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A THEORETICAL AND EXPERIMENTAL RESEARCH ON THE FATIGUE BEHAVIOUR OF REINFORCED SHEETS

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Abstract

Research into fatigue behaviour of stiffened panels as been carried out at the Institute of Aeronautics University of Pisa for the following purposes:

- to determine the effect of repeated buckling by compression loads on the fatigue strength of panels stiffened with hat section stiffeners.
- to establish criteria for "fail safe" design of stiffened panels under tension loads.

In the first case static and fatigue tests on a large number of panels of different sizes have been carried out as well as strain gauges measurements of the strain. These tests have allowed to study the causes of fatigue cracks and the effects of these cracks on the behaviour of such structures under loads.

The second problem has been faced evaluating by a theoretical point of view the stresses in cracked stiffened panels under tension loads. Particularly the effect of stiffener size and of the type of the stiffener-sheet junction on the stresses near the crack tip have been evaluated. The information so obtained can be utilized to determine the critical crack length and fatigue crack propagation rate, utilizing also data that will be obtained by tests now being performed with a new loading apparatus "ad hoc".

I. Symbols

A_c - stiffener area

C - σ_1/σ_o

C' - σ_c/σ'_o

D - sheet bending stiffness $D=\frac{1}{12}\frac{E h^3}{1-\nu^2}$

E - Young modulus

F - force acting on each rivet

N - number of load cycles

T_o - maximum force acting on the junction for unity of length

P - compression applied load

P_{cr} - critical compression load

P_{max} - maximum compression load

P_{min} - minimum compression load

a - panel length

b - stiffener pitch

c - stiffener lateral plate dimension

d - stiffener top plate dimension

e - mean compression strain$=\Delta a/a$(positive number)

e_{cr} - critical mean compression strain

f - stiffener flange dimension

h - sheet thickness

i - rivet-hole interference

l - half crack length

p - rivet pitch

r - radius of rivet holes

t - stiffener thickness

u,v - displacement components in the middle plane of the sheet

w - displacement component perpendicular to the middle plane of the sheet

α_c - A_c/bh

δ - distance of the more stressed rivet from the top of the buckle

δ_T - technological error

η - effective width of sheet

λ - halfwave lenght

ν - Poisson's ratio

ξ - post-buckling parameter $\xi=\frac{12}{7}\frac{e}{\pi^2}(\frac{b}{h})^2$

ξ_{cr} - critical value of ξ

σ_c - tension stress in the stiffener when the sheet is cracked

σ_{cr} - theoretical critical stress

σ_1 - longitudinal tensile stress at the tip of a crack in the sheet of a stiffened panel

σ_o - longitudinal tensile stress at the crack tip in the sheet without stiffeners

σ'_o - longitudinal tension stress in sheet or stiffener in absence of crack

σ_p - theoretical critical stress of simply supported plate $\sigma_p=3,62\,E(h/b)^2$

σ_x - longitudinal compression stress

σ_{ymax} - maximum tensile stress on the stiffener top plate

Δa - total contraction of the panel

Φ,Ψ,ϕ - adimensional functions

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$$

II. Introduction

Many aircraft structures consist mainly of panels where a sheet is stiffened by longitudinal stiffeners of various types, the panels being supported by ribs, frames, spars bulkheads and so on.

In usual loading conditions such panels are subjected to fatigue loads, substantially in compression or in tension; compression loads prevail in certain parts of the aircraft, tension loads in others.

The repeated application of compression loads, causing repeated buckling in the sheets, may provoke fatigue cracks, in the stiffeners or in the sheet, spreading in the longitudinal direction,(1), bringing considerable changes in the structural behaviour of the panel, lowering the critical compression load, the postbuckling stiffness and the ultimate load of the so weakened panel.

The optimum design criteria for stiffened panels in compression from the point of view of static loading and long fatigue life would appear to be quite different.While plenty of information is available about the optimum design of statically compressed panels,(2) (3) (4),more extensive data about fatigue behaviour of buckled panels ought to be collected by means of theoretical analysis of the phenomena observed and sufficient panel testing. Further research in particular is also required into the fatigue post-buckling behaviour of panels and about the diminution of static strength of panels weakened through fatigue cracks.

A contribution towards the solution of such design problems, obtained through research work performed at the Institute of Aeronautics - University of Pisa, is summarized in the following paragraphs.

In the stiffened panels in aircraft construction, subjected to prevailing tension fatigue loads, cracks may spread with a propagation rate depending not only on the distribution, form and size of the stiffeners and of their attachments to the sheet, but depending on the material used and on the loading spectra too.

Fatigue crack may be stopped through suitable design of the stiffeners and the way they are attached to the sheet.

A considerable amount of research work has recently been carried out (5) (6); but the insufficient information at present available prevents designers from solving correctly certain problems in the field of fail-safe fatigue design of aircraft structures.

Further investigation is therefore necessary; theoretical and experimental contributions to it, performed at the Institute of Aeronautics - University of Pisa, are described below.

III. Analysis of the postbuckling behaviour of stiffened panels subjected to compression loads.

The postbuckling bending of the sheet of compressed stiffened panels provokes tension stresses perpendicularly to the compression; the junction between the sheet and the stiffeners, in such conditions, can be subjected to tension or bending, that produce tension stresses in the stiffeners too. Such tension stresses, if repeatedly applied,may produce fatigue cracks, particularly in notched zones of the sheet and stiffeners.

When the junction between the sheet and the stiffeners is riveted or spot welded,fatigue cracks usually start from any rivet hole or spot weld, also depending upon the technological procedures adopted for riveting or spot-welding.

The form of the buckled sheet may be symmetrical or antisymmetrical with respect to the junction line with quite different effects on the stressing and fatigue behaviour of the panel (1).

For compressed panels with hat section stiffeners, the form of the buckled sheet

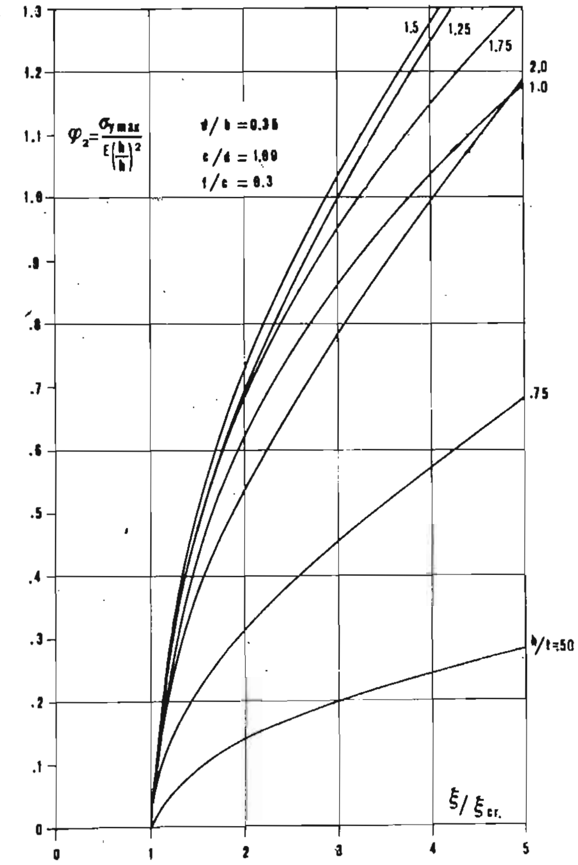

Fig. 1 - Theoretical values of $\phi_2 = \frac{\sigma_{ymax}}{E(h/b)^2}$ *versus* ξ/ξ_{cr} *for some values of* h/t.

