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DOWNWASH AT UNSTEADY MOTION OF A SMALL AEROPLANE AT LOW AIRSPEEDS. FLIGHT INVESTIGATION AND ANALYSIS.

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DOWNWASH AT UNSTEADY MOTION OF A SMALL AEROPLANE AT LOW AIRSPEEDS.

FLIGHT INVESTIGATION AND ANALYSIS.

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Abstract.

Measurements of the downwash were carried out on a small A 145 transport aeroplane not only at rectilinear steady flights but also at unsteady flights. The method of measuring the local downwash angle by means of vanes in front of the horizontal tailplane was supplemented by the determination of the effective downwash angle from the directly measured forces on the horizontal tailplane. There were applied identification methods and statistical tests of significance to judge the suitability of the mathematical models used.

The value of the downwash angle derivative measured at unsteady flights was significantly lower than that measured at steady flights. These phenomena have been analysed and a simple method of estimation of a weighted effective value of the downwash angle derivative has been proposed.

1. Introduction

On a small A 145 transport aeroplane there were gradually performed since 1964 different flight measurements of the longitudinal motion as e.g.: measurements of load spectra of the horizontal tailplanes [19], measurements of aerodynamic derivatives at longitudinal steady and unsteady motion [20], measurements of loads on horizontal tailplane passing through a single gust [15] and measurements of aerodynamic characteristics of tailplane and of stream characteristics in the point of the same [17]. From the comparison of measurements and analyses results followed that the stated values of the downwash derivative in the place of the tailplane, and of slow down coefficient of the flow are for the tailplane lift estimation at unsteady motion very unreliable.

In order to get a more extensive physical knowledge of phenomena in the air stream in the place of the tailplane the downwash measurements at steady flights were supplemented also by measurements at unsteady flights and the method of the local downwash measurement by vanes in front of the tailplane was supplemented by the statement of the effective downwash from a direct tailplane forces measurement.

Because the value of the aerodynamic time unit $\tau_A = l/V$ and therefore also of the kinematic similarity number $\omega^* = \omega l/V$ (the Strouhal number) is growing when the flight speed decreases at a constant wing chord, there was verified the significance of the effect of the unsteady flow on the airframe by an analysis of a hypothetic small transport aeroplane flying at low speed.

2. The downwash measurement in flight on the A 145 aeroplane

As an experimental aeroplane there was used the small A 145 transport aeroplane with two Walter M332 piston engines of 180 HP. Its view is shown in Fig.1. Its main mass and geometric characteristics are given in Tab. 1.

The horizontal tailplanes of the aeroplane were hinged by four elements that made possible to measure directly the resulting normal forces on the tailplane.

2.1 The method of measurement

Measurements was performed at gliding flights with engines out and feathered airscrews. Before the down wash measurements were done there was qualitatively judged the character of flow on the tailplane by means of taking motion pictures of filaments placed on a net in front of the tailplane. Quantitatively the character of flow in front of the tailplane was judged on

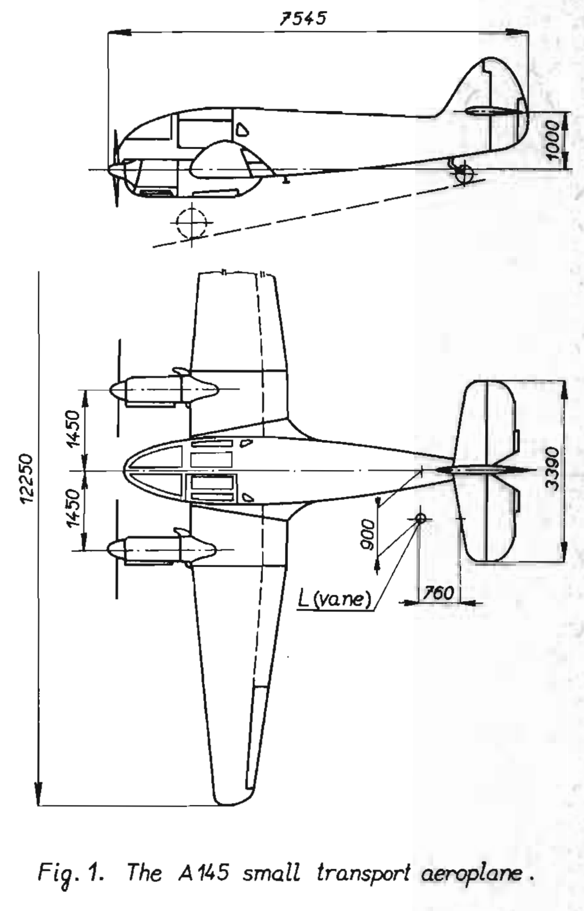


Fig. 1. The A145 small transport aeroplane.

Table 1. Basic characteristics of the A 145 small twin – engined aeroplane

F [m²]	17.09	F_H m²	3.31
b [m]	12.25	b_H m	3.39
l [m]	1.480	l_H m	1.030
λ	8.78	λ_H	3.47
r' [m]	4.880	$\tilde{r}_H$	3.339
m [kg]	1530	$\tilde{i}_y^{\,2}$	0.859
$\tilde{x}_s$	0.247	$\tilde{V}_H = F_H r_H / Fl$	0.647
$\tilde{\xi}_H$	2.374	$\tilde{\xi}_L$	1.741

ground of measurements by four Pitot-static tubes with vanes, that were placed gradually in three heights $2z/b_H$ = -0,156, 0,062 and 0,282 relating to the tailplane and at three side distances $2y/b_H$ = 0,236, 0,531 and 0,826 relating to the aeroplane symmetry plane. At steady rectilinear flights the measurements were done at three values of c.g. position $\tilde{x}_s$ = 0,214, 0,247, 0,289 using usual methods. At unsteady flights the measurements were performed at the mean value of c.g. position $\tilde{x}_s$ = 0,247 only and individual experiments were repeated several times at the same conditions. When flying in calm atmosphere the pulse inputs by elevator of 0,2 til 0,4 sec and informatively also harmonic imputs were used. The mean initial values of conditions from 11 experiments are given in Tab. 2. When measured informatively at gliding flights in turbulent atmosphere a constant flight speed was being maintained by elevator.

The results of repeated experiments to be comparable it was necessary to keep the similarity conditions: c_A = const., μ = const., τ = const., i_y = const., $\tilde{x}_s$ = const. From this it followed that at the constant initial flight speed there was necessary to increase the height of flight due to fuel mass diminuation in agreement with the relation

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} = 2\,\frac{\Delta V_e}{V_e} \qquad (1)$$

Table 2. Mean values of basic conditions for steady – state flights

Ref.	n_j	H [m]	V [m/s]	m kg	μ	τ s	τ_A s
17	11*	1390	54.20	1529	112.96	3.085	0.0273
21	1**	1020 820	52.81	1500	107.50	3.013	0.0280
20	5*	1796	54.73	1496	103.1	2.788	0.0270
* pulse input ; ** vertical gust input							

2.2 The measuring devices and the method of treatment.

The measuring devices were common for measurements at steady and unsteady flights. The measured parameters were recorded by oscillographs. The components of acceleration a_x, a_z and the pitching velocity ω_y were measured by the sensors SFIM with a root mean square error 0.1 to 0.4 per cent of their maximum range (inherent frequency without damping f_n = 20 to 30 Hz, relative damping ξ = 0,7). Angle of attack of the aeroplane and of the tailplane were measured by means of vanes ARTI with a root mean square error 0,3% (f_n = 8 Hz, ξ = 0,12). The maximum distorsion of the vanes fitting during flight was 0,05 deg. The elevator deflection was measured by means of ARTI sensor with the mean square error of 1%. Further there were measured the temperature and static pressure of the atmosphere and the total pressure.

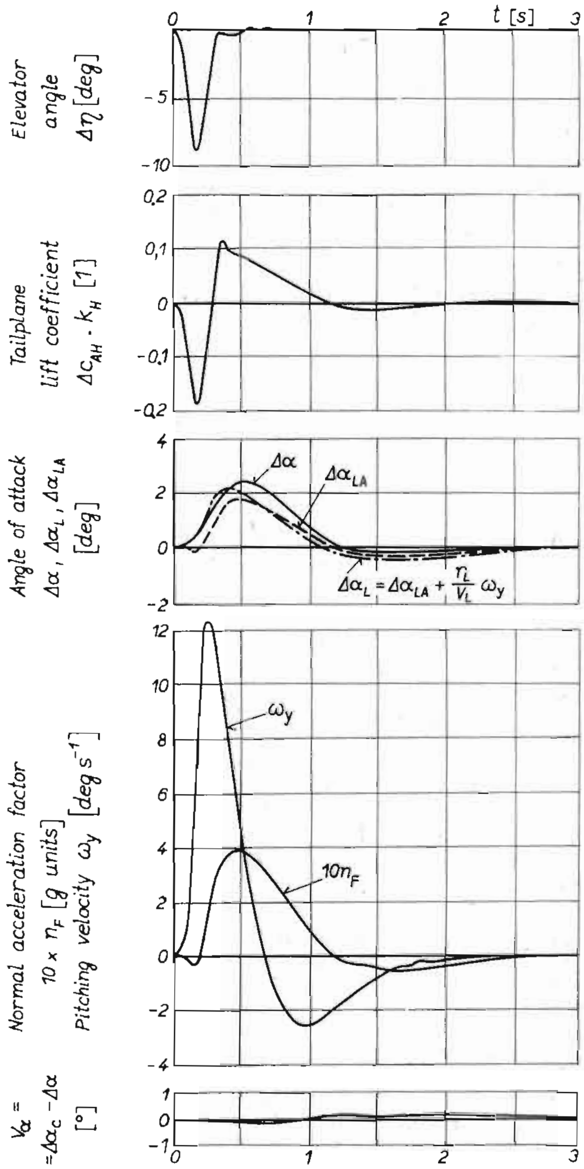


Fig. 2. Sample of measured input and output time histories of the A 145 small twin – engined aeroplane (pulse input , $\Delta\eta$ (t)) .

The resulting normal force on the horizontal tailplane was measured by means of semiconductor strain-gages on four short hinge parts which took just the component of the resulting force perpendicular to the tailplane. Electric signals from sensors proportional to loads were recorded by an oscillograph. The hinge parts were provided by termistor temperature sensors to enable to correct the strain-gages data by the temperature effect. Further there was measured the force in the last lever of the elevator control mechanism for to correct the measured normal force on the tailplane.

At unsteady measurements in calm atmosphere the measured data of instruments, as linear systems of the 2nd order, were corrected by the effect of their dynamic characteristics and of other systematic errors of various physical origin by means of a correcting programme, see (17). Then a checking of absence of systematic errors at corrected parameters $n_F = -a_{z_a}/g$, ω_y and α was made by the checking quantity

$$V_\alpha = \int_0^t \left(\omega_y - 57.3\,\frac{g}{V_0}\,n_F\right) d\tau - (\alpha - \alpha_0) \overset{!}{=} 0 \qquad (2)$$

by means of a computer programme. Some samples of corrected time histories of the main parameters are shown in fig. 2.

From the selected experiments the frequency characteristics $F_{\alpha.\eta}$, $F_{y.\eta}$, $F_{n.\eta}$, $F_{c_{AH}.\eta}$ and $F_{\alpha_{LA}.\alpha}$ were calculated, and then from them for individual levels ω_j the mean values of amplitude ratio R_{M_j} and phase angle φ_{M_j} were stated and at the same time there were estimated the experimental variances round the mean values $s^2_{E_j}$ and the total experimental variance s^2_E . The checking of the absence of substancial errors was done by the Grubbs test and of experimental variance homogeneity by the Bartlets test.
From the mean values there were estimated the transfer coefficients K_r , K_{qr} ($r = 0,1,2$; $q = \alpha$, y , n , c_{AH}) by the least square method and from them there were calculated points of fitted frequency response curves and their total variance s^2_S round the means values at levels ω_j . To test the suitability of the mathematical model for fitting in relation to the measurement precision the test parameter $F = s^2_S/s^2_E$ was used for $s^2_S > s^2_E$, see e.g. ref. (11), (20). The detailed identification analysis of $F_{\alpha_{LA}.\alpha}$ and $F_{c_{AH}.\eta}$ is in chapter 4.

