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ANALYSES OF JET AND BOUNDARY LAYER NOISE

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ANALYSES OF JET AND BOUNDARY LAYER NOISE (TURBULENCE NOISE)

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Summary

The problem exists of gaining information on the intensity and spectrum of turbulence-generated noise. This task is solved by assuming that: a homogeneous, isotropic and (quasi-)stationary turbulence field is subject to a canonical distribution.

With this postulate it is possible to apply the terms and solution techniques of statistical thermodynamics to a turbulence field. In particular, a "turbulence temperature", a "turbulence entropy" and laws which are analogous to the First and Second Laws of Thermodynamics will be introduced.

The analysis is limited to the acoustic component of turbulence and has the following results for the linearized case:

1. The sound intensity radiated by turbulence increases as the 8th power of a characteristic turbulence velocity.

$$J \sim v^8$$

2. The frequency f_M of the maximum of the spectral intensity increases in proportion to the second power of a characteristic turbulence velocity.

$$f_M \sim v^2$$

3. The intensity i_M of the maximum of the spectral intensity increases as the 6th power of a characteristic turbulence velocity.

$$i_M \sim v^6$$

4. The ratio between the spectral emission coefficient ε (f) and the spectral absorption coefficient α (f) is equal to the spectral intensity.

$$i(f) = \frac{\varepsilon(f)}{\alpha(f)}$$

The v^8 law is identical to the Lighthill law developed in another way. In optics, analogous relationships apply for the black body.

1. Introduction

1.1 Background

The turbulent mixture of a jet engine or rocket blast with the ambient air causes high acoustic energies to be released. Thus, aircraft engines with afterburner can release an acoustic power of 100 kW. The acoustic power of the known large-size rockets amounts to several MW. Thus, every increase in engine output results in an over-proportional increase in acoustic power.

The alternating pressures of such sound intensities are so high that with the lightweight construction used for aircraft, they are a danger to airframe and structure. A number of fatigue failures due to acoustic loads are known. The noise effect on the electronic and hydraulic airborne instruments is just as dangerous. At critical vibration levels, malfunctions occur, causing position and homing indicators as well as navigation and control systems to fail. It is therefore essential that knowledge be acquired regarding the nature and the laws of turbulence noise.

Research efforts concerning turbulence noise were started rather late ($\sim$ 1950). The actual impetus was provided by the use of jet and rocket engines. As Lighthill (7) mentioned in his paper, "On Sound Generated Aerodynamically", this problem had been ignored.

In this connection, it should be noted that the theory of turbulence itself is not yet complete. Of course, by applying statistical methods it was possible to develop valuable relationships which qualitatively describe the internal structure of turbulence, but which hardly provide any quantitive description. Only by limiting the subject to homogeneous and isotropic turbulence, was G. I. Taylor (2) able to obtain concrete results by means of this method. In particular, T. v. Kármán (3), A. N. Kolmogoroff (4), W. Heisenberg (5) and C. F. v. Weizsäcker (6) have furthered the development of turbulence statistics.

In 1951, J. E. Moyal (9) stated that turbulence is composed of two different components; by means of Fourier-analysis he identified a longitudinal (=acoustic) and a transverse (=eddy) element.

J. M. Lighthill (7, 8) proved that quadrupoles are the source of turbulent sound generation. The most important result of Lighthill's work, which states that the emitted acoustic power increases as the 8th power of a characteristic turbulence velocity, was subsequently examined and confirmed by several authors: E. J. Richards (10), H. M. Fitzpatrick, R. Lee (11), J. M. Tyler, E. C. Perry (13), L. W. Lassiter, H. H. Hubbard (14) and H. v. Gierke (15).

Particularly at English institutes it was possible to further modify the theory of turbulence noise. In the U. S. A., Powell (19, 23), H. S. Ribner (17) and D. M. Phillips (18) worked on this problem. Dyer and Heckel (30) have applied thermodynamic relationships to statistical vibration phenomena. By introducing "vibration temperature", it became possible to describe the vibration energy of systems with multiple degrees of freedom.

1.2 Analogy: Turbulence/Statistical Thermodynamics

The rigorous equations for a viscous, compressible medium are not linear, and therefore the mathematical treatment of turbulence is very complicated and laborious. In order to keep the work within reasonable bounds, simplifications, linearizations and omissions will be made so that results are valid only within a limited area.

On the other hand, a theory which is based on the micro-condition of turbulence, that is, which considers the fate of every individual particle in the medium, requires a high level of mathematical effort. It would correspond to the effort that would be required to obtain the pressure a gas exerts upon a wall by summation of the individual impacts of each molecule against the wall. For this purpose, with known position and velocity of every molecule, the force would have to be calculated by application of the impact laws. This would be hopeless for molecular systems from the very start, even if the Heisenberg uncertainty principle did not exist, and if an exact determination of position and velocity of the molecules were possible.