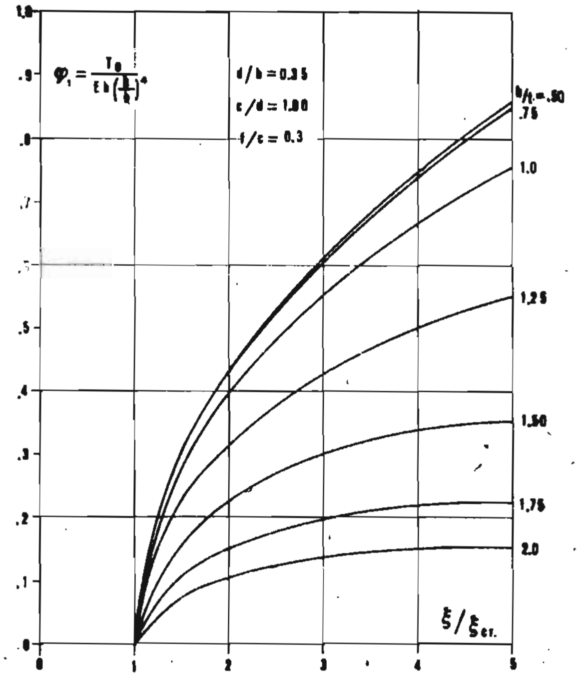

Fig. 2 - Theoretical values of $\phi_1 = \frac{T_0}{Eb(h/b)^4}$ versus ξ/ξ_{cr} *for several values of h/t.*

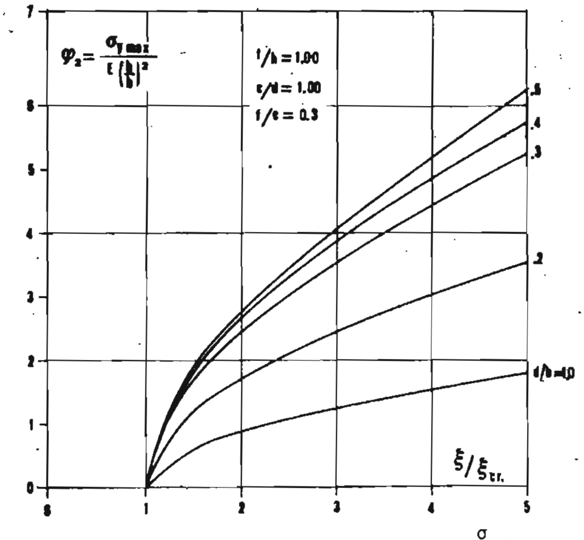

Fig. 3 - Theoretical values of $\phi_2 = \frac{\sigma_{ymax}}{E(h/b)^2}$ *versus* ξ/ξ_{cr} *for some values of d/b.*

is generally symmetrical (1) (7), and some junction rivets or spot welds are subjected, above all, to tension forces.

The following reference parameters may be used for the programming and analysis of compression fatigue test on such panels:

1) effective sheet width η.
2) Maximum T_o acting in the junction (produced by the buckling).
3) Maximum tension stress σ_{ymax} in the compressed stiffener, caused by the bond with the buckled sheet.

The reference parameters η, T_o and σ_{ymax} were evaluated in (1),(7) on the basis of the simplifying assumption of continuous bonding between the sheet and the stiffeners along the junction line; furthermore splitting of σ_{ymax} in the product of two independent functions was considered advisable; namely

$$\sigma_{ymax} = E\left(\frac{h}{b}\right)^2 \phi_2\left(\xi\ \frac{h}{t}, \frac{d}{b}, \frac{d}{c}, \frac{f}{c}\right) \Psi\left(\xi\ \frac{p}{\lambda}, \frac{\delta}{p}, \frac{i}{r}, \frac{r}{d}\right)$$

σ_{ymax}/Ψ can be calculated theoretically while Ψ can be deduced by test results, some of which are reported below.

σ_{ymax}/Ψ was calculated in (7) nullifying the first variation of the strain energy of the sheet stiffeners system, assuming suitable expressions for the displacement components u, v, w of the middle plane of the sheet; it is supposed that the top plate of the hat section stiffener behaves like a plate elastically built-in along its corner edges and the deflection surface of the middle plane of the stiffener top plate is assumed to satisfy equation:

$$\nabla^4 w = \frac{\sigma_x t}{D} \frac{\partial^2 w}{\partial x^2}$$

In Figgs. 1, 2, 3, 4, T_o and σ_{ymax}, namely in a adimensional form:

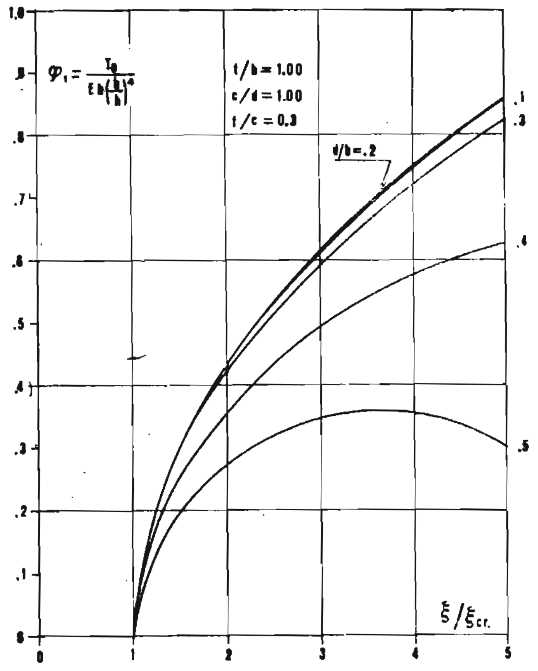

Fig. 4 - Theoretical values of $\phi_1 = \frac{T_o}{Eb(h/b)^4}$ *versus* ξ/ξ_{cr} *for several values of d/b.*

$$\phi_1 = \frac{T_o}{Eb(h/b)^4} \quad \text{and} \quad \phi_2 = \frac{\sigma_{ymax}}{E(h/b)^2}$$

are given in function of ξ/ξ_{cr} for several values of h/t and d/b; d/c and f/c have seldom any appreciable influence above the indicated functions of ξ/ξ_{cr}. The results here reported concern the rather frequent values d/c = 1, f/c = 0.3. However fuller information is given in (7).

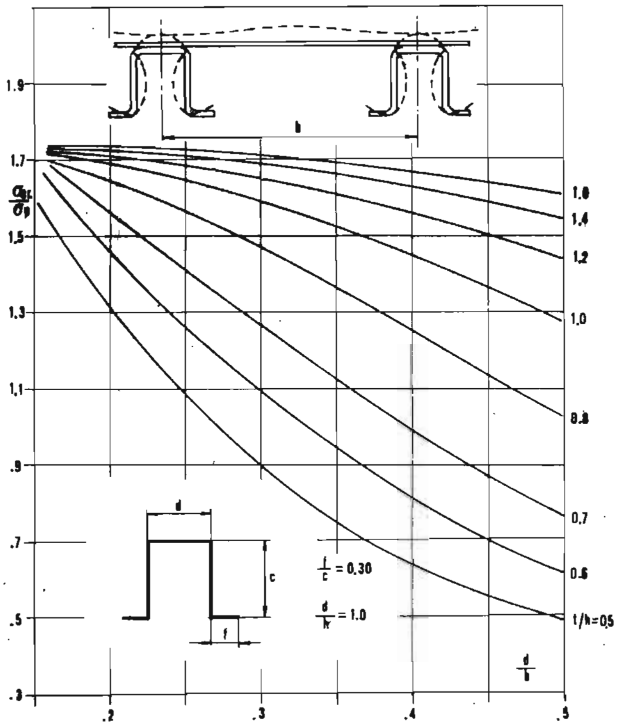

Fig. 5 - Values of the critical stress σ_{cr} versus d/b, t/h, for the deformation, shown, in enlarged way, on the top of the figure.