In a turbulent atmosphere some samples were measured informatively, which with respect to the operating limitation following from flight with feathered prolellers were relatively short, 80 sec maximum. Corrections of the dynamic data distortions were made by time shifting of the records by values of time constants T_1 of individual instruments. The effect of unsteadiness of random process in the samples was limited both by a filtration and by the method of linear regression, see (21).

2.3 The results of measurements at steady flights.

Because the measurements at steady flights were performed in the linear extent of the lift coefficient and of the pitching moment coefficient, the relations (c_A, α); (c_W, c_A^2); (c_A, η); ($c_{AH}.k_H, c_A$) could be fitted by straight lines simultaneously for all the three c.g. positions as a whole with the adjoining conditions by the method of least squares. The slopes of fitted lines that are shown in tab. 3 were used to estimate the aerodynamic derivatives and other data.
The measuring of derivative $dc_{AH}.k_H/dc_A$ made possible to estimate besides usual derivatives also the tailplane volume $\overline{V}$, the wing lift derivative a , the a.c. position of the wing with fuselage $\tilde{x}_{N_F}$ and the derivative $a_2 k_H$. A survey of expressions derived by Tichopád in paper (17) is in appendix.

Table 3. Basic results of steady flight measurements

(a)	$\frac{dc_A}{d\alpha}$	$\frac{dc_W}{dc_A^2}$	$\left(\frac{d\alpha_H}{d\alpha}\right)_{Left}$	$\left(\frac{d\alpha_H}{d\alpha}\right)_{Right}$	$\left(\frac{d\alpha_a}{d\alpha}\right)_{ef}$
Value [1]	4.92	21.5	0.716	0.750	0.304
R( ) [1]	± 0.18	± 1.2	± 0.028	± 0.038	—
R_0 %	± 3.7	± 5.5	± 3.9	± 5.1	—

(b)	$\frac{d\eta}{dc_A}$ [deg]			$\frac{dc_{AH}.k_H}{dc_A}$ [1]		
$\tilde{x}_S$ [1]	0.214	0.247	0.289	0.214	0.247	0.289
Value	-9.60	-8.00	-5.60	0.167	0.202	0.270
R( )	± 0.17	± 0.18	± 0.18	± 0.011	± 0.012	± 0.011
R_0 %	± 1.8	± 2.3	± 3.1	6.6	6.0	4.1
$(k_H)_{ef}$ = 0.93						

2.4 The results of unsteady flight measurements.

The frequency response function for the local angle-of-attack with α input, $F_{\alpha_{LA}.\alpha}$, is shown in figs. 4 and 8, and the function for tailplane lift coefficient with η input, $F_{c_{AH}.\eta}$, is shown in fig. 3. For the frequency response functions of the short-period motion of the aeroplane $F_{q.\eta}(i\omega)$ ($q = y, \alpha, c_{AH}$) there are given in tab. 4 the values of the transfer coefficients K_r , K_{qr} ($r = 0,1,2$) which were estimated by the least squares method for a quasi-steady model of the type

$$F_{q.\eta}(i\omega) = \frac{K_{q0} + i\omega K_{q1} + (i\omega)^2 K_{q2}}{K_0 + i\omega\, K_1 + (i\omega)^2} \qquad (3)$$

Table 4. Transfer function coefficients of the A 145 aeroplane estimated by the FRCF method, the variance estimates and the test characteristics.

$F_{q.\eta}$	F_{SE}/F_α [1]	$\hat{K}_0$ * $[s^{-2}]$	t ** [1]	$\hat{K}_1$ $[s^{-1}]$	t	$\hat{K}_{q0}$ $[s^{-2}]$	$\hat{K}_{q1}$ $[s^{-1}]$	$\hat{K}_{q2}$ [1]
$F_{y.\eta}$	1.652	11.99 ±0.25		4.57 ±0.04		-19.30† ±0.66	-14.10‡ ±0.13	—
			2.66		1.96			
$F_{\alpha.\eta}$	$s_S^2 < s_E^2$	13.34 ±0.44		4.95 ±0.19		-15.15 ±0.54	-0.103 ±0.082	—
			0.44		0.10			
$F_{c_{AH}.\eta}$	$s_S^2 < s_E^2$	13.81 ±0.98		4.98 ±0.25		-12.52 ±1.24	+4.32 ±0.65	+1.798 ±0.035
$*\hat{K}_r \pm {}^RS(\hat{K}_r)$; † $[s^{-3}]$; ‡ $[s^{-2}]$	$** t \cong \frac{\lvert\Delta\hat{K}_r\rvert}{\sqrt{\Sigma\, {}^RS^2(\hat{K}_r)}}$; $t_{0.05} = 2.00$							

From this table it follows that in relation to the measurements precision this model does not satisfy for the $F_{y.\eta}$ transfer function and that the differencies ΔK_0, ΔK_1 between the coefficients K_0, K_1 of the common polynomial in the denominator of eq. (3) are significant in the functions $K_{y.\eta}$ and $K_{\alpha.\eta}$. Therefore for further identification estimates of the weighted effective value of the down-wash derivative, derived from forces on the tailplane, there were used the mean measured values of the frequency responses $F_{y.\eta}$ and $F_{\alpha.\eta}$ on individual ω_i levels.

From informative measurements in turbulent atmosphère there is shown in fig. 5 the measured spectrum of the vertical turbulence component and in fig. 6 there is plotted a sample of the amplitude characteristic of the normal acceleration transfer function, $|F_{a_z.w_H}(i\omega)|$.

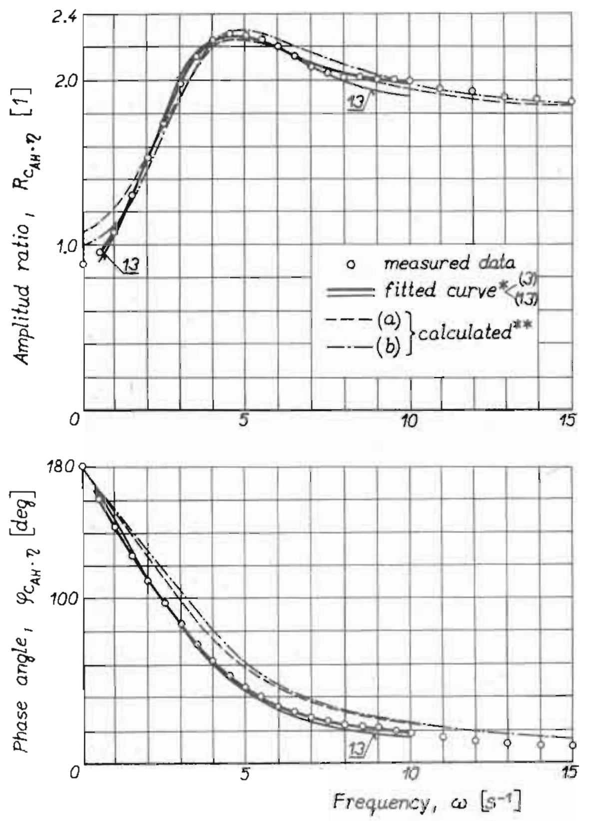
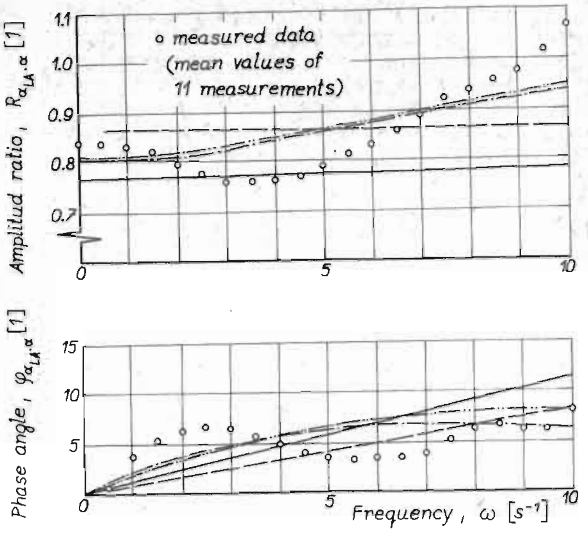


Fig. 3. Frequency response curve of the A 145 aeroplane relating the tailplane lift coefficient to elevator deflection. * The FRCF method; pulse input, mean values from four measurements. **Quasi-steady state model, aerodynamic derivatives from steady state measurements: (a) $\left(\frac{d\alpha_a}{d\alpha}\right)_{steady} = 0.304$; (b) the effective dynamic value $\left(\frac{d\alpha_a}{d\alpha}\right)_{ef.w.} = 0.233$ (weighted mean value).



Eq.	Marking	T_1 [s]	T_2 [s²]	$\frac{d\alpha_a}{d\alpha}$	τ_{Lx} [s]	F_{SE}/F_α
5	*	0	0	0.234	0.068	—
7	**	0.127	0.0026	0.202	0	1.53
7	**	0.075	0	0.190	0.049	1.55
5	**	0	0	0.145	0.084	3.07

* Calculated for the tailplane $\quad \omega^* = \frac{\omega}{36.62}$ [1]
** Estimated by the FRCF method

Fig. 4. Amplitud and phase characteristics of frequency transfer function $F_{\alpha_{LA}.\alpha} = \frac{\Delta\alpha_{LA}}{\Delta\alpha}$ for the L point at the left-hand side before the tailplane of the A 145 aeroplane.

3. The identification analysis of results

3.1 The supplement models

It is possible to express analytically the frequency response functions between the angle-of-attack deflection $\Delta\alpha_{LA} = \Delta\alpha_L - \frac{\omega_y r_H}{V_H}$ in point L before the tailplane which was corrected by the effect of pitching motion of the aeroplane relating to its c.g., and between the angle-of-attack deflection of the wing, $\Delta\alpha_F \equiv \Delta\alpha$, by means of approximative expressions which are given in chap.5, that is

$$F_{\alpha_{LA}.\alpha}(i\omega) = {}^{o}F_{\alpha_a.r}(i\omega) \cdot {}^{o}F_{r.\alpha_F}(i\omega) \qquad (4)$$

This transfer functions includes 13 constants. With respect to the attainable precision of the angle-of-attack α_{LA} measurements, and also with respect on the limitations following from it when using identification methods, there must be found for the identification aim some simpler models than that one in (4).

The simplest generally used expression for quasi-steady aerodynamic model is:

$$F_{\alpha_{LA}.\alpha}(i\omega) = 1 - \frac{d\alpha_a}{d\alpha}(1 - i\omega\tau_{Lx}) \qquad (5)$$

This expression is a relatively rough approximation of the expression with transportation lag:

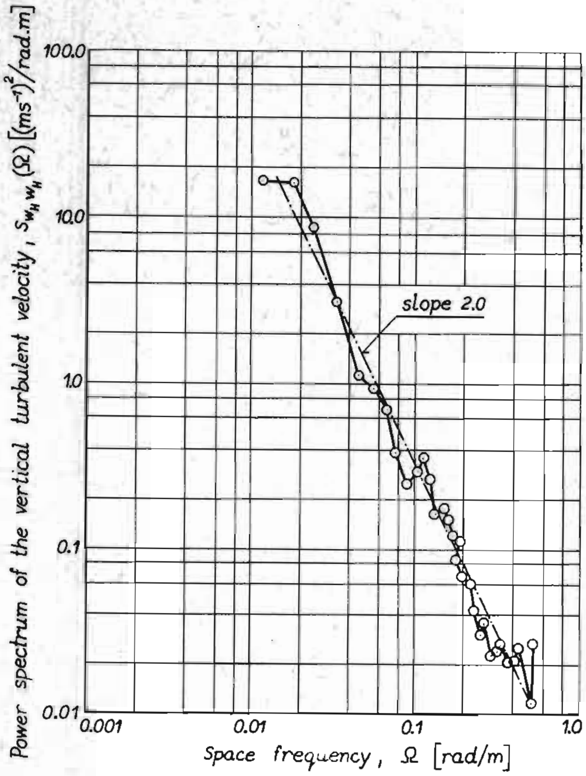
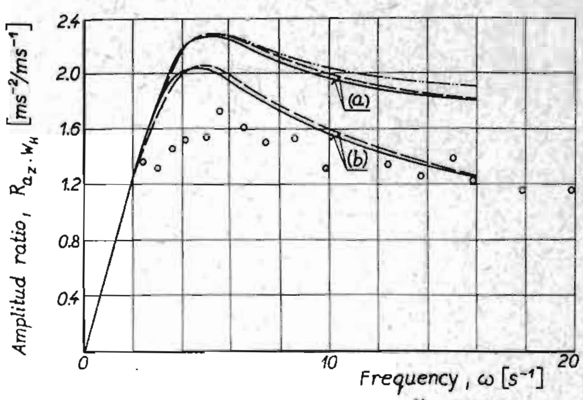


Fig. 5. Measured power spectrum of the vertical turbulent velocity, $w_H(t)$, by means of the A 145 aeroplane (height H_p =750m, centre Bohemia).



