ICAS PAPER No. 72 – 03

A SEMI-EMPIRICAL METHOD FOR THE EVALUATION OF AEROTHERMODYNAMIC PROPERTIES IN THE INTERMEDIATE HYPERSONIC FLOW REGIMES

by
Ernesto Vallerani
Head of the Aerothermodynamic Department
FIAT - Aviation Division
Torino, Italy

The Eighth Congress of the International Council of the Aeronautical Sciences

INTERNATIONAAL CONGRESCENTRUM RAI–AMSTERDAM, THE NETHERLANDS
AUGUST 28 TO SEPTEMBER 2, 1972

Price: 3. Dfl.

A SEMIEMPIRICAL METHOD FOR THE EVALUATION OF AEROTHERMODYNAMIC PROPERTIES IN THE INTERMEDIATE HYPERSONIC FLOW REGIMES

E. VALLERANI
Professor of Gasdynamics
Head of the Aerothermodynamic Department
FIAT Aviation Division
TORINO - Italy

ABSTRACT

A method enabling the prediction of the aerothermodynamic properties all over the entire range of flow regimes ranging between the continuum and the free molecular flow without a solution of continuity has been developed, by suitably combining the available results valid for the extreme flow regimes on the basis of a probability function defined on an appropriate intermediate flow Knudsen number.

In the definition of the probability function a parameter is introduced to allow an improved capability of the correlation formula to generate results in better agreement with the experimental data, whenever available, or with the results of complex and accurate theories.

The method has been applied with success to the evaluation of sphere drag and stagnation point heat fluxes, the results are compared with the available experimental data and the theoretical predictions.

INTRODUCTION

The design engineer facing the problem of predicting the aerothermodynamic actions exerted on the surface of a space vehicle by the medium, during the crossing of the upper atmosphere, has but very limited methods of analysis of general validity at his disposal due to the inherent complexity of the solutions, up to now formulated, for the flow regimes ranging between the continuum flow and the free molecular flow regimes.

Reference is made to Probstein and Kemp [1] for the relevant definitions of the regimes such as: vorticity interaction, viscous layer, incipient merged layer, fully merged layer, transitional layer, first order collision, in which according to the degree of rarefaction what we are calling "intermediate regimes" can be subdivided.(fig. 1)

For several of the above sub-regimes appropriate methods have been developed to solve, time by time, specific problems; the very involved physical situation has led to sophisticated and complex mathematical models generally unpractical for engineering purposes

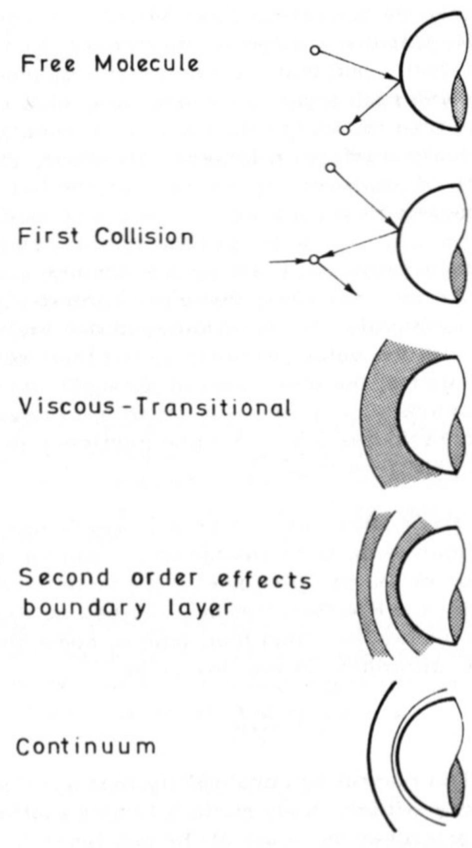

Fig. 1 - Hypersonic Flow Regimes

In order to provide for the lack simple methods able of predicting the aerothermodynamic actions all over the entire range of flow regimes without solution of continuity, the author has developed the present semi-empirical method that by suitably combining the results valid for the extreme flow regimes (the continuum flow and the free molecular flow, the only ones for which well established theories provide sufficiently simple methods of evaluation of the basic thermodynamic actions) allows a first order approximate evaluation of the actions exerted on simple bodies in the "intermediate flow regimes".

OUTLINE OF THE METHOD

The need to obtain at least engineering results has suggested, since long time, to rely on experimental data for constructing empirical correlations; very promising is the correlation formula originally proposed by Martino [2] for predicting the temperature recovery factors on circular cylinders, successively adopted by Reeves and Van Camp [3] and more recently by Blick [4] for evaluating some aerothermodynamic coefficients in the intermediate flow regimes.

Starting from the consideration that in the continuum flow a gas layer is formed around the body so that the incoming molecules do not strike directly the surface but collide with molecules whose state is altered by the presence of the body itself, and that increasing the degree of rarefaction such a gas layer weakens, disappearing in the free molecular flow regime dominated by gas-solid surface molecular collisions, the evaluation of whatever aerodynamic action in the intermediate flow regimes can be performed assuming that part of the molecules passes through the gas layer adjacent the body and impinges on it without any collision, therefore contributing to the global value of the property under investigation with the value pertinent to the free molecular flow regime; the other part of the molecules, on the contrary, collides with those in the gas layer and contributes with the value pertinent to the continuum flow regime.

The number of particles acting in one or in the other mode is connected to the degree of rarefaction of the gas that can be characterized in turn by the Knudsen number, defined as the ratio of the molecular mean free path to some characteristic dimension of the flow field:

$$K_n = \lambda / d \qquad (1)$$

The normalized probability that a molecule collides with the body surface before colliding with any other molecule of the gas layer is assumed to be a continous, monotonic function of the degree of rarefaction of the gas, and therefore of the Knudsen number P=P (Kn); the resultant basic expression for any assigned aerothermodynamic property or variable in the intermediate flow regimes is therefore:

$$X^* = P X_{FM} + (1-P) X_c \qquad (2)$$

being: X_{FM} the value computed according to free molecular flow theories

X_c the flow value computed according to continuum theories.

For the normalized probability function P the simplest expression satisfying the correct limiting conditions:

$P = 0$ for $K_n \to 0$ (continuum flow)

$P = 1$ for $K_n \to \infty$ (free molecular flow)

that is to say:

$$P = K_n \, (1 + K_n)^{-1} \qquad (3)$$

has been assumed by the previous investigators (2) (3) (4)

The difficulty in applying formulas such as the Martino one (eq. 2) lies in the appropriate definition in the intermediate flow regimes of a Knudsen number, that is to say in the correct choice of the significant dimension d connected with the flow field around an assigned body. As outlined by Tsien [5] such a characteristic dimension d is, for the continuum flow, the boundary layer thickness δ, while for the free molecular flow is the body dimension R itself.