The values calculated in this way and the corresponding test results agree satisfactorily as far as T_o is concerned (7), while as to the effect of the rivet spacing concern a difference is found between calculated and measured values of σ_{ymax}. A theoretical method may be developed taking into account the effect of the rivet spacing, but the calculations involved are very laborious and probably inadequate and an experimental determination of Ψ function may therefore be considered more reliable.

Remarks on Ψ function will be found below.

A method of assessment of σ_{cr} and ξ_{cr} for panels with hat section stiffeners, subjected to symmetrical buckling of the sheet, has been also devised in (8).

This is based on the assumption that the corner edges of hat section stiffeners remain rectilinear during the deformation even beyond the compression critical load; the surfaces of the stiffeners and the sheet are subjected to the postbuckling deformation indicated in the top of Fig. 5. The assumption of continuous bonding between sheet and stiffener is also made.

Calculations reported in (8) provide an accurate evaluation of critical compression stress in the panels, since the effect produced on the critical compression stress by the particular form of the bonding between the sheet and the stiffeners is negligible.

The values of σ_{cr} calculated in this way are also reported in Fig. 5. A discussion concerning these values as compared to the experimental ones are reported in (8). The agreement between the theoretical and the experimental values of σ_{cr} is satisfactory for relatively small values of d/b. For large values of d/b an improved method for calculating σ_{cr} is suggested in (8).

IV. The stressing of cracked stiffened panels subjected to tension.

The stress in a cracked stiffened panel and in a simple plate, both with a crack of given lenght perpendicular to the applied tension stress, was worked out theoretically in (9) (10) using the matrix method of displacements (11). The general layout of the stiffened panels considered in (9) (10) is shown in Fig. 6.

On this basis the C ratio between the maximum stress at the crack end for the stiffened panel and for the corresponding plate was calculated in (9) (10). Panels with cracked sheet only or with disrupted stiffeners too were also considered.

A simple cracked plate is considered corresponding to a given stiffened panel when it is equal to the cracked sheet of the panel.

Since the Stress Intensity Factor (SIF) as defined in (14), as well as the factor K_T indicated in (5) as the theoretical factor of stress concentration for cracked sheets in elastic range, are known for the plates with cracks of given lenght, assessment of the C ratio allows us to determinate the SIF, or K_T for a given cracked stiffened panel (10) (12).

The SIF or K_T are generally assumed as basic parameters for the evaluation of the residual static strength and Fatigue Crack Propagation Rate (FCPR).

The method of calculation described in (9) enable us also to determinate the overloading coefficient C' for the stiffeners and the stressing of the bonding between the cracked sheet and the stiffeners, both caused by a given crack.

The results are summarized in Figgs. 7, 8, 9, 10.

These figures in particular show the stiffeners have a considerable influence on C ratio only when cracks tip is in the neighbourhood of the stiffener.

The influence of stiffeners on the C and C' ratios is greater for integral stiffened panel than for riveted ones, and for the latter such an influence diminishes, increasing the rivet spacing (Figgs. 7 and 9).

Further calculations are being made with various values of α_C, for the assessment of the stressing caused by the cracking of the panel sheet in the rivets bonding the sheet to the stiffeners too.

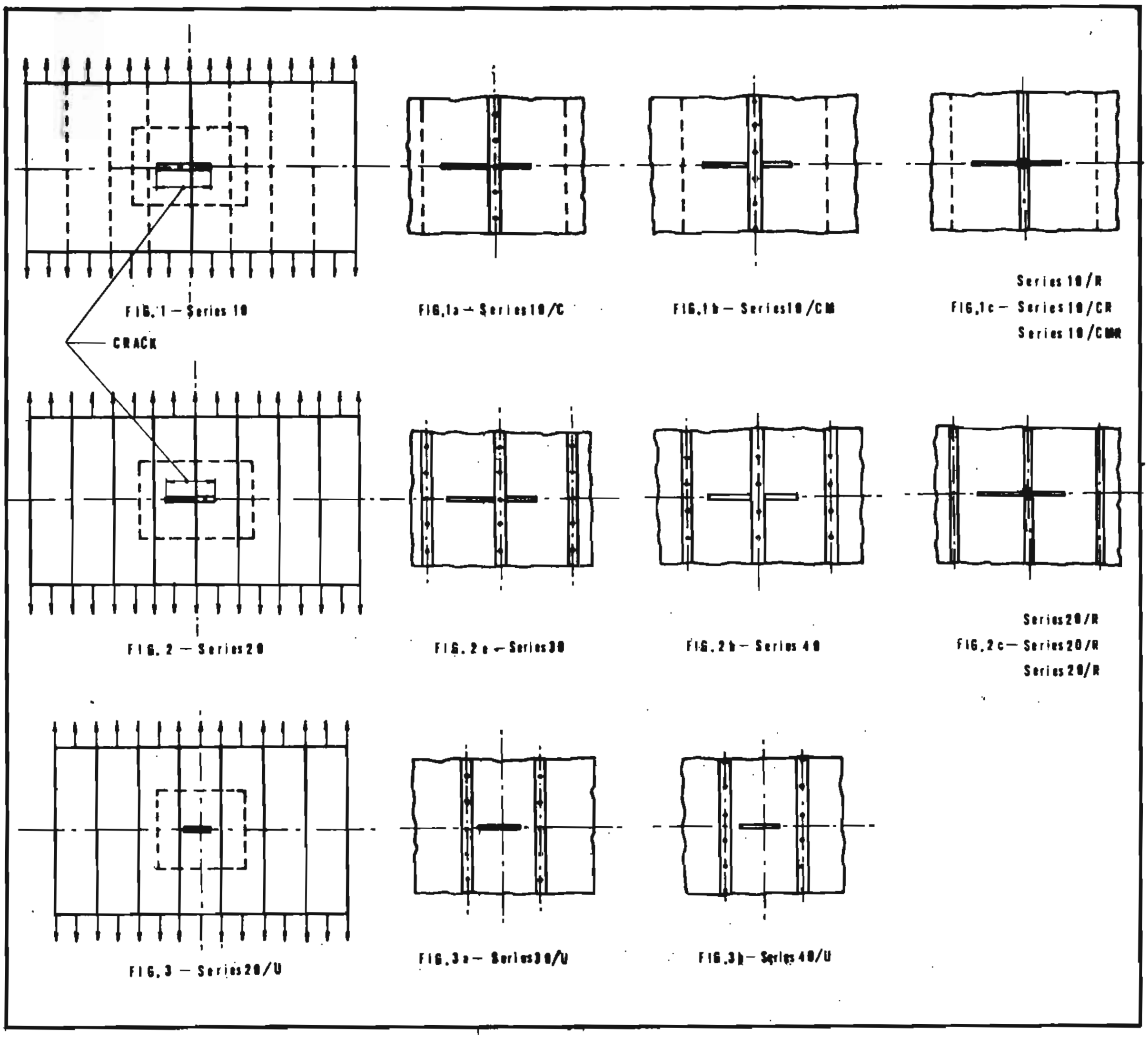

Fig. 6 - Examples of structures that may be analyzed through the program described in(9). The structures of the series 10 have only one stiffener, riveted (series 10/C, 10/CM) or continuously bonded to the sheet (Fig. 1,C). Series 20 and 20/U relates to structures with many stiffeners both riveted and continuously attached to the sheet with various positions of the cracks as indicated.