V_A = 52.81 [m/s], τ = 3.013 [s], μ = 107.50, $\tilde{x}_s$ = 0.247,
L = 133 [m], σ_w = 0.765 [ms^{-1}], l/V = 0.0273 [s]

Exple	Marking	Aerodynamic model	$\frac{d\alpha_a}{d\alpha}$
1.0	——— (a)	quasi - steady	0.304
1.1	– – – (a)		0.264
2.0	═ ═ ═ (b)	$(R_{a_z \cdot w_H}(\omega))_{unst.} = \lvert\phi(i\omega)\rvert \cdot (R_{a_z \cdot w_H})_{steady}$ *	
2.2	—··—··	unsteady downwash only	
—	o	measured data (for $\eta(t)$ corrected)	
*	$\lvert\phi(i\omega)\rvert^2 = \dfrac{1+\omega\, 7.5\, l/V}{1+\omega\, 7.5\, l/V \cdot [1+\omega\pi\, l/V]}$		

Fig. 6. Amplitud frequency response curve $R_{a_z \cdot w_H}(\omega)$ from the power spectra $S_{w_H \cdot w_H}(\omega)$, $S_{a_z \cdot a_z}(\omega)$ estimated from the acceleration response $a_z(t)$ of the A 145 aeroplane to random vertical gusts $w_H(t)$, see ref [21].

$$F_{\alpha_{LA} \cdot \alpha}(i\omega) = 1 - \frac{d\alpha_a}{d\alpha}\,(\cos\omega\tau_{Lx} - i\sin\omega\tau_{Lx}) \qquad (6)$$

From a more detailed analysis it followed that the physical nature of the approximative relation (4) is represented by the expression:

$$F_{\alpha_{LA} \cdot \alpha}(i\omega) = 1 - \frac{d\alpha_a}{d\alpha} \cdot \frac{\cos\omega\tau_{Lx} - i\sin\omega\tau_{Lx}}{1 + i\omega T_1 + (i\omega)^2 T_2} \qquad (7)$$

From the fig. 2 it can be seen, however, that the vane data are affected by the elevator deflection, which changes the tailplane circulation. For this case there are shown in fig. 7 various effects on the vane deflection in front of the tailplane. The frequency response, which expresses the effect both of the wing and of the tailplane, is given by the full expression:

$$F^T_{\alpha_{LA} \cdot \alpha}(i\omega) \doteq F_{\alpha_{LA} \cdot \alpha}(i\omega) + F_{\alpha_{aH} \cdot \alpha_H}(i\omega) \cdot \Big[F_{\alpha_{LA} \cdot \alpha}(i\omega) + \frac{r_H}{V_H}\left(\frac{F_{y \cdot \eta}}{F_{\alpha \cdot \eta}}\right)_E + \frac{a_2}{a_1}\left(\frac{1}{F_{\alpha \cdot \eta}}\right)_E\Big] \stackrel{!}{\doteq} F^T_{\alpha_{LA} \cdot \alpha E}(i\omega) \qquad (8)$$

where $F_{\alpha_{LA} \cdot \alpha}$ is described approximatly by eq. (7) and

$$F_{\alpha_{aH} \cdot \alpha_H}(i\omega) = \frac{d\alpha_{LA}}{d\alpha_H} \cdot \frac{1}{1 + i\omega T_3 + (i\omega)^2 \cdot T_4} \qquad (9)$$

The suffix E denotes parameters or functions that were measured experimentally in flight, and $\frac{d\alpha_{LA}}{d\alpha_H} \equiv T_5$, $\frac{d\alpha_a}{d\alpha} = T_6$ and $\tau_{Lx} \equiv T_7$. To estimate the constants T_r (r = 1, ... , 7) there can be used the least squares method.

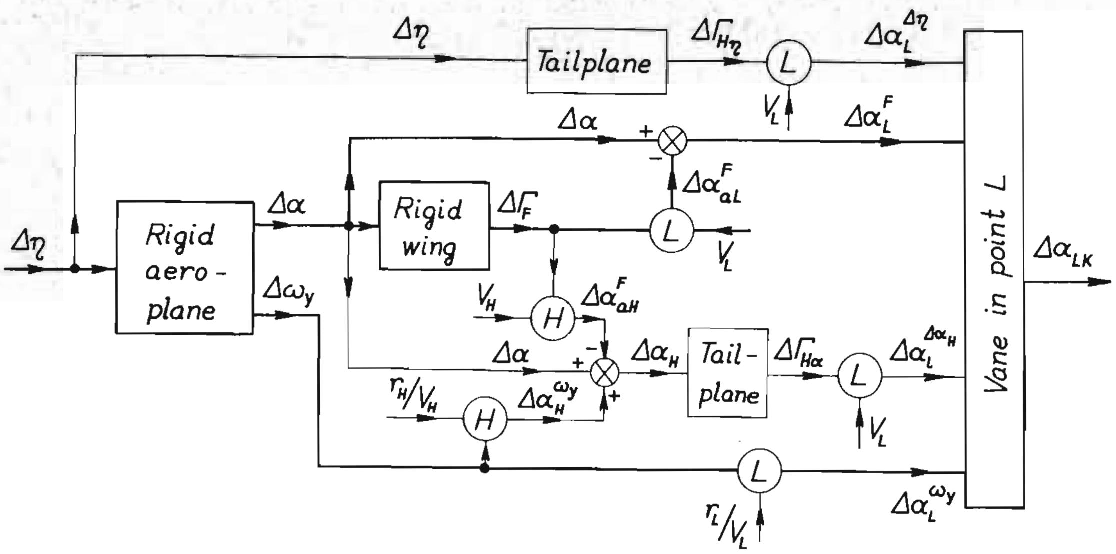
The complex deviation $w(i\omega_j) = F^T_{\alpha_{LA} \cdot \alpha}(i\omega_j) + - F^T_{\alpha_{LA}\alpha_E}(i\omega_j)$ follows from the eq. (8). From the condition $\sum_j w^2(i\omega_j) \Rightarrow$ min there can be derived a system of n_{K_r} normal equations generally in the form:

$$\sum_j (i\omega_j)\,\frac{\partial w(i\omega_j)}{\partial T_r} = 0\,, \qquad r = 1, 2, \ldots, n_{K_r} \qquad (10)$$

As the equation for deviations w_j in this case is a nonlinear function for the sought constants it is necessary to estimate the constants T_r by an iterative method. For the s[th] step there is:

$$T_{r(s)} = T_{r(s-1)} + \Delta T_{r(s)} \qquad (11)$$

The estimate of the initial values T_{r0} for the first step, which can be usually done by the equation-of-motion method TEM, see ref. [11] , in this case for the complete equation (8) fails. It is therefore necessary to start from the supposition that the effect of the tailplane can be neglected, i.e. $T_5 \equiv 0$, and to estimate the constants T_1 , T_2 only and eventually the constants T_6 , T_7 by a minimalisation of deviations according



influence of aeroplane response $\Delta\alpha$, $\Delta\omega_y$ on the vane.

influence of tailplane on the vane.

(L) point L in front of tailplane in which the vane is placed.

(H) point H in which the aerodynamic centre of the tailplane is situated.

Fig. 7. Chart of the origin of the angle of incidence, $\Delta\alpha_{LK}(t)$, due to the elevator deflection, $\Delta\eta(t)$.

to eq. (7). If there is found out by F-test that the variance s_S^2, stated from deviations of the mean experimental values on ω_j levels from values of the fitted curve (7), is not significant in relation to the experimental variance s_E^2 of the measured points round their mean values on ω_j levels, then it is not necessary to estimate the constants T_3, T_4 of the complete equation (8). The identification process then breake into two groups:
the froup A, when $T_5 = \frac{d\alpha_{LA}}{d\alpha_H} \equiv 0$ and
the group B, when $T_5 \neq 0$.
For the solution of the both groups there was worked out a digital computer programme, see [22].

To make easier the analysis of the identification results from the physical point of view there can be used expressions for the derivative $\frac{d\alpha_{aL}}{d\alpha}$, calculated for a rectilinear steady flight, when $\omega \equiv 0$ and $\omega_y \equiv 0$ and for quasi-cirlular flight, when $\omega \equiv 0$ and $\omega_y \doteq$ const. Besides there is possible to estimate informatively the derivative $\frac{d\alpha_L}{d\alpha_H}$ in point L for a quasi-steady aerodynamic model from the measured values $\alpha_{LA}(t_\eta)$ and $\alpha(t_\eta)$ for the time t_η, when $\dot{\eta}=0$. To judge roughly the relative relation of the derivatives $T_6 = \left(\frac{d\alpha_{aL}}{d\alpha}\right)_{Steady}$ and $T_5 = \left(\frac{d\alpha_L}{d\alpha_H}\right)_{Steady}$ in a rectilinear steady flight then can be used the expression:

$$T_6 = 1 - \frac{\left(\frac{d\alpha_{LA}^T}{d\alpha}\right)_{Steady} - \frac{a_2}{a_1}\left(\frac{d\eta}{d\alpha}\right)_{Steady} \cdot T_5}{1 + T_5} \qquad (12)$$

An estimate of the effective value of the derivative $\left(\frac{d\alpha_a}{d\alpha}\right)_{ef}$ for the whole tailplane was done by Tichopád in paper [17] with using a quasi-cteady model for the frequency transfer function $F_{(k_H c_{AH})\cdot\eta}$ which was calculated directly from the measured values of forces on the tailplane:

$$F_{(k_H c_{AH})\cdot\eta}(i\omega) = (\beta_2 + i\omega\beta_1)\, F_{\alpha\cdot\eta}(i\omega) + \beta_3 F_{y\cdot\eta}(i\omega) + \beta_4 \qquad (13)$$

where

$$\beta_1 = a_1 k_H \frac{d\alpha_a}{d\alpha}\frac{r_H}{V_H} = a_1 k_H \frac{d\alpha_a}{d\alpha}\frac{\tilde{r}_H}{\sqrt{k_H}}\frac{l}{V}\ [s] \qquad (14)$$

$$\beta_2 = a_1 k_H \left(1 - \frac{d\alpha_a}{d\alpha}\right) \qquad (15)$$

$$\beta_3 = a_1 k_H \frac{r_H}{V_H} = a_1 k_H \frac{\tilde{r}_H}{\sqrt{k_H}}\frac{l}{V} \qquad (16)$$

$$\beta_4 = a_2 k_H = a_1 k_H \left(\frac{a_2}{a_1}\right) \qquad (17)$$

As the constants β_1 to β_4 are linearly dependent on each other it is necessary, when estimating the constants by the least squares method, to estimate one of them,best being β_4 from measurements at steady flights. Because the eq. (13) is linear for the sought constants, it is possible to calculate directly their most probable values for the measured values of the frequency characteristics $F_{c_{AH}\cdot\eta E}$, $F_{y\cdot\eta E}$ and $F_{\alpha\cdot\eta E}$.
From the estimates $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$ it is possible to calculate estimates

$$\left(\frac{d\hat{\alpha}_a}{d\alpha}\right)_{ef} = \frac{\hat{\beta}_1}{\hat{\beta}_3} \qquad (18)$$

$$\hat{k}_H = \left(\frac{\hat{\beta}_2}{\hat{\beta}_3 - \hat{\beta}_1}\right)^2 \cdot \left(\frac{r_H}{V}\right)^2 \qquad (19)$$

$$\hat{a}_1 = \frac{\hat{\beta}_2\,\hat{\beta}_3}{\hat{\beta}_3 - \hat{\beta}_1} \cdot \frac{1}{k_H} \qquad (20)$$

The value of $\hat{a}_1$ should not be significantly different from the value a_1 which was stated from measurements at steady flights.