Evaluation of the Knudsen Number

From the kinetic theory of gases, the Knudsen number can be expressed in terms of the Mach number and the Reynolds number based on the characteristic flow field dimension

$$(K_n)_d \equiv \lambda/d = 1.25\sqrt{\gamma}\,[M/(R_e)_d]_\infty \qquad (4)$$

By introducing the characteristic body dimension R, the Knudsen number can be rewritten as:

$$(K_n)_R \equiv \lambda/R = 1.25\sqrt{\gamma}\,[M/(R_e)_R]_\infty \, R/d \qquad (5)$$

In the extreme case of free molecular flow regime, being as recalled d=R, the Knudsen number is therefore defined as:

$$(K_n)_{FM} \equiv \lambda/R = 1.25\sqrt{\gamma}\,[M/(R_e)_R]_\infty = C_{FM}/\xi \qquad (6)$$

In the other extreme case of continuum flow regime, the characteristic dimension being the boundary layer displacement thickness that according to classical treatments can be estimated to be:

$$\delta/R \div [M/(R_e)_R]^a_\infty \, M^b_\infty \qquad (7)$$

the Knudsen number is defined as:

$$(K_n)_c \equiv \lambda/\delta = C_c \, \xi^{(a-1)} M_\infty^{-b} \qquad (8)$$

For both the two extreme cases, the Knudsen number can therefore be defined, on the basis of the characteristic body dymension R and free stream conditions, as a function of the parameter $\xi = (Re/M)_\infty$ lumping the Mach and Reynolds numbers and at maximum the Mach number M_∞. The values of the constants C_c, a, b differ according to the body shape and the free stream velocity analized.

Even if the continuum flow regime and the free molecular flow regime are strictly attained only for values of the Knudsen number limiting respectively zero $(K_n)_c \to 0$ and infinity $(K_n)_{FM} \to \infty$, it is well known that the dominion of the classical gasdynamic from one side and of the kinetic theory of gases from the other are to be considered wider.

For the lower boundary of the free molecular flow regime we adopt the value originally proposed by Tsien [5] $(Kn_\infty)_{FM} = 10 = K_1$ while for the upper boundary of the continuum flow regime we accept the value more recently proposed by Reeves [6] $(Kn_\infty)_c = 0.05 = K_2$ different values could be adopted.

To account for the effective boundaries between the continuum and the lower region of the intermediate flow regimes (slip flow) and between the free molecular flow and the upper region of the intermediate flow regimes (near free molecular flow), two modified Knudsen numbers are introduced in the present investigation:

for the continuum flow regime:

$$(\bar{K}_n)_c = (K_n)_c - K_2 \qquad (9)$$

for the free molecular flow regime:

$$(\bar{K}_n)_{FM} = [(K_n)_{FM}^{-1} - (K_1)^{-1}]^{-1} \qquad (10)$$

These modified Knudsen numbers are determined in such a way that the upper boundary of the continuum flow regime is defined by

$$(\bar{K}_n)_c \rightarrow 0 \qquad (11)$$

and the lower boundary of the free molecular flow regime by:

$$(\bar{K}_n)_{FM} \rightarrow \infty \qquad (12)$$

The leading idea, on which the present correlation method is based, that circumvents the recalled difficulty of defining an appropriate Knudsen number to use in the correlation formula, is the consideration that the Knudsen number itself, as whatever other quantity defining any aerothermodynamic action, has to be determined, in the intermediate flow regimes, by applying the above recalled Martino formula, thus obtaining:

$$(\bar{K}_n)^* = P(\bar{K}_n)_{FM} + (1-P)(\bar{K}_n)_c \qquad (13)$$

The Probability Function P

By definition the function P represents the probability that an incoming molecule collides directly with the body surface be fore colliding with any other molecule of the gas layer formed around the body, that is to say it represents the fraction of molecules passing through the gas layer without collisions before stiking on the body surface:

$$P = N_s/N_\infty = N_s/(N_s + N_i) \qquad (14)$$

being: N_∞ the total number of incoming molecules in the free stream.

N_s the number of molecules undergoing surface collisions.

N_i the number of molecules undergoing an intermolecular collision.

The probability function P has obviously to satisfy, at the two extreme regimes, to the boundary conditions:

lim. P. = O for $Kn \rightarrow 0$ Continuum flow

lim. P. = I for $Kn \rightarrow \infty$ Free molecular flow

in the previous investigations [2] [3] [4] it has been assumed

$$P = K_n/(1+K_n) \qquad (15)$$

for no other reason, up to our knowledge, but for the fact that eq. 15 is the simplest mathematical expression satisfying the above conditions.

A different form of the probability function has been developed, in the frame of the present investigation, starting from the simple model of flow proposed by Whitfield [7] that is based on the following four assumptions:

- the flow is hypersonic, therefore the number flux of the free stream molecules having a chance of colliding with the body surface is given by the product of the free stream number density and velocity ($n_\infty V_\infty$),
- all the molecules colliding with the body are re-emitted normal to the surface,
- each re-emitted molecule experiences only one collision with a free stream molecule,
- a collision surface is postulated on which all the intermolecular collisions occur.

According to the above simplified model the total number, per unit time, of incoming free stream molecules that can be involved in collisions with the body is:

$$N_\infty = n_\infty V_\infty A \qquad (16)$$

being A the frontal surface area of the body.
The number of molecules, per unit time, undergoing directly a surface collision is given in terms of the number density of the molecules behind the collision surface n_s by the relation:

$$N_s = n_s V_\infty A = N_\infty - N_c \qquad (17)$$

being N_c the number of molecules, per unit time, that are removed from the free stream by the intermolecular collisions occurring at the collision surface, with the re-emitted molecules.

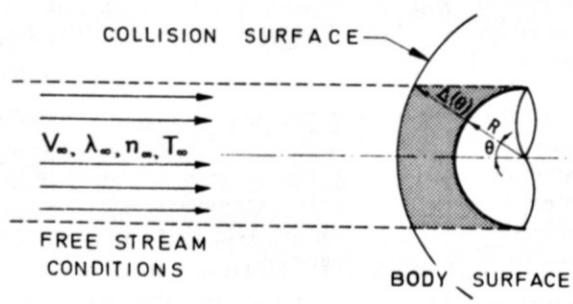

Fig.2 - Scheme of Collision Surface Model

In view of the assumptions made such a number is equal to the number of molecules re-emitted from the body surface within the angle θ (see fig. 2), that is:

$$N_c = n_\infty V_\infty A \sin^2\theta \qquad (18)$$

The ratio of the number density of the molecules colliding with the surface of the body, to the number density of the free stream molecules can easily be derived from the above equations and turns out to be:

$$\frac{n_s}{n_\infty} = (1 + \sin^2\theta)^{-1} \qquad (19)$$

From simple geometrical considerations evident in fig. 2, the relation

$$\sin\theta = [1 + \Delta(\theta)/R]^{-1} \qquad (20)$$

involving the collision surface distance Δ is obtained.

