THEORETICAL DETERMINATION OF
THE AERODYNAMIC FORCES ON SATELLITES

by
Silvio Nocilla, Professore ordinario
Istituto di Meccanica Razionale
del Politecnico di Torino, Italy

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Silvio Nocilla
Politecnico di Torino - Italia

Abstract

Drag and lift on artificial satellites chiefly depend, as it is well known, on the gas temperature, on the velocity, tempera= ture, chemical and physical conditions of satellite surface and on the interaction process between this surface and molecules impinging on and subsequently scattered from the surface. In the present paper it is made, first of all, a brief critical survey of the theoretical methods used to study those interaction phenomena. Succes= sively, results are reported about lift and drag on cones, cylinders and spheres, ob= tained by the Author and cowerhers accord= ding to a recent spatial impulsive intera= ction model by them proposed and studied. Comparisons with other theoretical results are also reported.

1. Introduction

The drag and lift calculation for satel= lites flying at the altitudes between 150 and 400 Km about requires, as it is well known, a good knowledge, of the surface interaction phenomena. When this knowledge is not acquired, as it happens today, and the exact laws of the gas surface interac= tion process are unknown, we cannot care= fully compute the forces exerted on the sa= tellites by the surrounding highly rarefied atmosphere, or the energy exchanges between stream and body. In this connection I must remind that though the drag forces acting on a missile at the above mentioned alti= tudes are very weak, yet these forces are very important for the determination of the artificial satellites life-time, of the at= mospheric density etc. On the ground of these considerations, in the present paper concerning aerospace applications, I will, first of all, dedicate a special emphasis on the basic studies about the surface in= teraction. Today the kinetic theory of gases is not able to give us desired laws in this field; on the other hand careful studies are carried on for several decennia about some scientific subjects as the ad= sorption process and molecular beams which are indeed so strictly related to the sur= face interaction phenomena, to be an essen= tial basis for these ones. Then this sub=

ject must be regarded as the starting point for the following considerations.

As regard the adsorption, I want first of all remind that the outside surfaces of missiles have a chemical and physical structure which is very complicated and perhaps not well known at the molecular range with chemically and physically ad= sorbed substances. Moreover, this chemical and physical structure may change when the missile passes from the altitude zero, at the ordinary pressure, to those ones in which the atmosphere is highly rarefied. Now, it is well known that the presence of mono-or plurimolecular layers chemically or physically adsorbed greatly affects the surface-force fields. In addition to this first aspect of the adsorption, and chief= ly the chemical one, as a suitable element characterizing the chemical and physical state of the surface, there is a second and equally important one, which is more specifically related to the physical adsor= ption. It consists in an alternative oc= currence of collisions between gas molecu= les and surface atoms, of permanence of these molecules over the surface for a time called "sitting time", and at last of reemission of the same molecules from the surface. From a statistical point of view, the phenomenon is stationary, and in fact we consider the number of adsorbed molecu= les for unit surface, even if a continuous alternation of these molecules actually occurs. The ammount of adsorbed molecules is depending, as it is well known, on the surface temperature T_w and on the surroun= ding rarefied gas pressure p. As an examp= le , we report from De Boer [4] one of the Langmuir adsorption isotherms,(fig. 1) a* is the ratio between the number of the ac= tually adsorbed molecules per cm^2 surface, and the number of molecules per cm^2 which
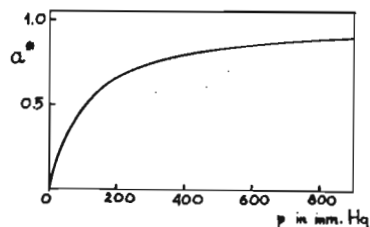

Fig. 1 - Adsorption isotherm

would form a completely filled unimolecu=
lar layer. It is known too that the adsor=
ption heat Q^* is defined as the amount of
heat which whether is lost by the gas when
its molecules are adsorbed or "condensed"
over the surface, or has to be provided to
the adsorbed molecules for desorbing or
"re-evaporating" them.

The same process of incidence and sit=
ting of the molecules on the surface, to=
gether with their subsequent reemission,
occurs when a molecular beam impinges on a
rigid surface and is subsequently reemit=
ted.

The same process occurs also when a mis=
sile moves within the high atmosphere at
the above mentioned altitudes, if we con=
sider the motion of the gas molecules re=
lative to the missile. We have so pointed
out a standard basic phenomenon which ori=
ginates from the behaviour of the gas mo=
lecules interacting with the atoms of a ri=
gid surface and which, on account of this,
must be studied in the light of this inte=
raction process. Evidently, the same ba=
sic phenomenon takes very different aspects
specially in the experimental field,whether
adsorption,or molecular beams, or aerospa=
ce applications are concerned. However, I
deem it is equally evident that the unita=
ry ground of these three and apparently so
different fields, on the one hand is an
effective instrument for studying them, on
the other hand it requires that the know=
ledges acquired about each one must be ta=
ken into account in studying the other ones.
Moreover, this consideration explains the
fact that for several years theoretical
and experimental studies about molecular
beams have been considerably developed even
by aerodynamic and aerospace scientists.

As regards the problem of the gas-surfa=
ce interaction we recommend to the reader
the recent surveys of Hurlbut [4], Trilling
[2] and Goodman [3].

As regards the fundaments for molecular
beams we recommend to the reader the books
[5] and [6].

2. Accomodation coefficients and sitting time of the molecules on the surface

From a long time it is customary to con=
sider as basic concepts in the gas-surface
interaction the classical "total"-accomo=
dation coefficients, respectively for the
energy, tangential and normal momentum:

$$\alpha = \frac{E_i - E_r}{E_i - E_w} \; ; \; \sigma = \frac{\tau_i - \tau_r}{\tau_i} \; ; \; \sigma' = \frac{p_i - p_r}{p_i - p_w} \qquad (2.1)$$

(the meaning of the symbols is well known:
see for inst. [7]), or the most recently

introduced "partial"-accomodation coeffi=
cients for the energy (see for inst. Hur=
lbut-Sherman [8] where the symbols are
defined in detail):

$$q\left(\theta_i, \theta_r\right) = \frac{\varepsilon_i(\theta_i) - \varepsilon_r(\theta_r)}{\varepsilon_i(\theta_i) - \varepsilon_w}$$

$$q_\ell(\theta_i) = \frac{\varepsilon_i(\theta_i) - \varepsilon_r}{\varepsilon_i(\theta_i) - \varepsilon_w} \qquad \qquad (2.2)$$

The first group (2.1) can be related both
to adsorption phenomena, molecular beams
and aerospace applications.
The second group (2.2) can be applied only
to molecular beams, or aerospace applica=
tions in hyperthermal conditions, that is
when the speed of the missile $\vec{U}$ is very
greater than the thermal velocity c of the
gas, that is

$$s \equiv u/c \gg 1 \qquad \text{(hyperthermal} \qquad (2.3)$$
$$\text{conditions)}$$

In these cases all the molecules impinging
on an elementary surface element dA posses
in the motion relative to dA the same di=
rection. In the aerospace case, as well
as in the case of a "monoenergetic" beam,
such molecules posses also a common velo=
city $\vec{U}_i = -\vec{U}$ (see fig. 2). In the case
of a non-monoenergetic beam, as for instan=
ce in a "maxwellian" beam (see for inst.
Esterman [5] or Ramsey [6]) $\vec{U}_i$ represents
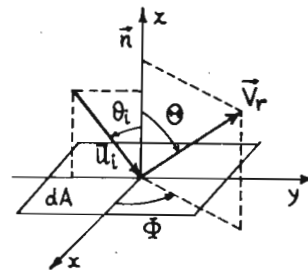a suitable "mean" velocity of the beam.

