simplicity and numerical stability. Below we shall describe a method based on the application of elementary hermitian matrices and analogous to the Householder method, of reducing a symmetric matrix to the tridiagonal form.

Let us suppose that the matrix $[\bar{E}]$ constitutes the first $n-N$ columns of an orthogonal matrix $[H]^T$ of degree n

$$[\bar{E}] = [H]^T \begin{bmatrix} I \\ \hdashline 0 \end{bmatrix} .$$

From the condition $[\bar{E}]^T[\bar{R}] = [0]$ it follows that

$$[H][R] = \begin{bmatrix} 0 \\ \hdashline T \end{bmatrix} \qquad (13)$$

where $[T]$ is a nonsingular matrix of degree N.

The matrix $[H]$ is sought for in the form of the product

$$[H] = [Q_1][Q_2] \cdots [Q_N] \qquad (14)$$

every factor of which, $[Q_r]$, is an elementary hermitian orthogonal matrix set up according to the scheme

$$[Q_r] = [I] - 2\{w_r\}\{w_r\}^T \quad \text{where} \quad \{w_r\}^T\{w_r\} = 1 \qquad (15)$$

From (13) we obtain the following condition for the determination of each particular matrix $[Q_r]$

$$([Q_1]([Q_2](\cdots([Q_N][R])\cdots)) = \begin{bmatrix} 0 \\ \hdashline T \end{bmatrix} \qquad (16)$$

The matrices $\{w_N\}, \{w_{N-1}\}, \ldots, \{w_1\}$ are selected in such a manner that if multiplication is performed in the order determined by the parentheses in (16), the elements above the diagonal of the block $[T]$ become zero at each stage. This means that the matrix $[T]$ is assumed to be in a lower triangular form. Let us consider the result of multiplication by the matrix $[Q_r]$. As a result of the preceding steps, the matrix $[R]$ has been transformed to

$$[Y_r] = \begin{bmatrix} F_r & 0 \\ \hdashline B_r & G_r \end{bmatrix} ,$$

where $[F_r]$ is a $(n-N+r) \times r$ matrix and $[G_r]$ is a lower triangular matrix of the degree $N-r$. We have, of course, $[F_N] = [\bar{R}]$ and $[G_0] = [T]$. If the matrix $[Q_r]$ is selected in the form

$$[Q_r] = \begin{bmatrix} [I] - 2\{v_r\}\{v_r\}^T & 0 \\ \hdashline 0 & I \end{bmatrix}$$

(where $\{v_r\}^T\{v_r\} = 1$), we obtain

$$[Y_{r-1}] = [Q_r][Y_r] = \begin{bmatrix} ([I] - 2\{v_r\}\{v_r\}^T)[F_r] & 0 \\ \hdashline B_r & G_r \end{bmatrix}.$$

In this connection the matrix $[F_r]$ is the only one to be modified. Let us denote it last column by $\{a_r\}$. The elements of the matrix $\{v_r\}$ should be selected in such a manner that

$$([I] - 2\{v_r\}\{v_r\}^T)\{a_r\} = \{e_r\} S_r ,$$

where $\{e_r\}$ is a column matrix, the last element of which is unity and the remaining ones zero. If the last element of $\{a_r\}$ is the only one different from zero, we have, of course, $[Q_r] = [I]$. In the opposite case we find

$$S_r^2 = \{a_r\}^T\{a_r\}.$$

Let us denote

$$\{u_r\} = \{a_r\} - S_r\{e_r\}. \qquad (17)$$

Then, we obtain

$$2\{v_r\}\{v_r\}^T = \{u_r\}\{u_r\}^T / (2K_r^2) ,$$

where

$$2K_r^2 = \{u_r\}^T\{a_r\} = S_r^2 - S_r\{e_r\}^T\{a_r\}.$$

The sign of the number S_r must be determined, in order to preserve numerical stability in such a manner that $S_r\{e_r\}^T\{a_r\} \leq 0$ (the product $\{e_r\}^T\{a_r\}$ is equal to the last element of the matrix $\{a_r\}$).

The transformed matrix

$$([I] - 2\{v_r\}\{v_r\}^T)[F_r]$$

is conveniently determined in two steps. We obtain first the auxiliary matrix

$$\{p_r\}^T = \{u_r\}^T \, [F_r] \, / \, (2.K_r^2) \, ,$$

and then

$$([I] - 2\,\{v_r\}\{v_r\}^T)\,[F_r] = [F_r] - \{u_r\}\{p_r\}^T .$$

For further calculations the matrices $\{u_r\}$ with $n - N + r$ elements, and the coefficients $2\,K_r^2\ (r = 1, 2, \ldots, N)$ are the only quantities necessary, because

$$2\,\{w_r\}\{w_r\}^T = \{w_r'\}\{w_r'\}^T / (2\,K_r^2) \quad \text{where}$$

$$\{w_r'\} = \begin{Bmatrix} u_r \\ \hline 0 \end{Bmatrix} \, .$$

In view of the method for the determination of the matrices $\{u_r\}$ (17), it is convenient to store them in the same space of the computer memory, in which columns of the matrix $[\bar{R}]$ which are no more needed were located. The Householder method described above is numerically unconditionnaly stable [3].