Whitefield and Stephenson [8] have correlated the collision surface distance to the free stream mean free path, in the form:

$$\Delta/R = B\,(K_n)_{FM} \qquad (21)$$

In conclusion we can state that, on the basis of the definition of the probability function (eq. 14), in virtue of eq. 19, accounting for eq. 20 and 21 we obtain:

$$P \equiv N_s/N_\infty = n_s/n_\infty = [1 + (1+\Delta/R)^{-2}]^{-1} = \frac{(C+K_n)^2}{C^2 + (C+K_n)^2} \qquad (22)$$

being:

$$C = 1/B$$

The new expression found for the probability function, being derived from a model valid for a rarefied gas, satisfies the limiting condition of the free molecular flow regime P=1 for $Kn \to \infty$, but not the one of the continuum flow regime; it cannot therefore be applied as it stands, but it suggests a new form for the probability function namely:

$$P = K_n^{\,2}/(\alpha + K_n^{\,2}) \qquad (23)$$

that not only satisfies the two boundary conditions but enables also to satisfy to the smooth limiting condition at both the boundaries being:

$$P' = 2\alpha K_n (\alpha + K_n^2)^{-2} = 0 \qquad (24)$$

for $Kn \to 0$ and for $Kn \to \infty$, a feature not possessed by the original form of the probability function (eq. 3).

In the definition of the new probability function a parameter α is directly introduced to allow an improved capability of the correlation formula to generate results in better agreement with the experimental data, whenever available, or with the results of accurate theories.

The Knudsen number to adopt in the expression of the probability function is, for the reasons discussed in the previous paragraph, the intermediate Knudsen number $\bar{K}_n^*$.

By suitably combining the equation defining the new probability function (eq. 23) with the one defining the modified intermediate flow regime Knudsen number (eq. 13), the following cubic equation is obtained

$$aP^3 + bP^2 + (c+\alpha)P + d = 0 \qquad (25)$$

with

$$a = [(\bar{K}_{n\infty})_{FM} - (\bar{K}_{n\infty})_c]^2$$

$$b = 2(\bar{K}_{n\infty})_c[(\bar{K}_{n\infty})_{FM} - (\bar{K}_{n\infty})_c] - [(\bar{K}_{n\infty})_{FM} - (\bar{K}_{n\infty})_c]^2$$

$$c = [(\bar{K}_{n\infty})_c]^2 - 2(\bar{K}_{n\infty})_c[(\bar{K}_{n\infty})_{FM} - (\bar{K}_{n\infty})_c]$$

$$d = -[(\bar{K}_{n\infty})_c]^2$$

that admits always a solution $0 \leq P \leq 1$.

The value of the modified probability function turns out to be therefore a rather complex but algebric function of the modified Knudsen numbers $(\bar{K}_n)_c$ (eq. 9) and $(\bar{K}_n)_{FM}$ (eq. 10), that is to say in general the probability function is dependent on the quantities ξ ratio of the Reynolds over the Mach number, the Mach number, the parameters K_1, K_2 defining the limits of the two extreme flow regimes, the free parameter α and the constants C_c, a, b through which the geometry and the free stream speed turn out to affect the value of P, preventing to derive once for all a probability function diagram possessing an universal character.

For the particular case of the sphere in hypersonic flow conditions treated in detail in the present note $C_c = 6.08$, $a = 1/2$, $b = 0$; the probability function P dependent only upon $\xi = (R/M)_\infty$ and the free parameter α is reported in figure 3 together with the boundaries of the two extreme regimes obtained from eq. 9 and 10 on the basis of the values of eq. 6 and 8 respectively.

$\bar{\xi}_c = 1{,}44\ 10^4$ upper boundary of the continuum flow

$\bar{\xi}_{FM} = 1{,}48\ 10^{-1}$ lower boundary of the free molecular flow.

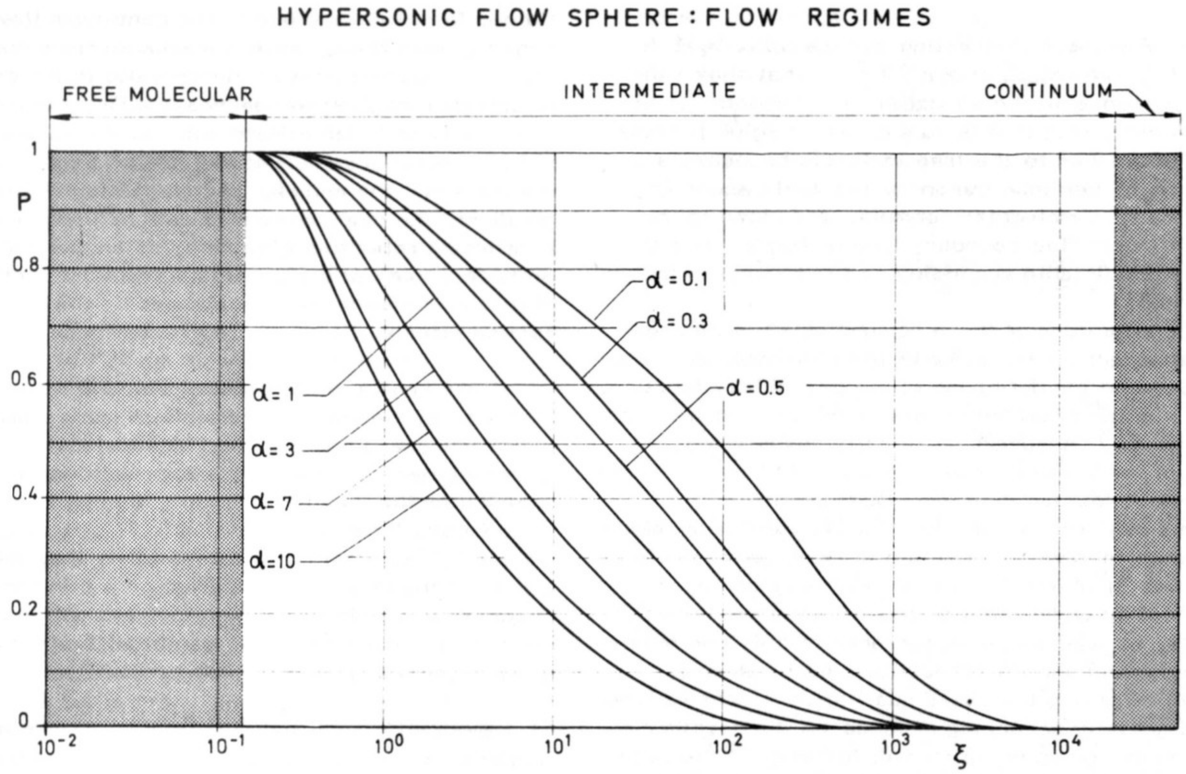

Fig. 3 - Probability Function $P = P(\xi)$

BRIDGING FORMULAS AND "OVERSHOOTS"

The bridging formulas capable of predicting, on the basis of theories valid in the extreme regimes, the aerothermodynamic actions all over the entire range of intermediate flow regimes, without solution of continuity, can be presented in a normalized form which evidentiates some important features later on discussed.