Fig. 2 - $\vec{V}_r$ is the velocity of a reemitted
molecules.

Much of the researches on the gas-surfa=
ce interaction consists in the theoretical
calculation of the above accomodation coef=
ficients. In this connection however I
point out that, in my opinion, both the
classical accomodation coefficients (2.1),
and the new ones (2.2)do not present to=
day much interest and importance, because:
i) the conception of a partly specular and
   partly diffused reemission, that is the
   basis of the accomodation coefficient
   conception, isn't valid. In fact, ac=
   cording to this hypothesis, the lobes

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related to the reemitted molecules should have the shape showed in fig. 3b), where the circular part should be corresponding to the diffuse reemission, and the remaining part to the specular one. On the contrary, experimental lobes have the shape reported in fig. 3a).
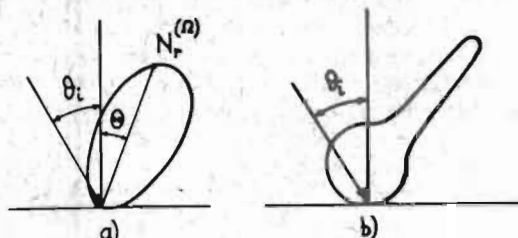

Fig. 3 - a) Actual phenomenon
b) Unreal phenomenon

ii) both on the ground of experimental measurements and of theoretical calculations, these coefficients can assume values out of the range $0 \mapsto 1$ in which its would be included, as pointed out for instance by Goodman [9] and Nocilla-Chiadò [10].

iii) each coefficient cannot be considered independent from the other ones, but they must be used in unitary way because they are correlated through the surface interaction mechanism.

A situation somehow opposite to the one concerning the accomodation coefficient arises, in my opinion, with regard to the sitting time of the molecules on the surface. Indeed, I think this quantity has been rather neglected in the researches on surface interaction, despite the importance I deem it holds. In fact, this quantity is typical of the phase in which molecules sit on the surface, and which is so important for the following reemission phase. It is certain that sitting time is strictly related to the whole interaction process, whose it may be so a very significant index, although that relationship is today unknown. A foundamental study on sitting time was carried out in 1924 by Frenkel [11] who, in studying adsorption by applying the methods of classic statistic mechanics, obtained the following expression of the sitting time τ^*:

$$\tau^* = \tau^{**} \exp \, (Q^*/RT_w) \qquad (2.4)$$

where Q^* is the adsorption heat, R is the adsorbed gas constant, T_w is the solid temperature, and τ^{**} is a time-dimensioned parameter, whose value is $10^{-12} \div 10^{-14}$ sec about, directly related to the vibration period of the adsorbing surface atoms. The magnitude of sitting time τ^* is greatly

variable, at the same temperature, for various adsorbed gases. For instance(see De Boer [4] page 25) at the ambient temperature it is 10^{-12} sec. about for hydrogen H_2 on several surfaces; and 10^{-10} sec. about for argon Ar, oxigen O_2, nitrogen N_2, carbon monoxide CO on several surfaces (and therefore about a thousand times higher than the oscillation period τ^{**}). For heavier gas molecules, the magnitude of sitting time may be much higher, till to 10^{-2} sec. Formula (2.4) is valid when the gas molecules are only in thermic motion; but I extended it ([12] part III and [13]) to the case of both thermic and macroscopic motions. Obtained results are written in the form:

$$\tau^* = \tau^{**} \exp(Q^*/RT_w) \sqrt{\frac{T^{**}}{T_w}} \; 1/\chi(s) \qquad (2.5)$$

where T^{**} is suitable temperature which must be related to the adsorbed molecules (calculations showed that practically it may be assumed either $T^{**} = T$ or $T^{**} = T_w$) and $\chi(s)$ is a well-known function of the ratio s between the drift velocity U of gas and the most probable velocity c in its thermic motion. Its value is:

$$\chi(s) = \exp(-s^2) + \sqrt{\pi} \; s(1 + \mathrm{erf} \; s) \qquad (2.6)$$

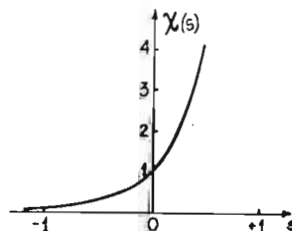with $s = U/c$

and its behaviour is indicated in fig. 4.

Fig.4 - Function $\chi(s)$ [see formula (2.6)]

I don't talk, in the present paper, over the assumptions made for deducing expression (2.5): I want merely to point out that, as far as it is within my knowledge, the before mentioned research is the only one examining the dependence of the sitting time τ^* on the drift velocity of molecules which is essential both for studying molecular beams and for aerospace applications. Since, as fig. 4 shows, function $\chi(s)$ highly increases with s, sitting time τ^* supplied by (2.5) is highly decreasing as s increases. I think it might be very helpful to study more accurately the dependence of τ^* on the parameter s, and therefore on the energy E_1 of

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incident molecules, as well as on the so=
lid temperature T_w. In one of his papers
[14] Pyarnpuu explicitly remarks that at
velocities of 8.000m/sec adsorption phe=
nomena vanish; this fact is in agreement
with the value near zero of sitting time,
deducible from (2.5) under the same con=
ditions. However, several experimental
results about molecular beams, as for in=
stance those ones deduced by Datz et alii
[19] , clearly show that as the solid tem=
perature T_w is about 0°C, even if the e=
nergy of incident molecules is high, yet
we obtain, for the number of reemitted mo=
lecules, the nearly circular lobes which
t'll today had been considered as indica=
tive of a substantially diffuse reemission
that is with a great surface interaction
and (physical) adsorption of the incident
molecules and therefore with a considera=
ble sitting time. These considerations
are sufficient to show that our opinions
about interaction phenomena are to be re-
examined in a critical way; to my mind

in this re-examination the sitting time
should take a very prominent part.