From the physical point of view the expression of β_1 by eq. (14) is just approximate. It would be more correct for to express it to use an unknown transportation lag τ_{Hx} so that

$$\beta_1 = (a_1 k_H) \cdot \left(\frac{d\alpha_a}{d\alpha} \cdot \tau_{Hx}\right) \quad [s] \qquad (21)$$

and to suppose that the values of derivatives a_1 , a_2 are at unsteady flights in a certain interval of circular frequencies the sames as at steady flights. The solution of normal equations, given by eq. (13), then should be repeated for values β_4 getting more accurate. From in this way calculated estimates $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$ it would be then possible to express:

$$\hat{k}_H = \left(\frac{\hat{\beta}_3}{a_1 \tilde{r}_H\, l/V}\right)^2 \qquad (22)$$

$$\left(\frac{d\hat{\alpha}_a}{d\alpha}\right)_{ef} = 1 - \frac{\hat{\beta}_2}{a_1 \hat{k}_H} \qquad (23)$$

$$\hat{\tau}_{Hx} = \frac{\hat{\beta}_1}{a_1 \hat{k}_H \left(\frac{d\hat{\alpha}_a}{d\alpha}\right)_{ef}} \quad [s] \qquad (24)$$

In the case when in eq. (13) the constant downwash derivative $\left(\frac{d\alpha_a}{d\alpha}\right)_{ef}$ is supplemented in a certain interval of circular frequencies ω_j by the frequency response function $F_{\alpha_H \cdot \alpha} = 1 - F_{\alpha_a \cdot \alpha}$, there it is possible to calculate this function from the expression:

$$F_{\alpha_H \cdot \alpha}(i\omega) = \left[\frac{F_{(k_H c_{AH}) \cdot \eta}(i\omega)}{a_1 \hat{k}_H} - \frac{\tilde{r}_H}{\sqrt{\hat{k}_H}} \cdot \frac{\tau}{\mu} F_{y \cdot \eta}(i\omega) - \frac{a_2}{a_1}\right] \cdot \frac{1}{F_{\alpha \cdot \eta}(i\omega)} \qquad (25)$$

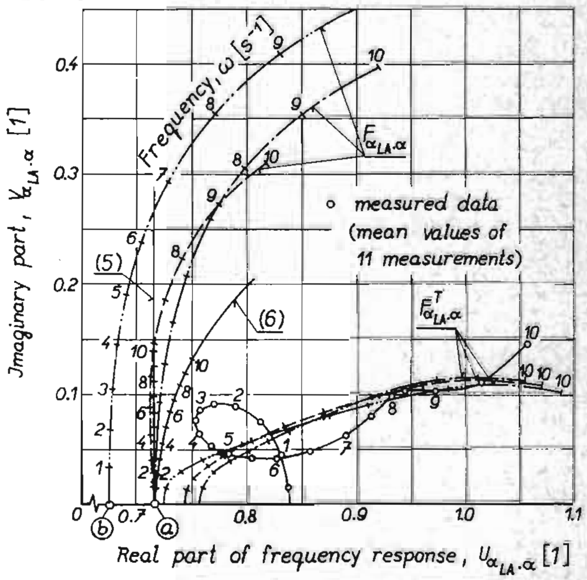
where $\hat{k}_H$ is the estimate of the effective value at unsteady flight. Then it is possible to identify the estimate of the frequency response function $\hat{F}_{\alpha_H \cdot \alpha}$ by the eq. (7).

However the calculations of estimates $\left(\frac{d\alpha_a}{d\alpha}\right)_{ef}$ and $F_{\alpha_H \cdot \alpha}$ based on expressions (21) and (25) require very high accuracy of measurements of the used transfer functions $F_{(k_H c_{AH}) \cdot \eta}$, $F_{\alpha \cdot \eta}$ and $F_{y \cdot \eta}$.

3.2 Identification results.

The effect of horizontal tailplane on the vane being not considered (group A, for $T_5 = d\alpha_{LA}/d\alpha_H = 0$) there were used for the identification of the local angle-of-attack dependence on the aeroplane angle-of-attack two methematical models: the quasi-steady model (5) with only approximatively expressed transportation lag and the model (7), for which 2 to 4 constants were gradually optimalised. The main results are given in fig. 4, where in a table the estimates of constants T_1 , T_2 and $T_6 = d\alpha_a/d\alpha$ and the value of the test parameter F_{SE} related to its critical value for the significance degree 5% are given. This ratio F_{SE}/F_α gives an information about the "closeness" of the fitted curve to the measured points with respect to the measurement precision.

From the table it is evident that from the point of view of closeness of the fitted curve none of the models used in the group A satisfied and that the worst one is the model (5). For the model (7) the least ratio F_{SE}/F_α = 1.55 was when optimalising three constants, T_1 , T_2 and T_6 for the given values of transportation lag $\tau_{Lx} = 0$ and 0.049 secs. It is interesting that for both values of the transportation lag the sum $T_1 + \tau_{Lx}$ is equal, i.e. 0.127 and 0.124 secs. The mean square errors of the constants are the following: $S(\hat{T}_1)$ 14 and 19%, $S(\hat{T}_2)$ = 90% and $S(\hat{T}_6)$ = 9 and 7%. Therefore it is useless with respect to the measurement precision to calculate istimates of the constant T_2 . It also follows from the table that the effect of the transportation lag is not significant. In the groups A always 5 iterations were sufficient.



Eq.	Marking	T_1 [s]	T_2 [s²]	T_3 [s]	$\frac{d\alpha_L}{d\alpha_H}$ [1]	$\frac{d\alpha_a}{d\alpha}$	τ_{Lx} [s]	F_{SE}/F_α
5	* ———	—	—	—	—	0.284	0.049	—
6	* ———	—	—	—	—	0.284	0.049	—
8	— — —	0.068	0.0060	0.056	0.083	0.284	0	0.86
	—·—·—	0.054	0.0056	0.148	0.124	0.284	0.049	0.75
**	—··—··	0.054	0.0056	0.154	0.162	0.325	0.049	1.02

*Calculated for the point L. ** Estimated by FRCF method. (a) Steady state value. (b) Steady state value corrected for $\frac{d\alpha_L}{d\alpha_H}$ = 0.162.

Fig. 8. Measured and fitted frequency response curves $F_{\alpha_{LA} \cdot \alpha}(i\omega)$ for the L point at the left-hand side before the tailplane of the A 145 aeroplane. (the influence of the tailplane considered)

With respect to the unsatisfactory "closeness" of the curves in the group A there was necessary to calculate estimates using the model (8) in the group B with consideration of the tailplane effect on the vane. The results are given in fig, 8, where both the frequency response functions $F^T_{\alpha_{LA}\cdot\alpha}$, fitting the measured points including the tailplane effect, and the frequency response functions $F_{\alpha_{LA}\cdot\alpha}$, calculated for the constants values estimated when optimalising by means of the model (8), are given. Besides in the fig. 8 there are shown for information the curves calculated by means of the models (5) and (6) for the downwash derivative values, which were found out from steady flights. To optimalise by means of the model (8) there was selected for the constant T_6 both the value stated at steady flights $d\alpha_a/d\alpha = 0.284$, and the value from steady flights, which was corrected by the tailplane effect, $d\alpha_a/d\alpha = 0.325$. The transportation lag was estimated as $T_7 \equiv \tau_{Lx} = 0$ and 0.049 secs.

From the table in fig. 8 one can see that in group B when considering the "closeness" of the fitted curve two cases for the derivative $d\alpha_a/d\alpha = 0.284$ and for the transportation lag $\tau_{Lx} = 0$ and 0.0495 secs satisfied and one case was on the significance boundary. In all these cases there was $T_4 \equiv 0$, because for $T_4 \neq 0$ the results were not physically acceptable ($T_5 < 0$). There was also not acceptable $T_2 = T_4 \equiv 0$ for too high values of T_3. The mean square errors of constants in the order of rank given in the table are high: $S(\hat{T}_1)$ 22, 23 and 28%, $S(\hat{T}_2) = 48$, 18 and 14%, $S(\hat{T}_3) = 115$, 59 and 47%, $S(\hat{T}_5) = 25$, 26 and 20%. Therefor it is not too useful to consider in the model (9) the constant T_3 but just to consider the constant $T_5 = d\alpha_L/d\alpha_H$. In the group B it was necessary to use 10 to 15 iterations. The third case in the table, though being the worst gives the estimate of the constant $T_5 = 1.162$, which is the closest to the values from the check calculation from time histories for t_η. From this calculation there followed that $d\alpha_L/d\alpha_H = 0.170 \pm 0.009$ and $(d\alpha_{aL}/d\alpha)_{steady} = 0.328 \pm 0.002$.

The curves which wěre fitted according to the equations (5) and (6) show in comparison with the curves $F_{\alpha_{LA}\cdot\alpha}$ fitted according to the eq. (8) that for the usually used model the transportation lag value 0.049 secs is small. This value was calculated from the distance of the point L behind the trailing edge of the wing. This agrees with notion given in paper(3).

As for the effective value of the downwash for the whole tailplane an analysis in accordance with the equatons (13) to (20) has given the following estimates: $(d\alpha_a/d\alpha)_{ef} = 0.234$ and $(k_H)_{ef} = 0.92$. These values which are the mean values from the given interval $\bar{\omega}_i$, are smaller then the values, which were measured at steady flights, i.e. $(d\alpha_a/d\alpha)_{ef} = 0.304$ and $(k_H)_{ef} = 0.93$. It is noteworthe that frequency response curve shape $F_{C_{AH}\cdot\eta}$, calculated backword from estimates of these constants according to the eq. (13), agrees relatively well with the measured points in fig. 3 ($s_S^2 < s_E^2$). The results of calculations according to equations (22) to (25) are not given here because the necessary data have not been measured with accuracy required for this purpose.

4. Analytical example

The aim of the example is to prove the effect of the Strouhal number on the longitudinal short period motion at low velocities of a small aeroplane. The analysis was

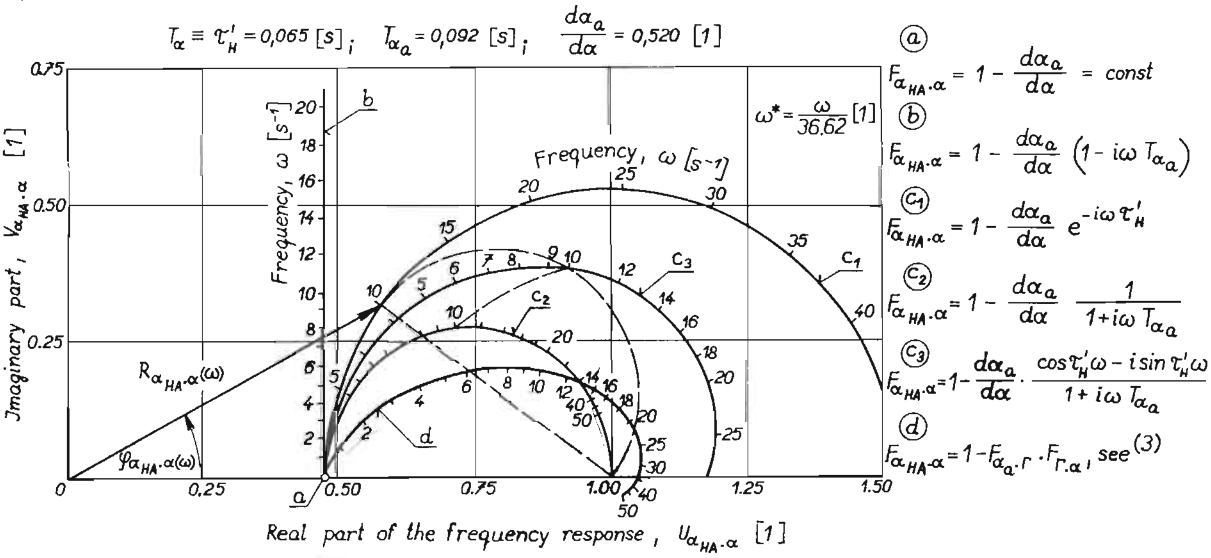


Fig. 9. Frequency response curves $F_{\alpha_{HA}\cdot\alpha} = \frac{\Delta\alpha_{HA}}{\Delta\alpha}(i\omega) = 1 - F_{\alpha_a\cdot\alpha} = U_{\alpha_{HA}\cdot\alpha} + iV_{\alpha_{HA}\cdot\alpha}$ relating the angle of attack at tailplane, $\Delta\alpha_{HA} = \Delta\alpha_H - \frac{\omega_y r_H}{V_H}$, to these of aeroplane, $\Delta\alpha$ (hypothetical aeroplane).

done for a hypothetical aeroplane, which has the same main geometric and mass characteristics as had the A 145 aeroplane, the aerodynamic characteristics of which being a little different.