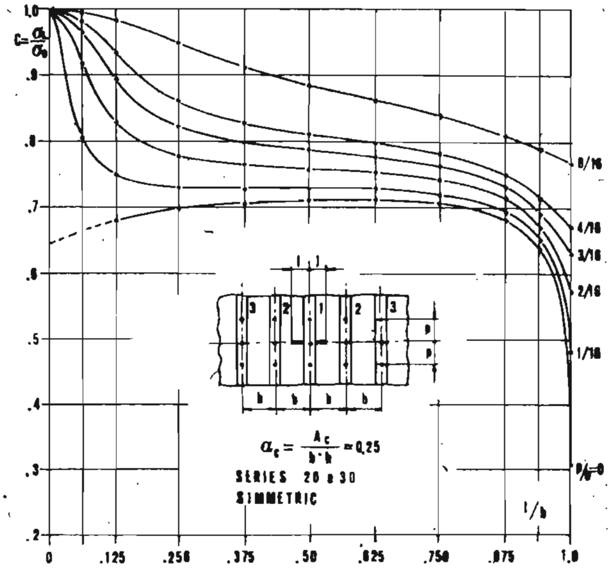

Fig. 7 - Effect of the rivet pitch on the coefficient C for the panel shown in the figure. Crack starts in the sheet from a rivet hole. The limit to which C tends when l tends to zero must be different for p/b = 0 and p/b ≠ 0 (10).

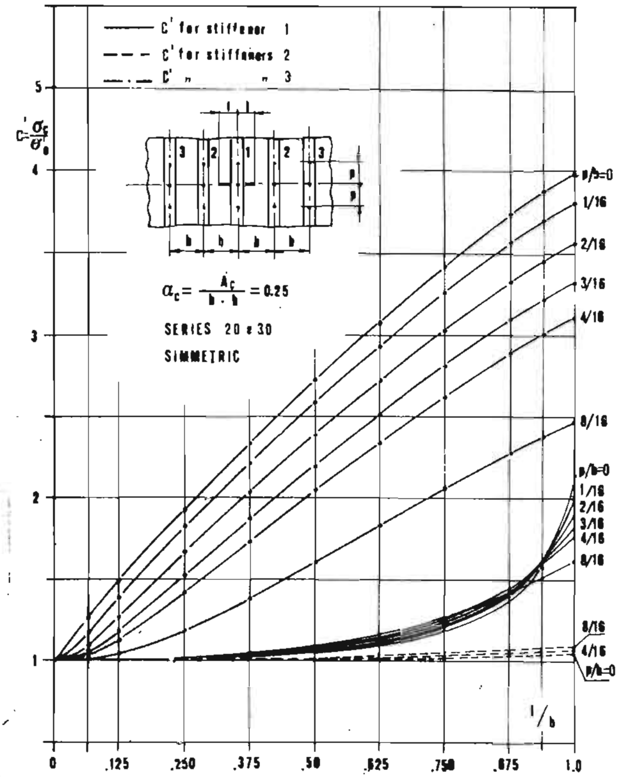

Fig. 9 - Effect of the rivet pitch on the coefficient C' = σ_c/σ_0' for the panel shown in figure. Crack starts in the sheet from a rivet hole.

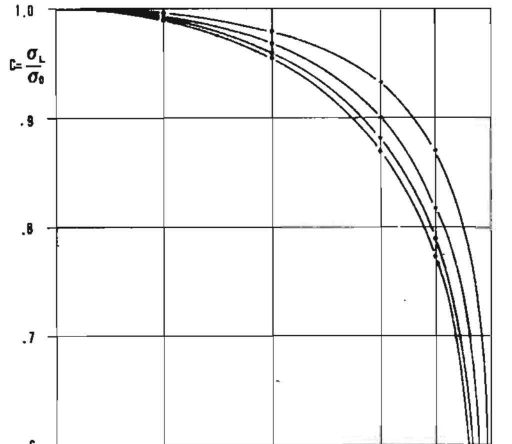

Fig. 8 - Coefficient C = σ_l/σ_o versus l/b and α_c for a panel with a continuous bonding between stiffeners and sheet. The crack starts between two stringers.

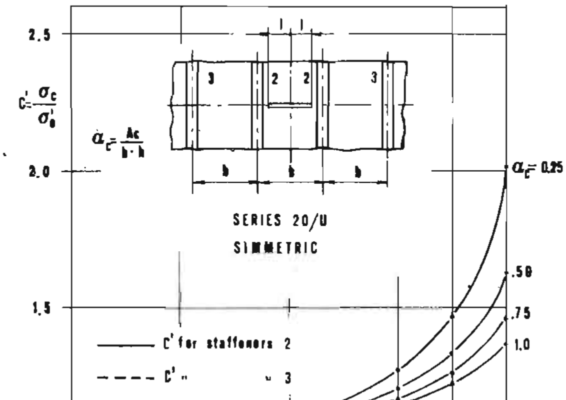

Fig. 10 - Coefficient C'=σ_c/σ_0' versus l/b and α_c for a panel with a continuous bonding between stiffeners and sheet. The crack starts between two stringers.

V. The test apparatus and the test programme for the study of fatigue behaviour of stiffened panels.

The test apparatus for fatigue testing of compressed panels, designed and built in the Institute of Aeronautics - University of Pisa, was developed from that described in (1), (13).

It consists in the following main parts:

- A frame supporting the test panel mounting and the 60 tons loading jack(Fig.10).
- A Losenhausenwerk SBE machine, sometimes substituted by a 300 cm^3 pulsator, completed by a programming load system providing the jack with the foreseen load programme through suitable periodic variations of the oil pressure in the jack (30 ÷ 70 cycles per minute for the SBE machine, 200÷ 300 cycles per minute for the pulsator).
- Measuring equipment, capable of supplying adequate information about σ_x and σ_y in four or six points of two central stiffeners, near the rivet holes where maximum stressing existes.

The shearing forces acting on the bonding between the buckled sheet and the stiffeners can also be obtained starting from the strains measured on the two apposite surfaces of the sheet near the bonding. The forces acting on the rivets may be deduced from

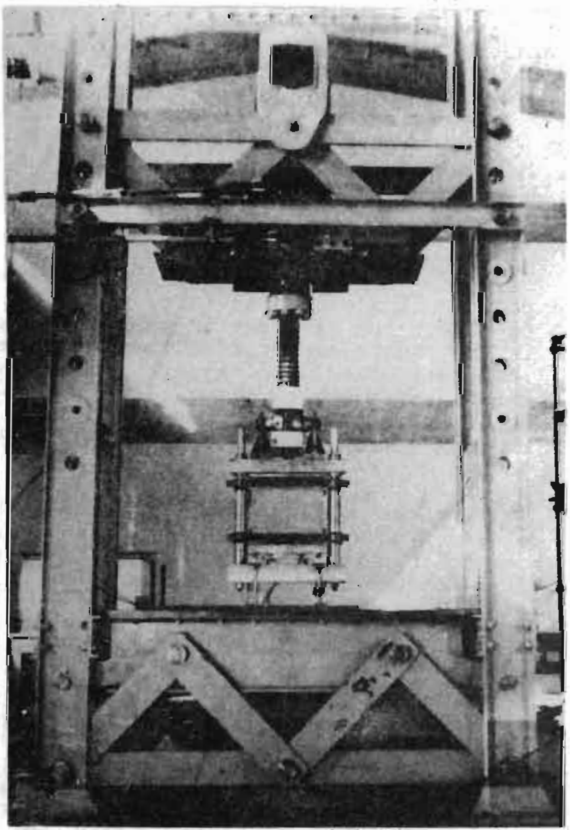

Fig. 11 - Test apparatus with the panel mounting for compression tests.