3. Flux statistic as basis of the studies on gas surface interaction

For a more advanced study of the gas-
surface interaction I think useful, first
of all, to distinguish between "re-emis=
sion" and "interaction" models. I call
"re-emission model" a suitable, unitary
description of the physical properties of
re-emitted molecules. With the expression
"interaction model" I call a much more so=
physticated and complete thing, that is a
description of the gas-surface interaction
mechanism which allows to determine the
physical properties of reemitted molecules
as depending on the physical properties of
the impinging molecules and on the geome=
trical (lattice constants etc.), physical
(surface force felds etc.), and chemical
(presence of adsorbed substances etc.)
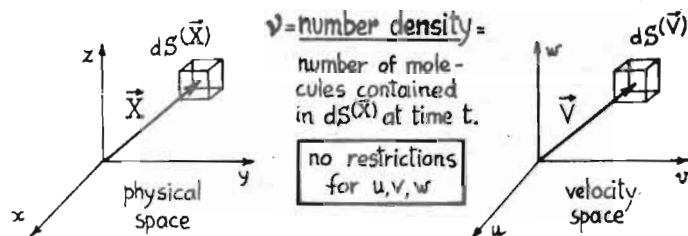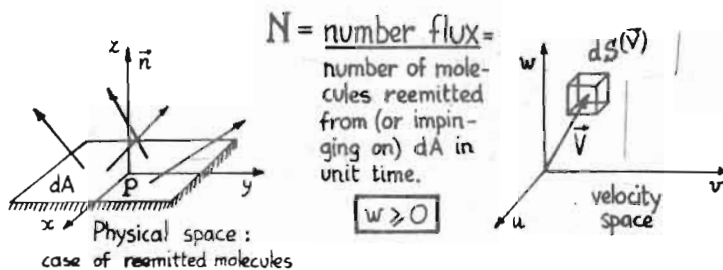properties of the solid.

Fig. 5 - Statistics on density

Fig. 6 - Statistics on flux

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Because of the very great number of the gas molecules occurring in the surface interaction, statistical methods are requested. Yet this statistic must be made upon the gas molecules flux, not upon the density. This very simple statement don't appear clear in the literature, so that some confusion or mistakes often occur. For the sake of clearness we remember that the classical velocity distribution function $f(\vec{X},\vec{V},t)$, ($\vec{X}$ = vector in the physical plane $\vec{V}$ = velocity; t = time, see fig. 5) arising as unknown quantity in the Boltzman equation in the phase-space implies the consideration of the molecules contained at time t in an elementary volume $dS^{(X)}$, and is defined in the whole velocity space. Calling ν the number density in ($\vec{X}$,t) and $d\nu^{(V)}/\nu$ the fraction of these $\nu = \nu(\vec{X},t)$ molecules contained in the elementary volume $dS^{(V)}$ of the velocity space, we have:

$$\frac{d\nu^{(\vec{V})}}{\nu} = f(\vec{X},\vec{V},t)\frac{dS^{(\vec{V})}}{c^3} \qquad (3.1)$$

where c is a suitable normalization velocity. In a quite different way must be performed the statistic in surface interaction phenomena: we must consider the molecules leaving (or impinging) in a time range dt the elementary surface dA. Calling N the number flux through dA in unit time, and $dN^{(V)}/N$ the fraction of these N = N(P,t) molecules contained in the elementary volume $dS^{(V)}$ of the velocity space, we introduce the new velocity distribution function $g(P,\vec{V},t)$ as follows:

$$\frac{dN^{(\vec{V})}}{N} = g(P,\vec{V},t)\frac{dS^{(\vec{V})}}{b^3} \qquad (3.2)$$

(P = point on the surface)

where b is a suitable normalization velocity. The function $g(P,\vec{V},t)$, differently from $f(\vec{X},\vec{V},t)$, is defined (see fig. 6) for every point P on tne surface and every t, in the half-space w≥0 of the velocity space, corresponding to the external normal to the surface element dA relative to point P, when we consider the molecules leaving the surface; in the half-space w ≤ 0, corresponding to the internal normal when we consider the impinging molecules.

These concepts was by me introduced in [15] with the denomination of "free molecule surface sources and sinks". By means of the function $g(P,\vec{V},t)$ the distribution functions of the number, momentum and energy flux in the unit solid angle may be calculated in unitary way as follows:

$$N_t^{(\Omega)} = N_r \int_0^\infty g_r(Q)\frac{v^2 dv}{b^3}$$

$$\vec{Q}_r^{(\Omega)} = mN_r \int_0^\infty g_r(Q)\frac{v^3 dv}{b^3}\cdot\vec{\omega} \qquad \left.\right\} \quad (3.3)$$

$$E_r^{(\Omega)} = mN_r \int_0^\infty \left(\frac{v^2}{2}+e^*\right)g_r(Q)\frac{v^2 dv}{b^3}$$

(e^*=internal molecular energy)

By means of integrations over the whole half-solid angle, the tangential τ_r and normal p_r pressures, as well as the total energy flux E_r through dA may be calculated.

As an example of the above concepts I recall my reemission model [16] according to which the reemitted molecules behave as that part of a gas in Maxwellian equilibrium, having most probable velocity c_r and drift velocity $\vec{U}_r$, whose velocity is directed towards the same side of the considered surface elemnt dA. The velocity distribution function $f_r(\vec{X},\vec{V},t)$ of this quite hypotetical gas is therefore:

$$f_r(\vec{X},\vec{V},t) = \exp\left(-\left|\vec{V}-\vec{U}_r\right|^2/c_r^2\right) \qquad (3.4)$$

($\vec{V}$ whatever)

and is defined on the whole space velocity (u,v,w). The velocity distribution function of molecules reemitted from dA according to this model is:

$$g_r(P,\vec{V},t) = \frac{2}{\pi}\frac{1}{\chi(\sigma_r)}\frac{V\cos\Theta}{c_r}\exp\left(-\left|\vec{V}-\vec{U}_r\right|^2/c_r^2\right) \quad (3.5)$$

($\vec{V}\times\vec{n} \geq 0$)

an is defined only in the half-space w≥0.

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The symbols have the following meaning (see fig. 7):

$$\left.\begin{array}{l}
\Theta = \text{angle between } \vec{V} \text{ and the normal} \\
\quad \vec{n} \text{ to the considered surface elem=} \\
\quad \text{ent } dA = \text{angle between } \vec{V} \text{ and w} \\
\quad \text{axis in velocity space} \\
\theta_r = \text{angle between velocity } \vec{U}_r \text{ and the} \\
\quad \text{same normal } \vec{n} \\
s_r = U_r/c_r \\
\sigma_r' = s_r \cos \theta_r'
\end{array}\right\} (3.6)$$

The function $\chi(\sigma)$ is defined by eq.(2.6) and plotted in fig. 4. Velocities U_r and c_r and the angle θ_r become arbitrary
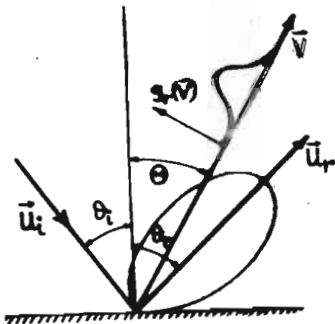

Fig. 7 - Qualitative picture of the Aut=
hor's re-emission model [16]

parameters, which in the same paper [16] and in the following [17] and [15] part II, I de= termined by means of an attempt numerical procedure, in order that the reemission model were in agreement with the various experimental results obtained for $N^{(\Omega)}$ by Hurlbut [18], Datz et alii [19], Jawtusch [20], Smith and Fite [21], by means of expreriments carried out with molecular beams. The agreement with those experi= mental values was very good. It resulted that the values of parameters s_r and θ_r' are depending on the angle of incidence, on the incident gas temperature and on the target temperature. However, in the abo= ve mentioned works I did not investigate this dependence. This was accomplished later by Hurlbut and Sherman [8].