4.1 The basic relations

The longitudinal quasi-steady motion of a rigid aeroplane at a constant value of airspeed, which was excited by an elevator deflection in flight in the calm atmosphere is described by a system of two differential equations, which can be expressed after a Fourier transformation is done in the matrix form:

$$\left\{-\omega^2\begin{bmatrix}0, & 0\\ 0, & -\tilde{i}_y^2\tau^2\end{bmatrix}+i\omega\begin{bmatrix}\tau, & -\tau\\ 0, & 0\end{bmatrix}+\begin{bmatrix}c_{W0}, & c_{W0}\\ 0, & 0\end{bmatrix}+\right.$$

$$\left.+\begin{bmatrix}c_{A.\alpha}+i\omega\frac{\tau}{\mu}c_{A.\dot{\alpha}*}, & i\omega\frac{\tau}{\mu}c_{A.y}{}^*\\ \mu\left(c_{M.\alpha}+i\omega\frac{\tau}{\mu}c_{M.\dot{\alpha}*}\right), & i\omega\,\tau\,c_{M.y}{}^*\end{bmatrix}\right\}\times\begin{bmatrix}\Delta\bar{\alpha}\\ \Delta\bar{\vartheta}\end{bmatrix}=$$

$$=-\begin{bmatrix}c_{A.\eta}+i\omega\frac{\tau}{\mu}c_{A.\dot{\eta}}{}^*\\ \mu\left(c_{M.\eta}+i\omega\,\tau\,c_{M.\dot{\eta}}{}^*\right)\end{bmatrix}\times\Delta\bar{\eta} \qquad (26)$$

where $i\omega\,\Delta\bar{\vartheta} \doteq \bar{\omega}_y(i\omega)$ and $i\omega\,\Delta\bar{\alpha} = \Delta\bar{\dot{\alpha}}(i\omega)$

The first three square matrices relate to non aerodynamic forces and the last square matrix relates to response aerodynamic forces and the last column matrix relates to the input control aerodynamic forces. In case, that there reveals an aerodynamic effect of unsteady flow on the airframe, then the matrices with aerodynamic derivatives of forces and moments have to be replaced by matrices of aerodynamic frequency responses:

$$\begin{bmatrix}F_{c_A.\alpha}(i\omega), & i\omega\frac{\tau}{\mu}F_{c_A.y*}(i\omega)\\ \mu F_{c_M.\alpha}(i\omega), & i\omega\,\tau\,F_{c_M.y*}(i\omega)\end{bmatrix} \qquad (27)$$

$$\begin{bmatrix}F_{c_A.\eta}(i\omega)\\ \mu F_{c_M.\eta}(i\omega)\end{bmatrix} \qquad (28)$$

The matrices of aerodynamic derivatives in equation (26) followed from the linearisation of the aerodynamic lift coefficient c_A and moment coefficient c_M which immediately change their values with the changes of kinematic parameters $\Delta\alpha$, $\Delta\eta$, $\Delta\dot{\alpha}$, $\Delta\dot{\vartheta}$, $\Delta\dot{\eta}$.

The aerodynamic frequency functions are given by Fourier transforms of aerodynamic admitances of the response value $y(t)$ to a unit step input $x = 1$ by the expression:

$$F_{y.x}(i\omega)=i\omega\,\bar{A}_{y.x}(i\omega)=U_{y.x}(\omega)+i\,V_{y.x}(\omega) \qquad (29)$$

where the imaginary part can be expressed for $y = c_A$, c_M and $x = \alpha$, η in the form

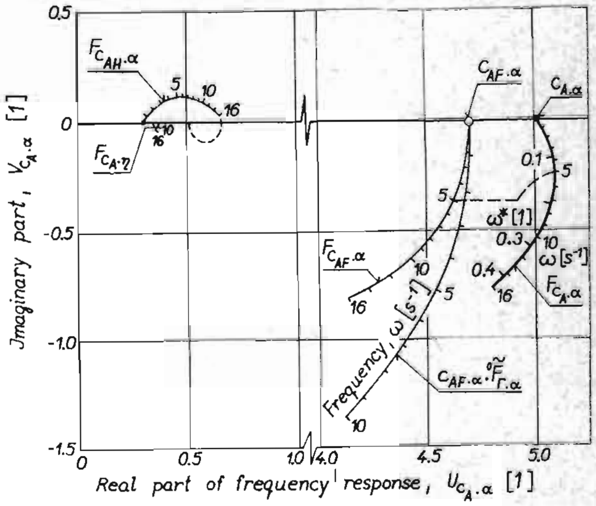
$$V_{y.x}(\omega)=i\omega\,V'_{y.x}(\omega)=i\omega\frac{l}{V}V_{y.\dot{x}*} \qquad (30)$$

To express these functions analytically the aproximative relations for step admitances by R.T.Jones [3] and J.A.Dritschler [6] were used, in which exponential functions are employed.

The aerodynamic frequency functions were estimated in paper [22] :

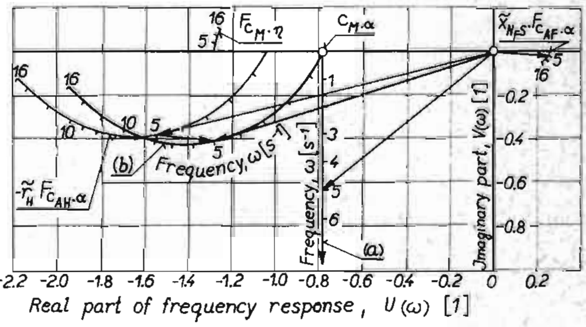
a) from the aerodynamic frequency function $F^{W}_{c_{AF}.\alpha_F}$ for an eliptical wing of the aspect ratio 6, for uncompressible flow and for a simultaneous change of angle of attack on the whole wing according to [6] ;

b) from the frequency function of a horizontal tailplane of aspect ratio 3 and for uncompressible flow according to [3]



$$F_{c_A.\alpha}(i\omega)=U_{c_A.\alpha}(\omega)+i\,V_{c_A.\alpha}(\omega)=F_{c_{AF}.\alpha}+F_{c_{AH}.\alpha}$$

Fig.10. Aerodynamic frequency responses of the lift coefficient of the wing , $F_{c_{AF}.\alpha}(i\omega)$, of the tailplane, $F_{c_{AH}.\alpha}(i\omega)$, and of the whole hypothetical aeroplane , $F_{c_A.\alpha}$, and the frequency response for downwash calculation , $c_{AF.\alpha}\cdot{}^{0}F_{\Gamma.\alpha}$, for the unsteady flight in calm atmosphere . (see [3],[6] and the d curve from Fig. 9.)



(a) $F_{c_M.\alpha}(i\omega) = c_{M.\alpha}+i\omega\frac{l}{V}c_{M.\dot{\alpha}}{}^*$

(b) $F_{c_M.\alpha}(i\omega) = U_{c_M.\alpha}(\omega)+i\,V_{c_M.\dot{\alpha}*}(\omega) = \tilde{x}_{N_FS}\,F_{c_{AF}.\alpha}-\tilde{r}_H\cdot F_{c_{AH}.\alpha}$

Fig. 11. Aerodynamic frequency responses of the pitching moment coefficient , $F_{c_M.\alpha}$ and $F_{c_M.\eta}$, of the hypothetical aeroplane for the unsteady flight in calm atmosphere .

both for a simultaneous change of angle of attack on the whole tailplane, $F^{W}_{c_{AH}\cdot\alpha_H}$, and for a gradual change of angle of attack from leading edge due to the down-wash change, $F^{K}_{c_{AH}\cdot\alpha_H}$;

c) from the frequency function for the down-wash angle of uncompressible airstream behind a wing of aspect ratio 6 in place of the tailplane according to (3) ,

$$F_{\alpha_a\cdot\alpha} = {}^{0}F_{\alpha_a\cdot r}\cdot {}^{0}F_{r\cdot\alpha}\ . \qquad (4)$$

These approximative expressions for the frequency functions involve constants and the expressions of the type of $\frac{a_i T_i}{1+i\omega T_i}$. The function $F_{\alpha_a\cdot r}$ involves moreover an integral exponencial function, which expresses the effect of wake vortex on the downwash in place of the horizontal tailplanes. Some aerodynamic transfer functions, estimated for the hypothetical aeroplane in accordance with the earlier given relations, are shown in fig. 9 (curve d), 10 and 11. The values of parameters μ, τ, τ_A correspond to parameters in the 1st line of table 2.

4.2 The Effect of Strouhal Number on frequency transform functions of the hypothetical aeroplane.

The Strouhal Number is a parameter of the kinematic similarity, which expresses the kinematic conditions on the model and on the aeroplane for a given flight condition. So e.g. for a given flight it defines the dimensionless time $\frac{t}{l/V}$ or the covered dimensionless flight path Vt/l . At a harmonic motion the Strouhal Number characterises the maximum velocity of a given aeroplane point in relation to the flight speed, or the maximum local deviation of the angle-of-attack at oscillations; so e.g. for a single wing or horizontal tailplane it equals the ratio $\frac{\omega l/2}{V} = k$ or for the whole aeroplane it equals the ratio $\omega r_H/V_H = \omega^*_H = 2k\,\tilde{r}_H$. This Number is usually called "reduced frequency" and in recent papers it is usually defined by $\omega^* = \omega l/V = 2k = \omega\tau_A$ (the circular frequency multiplied by the aerodynamic time unit). It is clear that for a given harmonic motion of an aeroplane the maximum local deviations of the angle-of-attack will be the greater the bigger is the reference geometric parameter of the aeroplane or the smaller is its reference speed.

The effect of the Strouhal Number at a slow flight of the hypothetical aeroplane was proved by a comparison of frequency response curves of short-period-motion with a constant value of airspeed. There were calculated three examples.

Example 1.0: The "quasi-steady" aerodynamic model. The aerodynamic derivatives in eq. (26) are constants and the frequency transfer function of the down-wash is given by the expression

$$F_{\alpha_a\cdot\alpha}(i\omega) = \frac{d\alpha_a}{d\alpha}\left(1 - i\omega\tau'_H\right)\ , \qquad (31)$$

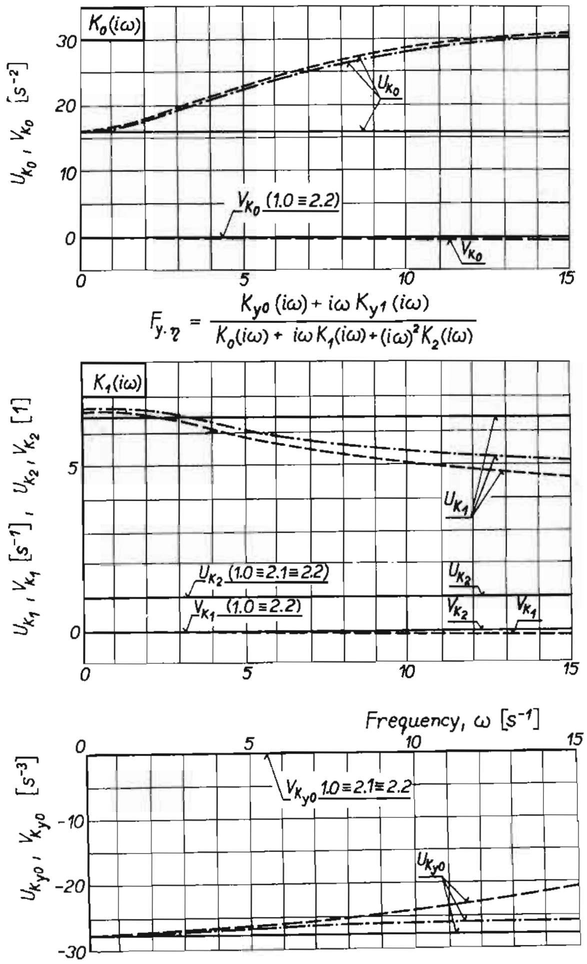
where the derivative $d\alpha_a/d\alpha$ relates to steady flights (fig. 9, curve d).