By normalizing the intermediate flow values, obtained according to the basic equation (eq. 3), with the value pertinent to the continuum flow regime, the form

$$\psi_x = X^*/X_c = P\, X_{FM}/X_c + (1-P) \qquad (26)$$

is obtained while, by normalizing with the value pertinent to the free molecular flow regime, the form

$$\Gamma_x = X^*/X_{FM} = P + (1-P)X_c/X_{FM} \qquad (27)$$

is obtained.

In general the aerothermodynamic properties in the continuum as well as in the free molecular flow regimes depend on the Mach and the Reynolds numbers, therefore the value of X_{FM}/X_c or X_c/X_{FM} turns out to be a function of the parameter ξ and at maximum of the Mach number, as the value of the probability function previously defined, accordingly:

$$\psi = \psi[\xi, M, P(\xi, M, \alpha)]$$
$$\Gamma = \Gamma[\xi, M, P(\xi, M, \alpha)]$$

For the particular case of the sphere in hypersonic flow, simplier it results:

$$\psi = \psi[\xi, P(\xi, \alpha)]$$
$$\Gamma = \Gamma[\xi, P(\xi, \alpha)]$$

To depict the possible trend of the parameter ψ (the situation for Γ is analogous) let us examine the three possible cases for the function X_{FM}/X_c assumed monotically increasing in the intermediate flow regime range as schematized in figure 4.

For case a) $X_{FM}/X_c > 1$ all over the range, the adoption of the free molecular formula in the intermediate flow regime would introduce an over-extimate of the aerothermodynamic property, while the adoption of the continuum flow formula would introduce an under-extimate. The bridging formula on the contrary provides a smooth passage from one extreme regime to the other.

For case c) $X_{FM}/X_c < 1$ all over the range, the situation is reversed and the bridging formula still provides a smooth passage from the free molecular to the continuum flow regime.

Far more interesting is case b) $X_{FM}/X_c > 1$ for $\xi < \bar{\xi}$ and $X_{FM}/X_c < 1$ for $\xi > \bar{\xi}$, that shows the occurrence of the so called "Overshoots" at the two extremes; that is to say the bridging formula gives, close to the free molecule boundary a value higher than the one predictable according to the free molecular formula, and close to the continuum flow boundary a value higher than the one predictable according to the continuum theory formula.
The magnitude of these "overshoots" is strongly dependent on the value of the parameter α introduced in the definition of the probability function.
A similar situation occurs in the case of the function X_{FM}/X_c assumed monotically decreasing, except for the occurrence in case b) of two "Undershoot" instead than two "Overshoots".
The case of the function X_{FM}/X_c exhibiting maxima or minima gives origin to more complex situations in which the presence of only one overshoot or undershoot at one of extremes is possible, as well as the occurrence of only one overshoot and one undershoot at either extreme.
Which among the cases qualitatively schematized is actually occurring depends on the aerothermodynamic property analyzed; for most of the usual properties the case depicted as case b) of X_{FM}/X_c monotically increasing occurs.
For such cases the bridging formula brings in evidence the existance of two overshoots at the extremes of the intermediate flow regimes. Some theoretical predictions sustained as well as by experimental data support indeed the occurrence of these overshoots not predictable by any other correlation formula.
Without entering too much into the details we recall some basic considerations qualitatively supporting the formation of the "overshoots" previously denounced.

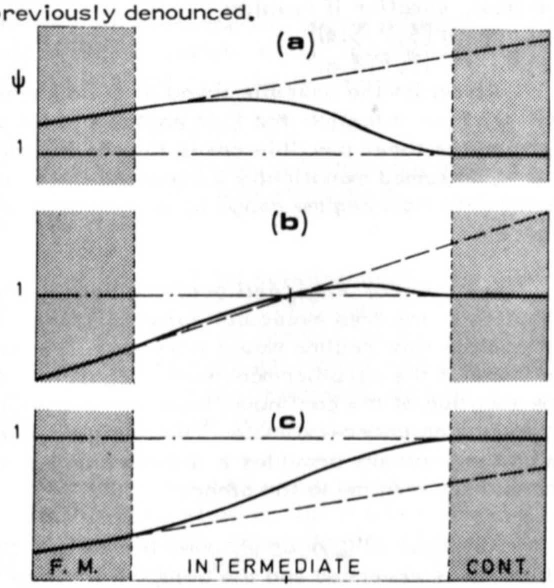

Fig. 4- Schematic Trend $\psi = \psi(\xi)$

Theories accounting for the so called second order boundary layer effects (9) (10) have evidentiated, for regimes close to the continuum flow regime, deviations, quite remarkable from the classical boundary layer theory valid in the continuum flow regime; for decreasing Reynolds numbers, second order effects such as the longitudinal and transverse curvature, the slip and temperature jump, the entropy and stagnation entalpy gradient, the displacement effects begin to ply conflicting roles that give rise, as an over all, to an "overshoot" at the departure from the continuum flow successively counterbalanced by the rarefaction effects.

Less known are the phenomena that make the situation at the other extreme much more complex; up to few years ago the near free molecular flow theories were giving results approaching the free molecular flow limit from below, that is to say without any "overshoot" (11) (12)(13). It has been Kogan (14) to point out the possibility that the drag and the heat flux of a plate or a thin cone at small angles of incidence, calculated on the basis of a first collision theory, can be higher than the free molecular regime limiting values.

Basically the model adopted for extimating the actions exerted on the body surface assumes that for all the collisions of any molecule with the surface a complete transfer of its momentum to the object occurs.
Three classes of molecules are assumed to exist in the near free molecular flow regime: (o) the oncoming free stream molecules (e) the emitted molecules, diffusively re-emitted from the body surface, (s) the scattered molecules resulting from the encounters of oncoming and emitted molecules.
The free stream molecules (o) suffer collisions with the re-emitted ones (e) and a change in the flux of molecules at the surface of the body with respect to the free stream value occurs due to two conflicting phenomena: a shielding of the body from some oncoming free stream molecules due to collisions with re-emitted molecules that tends to reduce the value of whatever property below its free stream value, and a scattering onto the surface of class (o) molecules that other wise would not have reached the body surface that tends to increase the value of whatever property over its free stream value.