4. Recent studies on the gas surface interaction

Let us now outline the theoretical and experimental methods used for studying the surface interaction, chiefly by taking into account the most recent ones. As regards the theoretical ones, I think that, certainly without any pretence to com= pleteness, the very numerous and interes= ting works to-day existing about this subject may be roughly classified in the following way:

- researches where the interaction between a single gas molecule and a single atom of the lattice which constitutes the solid is considered, and studied with the methods of quantum mechanics. We quote, for instance, one of the numerous papers by Lennard-Jones and others [22] and a recent paper by Nikuradse [23]

- researches where the same kind of the above mentioned interaction between atoms and molecules is studied with the methods of classical mechanics. Parti= cularly topical are, in this connection, the researches where the interaction is considered as an impulsive phenomenon. That is each incident molecule is sup= posed to be subjected to a collision with one atom of the lattice, which is considered as a sphere [24], or a rigid cube [9] and [25], or a prism having a tra= peziodal cross-section [27], [28] and [10]. There are also many variants in these researches, according to whether the so= lid atom is considered as fixed or in thermic motion and according also to the law assumed for describing the collision. Results obtained with the aim of this interaction models are very interesting, because are very apt to be directly com= pared with the experiments. However, we must not forget that all these models involve the hypothesis that the value of sitting time is zero, whereas this as= sumption is generally questionable. Moreover, as regards the use of the classical instead of quantum mechanics, I think that today it is not certain what are the limits of validity of the former for studying these surface interaction phenomena, and whether the latter is apt to a real practical application.

- researches taking into account the action of not merely one atom of the solid sur= face, but also of a certain number of other lattice atoms surrounding it. Between these, for instance, the resear= ches of Oman and Goodman, quoted in the above mentioned papers by Hurlbut [1] and

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Trilling [2] and the paper by Erofeev[19] which takes into account also the pene= tration of the gas molecule into the lattice with consequent multiple colli= -sions.

A new and very promising idea in this feld seems to us the one developed by Goodman in a recent paper[30] which, with a rich bibliography on the gas surface interaction, contains also the indica= tion of previous papers of the same Author. The idea consists in the assum= ption that the three-dimensional lattice of the solid surface is separated from the surrounding gas by an onedimensional potential well in the direction of the outward normal, into which the impinging molecules are trapped if their energy doesn't exceeds a suitable value. In this case the trapped molecules undergo very many collisions with the lattice atoms and, after a relatively long : . . stay on the surface, are reemitted almost randomly. If, on the contrary, the same molecules posses a greater e= nergy, the trapping phenomenon cannot occur, and they are reemitted almost as predicted by the impulsive models. The above outlined picture of the phe= nomenon seems to us to give a good and unified explanation of the actual be= haviour of the scattered molecules, as shown by the experimental results we will briefly summarize later.
- researches calculating the path of each incident molecule, under the action of differently sketched surfa= ce force fields, Rogers[34] and Hurlbut [32] . I think these researches have the merit, at least from the conceptual po= int of view, of taking into account the fact that the interaction lasts some time; and this concept is very similar to the one of sitting time. However, they present the disadvantage of requi= ring very arduous calculations, which are usually performed with the aim of computers whose results, besides, can= notbe easily compared directly with the experiments.
- researches studying the effects produced on the surface interaction by the pres= ence of chemically or physically adsor= bed substances, Stickney [34] . This last kind of researches as I said befo= re,acts very close to the real condi= tions of the surfaces of missiles mo= ving in rarefied atmosphere; from this point of view they are particulary in= teresting for aerospace applications. However I think that, quantitatively, quite satisfactory results have not been yet obtained.

- At last, I want to point out an idea that is quite different from the one as= sumed in the above researches that is the idea suggested by Karamketi and Sen= tman [35] who studies the surface interac= tion with the aim of the Boltzman equa= tion, by introducing the surface effect by means of suitable external forces.

In all the above mentioned researches, two parameters play an essential role and namely the wall temperature T_w and the in= cident gas energy E_i.

As regards the experimental researches I proceed now to brief review of those ones performed with molecular beams tech= nique. Fist of all we want to point out the above mentioned systematic researches carried out by Hurlbut [18], Datz et alii [19] , Jawtusch [20] , Smith and Fite[21] and others, calculating the space distribution law of the number of molecules reemitted by a target hit by a molecular beam pro= duced either from an oven, or by means of other techniques, apt to produce a "mono= cromatic" beam, that is a beam where all molecules have the same velocity. By plotting in polar coordinates this number of molecules, that is the magnitude we in= dicated with $N_r^{(\Omega)}$ in the first of eq. (3.3), "lobes" which are qualitatively reported in fig. 3a) have been obtained. This re= sult is valid when the energy of the in= cident molecular beam is not too great, that is lower than 10 eV about. There are also well-known and very interesting ex= tensions, made by Devienne and his cower= kers [36] , to the case of molecular beams of much greater energy, where the above mentioned polar diagrams have more than one lobe. However, as regards only the cases of lowers energy, which concern the aerospace applications, the following re= sult may be considered as acquired. The lobes themselves have an almost circular shape if the surface has many impurities instead of being a pure cristal, or if the same surface is kept at a temperature of 300 - 400 °K about. On the contrary, if the target is heated at temperatures of 1000 °K about or more, then the shape of the lobes is more and more lengthened in a direction near to the one corresponding to the specular direction relative to the incident beam. The sametendency to len= gthened instead of circular lobes arises if the target is a crystal,that is without impurities. It is very important the fact that these results are substantially indi= pendent from the energy, or the tempera= ture of the incident molecular beam. In conclusion these experiments supplied a very wide and exact picture of the func= tion $N_r^{(\Omega)}$, as depending on the values of

the angle of incidence of the beam, on the beam and target temepratures and, at last, on the physical and chemical condi= tions of the target itself. On the con= trary, till today we cannot have the same experimental informations as regards the space distribution law of momentum and energy flux of reemitted molecules in each solid angle, idest of the magnitudes I indicated respectively with $Q_r^{(\Omega)}$ and $E_r^{(\Omega)}$. Other remarkable experimental results have been obtained by Stickney and Hurlbut [37] and [38] , who measured the normal pressure p_r, exerted by a molecular beam normally incident on a target. In this connection we point out that, in agreement with my opinion on the accomodation coefficients explained in n. 2, these results are sig= nificantly presented through the ratio p_r/p_i instead of σ'. Finally I remember the very interesting experimental studies concerning the determination of the velo= city distribution function, or related magnitudes, of the molecules reemitted in every direction by the target hit by a molecular beam. I quote, for instance, the recent papers by Knuth at alii [39] , Klapier [40] , Moran [41] , Jakus [42] and others. I think that these experiments are foun= damental for understanding the surface in= teraction phenomenon and for carrying out that unitary formulation of the reemission law I synthetized by means of the only fun= ction $g_r(Q)$, on which, however, we are not able, to-day, to draw definitive conclus= ions.