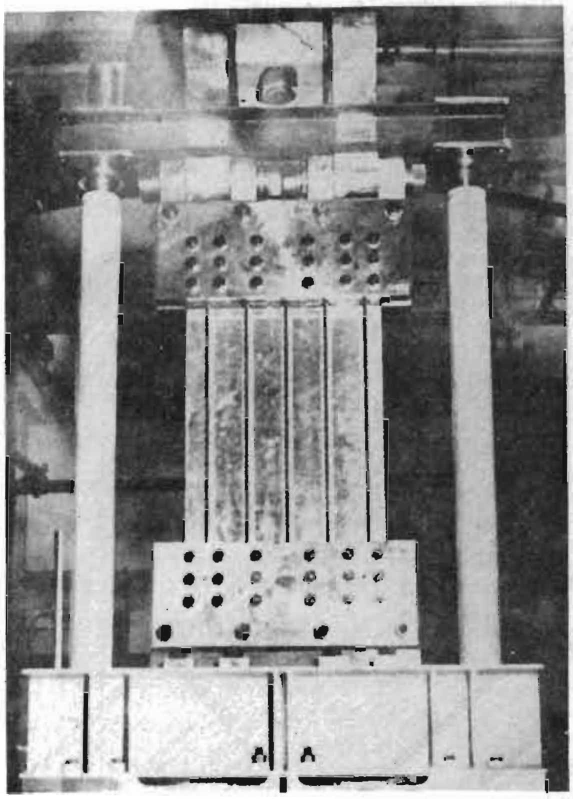

Fig. 12 - Panel mounting for traction tests of stiffened panels.

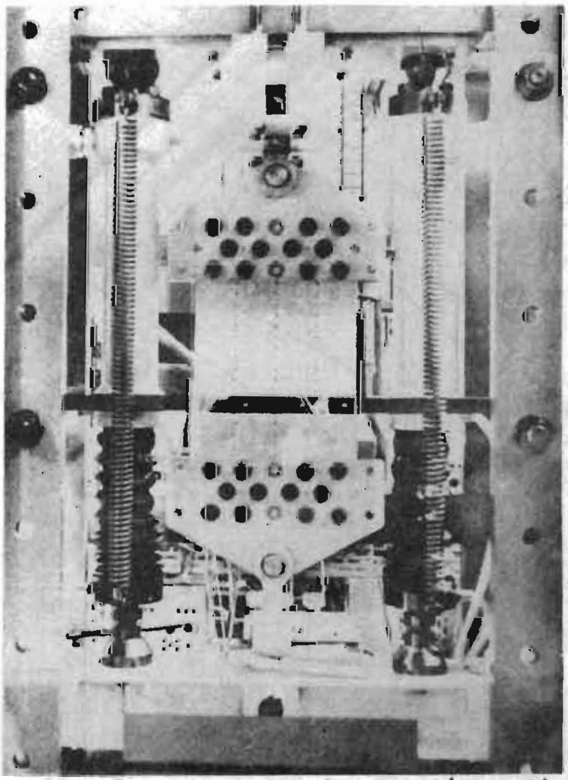

Fig. 13 - Test apparatus for traction testing of simple plate and strap stiffened sheets.

integration of such shearing forces in the inter rivet bonding length.

The test apparatus above described was modified to also perform tests on fatigue crack propagation and crack stopping in sheets and stiffened panels subjected to fatigue tension loads.

The main modification consists in the substitution of the panel mounting by another suitable for tension fatigue tests, also designed and built at the Institute of Aeronautics University of Pisa,as described in (13) (Fig. 12).

Another test apparatus (Fig. 13)was built for the fatigue testing of simple plates and sheets stiffened with straps and for the determination of the crack propagation rate of adopted material. Such test apparatus,capable of frequencies up to 1000 cycles per minute,allows faster testing.

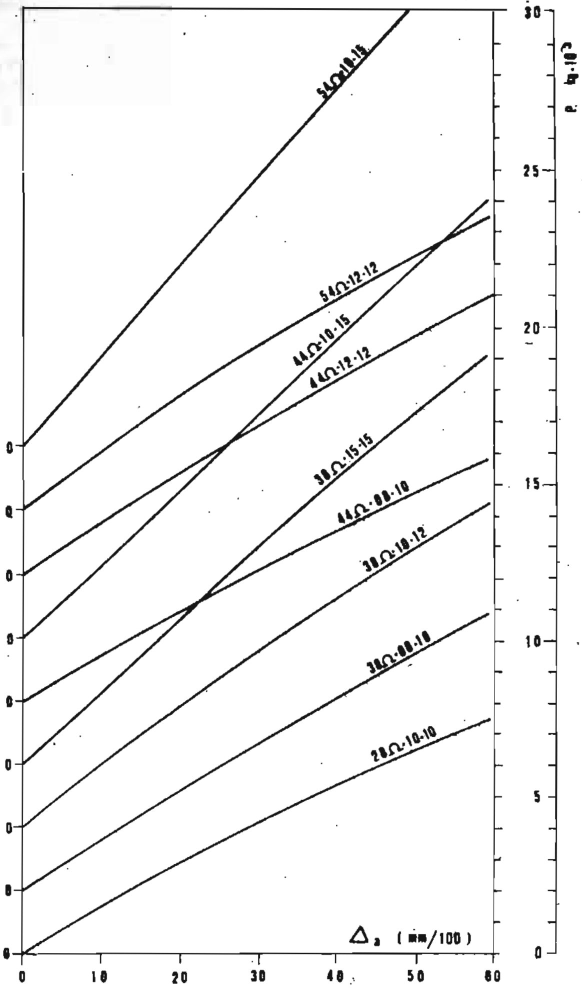

Fig. 14 - Δα as function of applied load for the different panels reported in Tab. I.

PANNELLI	t/h	t/b	a/c	f/c	d/b	Pcrit.
28Ω·10·10·B	1	0.0083	0.60	0.47	0.23	3240
38Ω·08·10·A	1.25	0.01	0.80	0.47	0.32	3700
38Ω·08·10·B	1.25	0.01	0.80	0.47	0.32	3700
38Ω·10·12·A	1	0.01	0.80	0.47	0.32	5500
38Ω·10·12·B	1	0.01	0.80	0.47	0.32	5500
38Ω·15·15·A	0.67	0.0083	0.80	0.47	0.32	6400
38Ω·15·15·B	0.67	0.0083	0.80	0.47	0.32	6400
44Ω·08·10	1.25	0.01	0.94	0.47	0.37	3550
44Ω·10·15·A	1	0.0125	0.94	0.47	0.37	10400
44Ω·12·12·A	0.83	0.01	0.94	0.47	0.37	4350
54Ω·10·15·A	1	0.0125	1.15	0.47	0.45	9850
54Ω·10·15·B	1	0.0125	1.15	0.47	0.45	9850
54Ω·10·15·C	1	0.0125	1.15	0.47	0.45	9850
54Ω·12·12·A	0.83	0.0083	1.15	0.47	0.45	4000
54Ω·12·12·B	0.83	0.0083	1.15	0.47	0.45	4000

Tab. I - Principal geometric characteristics and critical loads of the first lot of panels tested.