Kogan (14) uses order of magnitude arguments to derive for the first collision regime formulas with undetermined numerical coefficient that he himself has successively computed resorting to the numerical Monte Carlo method (15). Successively Laurmann (16) has extended the analysis proving that for cone semivertex angles less than about 9°, near free molecular flow theory predicts, for a diffusively reflecting surface, a rise

of drag over the free molecular value.
From the analysis of Kogan the occurrence of overshoots in the near free molecular flow regime seems confined to flat surfaces at small incidence, being excluded for bluff shapes Bird [17] on the contrary using the Monte Carlo technique has recognized that for cylinders and spheres the flux of molecules colliding with the body surface is, for Knudsen numbers Kn_{∞} between 3 and 30, higher than the value relative to the free molecular flow condition, so that under the condition of cold wall $T_w=T_{\infty}$ the drag coefficient rises above the free molecular flow value before falling towards the continuum flow value, due to an overcompensation of the fall of the streamwise momentum by the above mentioned increase of the flux of molecules striking the surface. For different wall conditions, hot wall $T_w=T_{st}$, the decrease in the streamwise momentum is almost exactly compensated by the increase in the flux of molecules so that the overshoot in the drag coefficient is not present.

Bird has treated as well the case of the flat plate at zero incidence discovering that the increase in molecular flux to the surface, induced by the collisions, is so great that for both cold and hot surfaces the shear stress increases above the free molecular value.
A situation similar to the one recalled for the drag coefficient has been found by Bird for the Stanton number.

APPLICATIONS OF THE METHOD

The present semiempirical method has been originally applied by the author with success to evaluate the sphere stagnation point heat flux [18]; successively it has been extended also to the evaluation of the sphere drag and of other aerothermodynamic properties of spheres and plates in the supersonic intermediate flow regimes.
Being in the present note more concerned in illustrating the basic features of the method than in reporting the results obtainable with it, we will discuss only two typical applications for the supersonic sphere: the stagnation point heat flux and the drag.

The sphere configuration has been selected because, as pointed out before, for the sphere the probability function P is dependent only upon the parameter $\xi = (R_e/M)$ and the free parameter α . Values of the probability P for the sphere have been reported in fig. 3.

Stagnation Point Heat Flux

For the evaluation of the stagnation point heat flux simplified formulas have been used:
for the free molecular flow regime it is assumed according to Eckert [19]:

$$q_{FM} = Const_{FM}\, a\, 1/2\, \rho_{\infty} V_{\infty}^3 \qquad (28)$$

for the continuum flow regime the correlation formula derived by Vallerani [20] on the basis of theoretical and experimental results is assumed:

$$q_c = const_c \left(\sqrt{\frac{M}{R_e}}\right) \frac{1}{2} \rho_{\infty} V_{\infty}^3 \qquad (29)$$

Expressing the heat fluxes in Watt/m^2 the values of the constants result to be:
$Cost_{FM} = 1$ and $Const_c = 1,74$.
In the intermediate flow regimes, according to the method presented, the heat flux turns out to be therefore expressed by:

$$\psi_q = q^*/q_c = P\left(0.57\sqrt{\xi} - 1\right) + 1 \qquad (30)$$

Being $q_{FM} = q_c$ for $\bar{\xi} \simeq 3$ we notice that the situation previously depicted as X_{FM}/X_c monotonically increasing case b) occurs; therefore at both the boundaries of the intermediate flow regime an overshoot exists whatever the definition of the probability function is
For $\xi_{FM} < \xi < \bar{\xi}$ being $q_c > q_{FM}$ we obtain indeed $q_{FM} < q^* < q_c$ and for $\bar{\xi} < \xi < \xi_c$ being $q_c < q_{FM}$ we obtain indeed $q_c < q < q_{FM}$.
The results of the computations performed, for various values of the free parameter α , assuming the probability function previously postulated are reported in figure 5, that evidentiates the influence of the parameter α especially for the low values.

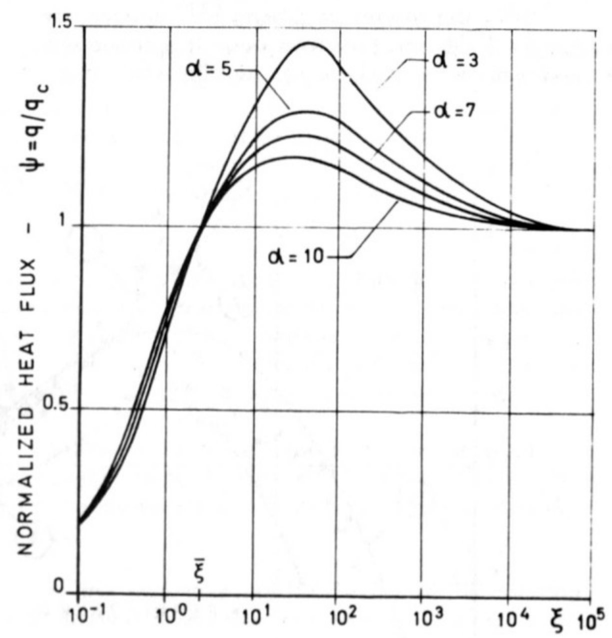

Fig. 5 - Intermediate Regime Stagnation Point Heat Flux

The choice of the value of the parameter α is linked as previously pointed out, to the comparison with the experimental data and the results of the theories available, that cover but limited ranges of the parameter ξ .

The presence for the stagnation point heat flux of an overshoot in the region of the slip flow has been established since long time being predictable by several theories, for instance Cheng (21), Ferri, Zakkay, Ting (22), Kao (23), Goldberg, Scala (24) and is confirmed by the experimental results Ferri, Zakkay (22) (25).

The existence of an overshoot also in the near free molecular flow region is on the contrary not so well documented, only Bird (17) has obtained, as previously recalled, by use of the Monte Carlo technique the evidence of a slight but definitive rise in the Stanton number above the free molecular flow value. Experimental evidence up to our knowledge is missing for the stagnation point heat flux in the near free molecular flow regime.

Several of the theories developed to cover the effects induced by the departure from the continuum flow investigate the heat flux in regimes ranging within the intermediate flow regimes range without covering the entire range up to the free molecular flow limit; the results reported by Cheng (21), Ferri, Zakkai, Ting (22), Kao (23), Goldberg, Scala (24), transformed in terms of the variable ξ introduced in the present note, are reported in figure 6 and compared with the experimental results of Ferri, Zakkay, Ting (22), and Ferri, Zakkay (25).

Only the theory of Cheng (21) covers the range $\bar{\xi}<\xi^{*}<\xi_c$ and foresees over it a trend with a maximum of the ratio q/q_c for $\xi \sim 40$, but provides results lower than the experimental data all over the range.

The other theories are limited to values of higher than 40 except the one of Goldberg and Scala and foresee only a monotonic increase of q/q_c giving results comparable among themselves and in agreement, as an over-all, with the experimental results of Ferri, Zakkay and Ting (22).