5. Drag and lift calculations accor= ding to a spatial impulsive sur= face interaction model

Some of the studies quoted in the pre= vious n. 4 was further developed up to the calculation of the forces acting on the satellites. Other researches were also developed specifically to the calculation of these forces. On these researches I will briefly relate in the following n. 6. In the present number I will indicate some numerical results of drag and lift calcu= lation carried out according to a recent spatial impulsive surface interaction model which can be related to the spatial model of Logan et alii[25] and [26] but it is sub= stantially different from this one, both because it is based on different physical assumptions, and for a different analytical methods. The model here applied derives from a bidimensional one introduced by Nocilla-Chiadò Piat [27] and [40] for normal incidence, and successively extended to a whatever incidence by Chiadò Piat [28].
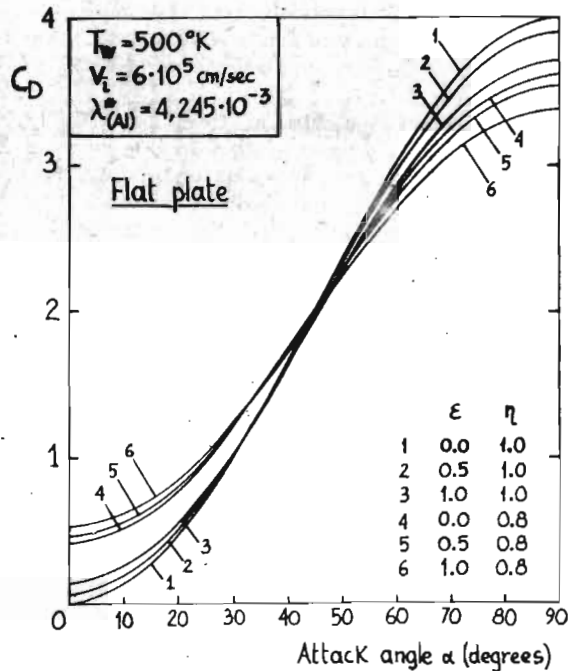
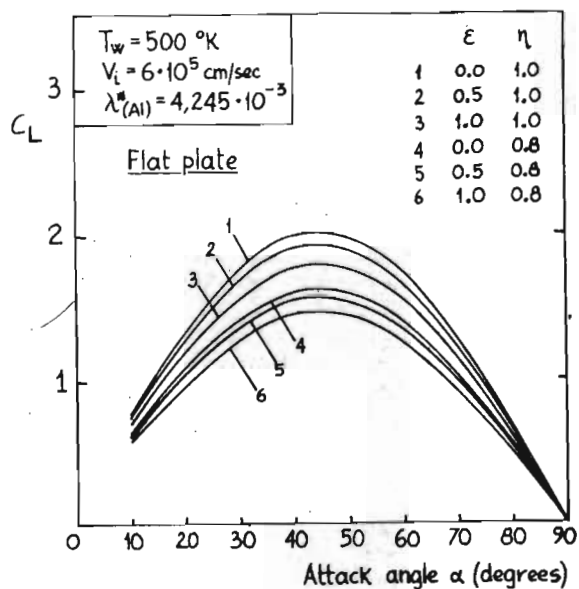Fig. 8 - Flat plate drag coefficients versus attack angle.

Fig. 9 - Flat plate lift coefficients versus attack angle

8

This spatial impulsive surface interaction model will be described in details in a paper of ChiadòPiat-Riganti[43] to be published. It relates directly the velocities before and after the collision and has the advantage of allowing simple and complete calculations of all the quantities related to the reemitted molecules in the case of a monoenergetic incident beams. The obtained velocity distribution law is defined as follows (see fig.2):

1) V_r is constant along any direction (ϕ, Θ) and is a well determined function $V_r(\phi, \Theta)$

2) the function $g(\vec{V})$ defined by the second of eq. (3.2) is:

$$\boxed{g(\vec{V_r}) = \frac{1}{J} \frac{\ell}{\varepsilon \pi^{3/2}} \left(1 + tg^2 \ell \alpha\right) exp\left[-\frac{tg \ell \alpha}{\varepsilon}\right]^2} \qquad (5.1)$$

where:

$$\begin{cases} J = J(\phi, \Theta; \eta) \\ \alpha = \alpha(\phi, \Theta; \eta) \end{cases}$$

are well determined functions of their arguments; ε is a suitable roughness parameter: for $\varepsilon = 0$ no roughness; for $\varepsilon = 1$ maximum roughness; η is another parameter related to the dissipation effects during the collision: for $\eta = 1$ no dissipation, for $\eta < 1$ dissipation occurs. According to the above reemission law the drag and lift coefficients for the flat plate are obtained respectively defined by:

$$C_D = \frac{D_i + D_r}{\frac{1}{2} \rho \mathcal{U}_i^2 \cos \Theta_i} \qquad (5.2)$$

$$C_L = \frac{L_i + L_r}{\frac{1}{2} \rho \mathcal{U}_i^2 \cos \Theta_i} \qquad (5.3)$$

where:

$$\left. \begin{aligned} D_i &= p_i \cos \Theta_i + \tau_i \sin \Theta_i \\ L_i &= p_i \sin \Theta_i - \tau_i \cos \Theta_i \end{aligned} \right\} \qquad (5.4)$$

$$\left. \begin{aligned} D_r &= p_r \cos \Theta_i - \tau_r \sin \Theta_i \\ L_r &= p_r \sin \Theta_i + \tau_r \cos \Theta_i \end{aligned} \right\} \qquad (5.5)$$