The present test program concerning the postbuckling fatigue behaviour of compressed test panels consists in a preliminary lot of 20 panels, used for the exploration of the phenomena involved, and in two lots, each of 15 panels to be tested varying accordingly the main parameters which influence the behaviour of such panels (h/t, d/b, d/c). The results obtained in the preliminary lot were reported in (1); results obtained from the first lot of 15 panels will be summarized later. The testing of the second lot is in progress.

Prior to the compression fatigue tests, postbuckling static tests were carried out, to assess the stresses in the buckled sheets, in the rivets and in the stiffeners near the rivet holes, as they are provoked by the buckling of the sheet. Such tests were repeated in absence and in presence of the forcing action of the rivets against the border of the rivet holes.

Inspection of the crack borders with the electron microscope has been planned in order to observe the direction and the growing modes of the crack propagation.

The main aim of test programme concerning the fatigue behaviour of stiffened panels in tension was to check the reliability of the theory summarized above. The reliability of the hypothesis that to equal SIF correspond equal FCPR and static residual strength will be checked in particular.

The possibility of application to stiffened panels of the CSA method,proposed by Kuhn in (5), will be also experimentally in vestigated.

The test programme in progress consists in the testing of a first lot of 30 panels with 5 or 7 riveted stiffeners, with a crack starting from a rivet hole on the central stiffener, and of a second lot of 10 panels with 6 stiffeners with of crack starting from a small hole on the middle line; the ratio A /bh varies between 0,2 and 0,5; the panels are built in 2024T3 and 7075T6 alloys. A lot of 15 corresponding simple cracked plates will be also tested.

A preliminary lot of 10 panels stiffened with straps has the aim of investigating the phenomenon in simplified specimens.

The FCPR and the residual static strength of such cracked panels or plates are assess experimentally, as well as the stresses in the stiffeners and in the bonding between the stiffeners and the sheets. A comparison will be made between the test results and the ones obtained theoretically as described above.

VI. Test results up to time of pubblication of the present paper.

Previous static and fatigue tests on buckled panels were performed on 44 panels.

The critical values of the applied compression load, as well the postbuckling values of σ_{ymax} were measured for the first lot.

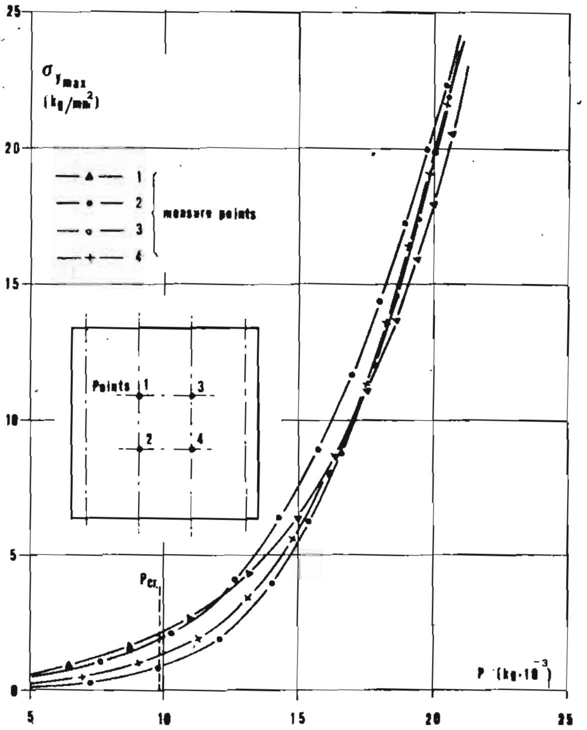

Fig. 15 – σ_{ymax} as function of applied load P measured at different points of of panel 54 Ω 10 15 A in absence of forcing action of rivets.

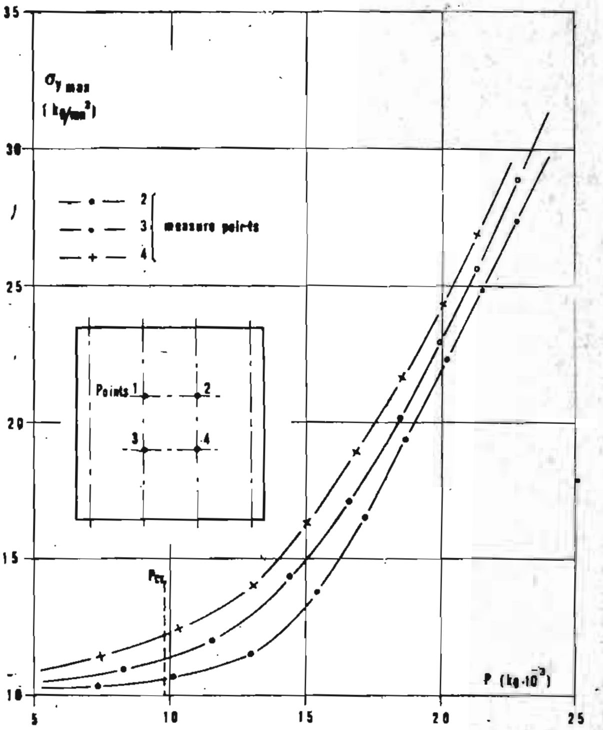

Fig. 16 – σ_{ymax} as function of applied load P measured at different points of the panel 54 Ω 10 15 A. The forcing action of the rivet provokes an initial circonferential stress $\sigma_{ymax} \approx 10$ Kg/mm².

Tab. I shows geometric characteristic and critical loads of such panels tested. Fig. 14 indicates Δa in relation to the applied load for panels tested.

The results obtained show that relevant σ_{ymax} values are also measured for loads slightly higher than critical ones (Figgs. 15 and 16).

When there is not forcing action of the rivets against the border of the rivet holes, the σ_{ymax} is smaller than when such forcing action is present only for loads near to the critical ones (Figgs. 15 and 16).

For higher loads no general conclusion may be got because such differences are influenced by the value of forcing action.

The forcing action indicated above has a considerable influence on the fatigue behaviour of panels. However further investigation is needed before such an influence can be fully understood.

The differences between the measured and the calculated values of σ_{ymax} have decreasing importance for loads increasing beyond the critical ones, because the influence of deviations from the theoretical conditions of the panels (caused by the usual technological procedures) diminishes for higher loads. Such differences also depend on the assumption of a continuous bonding between the sheet and the stiffeners.

Fig. 17 and Fig. 18 indicate the experimental values obtained for:

$$\Phi = \frac{\sigma_{ymax}}{E(h/b)^2} = \phi_2\Psi + \delta_t$$

in function of ξ/ξ_{cr}, for various values of h/t and d/b. Such values are compared with the corresponding values of the ϕ_2 function. Through such a comparison the Ψ function may be assessed.

However adequate knowledge of the Ψ function seems difficult to obtain, considering the number of parameters involved.

Only widespread panel testing may provide such a knowledge.

The results obtained up to now indicate however that the dependence of

$$\frac{\sigma_{ymax}}{E(h/b)^2}$$

from h/t and d/b, as can be obtained experimentally, is in accordance with the theoretical dependence of ϕ_2 from the two above indicated parameters.