The agreement among the two sets of experimental data (22) (25) is good for values of ξ higher than about 100 for lower values while the data of Ferri, Zakkay and Ting (22) report a continuous increase of q/q_c for decreasing values of ξ, the more recent results of Ferri and Zakkay (25) extending up to the near free molecular flow regime on the contrary exhibit a definite maximum thereafter decreasing for decreasing values of ξ.

The results derived with the presently proposed correlation formula, assuming a value of $\alpha = 7$ for the free parameter, are compared with the other predictions in figure 5; the trend of the curve is quite similar to the one of the Cheng results, but the correlation of the experimental data all over the range of the parameter ξ is improved.

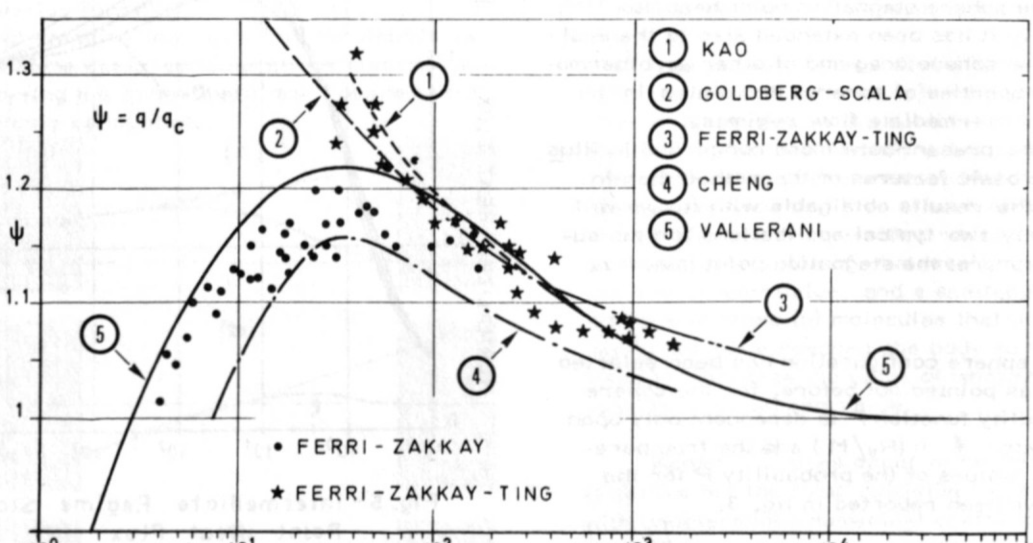

Fig. 6 - Intermediate Regime Stagnation Point Heat Flux Comparison Between Theories and Experimental Data

Drag Coefficient

The interest is correctly evaluating the drag coefficient for a sphere, all over the range of the Knudsen number from the continuum limit up to the free molecular flow limit, dictated apart from the pure speculation by the need of space applications, has stimulated several theoretical approaches and has promoted extensive experiments.

The sphere drag coefficient in low density continuum flow can be expressed in the general form suggested by Aroesty [26]

$$C_{D_c} = C_{D_i} + K_i \left(Re_2\right)^{-1/2} + K_2\left(Re_2\right)^{-1} \qquad (31)$$

as the sum of the infinite Reynolds number coefficient C_{D_i}, plus the contribution of the skin friction $\sim Re_2^{-1/2}$, plus a third term $\sim Re_2^{-1}$ representing the combined influence of the second order effects being Re_2 the Reynolds number based on properties behind the shock.

In the present analysis the theoretical results of Davis and Flugge Lotz [27] retaining only first order effects are adopted

$$(C_D)_c = 0.89 + 1.7\,\varepsilon = 0.89 + 2.8/Re_2^{1/2} \qquad (32)$$

being $\varepsilon = \left[(\gamma-1)M^2\right]^{\omega/2}/Re_\infty^{1/2}$ the perturbation parameter adopted in the analysis; the above formula, valids for $M_\infty = 10$ and cold wall conditions $T_w/T_\infty = .4, 2$ recast in terms of the parameter ξ is, for $\omega = 1/2$ and $\gamma = 1.4$:

$$(C_D)_c = 0.89 + 1.35/\xi^{1/2} \qquad (33)$$

The second order boundary layer effects have been found by Van Dyke [9] to give, as an overall, a positive effect on skin friction evaluated of the order of $0.47\,\varepsilon \simeq 0.37/\xi^{1/2} \simeq 0.8/Re_2^{1/2}$ that is to say a value of $K_2 = 2.24$ confirming the presence, also for the drag coefficient, of an overshoot over the classical boundary layer theory.

From the correlation of experimental data Bailey [29] has obtained

$$(C_D)_c = 0.88 + 1.4/Re_2^{1/2} + 0.8/Re_2 \qquad (34)$$

confirming the above conclusions while Kinslow and Potter [28] have deduced a negative value for the coefficient

$$(C_D)_c = 0.88 + 1.9/Re_2^{1/2} - 0.7/Re_2 \qquad (35)$$

The situation therefore is not yet completely clear and the evidence of the presence of an overshoot at the boundary of the continuum flow cannot be fully assessed; recent data of Geiger [30] and of Aroesty [26] seem to confirm the occurrence of an overshoot reporting values higher than the ones predictable according to Davis and Flugge Lotz [27].

The free molecular drag coefficient for the sphere in supersonic flow conditions is simply given by:

$$(C_D)_{FM} = 2 + f \qquad (36)$$

f=O for specular reflection, while for diffuse reflection f varies between zero and one as the thermal accomodation coefficient decreases from one to zero.

Theoretical values ranging from 2 to 2,8 depending on the reflection model choosen have been computed by Hurlbut and Sherman [31] using the Nocilla [32] reflection model.

The surface reflection laws as well as the energy accomodation laws are expected to be functions of the Mach number and of the wall temperature ratio, therefore the choice of the value to be given to the free molecular flow limit appears very difficult and delicate rendering the interpretation of the experimental data cumbersome and not definitive.

The possibility for the drag coefficient to exceed the free molecular flow value has been assessed on the basis of theoretical investigations as previously discussed by Bird [17] no other near free molecular theory, such as the one of Baker and Charwat [11], Rose [12], Willis [13], predicts this effect.

From the experimental side the situation is very controverse, from one side Phillips and Kultau [35] [40], Smolderen [37] Potter and Miller [38] Slattery, Hersh and Friichtenicht [39] Legge and Koppenwallner [41], who have focussed the attention on the near free molecular limit indicate no tendency to overshoot the free molecular limit, while on the other side recent results of Hussoy and Horstman [42, 43] seem to indicate for cold wall conditions an overshoot.

More accurate measurement, in well specified test conditions, are needed to solve the problem because being the scattering of the data available high and the discrepancy among different author results very large any definite conclusion seems for the moment being precluded.

In the present investigation the value of $C_{D_{FM}} = 2.3$ adopted by Potter [33] in a survey of cold wall hypersonic data is retained for sake of comparison.