Example 2.1: The "unsteady" aerodynamic model. In eq. (26) there were used aerodynamic frequency functions of which $F_{c_A\cdot\alpha}$, $F_{c_A\cdot\eta}$, $F_{c_M\cdot\alpha}$, $F_{c_M\cdot\eta}$ are with the imaginary part in accordince with (30) and $F_{c_A\cdot y^*}$, $F_{c_A\cdot\dot{\eta}^*}$ according with (29). There were neglected the imaginary parts of functions $V_{c_A\cdot y^*}$ and $V_{c_A\cdot\dot{\eta}^*}$. The frequency function of the down-wash is given by the expression

$$F_{\alpha_a\cdot\alpha}(i\omega) = F_{\alpha_a\cdot r}\cdot F_{r\cdot\alpha} \qquad (4)$$

see fig. 9, curve d and ref. (3) .

Example 2.2: The "unsteady" model for the downwash only. There were used the constant



Exple:	Marking	Aerodynamic model:
1.0	————	quasi - steady
2.1	— — — —	unsteady
2.2	—·—·—	unsteady downwash only

Fig. 12. Dependence of transfer function coefficients of hypothetical aeroplane calculated by means of three aerodynamic models on the frequency.

Corrections to the Preprint by V. Kočka

Page/Col.	Printed	Should read
$4/2_{23}$	r_H	r_L
$4/2_{17}$	chap. 5	chap. 4.1
4/2	(4)	$= 1 - {}^{o}F_{\alpha_a \cdot r}(i\omega)\, {}^{o}F_{r \cdot \alpha_F}(i\omega)$
5/2	(10)	$\sum_j W(i\omega_j) \frac{\partial W(i\omega_j)}{\partial T_r}$
$8/2^3$	$T_S = 1.162$	$T_S = 0.162$
$10/2^2$	curve d	curve b
$11/2^{26}$	$10\ s^{-1}$	$7\ s^{-1}$
$12/1_1$	resulting spectrum	spectrum sum
$12/2^1$	$C_{\alpha_H}(i\omega_j)$	$\sum_j C_{\alpha_H}(i\omega_j)$
12/2-(37)	$= \frac{(V_{\alpha_H \alpha})_W}{\alpha_W} =$	$= \frac{(V_{\alpha_H \alpha})_W}{\omega_W} =$
$12/2_{13}$	(6)	(b)
$13/1^{37}$	0.90	0.85
$14/2_{10}$	$n_j \cdot (S)$	$n_j (S_M)$
$14/2_{6,4}$	(S)	(S_M)
$15/1^1$	—	add: $(S) = n_j \cdot (S_M)$

aerodynamic derivatives as in the example 1.0, excepting $c_{M.\alpha}$ and $c_{M.\dot{\alpha}*}$ which were replaced by the frequency transfer function

$$F_{c_{M.\alpha}}(i\omega) = U_{c_{M.\alpha}}(\omega) + i\omega \frac{l}{V} V_{c_{M.\dot{\alpha}}*}(\omega), \qquad (32)$$

that involves the aerodynamic derivatives a, a_1 and the frequency transfer function $F_{\alpha_a.\alpha}(i\omega)$ like in the example 2.1 only.

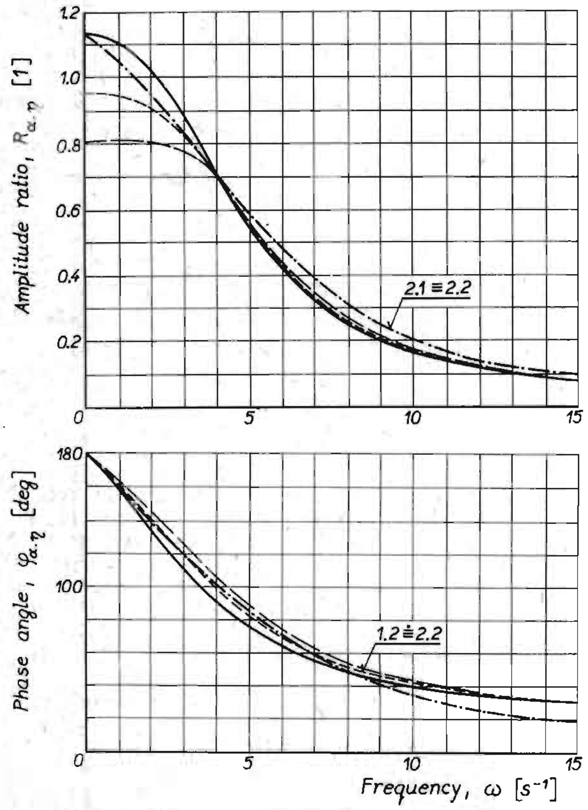
The frequency transfer functions $F_{\alpha.\eta}(i\omega)$, $F_{y.\eta}(i\omega)$ are given as ratios of polynomials of the 1^{th} and 2^{nd} degree, see the expression in fig. 12. In the example 1.0 the transfer coefficients K_r, K_{qr} are constants, in the example 2.1 they are functions of $i\omega$ and so they are in the example 2.2, excepting $K_{\alpha 0}$, $K_{\alpha 1}$ which are constants. The transfer coefficents are plotted for individual examples in fig. 12. The effect of the angular frequency revealed mostly in the real part $U_{K_0}(\omega)$ and then in $U_{K_1}(\omega)$ and $U_{K_{y0}}(\omega)$. Their differences in the example 2.1 and 2.2 are relatively small which proved that the unsteady aerodynamic effect in the circular frequency boundaries $\omega = 0$ to $15\ s^{-1}$ revealed mainly by means of the transfer function $F_{\alpha_a.\alpha}(i\omega)$.

The frequency transfer function $F_{c_{AH}.\eta}(i\omega)$ for the example 2.1 is given by the expression:

$$F_{c_{AH}.\eta}(i\omega) = a_1 k_H \left\{ \left[{}^0F^{W}_{c_{AH}.\alpha_H}(i\omega) - {}^0F^{K}_{c_{AH}.\alpha_H}(i\omega) \cdot F_{\alpha_a \alpha}(i\omega) \right] F_{\alpha.\eta}(i\omega) + \frac{\tilde{r}}{\sqrt{k_H}} \cdot \frac{\tau}{\mu}\, {}^0F^{W}_{c_{AH}.\alpha_H}(i\omega) \cdot F_{y.\eta}(i\omega) + \frac{a_2}{a_1}\, {}^0F^{W}_{c_{AH}.\alpha_H}(i\omega) \right\} \qquad (33)$$

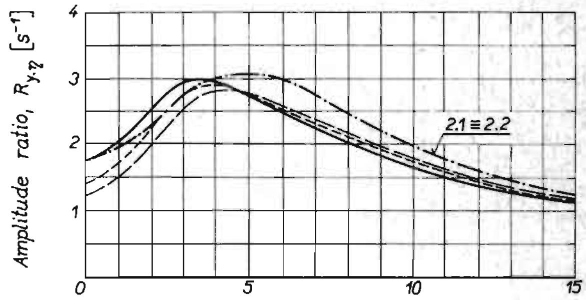
For the examples 1.0 and 2.2 there is ${}^0F^{W}_{c_{AH}.\alpha_H} = {}^0F^{K}_{c_{AH}.\alpha_H} \equiv 1$. The transfer function $F_{\alpha_a.\alpha}$ is defined for the example 1.0 by the expression (31) and for the examples 2.1 and 2.2 by the expression (4).

The calculated frequency fransfer functions $F_{\alpha.\eta}$, $F_{y.\eta}$ and $F_{c_{AH}.\eta}$ from the examples 1.0, 2.1 and 2.2 are plotted in fig. 13, 14 and 15. From all the diagrames it is evident that the unsteady aerodynamic effect reveals itself mainly by means of the down-wash frequency function $F_{\alpha_a.\alpha}(i\omega)$. This is true for the amplitude characteristics in the whole circular frequency boundaries $\omega = 0$ to $15\ s^{-1}$ and for the phase characteristics in the boundaries $\omega = 0$ to $10 s^{-1}$.



Exple:	Marking	Jnput	Aerodynamic model	$\frac{d\alpha_a}{d\alpha}$
1.0	———	—	quasi-steady	0.522
1.1	— — —	pulse		0.316
1.2	- - - - -	step		0.418
2.1	—·—·—	harmonic	unsteady	
2.2	—··—··—		unsteady downwash only	

Fig. 13. Calculated frequency response curve $F_{\alpha.\eta} = \frac{\Delta\alpha}{\Delta\eta}(i\omega)$ of the hypothetical aeroplane.



For marking see the table in the figure 13

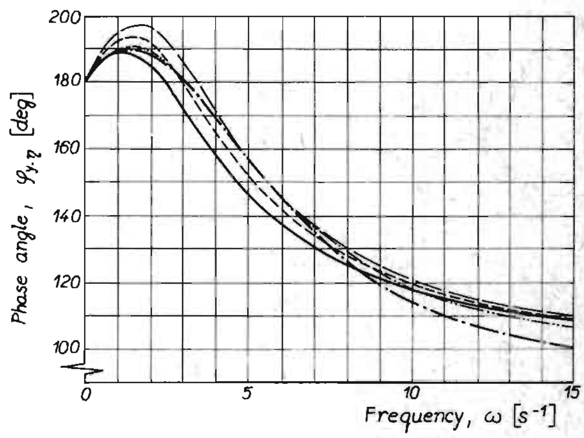
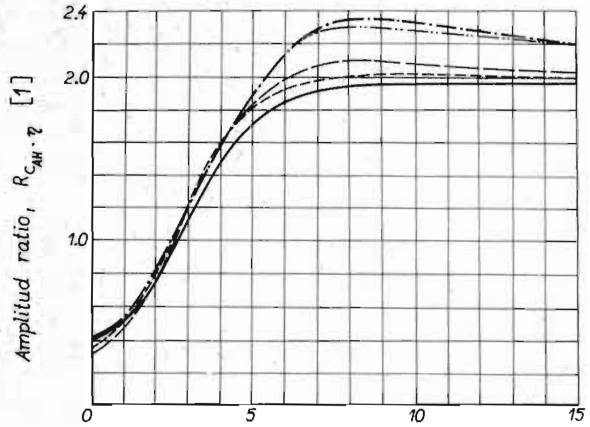


Fig. 14. Calculated frequency response curve $F_{y.\eta} = \frac{\Delta\omega_y}{\Delta\eta}(i\omega)$ of the hypothetical aeroplane.

In the amplitude characteristics in fig.13 and 14 the unsteady effect acts mainly by means of the transfer coefficient K_0 , as it is seen from fig. 16, where the effect of 1% error of coefficients K_0 , K_1 , K_{y0} , K_{y1} is shown. For $\omega = 4\ s^{-1}$ the error in K_0 does not show by any error in the amplitude characteristic, so that the curves of the example 1.0 intersect for this circular frequency the curves of the examples 2.1 and 2.2.



For marking see the table in the figure 13

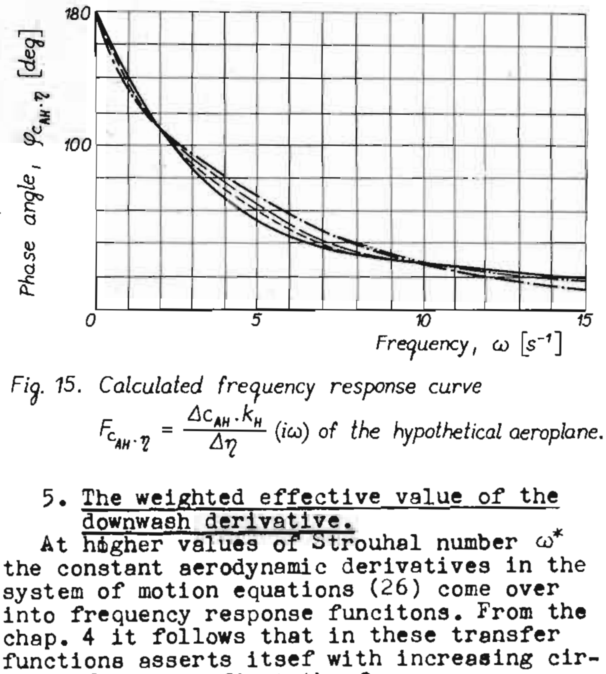


Fig. 15. Calculated frequency response curve $F_{C_{AH}\cdot\eta} = \frac{\Delta c_{AH}\cdot K_H}{\Delta\eta}(i\omega)$ of the hypothetical aeroplane.