The results of compression postbuckling fatigue tests on 13 panels of the first lot are reported in Fig. 19, where a Wöhler's curve is shown in relation to σ_{ymax}. The σ_{ymax} has been shown to be the more significant parameter as far as the postbuckling fatigue behaviour of compressed riveted panels is concerned. The determination of σ_{ymax} can im fact, for a given type of panel be sufficient to calculate in advance the fatigue behaviour of a compressed panel in postbuckling range. Since σ_{ymax} depends to a large extent on the forcing action of the rivets against the border of the holes, such forcing action was carefully checked while the riveting was being carried out.

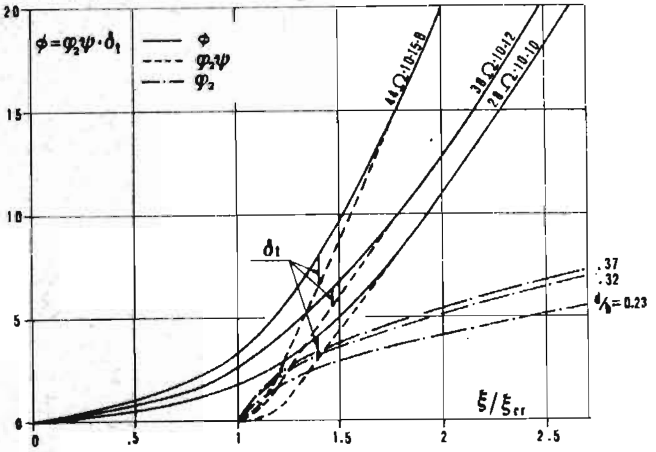

Fig. 17 - Φ, as obtained from experimental values of σ_{ymax}, and ϕ_2, as obtained from theoretical analysis, are compared to determine the Ψ function. $\Phi-\phi_2\Psi$ gives the technological error δ_T. $\Phi, \phi_2, \Psi\phi_2$ are reported versus ξ/ξ_{cr} for three values of d/b and for h/t = 1.

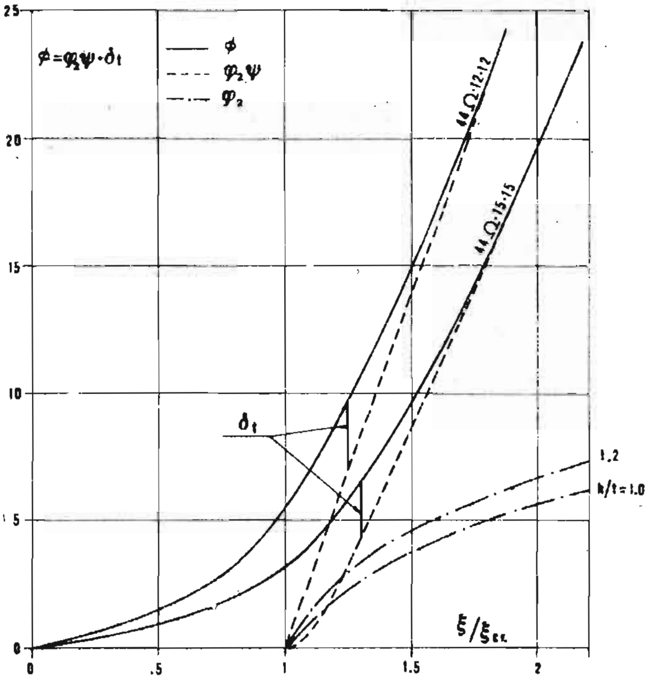

Fig. 18 - Φ as obtained from experimental values of σ_{ymax}, and ϕ_2 as obtained from theoretical analysis, are compared to determine Ψ function. $\Phi-\phi_2\Psi$ gives the technological error δ_T. $\Phi, \phi_2, \Psi\phi_2$ are reported versus ξ/ξ_{cr} for two values of h/t and for d/b = 0.37.

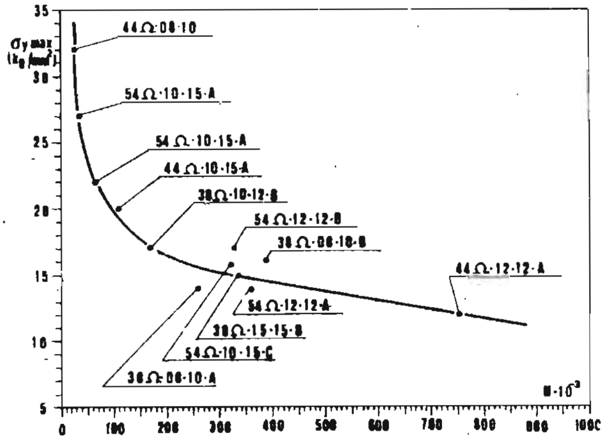

Fig. 19 - Results of fatigue tests reported on a Wöhler curve.

The presence of cracks involves a decrease in critical loads and in ultimate loads and an increase in the buckle amplitude and in plastic deformations, as is shown in a typical case in Fig. 20.

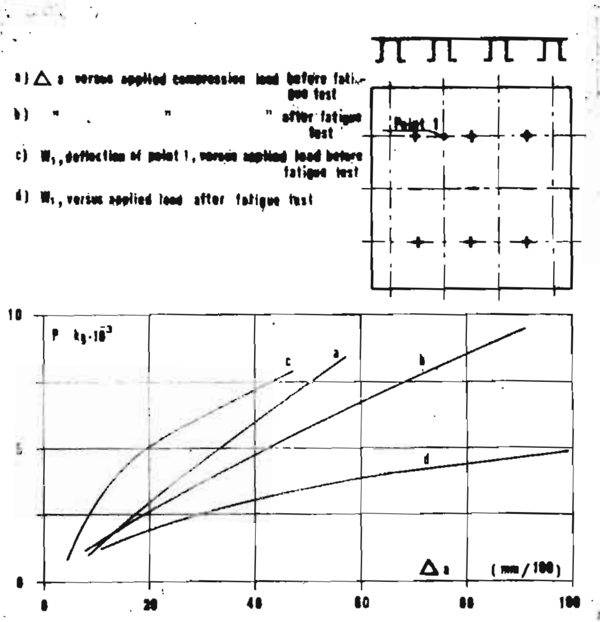

Fig. 20 - Example of damage provoked by fatigue cracks in compressed panel in the postbuckling range.

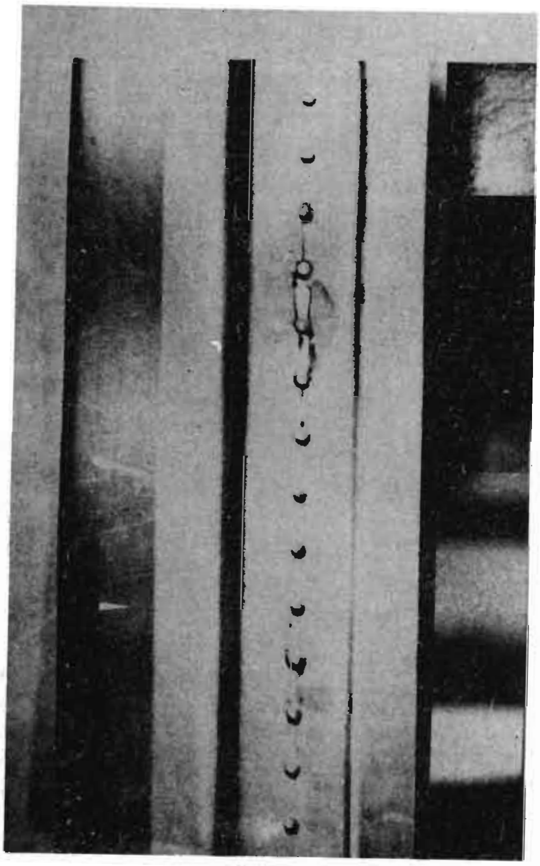

Fig. 21 - Typical pattern of a crack in the top plate of a stiffener.