In the intermediate flow regimes, according to the method presented the drag coefficient turns out to be therefore expressed by the formula:

$$\Gamma_{C_D} = C_D/C_{D_{FM}}$$

Recognizing that $C_{D_c} = C_{D_{FM}}$ for $\bar{\xi} = 1$ we notice that the same situation depicted for the heat flux occurs; at both the boundaries of the intermediate flow regime an over shoot exists whatever the definition of the probability function is.

The results of the computation performed for various values of the parameter α introduced in the probability function previously postulated, are reported in figure 7.

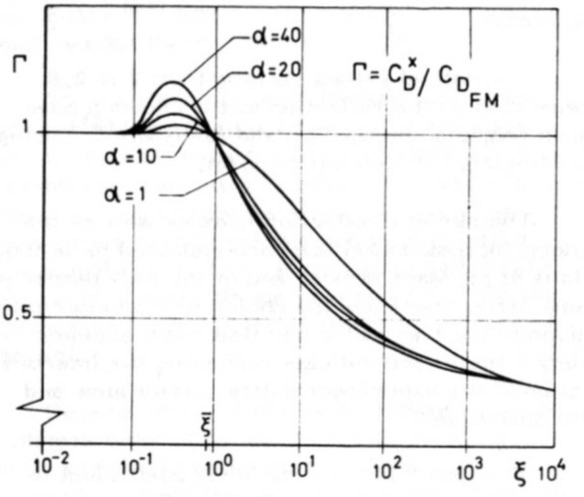

Fig.7 - Intermediate regime drag coefficient

Some of the experimental data previously discussed (26, 28, 29, 30, 34-43) are collected and compared in figure 8, the resulting scatter is quite high all over the intermediate flow regime range.

The results of the available theories Baker and Charwat [11] Rose [12] Willis [13] and Bird [17] relative to the near free molecular flow regimes and Davis and Flugge-Lotz [27] for the low density continuum flow regimes are compared in figure 8 with the results derivable from the existing correlation formulas: Blick [4] Whitfield [7] and the one presently proposed.
Values of $\alpha = 40$ and $Kn_c = 10$ and 100 have been selected to better correlate the experimental data and to approximate the theoretical predictions.

The formulas by Blick and Whitfield correlate reasonably well the data all over the intermediate regimes range, if no overshoot is admitted, otherwise only the present formula gives acceptable results all over the range the predicted values being close to the envelope of the experimental data and in good agreement with the theoretical predictions of Bird and Davis and Flugge-Lotz.

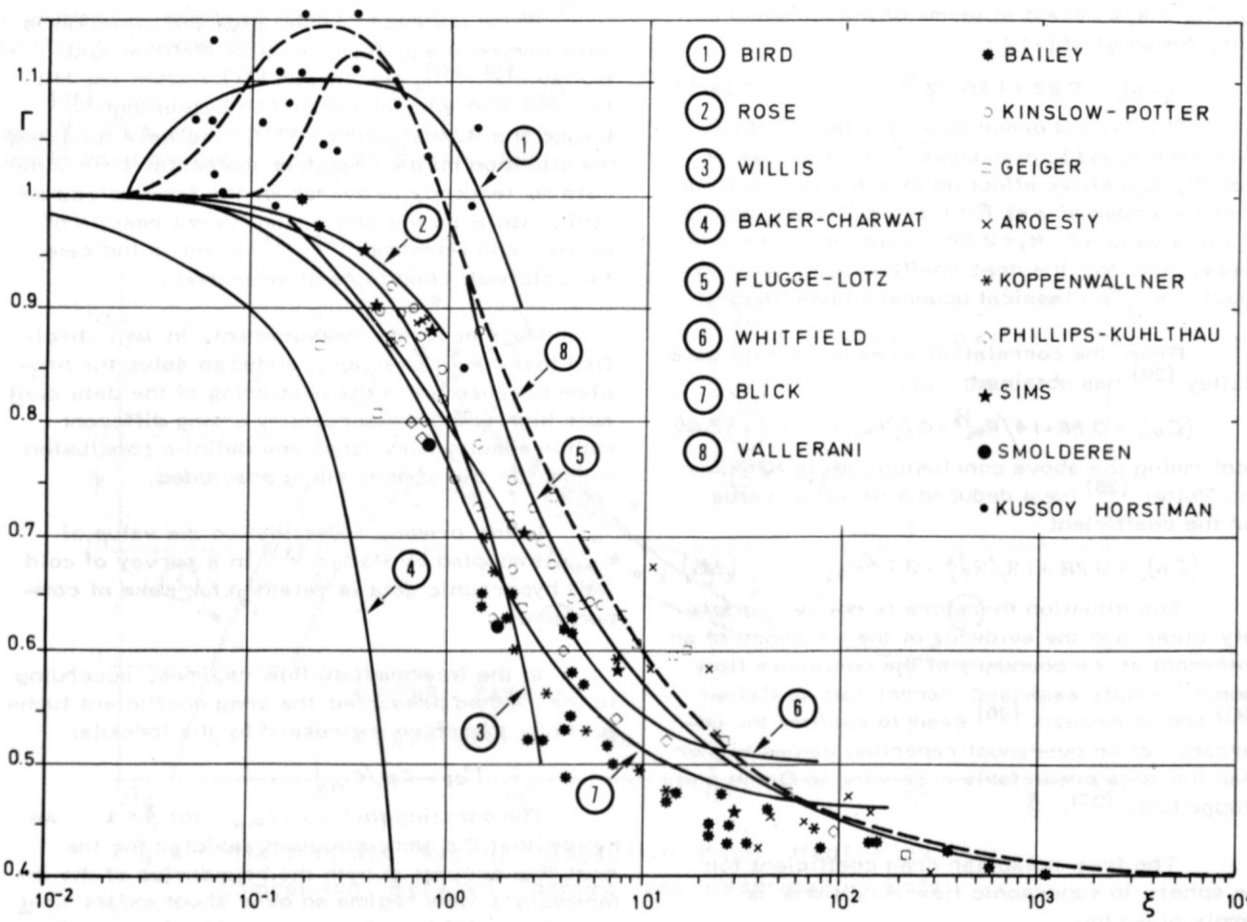

Fig. 8 - Intermediate Regime Drag Coefficient Theories and Experiments

CONCLUSIONS

The complex evaluation of the aerothermodynamic properties in the so-called "intermediate flow regimes" ranging between the free molecular flow and the continuum flow regimes has been led, in virtue of the proposed correlation formula to the evaluation of such properties in the two extreme regimes.

The proposed correlation formula enables the prediction of the "overshoots" at both the boundaries of the extreme regimes that, according to theoretical treatments supported by experimental data, have been recently evidentiated.

The results obtained for the supersonic sphere stagnation point heat flux and drag coefficient are encouraging, even if the large dispersion of the experimental data and the lack of theoretical information in some regimes prevent from deducing definitive conclusions on the accuracy of the proposed method that, at the level of engineering approximation, supplies anyway the information needed for the design of spacecraft and satellites travelling the upper layers of the atmosphere.