The normal and tangential stresses p_i and τ_i due to the impinging molecules have the values:

$$\left. \begin{aligned} p_i &= -\rho \mathcal{U}_i^2 \cos^2 \Theta_i \\ \tau_i &= \rho \mathcal{U}_i^2 \cos \Theta_i \sin \Theta_i \end{aligned} \right\} \qquad (5.6)$$
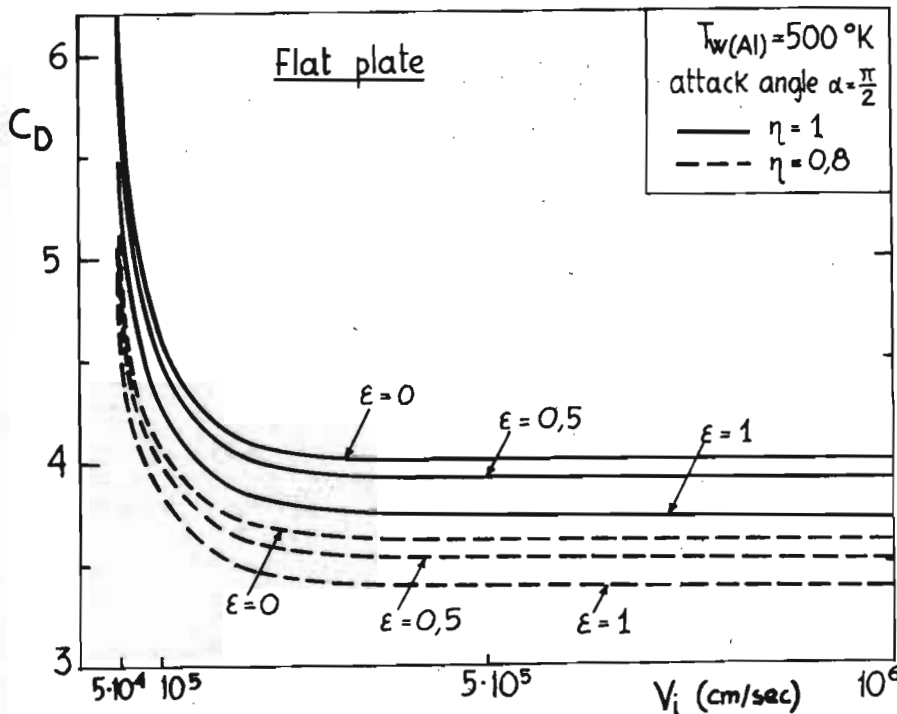
(ρ = density)

Fig. 10 - Flat plate drag coefficients versus incident velocity
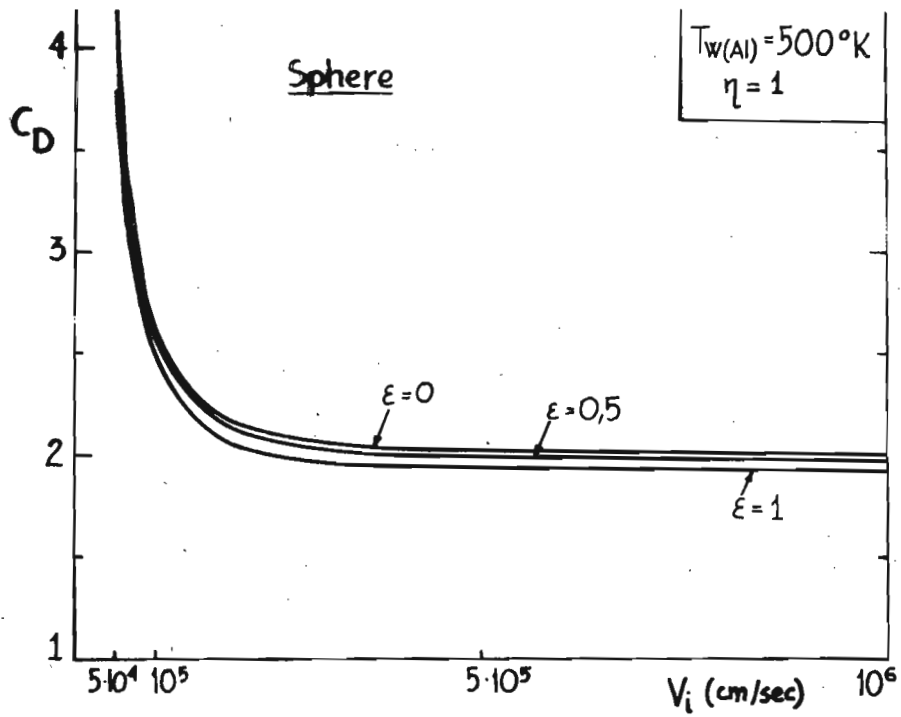
9

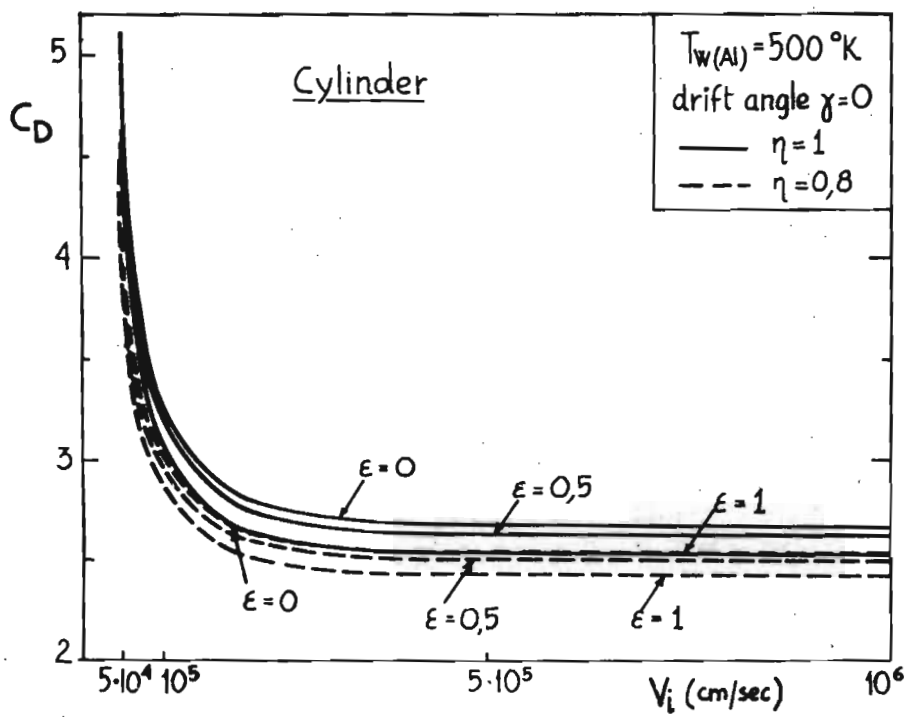Fig. 11 - Sphere drag coefficients versus incident velocity