After N steps

$$[C_{r-1}] = [Q_r]\,[C_r]\,[Q_r], \ (r = N, N-1, \ldots, 1) \tag{18}$$

we obtain on the basis of (14) and (11)

$$[D_D] = [I \,\vdots\, 0]\;[C_0] \begin{bmatrix} I \\ \hline 0 \end{bmatrix}, \ ([C_N] = [\bar{C}]) . \tag{19}$$

Taking into consideration the form (15) of the matrix $[Q_r]$ every transformation (18) is carried out according to the scheme

$$[C_{r-1}] = [C_r] - \{w_r'\}\{q_r\}^T - \{q_r\}\{w_r'\}^T \tag{20a}$$

where

$$\{q_r\} = \{p_r\} - \tfrac{1}{2}\{w_r'\}(\{w_r'\}^T\{p_r\} \, / (2\,K_r^2)) \tag{20b}$$

and

$$\{p_r\} = [C_r]\{w_r'\} \, / \, (2\,K_r^2) \, . \tag{20c}$$

Bearing in mind the form (19) of the matrix $[D_D]$, it sufficies, according to the formula (20) to determine only the elements belonging to the submatrix of degree $n - N + r - 1$ of the matrix $[C_{r-1}]$ and, for the matrix $\{q_r\}$ to calculate the first $n - N + r$ elements only.

IV. Example of calculations

In order to demonstrate a practical application of the previously discussed method, an example of natural vibration calculation for a glider is presented. Making use of the particular featurés of glider (and partially aeroplane) structures, we can introduce further simplifications to the procedure described above.

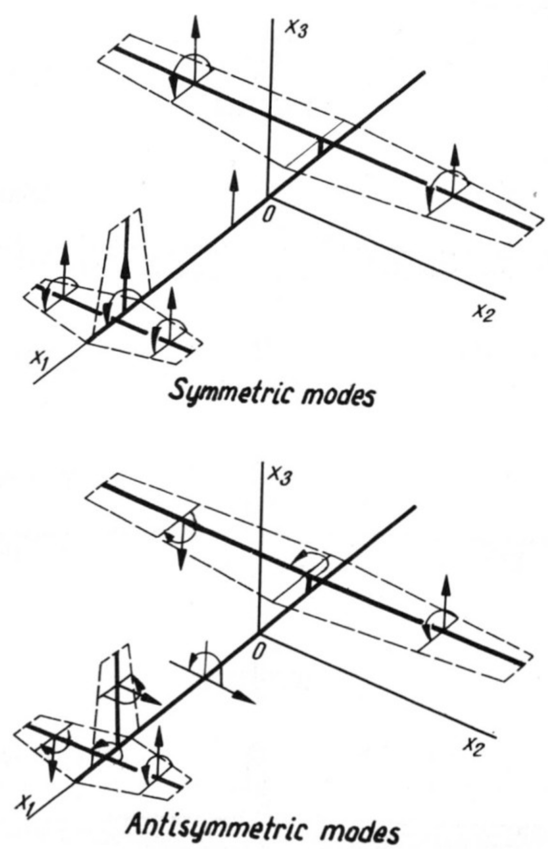

Figure 3. Computational scheme of a glider

Because of the symmetry of mass and stiffness distribution, the symmetric and antisymmetric modes may be calculated separatelly. The slenderness of the elements

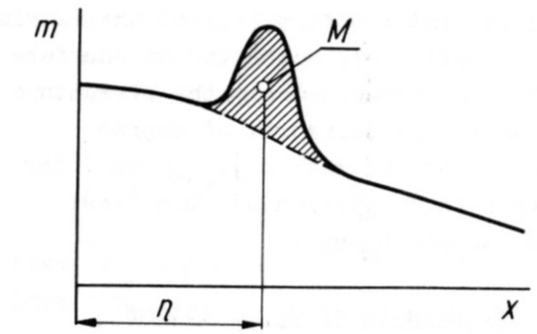

Figure 4. Smoothing of the mass distribution

allows the structure to be replaced by an approximate model consisting of beams as shown on Fig.3 . Wings and tails may be treated as rigid in their planes which implies that with satisfactory accuracy we can take into consideration only the displacements indicated by arrows in Fig.3. Direct replacement of the structure by lumped masses and inertia is not convenient. Better accuracy can be achieved by taking into account the continuous mass distribution and using numerical integra-