REFERENCES

1. R.F. Probstein and N.H. Kemp "Viscous Aerodynamic Characteristics in Hypersonic Rarefield Gas Flows" J. Aerospace Sci. 3. 1960.
2. R.L. Martino "Heat Transfer in Slip Flow" UTIA Rep. 35. 10. 1955.
3. B.L. Reeves W.M. Van Camp. "Aerodynamic Heating of Simple Geometries in the Slip and Intermediate Flow Regimes. J. Aerospace Sci. 12. 1959.
4. E.F. Blick. "Aerodynamic Coefficients in the Slip and Transition Regime". AIAA. J. 11. 1963.
5. H.S. Tsien "Supeaerodynamics, Mechanics of Rarefied Gases" J. Aeronautical Sci. 12. 1946.
6. B.L. Reeves Aerodynamic Heating in the Slip, Intermediate and Free Molecule Flow Regimes" Mc. Donnell Rept. 6859. 1959.
7. D.L. Whitfield "Analysis of Sphere and Cylinder Drag in Rarefied Flow" VII Symposium Rarefied Gas Dynamics.
8. D.L. Whitfield and W.B. Stephenson AEDC-TR-70-32. 1970
9. M. Van Dyke "Second order compressible Boundary Layer Theory" HYPERSONIC FLOW RESEARCH 1961.
10. W. Hayes and R.F. Probstein "HYPERSONIC FLOW THEORY" 1959.
11. R.M. Baker and A.F. Charwat "Transitional correction to the Drag of a Sphere in the Free Molecule Flow" Physics of Fluids 3-4. 1958.
12. M.H. Rose "Drag on an object in Nearly Free Molecular Flow" Physics of Fluids 8. 1964.
13. D.R. Willis "A study of some nearly free molecular flow problems" Princeton Univ. Aero. Eng. Lab. Rept. 440. 1959.
14. M.N. Kogan "Hypersonic Flows of Rarefied Gas" PMM 3. 1962.
15. M.N. Kogan "On the computation of flows at large Knudsen numbers" Astronautica Acta 1. 65.
16. J.A. Laurmann "First Collision Calculation of Cone Drag under Hypersonic Rarefied Flow Conditions" Astronautica Acta 4. 1967.
17. G.A. Bird "Aerodynamic Properties of Some Simple Bodies in the Hypersonic Transition Regime" AIAA. J. 1. 1966.
18. E. Vallerani "Rassegna dei Metodi per la Determinazione del Flusso di Calore incidente nel Punto di Ristagno di una Sfera in Regime Supersonico" Missili e Spazio 10-12. 1969.
19. E.R. Eckert "Survey of Boundary Layer Heat Transfer" WADC-TR-59-624.
20. E. Vallerani "Un metodo semiempirico per la valutazione delle azioni aerotermodinamiche nei regimi di flusso intermedi" Atti Società Ingegneri Torino 10. 1969.
21. H.K. Cheng "The Blunt Body Problem in Hypersonic Flows at Low Reynolds numbers" Cornell Rept. AF-1285-A-10 . 1963.
22. A. Ferri, V. Zakkay and Lu Ting "Blunt Body Heat Transfer at Hypersonic Speeds and Low Reynolds Numbers" J. Aerospace Sci. 12. 1961.
23. M.C. Kao "Hypersonic Viscous Flow Near the Stagna-

tion Point of a Blunt Body" AIAA.J. 11.1964.

24. L. Goldberg and S. Scala "Mass Transfer in the Low Reynolds Number Viscous Layer around the forward region of a Hypersonic Vehicle" General Electric Reprint 348.

25. A. Ferri and V. Zakkay "Measurements of Stagnation Point Heat Transfer at Low Reynolds numbers" J.Aerospace Sci. 7.1962.

26. J.Aroesty "Sphere Drag in a Low Density Supersonic Flow." III Symposium Rarefied Gas Dynamics.

27. R.T. Davis and I.Flugge-Lotz "Second Order Boundary Layer effects in Hypersonic flow past Axisymmetric Blunt Bodies" J.Fluid Mech 1964.

28. M. Kinslow and J.L. Potter "Drag of Spheres in Rarefied Hypervelocity Flow" AIAA.J. 1963.

29. A.B. Bailey "High Speed Sphere Drag in the Transition Flow Regime in an Aeroballistic Range" V Symposium Rarefied Gas Dynamics.

30. R.E. Geiger "Some Sphere Drag Measurements in Low Density Shock Tunnel Flows General Electric Rept. R63SD23 1963.

31. F.C. Hurlbut and F.S. Sherman "Application of the Nocilla Wall Reflection Model to Free Molecule Kinetic Theory" Physics of Fluids 3.1968.

32. S. Nocilla "The Surface Re-emission Low in Free Molecule Flow" III Symposium Rarefied Gas Dynamics.

33. J.L. Potter "The Transitional Rarefied Flow Regime" V Symposium Rarefied Gas Dynamics.

34. G. Koppenwalner "Drag and Pressure Distribution at Hypersonic Mach numbers in the range between Continuum and free molecular flow" VI Symposium Rarefied Gas Dynamics.

35. W.M. Phillips and A.R. Kuhlthau "Drag Measurements on Magnetically supported Spheres in Low Densities High Speed Flow" VI Symposium Rarefied Gas Dynamics.

36. W.H. Sims "Experimental Sphere Drag results in the Near Free Molecular Regime" VI Symposium Rarefied Gas Dynamics.

37. J.J. Smolderen, J.F. Wendt, J. Naveau and T.T. Bramlette "Sphere and Cone Drag in Hypersonic Transitional Flow" VI Symposium Rarefied Gas Dynamics.

38. J.L. Potter and J.T. Miller "Sphere Drag and Dynamic Simulation in Near Free Molecular Flow" VI Symposium Rarefied Gas Dynamics.

39. J.C. Slattery, A.S. Hersh and J.F. Friichtenicht "Drag coefficients of Microscopic Spheres in Free Molecular Flow" V Symposium Rarefied Gas Dynamics.

40. W.M. Phillips and A.R. Kuhlthau "Transition Regime Sphere Drag near to Free Molecule Limit" AIAA.J. 6.1971.

41. H. Legge and G. Koppenwallner "Sphere Drag Measurements in a Free Jet and a Hypersonic Low Density Tunnel" VII Symposium Rarefied Gas Dynamics.

42. M.I. Kussoy and C.C.Horstman "An experimental Study of Rarefied Gas Flow over Sharp Cones" VI Symposium Rarefied Gas Dynamics.

43. M.I. Kussoy, D.A. Stewart and C.C. Horstman "Sphere Drag in Near Free Molecule Hyper sonic Flow" AIAA.J. 11.1970.