5. The weighted effective value of the downwash derivative.

At higher values of Strouhal number ω^* the constant aerodynamic derivatives in the system of motion equations (26) come over into frequency response funcitons. From the chap. 4 it follows that in these transfer functions asserts itsef with increasing circular frequency first the frequency responseffunction of the down-wash angle. A harmonic input by elevator $C_\eta e^{i\omega t}$ causes a sinusoidal response in the angle of attack in place of the tailplane, $\Delta\alpha_H$, in the form:

$$C_{\alpha_H}\cdot e^{i\omega t} = F_{\alpha_H\cdot\alpha}(i\omega)\cdot F_{\alpha\cdot\eta}(i\omega)\cdot C_\eta\cdot e^{i\omega t} \qquad (34)$$

If the elevator motion is given in accordance with the Fourier development by a sum of harmonic motions with a spectrum $C_\eta(i\omega_j)$ then from the equality of the resulting spectrum

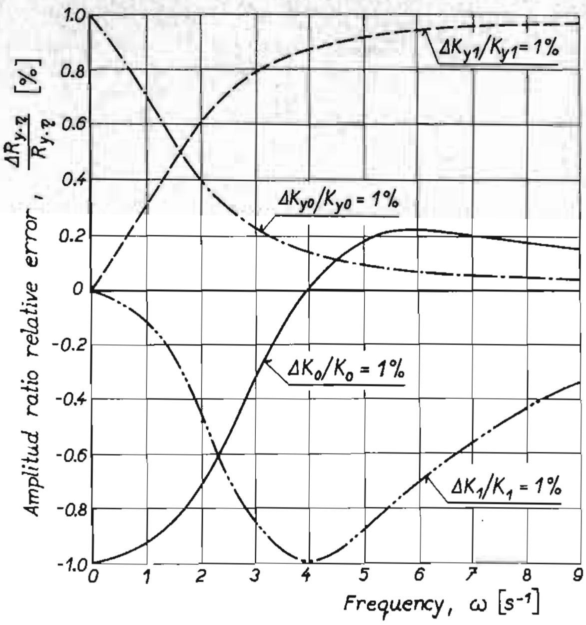


Fig. 16. Jnfluence of the 1% relative systematic error of transfer function coefficients, K_0, K_1, K_{y0}, K_{y1}, on the amplitude characteristics of the frequency response $F_{y\cdot\eta}$.

$C_{\alpha_H}(i\omega_j)$ both for the frequency response function $F_{\alpha_H\cdot\alpha}(i\omega_j)$ and for the weighted mean complex value $(K_{\alpha_H\cdot\alpha})_w$ there follows:

$$(K_{\alpha_H\cdot\alpha})_w = (U_{\alpha_H\cdot\alpha})_w + i\,(V_{\alpha_H\cdot\alpha})_w = \frac{\sum_{j=1}^{n} F_{\alpha_H\cdot\alpha}(i\omega_j)\cdot F_{\alpha\cdot\eta}(i\omega_j)\cdot C_\eta(i\omega_j)}{\sum_{j=1}^{n} F_{\alpha\cdot\eta}(i\omega_j)\cdot C_\eta(i\omega_j)} \qquad (35)$$

This expression represents a weighted mean value with the weights $F_{\alpha\cdot\eta}(i\omega)\cdot C_\eta(i\omega)$. This value makes possible to use the customary model (31) in which there is:

$$\frac{d\alpha_a}{d\alpha} \equiv \left(\frac{d\alpha_a}{d\alpha}\right)_w = 1 - (U_{\alpha_H\cdot\alpha})_w \qquad (36)$$

$$\frac{d\alpha_a}{d\alpha}\cdot\tau'_H = \left(\frac{d\alpha_a}{d\alpha}\cdot\tau_{Hx}\right)_w = \frac{(V_{\alpha_H\cdot\alpha})_w}{\alpha_w} = (V'_{\alpha_H\cdot\alpha})_w \qquad (37)$$

The weighted mean frequency ω_w can be substracted from the curve $V_{\alpha_H\cdot\alpha}(\omega)$ for the weighted mean value $(V_{\alpha_H\cdot\alpha})_w$. The expression (37) defines the unity length on the straight line (6) in fig. 9. If the derivatives of the type $c_{y\cdot\dot{x}^*}$ in eg. (26) are dimensionless then it is necessary to divide the value (37) by the aerodynamic time unity $\tau_A = l/V$. Further there are considered two teeoretical deterministic inputs:

a) a pulse input having the $C_\eta(i\omega_j) = 1$ spectru;

b) a step input having the $C_\eta(i\omega_j) = \frac{1}{i\omega_j} = -i\,\frac{1}{\omega_j}$ spectrum for $\omega > 0$.

From spahes of the spectra $C_\eta(i\omega)$ and $F_{\alpha\cdot\eta}(i\omega)$ it follows:

$$k_\alpha = \left(\frac{d\alpha_a}{d\alpha}\right)_w \Big/ \left(\frac{d\alpha_a}{d\alpha}\right)_{\omega=0} < 1 \qquad (38)$$

The value of this ratio, which was called the "dynamic down-wash efficiency", depends on the number of the circular frequency ω_i levels which were considered in the harmonic analysis of the input $\Delta\eta(t)$, and of course on the shape of the spectrum $C_\eta(i\omega)$.

The value of this ratio for a hypothetical aeroplane in the interval $\omega\langle 1,16\rangle$ sec^{-1} was for a pulse $\sim$ 0.60 and for a step $\sim$ 0.80 For the A 145 aeroplane by an analysis in accordance with (35) from fitted curves in fig. 8 in the interval $\omega\langle 1,10\rangle$ this value was for a pulse 0.70 to 0.80 and for a step 0.86 to 0.91. From measurements of the tailplane forces the value k_{α_a} for a pulse of a mean time basis 0.25 sec was 0.77 and by an analysis in accordance with (35) for the model (6) with τ_H 0.0 sec this value was 0.7 .

For the imaginary part in the interval $\omega\langle 1,10\rangle$ sec is valid $(V'_{\alpha_a\alpha})_w \doteq V'_{\alpha_H\alpha}(0)$.

The plots of frequency response curves calculated for weighted effective values of the down-wash derivative are in gig. 13 to 15 for a hypothetical aeroplane and in fig. 3 for the A 145 aeroplane. Similar discrepances, which are evident in these figures, were noticed at frequency characteristics measurements with various inputs in paper /20/, see e.q. fig. 17.

Analogically there was derived from power spectra an expression for the weighted mean value of down-wahh for flights in turbulent atmosphere. With respect to the shape of power spectrum of vertical gusts the value of dynamic efficienty for the A 145 aeroplane was higher than 0.90 and its effect on the frequency response function of the aeroplane, $F_{a_z.w_H}$, is relatively small, see fig. 6.

6. The conclusive discussions

From results of the flight measurements in chap. 2 it follows that the determination of the effective value of the down-wash derivative for the whole tailplane by a vane is unreliable. It is also difficult to find a representative position of the vane. This fact raises the significance of aerodynamic tailplane forces measurements which make possible to gain from measurements at steady flights very useful aerodynamic data, see appendix A.

An identification analysis of measurements at unsteady flights with deterministic inputs by elevator has shown substitutional model (7) of the frequency transfer function of the down-wash angle at the tailplane to be applicable. This model involves a transportation lag put in series with a frequency transfer function for a system of 2nd order. The gererally used expression $F_{\alpha_a.\alpha} = \frac{d\alpha_a}{d\alpha} + - i\omega\left(\frac{d\alpha_a}{d\alpha}\tau_H\right)$, which expresses roughly the transfer function with a transportation lag only, $\frac{d\alpha_a}{d\alpha}(\cos\tau_H\omega - i\sin\tau_H\omega)$, does not satisfy for identification purposes. The expression (7) for measurements by a vane had to be supplemented yet by the effect of tailplane circulation on the airflow in the vane place, see (8).

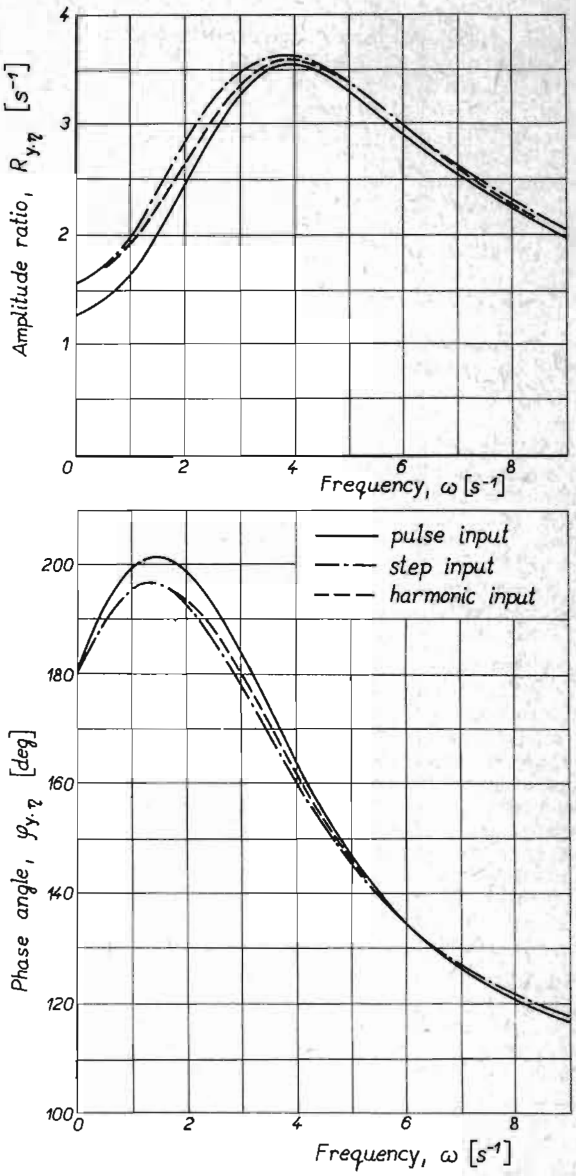


Fig.17. Fitted frequency response curve of the A 145 aeroplane relating the pitching velocity to elevator deflection (the FRCF method), see(20)

The measurements by a vane at unsteady flights make possible to get an information about the values of time constants and of transportation lag in the substitutional models (7) or (8) for the down-wash transfer function. Identification calculations of the frequency transfer functions of the effective down-wash from tailplane forces measurements according to (25) and of the mean value of the down-wash derivative according to (13) require high accuracy and precision of the frequency response functions of the short period motion of the aeroplane.

The causes of discrepances of the measured shape of the frequency transfer function $F_{\alpha_a.\alpha}$ with respect to the expected shape according to the approximative relation (4) were not yet explained. There offers hypothesis that under the consideration of a satisfactory rigid aeroplane the wake vortices in form of vortex quantities will flow from the wing or elementary wake vortices in a certain distance behind the wing will coil (Žádník, ARTI).

The calculations in chap. 4 have shown that at low flight velocities raises the importance of Strouhal number . By the hypothetical aeroplane there appeared the effect of nonsteady aerodynamics just by an unsteady down-wash in the interval of circular frequencies 0 to 7 sec^{-1}, where the generally used expression (5) does not satisfy. First at higher circular frequencies $\omega > 7\ sec^{-1}$ ($\omega^* > 0.19$) there starts to reveal gradually an effect of nonsteady aerodynamics of the wing and of the tailplane. The critical value of Strouhal number for this interval is determined just by requirements on the accuracy and precision of results.

From the identification analysis there has followed also that the effective value of the down-wash derivative, $\left(\frac{d\alpha_a}{d\alpha}\right)_{ef}$, from measurements at unsteady flights is significantly lower, 0.234, than that from measurements at steady flights, 0.304. The analysis in chap. 5 has explained this difference in that way that the mean value depends for a given aeroplane on the input parameter spectrum. When establishing a "weighted" effective value, which can be calculated according to eq. (35), it is possible to use in the interval of lower circular frequencies the simple expression (5) to estimate aerodynamic data. The difference between the value $d\alpha_a/d\alpha$ from steady flights and the wheighted mean value is mawimum for a pulse input by elevator, approx. 30%. Estimates of the tailplane lift using data for steady flight give therefore more favourable stresses on the tailplane than in fact.