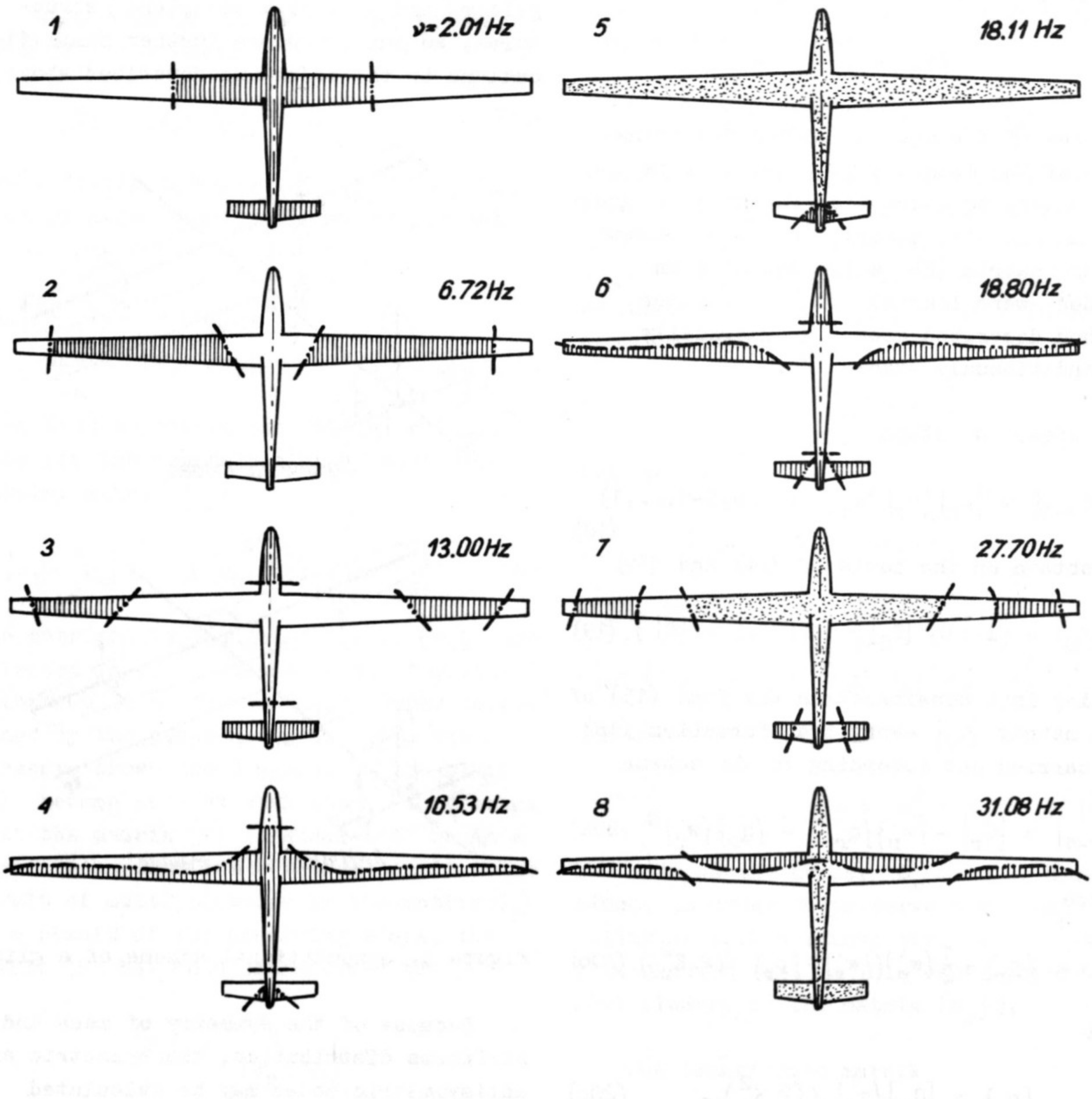

Fig.5 Symmetric vibration modes

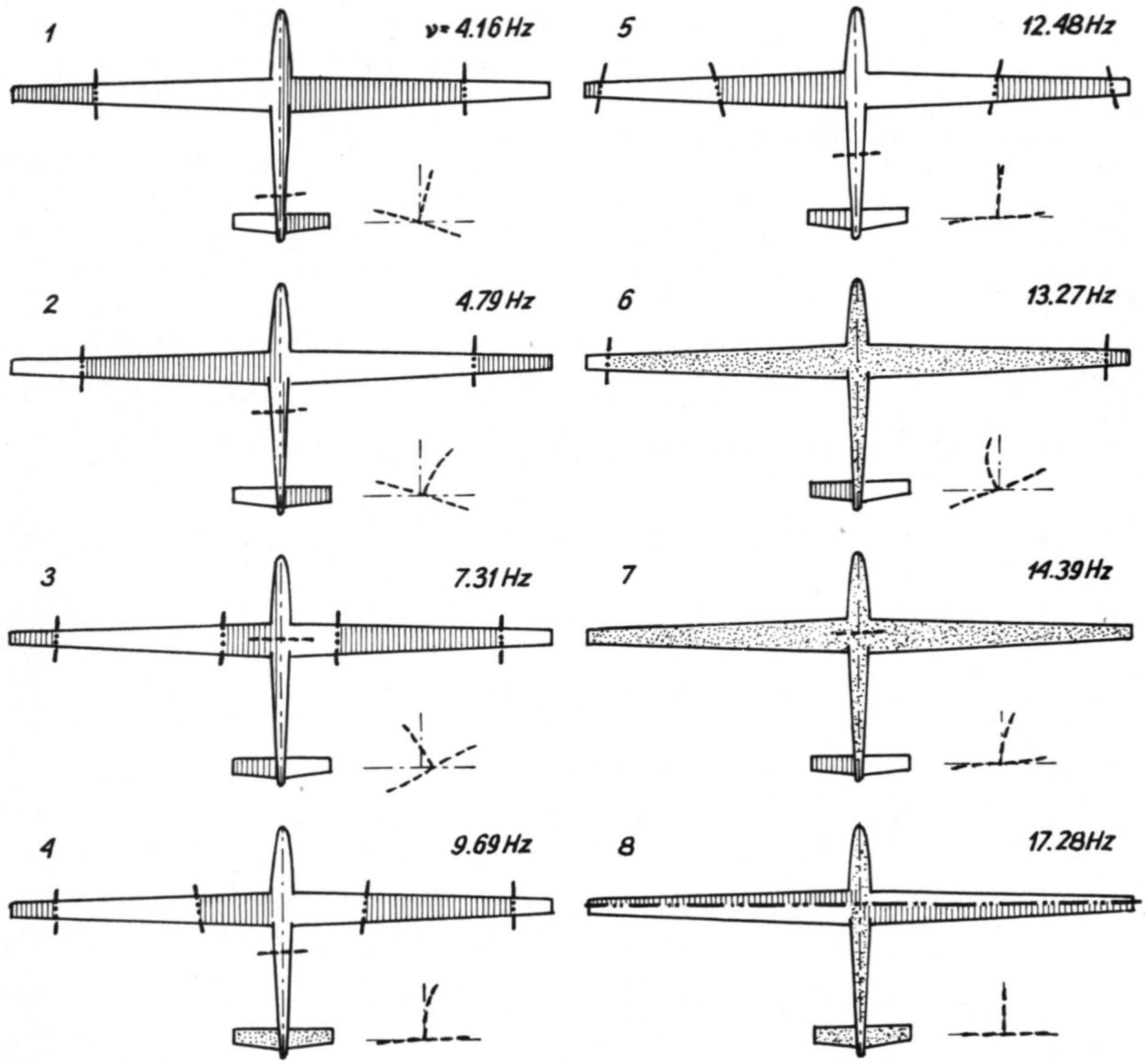

Fig.6 Antisymmetric vibration modes

tion methods to determine the lumped masses. However, we can get good accuracy this way, only if the mass distributions are smooth enough. In the case of irregular distributions as on Fig.4 , we may obtain sufficiently smooth distributions by introducing a concentrated mass with a value equal to the shaded field on Fig.4 , at the point with coordinate η . The remaining parameters of this lumped mass may be obtained from an analogous treatment of the diagrams for the static and inertia moments.

The rigid modes $\{\Psi_1\}$,$\{\Psi_2\}$,...,$\{\Psi_N\}$ are determined by the geometry and degrees of freedom of the structure. The influence coefficients matrix for a glider may be obtained very easily, if we can suppose that the beams have straight elastic axes.

Fig.5 and 6 show the result of calculations performed with 48 and 54 coordinates (degrees of freedom) in the symmetric and antisymmetric cases respectivelly. Despite the small number of coordinates, the principal modes were calculated with sufficient accuracy.

Because of the numerical stability of the method presented, nothing prevent the performance of the calculation with a larger number of coordinates too, as is needed in the case of complex aeroplane structures. It may be mentioned that this method is faster in computing time than those commonly used, which employ larger and unsymmetric matrices.

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