Fig. 12 - Cylinder drag coefficients versus incident velocity

10

The normal and tangential stresses p_r and τ_r due to the reemitted molecules are ob= tained through integration of $g(\vec{V}_r)$ in the half-velocity space $w \geq 0$ (see figg. 2 and 6):

$$p_r = \rho \, u_i \cos \theta_i \iint_{w \geq 0} g(\vec{V}_r) \, V_r \cos \Theta \, d\Theta \, d\phi$$

$$\tau_r = \rho \, u_i \cos \theta_i \iint_{w \geq 0} g(\vec{V}_r) \, V_r \sin \Theta \sin \phi \, d\Theta \, d\phi$$

$$(5.7)$$

In these integrations the polar coordinates Θ and ϕ vary in the ranges:

$$0 \leq \Theta \leq \pi/2 \quad ; \quad 0 \leq \phi \leq 2\pi$$

The results of the numerical calculations are shown in figg. 8 and 9 versus the at= tack angle:

$$\alpha = \pi/2 - \theta_i \qquad (5.8)$$

and in fig. 10 versus the velocity $V_i \equiv U_i$, for three values of the roughness parameter ε and for two values of the dissipative parameter η.
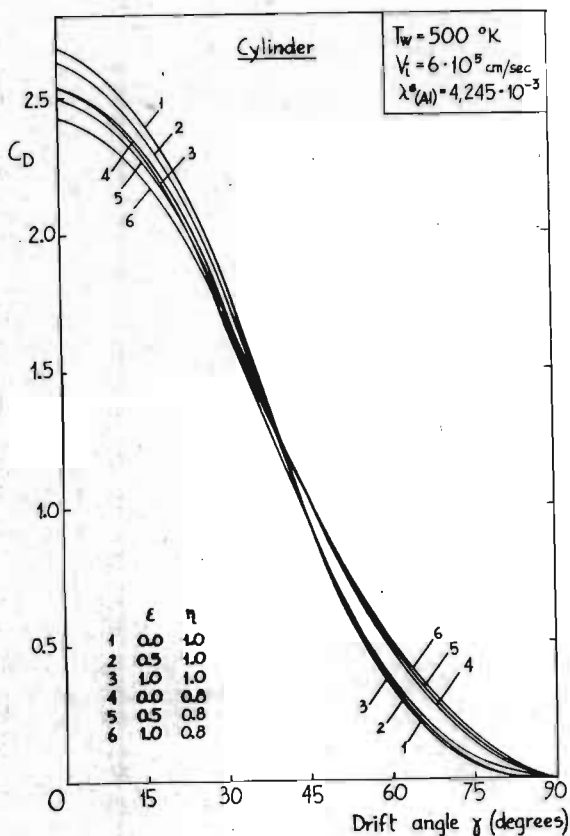

Fig. 13 - Cylinder drag coefficients versus drift angle γ

The above basic results relative to the flot plate may be applicated to calculate the drag and lift coefficients for convex bodies of whatever geometry:

$$C_D = \frac{\iint_A (D_i + D_r) \, dA}{\frac{1}{2} \rho \, u_i^2 \, A_o} \quad ; \quad C_L = \frac{\iint_A (L_i + L_r) \, dA}{\frac{1}{2} \rho \, u_i^2 \, A_o} \qquad (5.9)$$

A_o = cross section

Figg. 11 and 12 show the calculated values of C_D for the sphere and the transverse cylinder versus $V_i \equiv U_i$. Fig. 13 Shows the calculated values of C_D versus the drift angle γ for the cylinder with sidewind for a particular value of $U_i = V_i$, roughly corresponding to the or= bital velocity of satellites.

6. Other calculations of aerodynamic forces on satellites and comparisons

The calculation of the aerodynamic for= ces on bodies traveling the high rarefied atmosphere was for long time based on the classical conception of diffuse reemission, with suitable values of the accomodation coefficients, almost always assumed very close to the value1. Works concerning this subject are very numerous; "omptimal" geo= metric shapes, with regard to several as= pects of the missile flight, have been also studied. In this connection, we limit our= selves to remember the researches of Bell and Schaaf [45] and Tan [46] . In this connection I confirm my criticism already outlined in n. 2, mainly as regards the cal culation of the drag and lift coefficients as functions of the missile speed U_i. In fact, if we consider for instance the ener= gy accomodation coefficient α defined by the first of eq. (2.1), for such values of U_i to be $E_w \neq E_i$ as schematically is shown below:

either

$$\vdash_{E_w} \qquad \vdash_{E_r} \qquad \vdash_{E_i}$$

or

$$\vdash_{E_i} \qquad \vdash_{E_r} \qquad \vdash_{E_w}$$

that formula posses a physical meaning. But if we calculate the aerodynamic coefficient as depending on U_i, and hence on E_i, that formula becomes meaningless because the de= nominator vanishes for $E_i = E_w$, and it is not correct to assume a constant value for α . In agreement with these considera= tions I carried out in [12] and [13] the drag calculations for the flat plate, sphe=

re, cone and transverse cylinder by performing first of all an energy balance between the energy of the incident molecules, of the reemitted ones and some other kinds of energy as for instance radiation or thermal conduction which may be either received or lost by each elementary surface dA. Moreover, suitable "interaction" coefficients ζ_t and ζ_n was introduced which relate the drift velocity $\vec{U}_i$ and $\vec{U}_r$ of the impinging and reemitted molecules. Another interesting paper devoted to drag calculations is the one of Schamberg [47], which assumes a reemission model quite different from the traditional one. Namely, it is supposed that the molecules are reemitted within a suitable cone having an half-width angle ψ_0, where the number of molecules $N^{(\Omega)}_r$ contained within the solid angle $d\Omega_r$ is proportional to $\cos(\pi/2 \cdot \psi/\psi_0)$, where ψ is the angle between this solid angle and the cone axis. As regards the calculation of the velocities of reemitted molecules, it is reduced to the determination of a suitable energy accomodation coefficient. By following this model, Schamber calculates the drag coefficients for satellites of various shape and with a variable arrangement with respect to the flight direction, both fixed and toumbling, in hyperthermal conditions.

I end the brief review by quoting the older paper of Stalder et alii [48], where experimental values of the drag coefficients are obtained and compared with theory, and the very recent paper of Cercignani-Lampis [49], where a new mathematical model is proposed for the gas-surface interaction, and applied to the lift and drag calculation for simple shapes in free-molecule flow.

Finally I will compare the theoretical results shown in n. 5 with the ones obtained by Hurlbut and Sherman [8] already announced by Hurlbut [50]. For this comparison, as regards the flat plate I need calculate the quantities $C_D \sin\alpha$ and $C_L \sin\alpha$, shown in figg. 14 and 15, 16 and 17. Comparison shows a good agreement of my values for $\varepsilon = 0.0$; $\eta = 1.0$ (that is without surface roughness and without dissipation effects) with the Hurlbut-Sherman ones in the case of angle-independent parameters $S=100$, $S_r=10$ and $\alpha_2=0$. On the contrary, our results doesn't agree with Hurlbut-Sherman's calculations of the same coefficients, performed by assuming a dependence of those parameters on the incidence angle (see figg. 16 and 17). The same conclusions appear also from the behaviour of the ratio C_L/C_D shows in fig. 18.
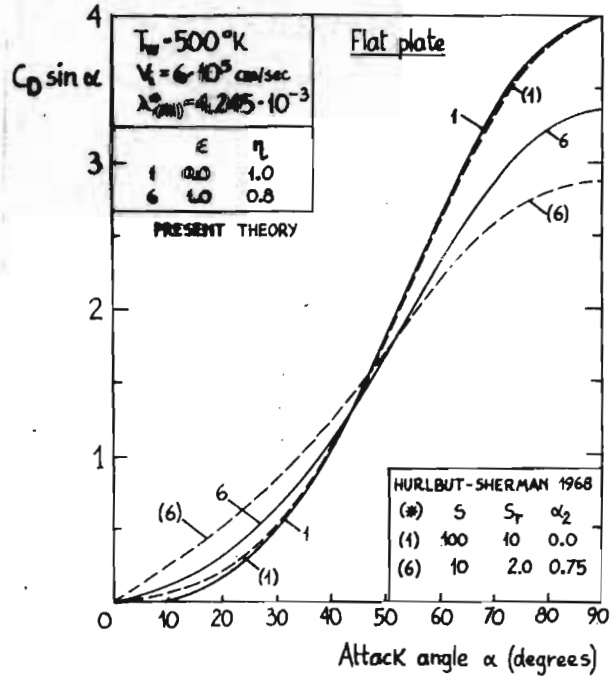
(*) angle-independent parameters S_r and α_2