Analogous conclusions as at deterministic input by elevator are valid also for flight in turbulent atmosphere. The power spectrum of vertical gusts, S_{ww}, according to Drydens model decreases for the A 145 aeroplane relatively little the dynamic down-wash efficiency i.e., to 0.88.

In conclusion it is necessary to emphasize that the aim of the paper was to point out problems at unsteady flights of a small aeroplane when flying at lower aerspeeds and to show on an example possibilities of their solution.

Acknowlegement

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Appendix A

Expressions employed for calculation of aerodynamic and other characteristics from derivatives $\frac{d\eta}{dc_A}$, $\frac{dc_A}{d\alpha}$, $\frac{dc_{AH}\cdot k_H}{dc_A}$ measured at steady flight. ($\tilde{x}_S = 0.247$)

Term	Expression	Value
$V_T k_H a_2$	$\Delta\tilde{x}_S/\Delta \frac{d\eta}{dc_A}$	1.088
X_{NS}	$-(V_T k_H a_2)\cdot\frac{d\eta}{dc_A}$	0.152
$\tilde{x}_N$	$\tilde{x}_S + X_{NS}$	0.397
$\bar{V}=\tilde{F}_H\tilde{r}$	$\Delta\tilde{x}_S/\Delta \frac{dc_{AH}\cdot k_H}{dc_A}$	0.675*
$\tilde{r}$	$\bar{V}/\tilde{F}_H$	3.483*
a	$\frac{dc_A}{d\alpha}\cdot\left[1-\frac{dc_{AH}\cdot k_H}{dc_A}\cdot\tilde{F}_H\right]$	4.735
$a_2 k_H$	$(V_T k_H a_2)\cdot\frac{dc_A}{d\alpha}/\left(\bar{V}.a + X_{NS}\frac{dc_A}{d\alpha}\cdot\tilde{F}_H\right)$	1.760
a_2/a_1	$-\frac{\Delta\delta}{d\eta}$	0.587*
$a_1 k_H$	$a_2 k_H/\frac{a_2}{a_1}$	3.00
V_T	$(V_T k_H a_2)/k_H a_2$	0.618*
$\tilde{x}_{N_F}$	$\dfrac{\left(\frac{dc_{AH}\cdot k_H}{dc_A}\right)_2\cdot\tilde{x}_{S1}-\left(\frac{dc_{AH}\cdot k_H}{dc_A}\right)_1\cdot\tilde{x}_{S2}}{\left(\frac{dc_{AH}\cdot k_H}{dc_A}\right)_2-\left(\frac{dc_{AH}\cdot k_H}{dc_A}\right)_1}$	0.106†
$\tilde{x}_{N_F S}$	$\tilde{x}_S - \tilde{x}_{N_F}$	0.141†
$\tilde{r}_H$	$\tilde{r} - \tilde{x}_{N_F S}$	3.342*
$\frac{d\alpha_a}{d\alpha}$	$a\left(\frac{a_2}{a_1}\right)\left[\left(\frac{d\eta}{dc_A}\right)-\bar{V}\frac{dc_{AH}\cdot k_H}{dc_A}/(V_T k_H a_2)\right]+1$	0.304
$c_{A.\alpha}$	$\frac{dc_A}{d\alpha}\left(1-c_{A.\eta}\frac{d\eta}{dc_A}\right)$	5.140
$c_{M.\alpha}$	$-X_{NS}\cdot c_{A.\alpha}$	0.781*
$c_{A.\eta}$	$(a_2 k_H)\cdot\tilde{F}_H$	0.341
$c_{M.\eta}$	$-(V_T k_H a_2)\left[1+\frac{c_{A.\alpha}}{a}X_{NS}/\tilde{r}\right]+k_H V_T\frac{c_{A.\alpha}}{a}\frac{\partial c_{MOH}}{\partial\eta}\cdot\tilde{l}_H/\tilde{r}$	-1.201
$c_{M.\dot{\alpha}}*$	$-\tilde{F}_H\cdot\tilde{r}_H(a_1 k_H)\cdot\frac{\tilde{r}_H}{\sqrt{k_H}}\cdot\frac{d\alpha_a}{d\alpha}$	-2.041**
$c_{M.y}*$	$-\tilde{F}_H\cdot\tilde{r}_H(a_1 k_H)\cdot\frac{\tilde{r}_H}{\sqrt{k_H}}\cdot k_{\omega y}$	-7.388

* Valuable for hypothetical aeroplane

** For weighted effective value, $\frac{d\alpha_a}{d\alpha} = 0.234$

† With respect to $c_{M.\alpha}$: $\tilde{x}_{N_F} = 0.126$, $\tilde{x}_{N_F S} = 0.121$.

Appendix B

Frequency response curve $F_{y.x}(i\omega)$ is given with amplitude ratio $R_{y.x}(i\omega)$ and phase angle $\varphi_{y.x}(i\omega)$, which are measured at discrete frequencies ω_j for $j = 1,2\ldots,$. The independent measured data $F_{E\nu}(i\omega_j)$ for $\nu = 1,2,\ldots, n_j$ have a normal distribution at every value of ω_j.

The sum of squares of residuals can be expressed as

$$\sum_j\sum_\nu\left|F(i\omega_j)-F_{E\nu}(i\omega_j)\right|^2 = \sum_j\sum_\nu\left|F_{EM}(i\omega_j)-F_{E\nu}(i\omega_j)\right|^2 + n_j\sum_j\left|F(i\omega_j)-F_{EM}(i\omega_j)\right|^2 = (E) + n_j(S)$$

The first sum from the right-hand side, (E), represents the variation of the measured points about the estimated mean value. The second sum (S) represents the variation of estimated mean values about the theoretical values. From the sum (S) after its linearization the normal equation was derived.

The estimate of the variances were calculated as follows:

(a) $s_E^2 = \frac{(E)}{k_E}$, $s_S^2 = \frac{(S)}{k_S}$,

where numbers of degrees of statistical freedom are $k_E = 2k(n_j - 1)$, $k_S = 2k - n_{K_r}$

(b) ${}^R S(\hat{K}) = s_R \sqrt{\frac{2}{n_j} \tilde{Q}_{K_r}}$,

where $s_R^2 = \frac{(E)+(S)}{k_E + k_S}$ and

$\tilde{Q}_{K_r}$ are the elements of the inverse matrix of normal equations.

The estimate s_E^2 represents the precision of measured data and the estimate s_S^2 represents the variation of estimated mean values of the measurements about the fitted curve. The weights $W_j = s_{Ej}^2 / s_E^2$ were not used because the assumption of absence of systematical errors defining the accuracy of measurements has not been fulfilled.

Symbols.

$a = \frac{\partial c_{AF}}{\partial \alpha_F}$ Wing lift curve slope

$a_1 = \frac{\partial c_{AH}}{\partial \alpha_H}$ Tailplane lift curve slope

$a_2 = \frac{\partial c_{AH}}{\partial \eta}$

A, A_F, A_H Lift or aeroplane, wing of tailplae respectively

$A_{y.x}(t)$ Unit step admittance (response $y(t)$ on the unit step $\Delta x = 1$)

$c_A = \frac{A}{qF}$ Lift coefficient of aeroplane

$c_M = \frac{M}{qFl}$ Moment coefficient of aeroplane

$c_{y.x} = \frac{\partial c_y}{\partial x}$ Aerodynamic derivative (y = A, M or A_F, A_H; $x = \alpha, \eta, \dot{\alpha}^*, \dot{\eta}^*, \omega_y^* \equiv y^*$, or α_F, α_H)

c_W Drag coefficient

F Wing area, m^2

F_H Horizontal tailplane area, m^2

$F_{SE} = \frac{s_S^2}{s_E^2}$ Random variable with the Snedecor s distribution (F-distribution)

$F_{y.x}(i\omega)$ Frequency response function (output y, input x)

$F_{c_{y.x}}(i\omega)$ Aerodynamic frequency response function ($y = A, M$, $x = \alpha, \eta, \dot{\alpha}^*, \dot{\eta}^*, \omega_y^* \equiv y^*$)

g Acceleration due to gravity, ms^{-2}

$i = \sqrt{-1}$

$\tilde{i}_y = \sqrt{J_y/ml^2}$ Nondimensional moment of inertia about the y-axis

k Number of measured values for different values of ω (number of ω_j levels)

k_R, k_S, k_E Number of statistical freedom degrees

$k_H = q_H/q$

k_{ω_y} Ratio of wing plus fuselage damping to horizontal tail damping

K_r, K_{qr} Transfer function coefficients ($r = 0,1,2$; $q = \alpha$, $\omega_y \equiv y$, $n_F \equiv n$)

l Lenght of aerodynamic mean chord, m

L Scale of turbulence, m

m Aeroplane mass, kg

M Pitching moment about the c.g. of aeroplane

$n_F = -\frac{a_{z_a}}{g}$ Normal acceleration factor

n_j Number of repeated measurements for a given ω_j

n_{K_r} Number of unknown frequency response coefficients

$q = \frac{\rho V^2}{2}$ Kinetic pressure

$q_H = \frac{\rho V_H^2}{2}$

r Distance between aerodynamic centres of wing plus fuselage and of tailplane, m

r_H Distance between c.g. and tail aerodynamic centre, m

$R(\hat{K}) = 0.6745\, S(\hat{K})$

s^2, s_S^2, s_E^2 Unbiased estimates of the variances

$S(\hat{K})$ Root-mean-square error estimate of transfer function coefficient

t a) Time, secs
b) Random variable with Student's distribution

$U_{y.x}(\omega)$ Real part of frequency response

V True velocity of an aeroplane, ms^{-1}

$\tilde{V} = F_H r / Fl$ Tail volume

$\tilde{V}_H = F_H r_H / Fl$

$V_T = \tilde{V}/1+k$ Reduced tail volume, where $1 + k = c_{A.\alpha}/a$

$V_{y.x}(\omega) = \omega V'_{y.x}(\omega) = \omega l/V\, V_{y.\dot{x}^*}$ Imaginary part of frequency response

x_S C.g. position on aerodynamic chord, m

x_{N_F} Position of aerodynamic cettre of wing plus fuselage on aerodynamic chord, m

X_{NS} Static margin

α a) Angle of attack of an aeroplane
b) Level of significance

α_a Downwash angle (positiv in oposite sign of α)

α_F Angle of attack of wing

α_H, α_L Angle of attack at tail or at L point respectively

$\alpha_{LA} = \alpha_L - \frac{\omega_y r_H}{V_H}$

η	Elevator angle
ϑ	Pitch angle of an aeroplane
λ	Aspect ratio
$\mu = \frac{2m}{\rho F l}$	Relative mass parameter
ρ	Air density, kg/m^{-3}
$\sigma^2(x)$	Variance for measurement of x
$\tau = \frac{2m}{\rho F V} = \mu \frac{l}{V}$	Dynamic time unit, secs
$\tau_A = l/V$	Aerodynamic time unit, secs
$\tau_H = \frac{r_H}{V_H}$, $\tau_L = \frac{r_L}{V_L}$	Time lag, secs
$\tau'_H = \frac{\xi_H}{V_H}$, $\tau'_L = \frac{\xi_L}{V_L}$	Time lag, secs
τ_{Lx}	Estimate of time lag τ_L, secs
$\varphi_{y.x}(\omega)$	Phase angle
ω	Cirlular frequency, $/s^{-1}/$
$\omega^* = \frac{\omega l}{V}$	Reduced circular frequency = = Strouhal number for the harmonic motion
ω_y	Pitching velocity, $rad.s^{-1}$
ξ_H, ξ_L	Distance of leading edge of tail or of the L point after wing trailing edge, m

Denominations.

$\bar{x}$	Fourier transform of x variable
$\tilde{x}$	Nondimensional quantity of type $\tilde{x} = \tau \dot{x} = \mu \dot{x}^*$
$\dot{x}^* = \frac{l}{V}\dot{x} = \frac{\tau}{\mu}\dot{x}$	
$\hat{x}$	Estimate of x by least square method
${}^0x = x/x(0)$	Normalized quantity
FRCF	Frequency response curve fitting method
TEM	Transformed-equation-of-motion method

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