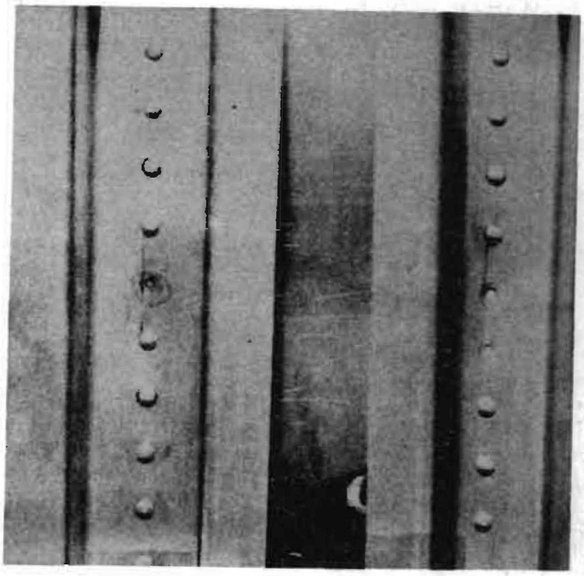

Fig. 22 - Another typical pattern of two cracks developed in two stiffeners provocking also the rupture of some rivets.

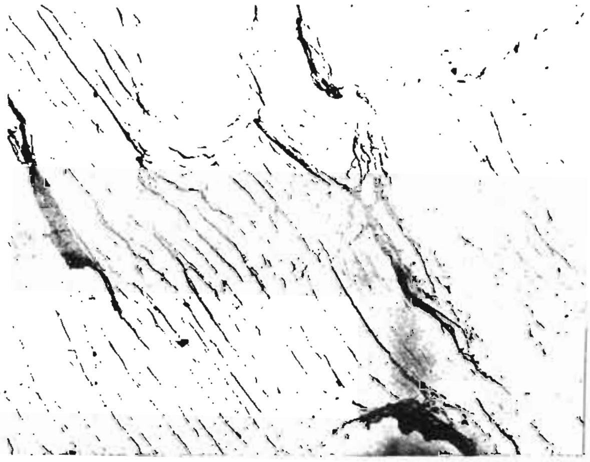

Fig. 23 - Crack propagation pattern in the neighbourhood of a rivet hole as revealed by electron microscope.

Figgs. 21 and 22 show typical fatigue cracks in the stiffeners of panels subjected to compression fatigue loads in postbuckling range.

Fig. 23 shows an electron microscope picture of a typical fatigue crack in a hat section stiffener; the crack starts from a rivet hole; the regular spread of the fatigue crack is evident, as well as the disturbances provoked by precipitates which cause interruptions in the crack propagation front.

Figgs. 24 and 25 shows preliminary results obtained through the test program in progress, summarized above, concerning the fatigue behaviour of stiffened panels in tension.

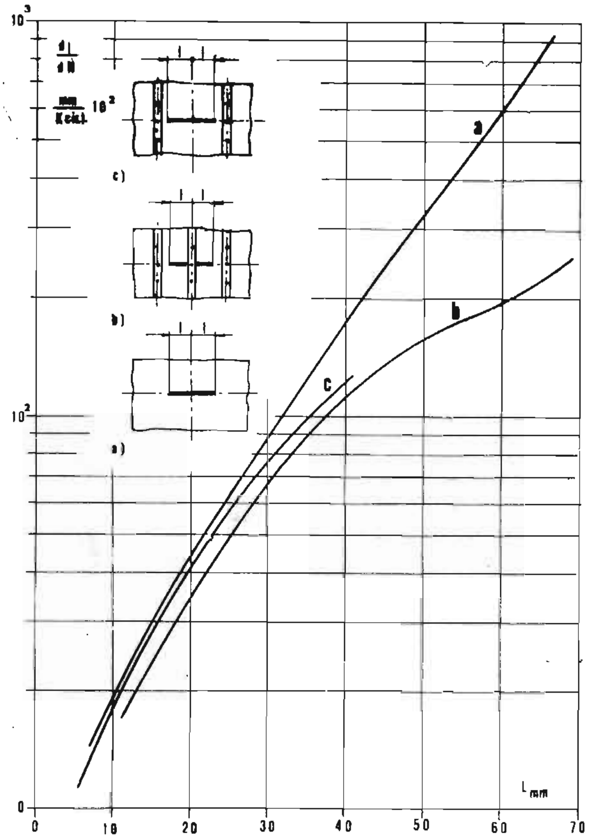

Fig. 24 - FCPR = dl/dN versus l for three different types of panels shown in figure.

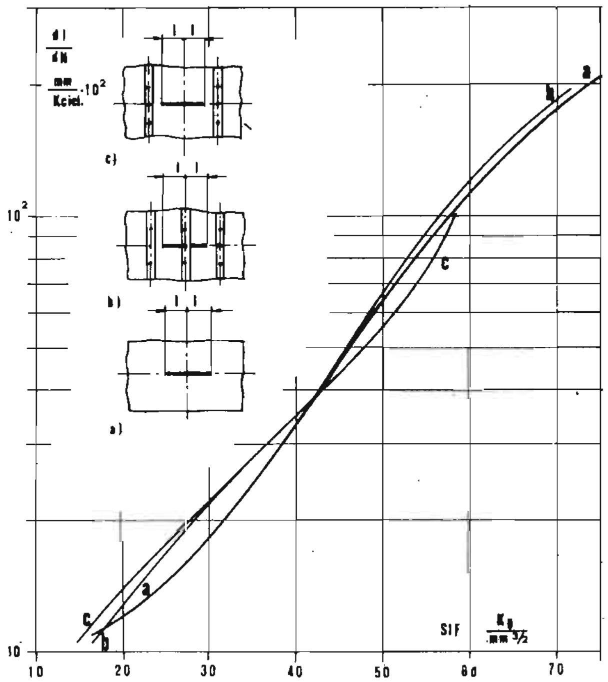

Fig. 25 - FCPR = dl/dN versus SIF for three different types of panels shown in figure.

Fig. 24 shows the influence of stiffeners in diminishing the FCPR. Fig. 25 shows that the FCPR is substantially the same when the SIF are equal for plates and stiffened panels.

VII. Conclusion.

The fatigue loads in buckled stiffened panels produce considerable tension stresses in the stiffeners as well as in the buckled sheet; such stresses can produce longitudinally propagating fatigue cracks. These cracks induce a decrease both of critical compression load and of ultimate strength of the panel.

The research work performed up to the present time allows us to conclude that the fatigue behaviour of such panels depends substantially on the maximum tension stress $\sigma_{y\max}$, produced by the buckling of the sheet. A method for the assessment of $\sigma_{y\max}$ is proposed here and a test programme is in progress to check this method experimentally.

Another test programme is being carried out to check the possibility of foreseeing the fatigue behaviour of stiffened panels in tension, on the basis of test results obtained from simple cracked plates and using assessment of the SIF or K_T, that can be obtained through theoretical methods developed in (9)(10).

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