Fig. 14 - $C_D \sin\alpha$ plotted versus α: comparison with Hurlbut-Sherman's results for angle independent parameters.
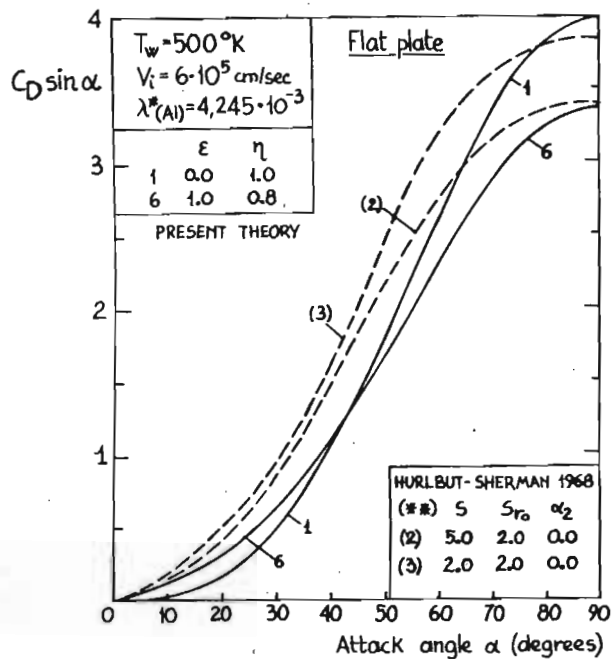
(**) angle-dependent parameters S_r and α_2

Fig. 15 - $C_D \sin\alpha$ plotted versus α: comparison with Hurlbut-Sherman's results for angle-dependent parameters.
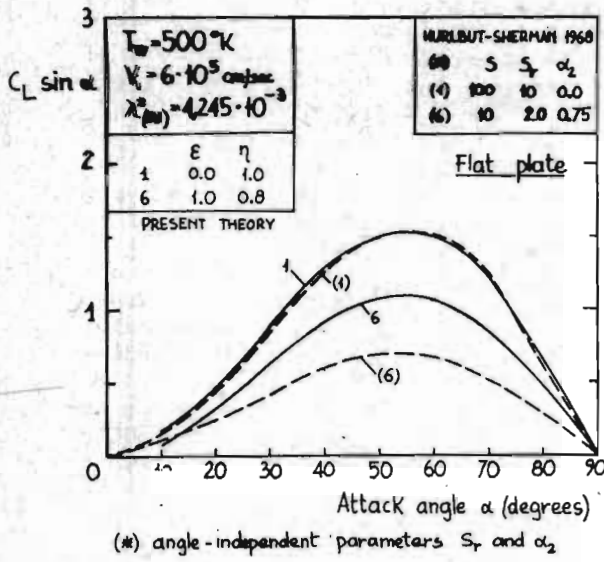
12

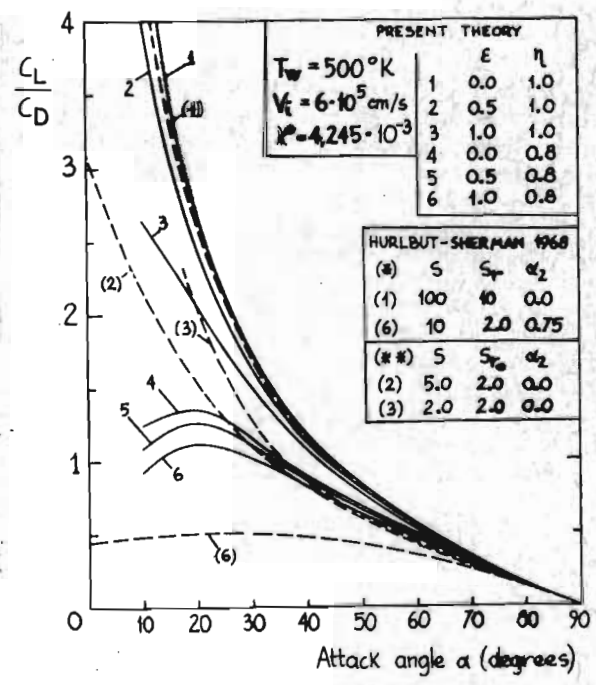**Fig. 16 – $C_L \sin \alpha$ versus α: comparison with Hurlbut-Sherman's results for angle-dependent parameters**

(*) angle-independent parameters S_r and α_2

(**) angle-dependent parameters S_r and α_2

Fig. 18 – Flat plate lift-drag ratios versus attack angle α.
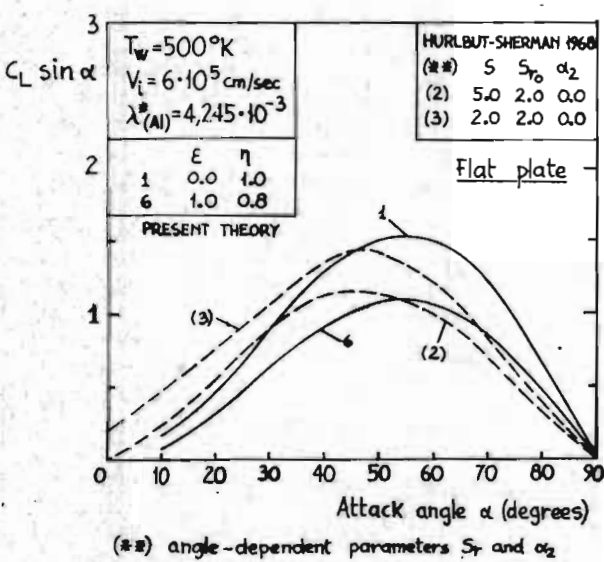

Fig. 17 – $C_L \sin \alpha$ versus α: comparison with Hurlbut-Sherman's results for angle-dependent parameters

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