The analogy between random statistical molecular movement in thermodynamics and random statistical turbulence movement justifies the attempt to apply the Laws of Thermodynamics to turbulence. This analogy becomes even more striking if the turbulence volume is thought of as being subdivided, and if these partial volumes are assumed to be the "molecules" of turbulence. These "molecules" exert forces against each other and mutually transfer potential and kinetic energy. Qualitatively, their behavior corresponds, for instance, to that of H_2O molecules of water in the liquid state.

Based on this conformity, it is assumed that a "turbulence temperature", a "turbulence entropy" and a "first and second law of turbulence dynamics" may be postulated for turbulence, analogous to temperature, entropy and the First and Second Laws of Thermodynamics, respectively.

Independently of the simple idea of assuming that a turbulence quantity consists of partial volumes, i. e., of "molecules", a more rigorous analogy may be presented. As is known, a turbulence volume may be assumed to be a sum of longitudinal (=acoustic) and transverse (=eddy) waves through Fourier-analysis. Planck (35) has shown that waves may also be assigned temperature and entropy. In addition, it may be assumed that it would be reasonable to assign a "turbulence temperature" and a "turbulence entropy" to the waves which form turbulence.

The question then arises as to how this "turbulence temperature" may be clearly demonstrated. For this purpose, it is necessary to return to the idea that the volume of turbulence is divided into small partial volumes. As a result of the turbulence motion, these partial volumes have a certain velocity, for instance, similar to the thermal molecular velocity of H_2O molecules. The magnitude of the velocity, that is, the mean value of the velocities of the individual partial volumes is a measure of the "turbulence temperature". Thus, a volume with no turbulence will have a "turbulence temperature" of zero, and the greater the turbulence motion is, the higher the turbulence temperature will be.

Unfortunately, it is not possible to provide a similarly clear demonstration for "turbulence entropy". Let it merely be mentioned that "turbulence entropy" may be attributed to the probability of the turbulence condition in the same way as thermodynamic entropy is related to probability.

2. Problem

It is the purpose of this paper to investigate the acoustic component of turbulence. This limitation was made, since only the acoustic energy portion of turbulence can be radiated to the outside, whereas the non-acoustic portion has no remote effect. Using the terms "turbulence temperature" and "turbulence entropy" as defined in Section 3, an attempt will be made to acquire information on the intensity and spectrum of turbulence noise.

A theoretical approach analogous to that which led to the determination of the laws of thermal radiation will be applied to a turbulence volume. This will result in the determination of relationships for turbulence noise which are analogous to the Stefan-Boltzmann law of radiation, Wien's displacement law and Kirchhoff's law of absorption. First of all, it should be stated that the law of radiation, determined in this way is identical to the Lighthill law determined in another way. It will not be possible to clarify the complete spectral law of turbulence noise. This is similar to the case of thermal radiation whose spectrum also cannot be determined solely by means of thermodynamics, but first requires application of the quantum theory. Nevertheless, it will be possible to again determine the law of radiation by means of the hypothesis upon which this paper is based, and to determine for the first time a law of similarity of turbulence noise, information on the position and level of the maximum spectrum, and a relation between the emissivity and absorptivity of turbulence fields.

3. Working Hypothesis

3.1 Assumptions

The dimensions of the investigated turbulence volume are presumed to be large with respect to the prevailing sound wavelengths in order to eliminate the influence of natural resonances. The turbulence in this volume is to be homogeneous, isotropic and stationary. In addition, the derivation will be limited to low turbulence velocities.

In the case of homogeneous turbulence, at all points in the volume, the mean turbulence velocity, the mean pressure, that is, in fact, all mean values of turbulence magnitudes are to be equal. Fields of turbulence occurring in jet engine streams are not homogeneous in this sense, since they have a high turbulence at the exhaust nozzle which decays downstream and finally disappears completely. However, such non-homogeneous fields can partially be regarded as homogeneous portions.

Isotropic turbulence is understood to mean that all directions in the turbulence volume under consideration are equally legitimate. The turbulence field of the jet stream is not isotropic, since it has a preferential direction, i.e., the direction of flow. By means of a system of coordinates moving with the flow, i.e., through a Galileen-transformation, such a field can be made isotropic.

In the case of stationary turbulence, the mean turbulence velocity and mean turbulence pressures are equal during the observation period. Notwithstanding this strict regulation, the investigation can also be extended to a quasi-stationary field of turbulence, the decay rate of which is low with respect to the speed at which turbulence equilibrium is attained.

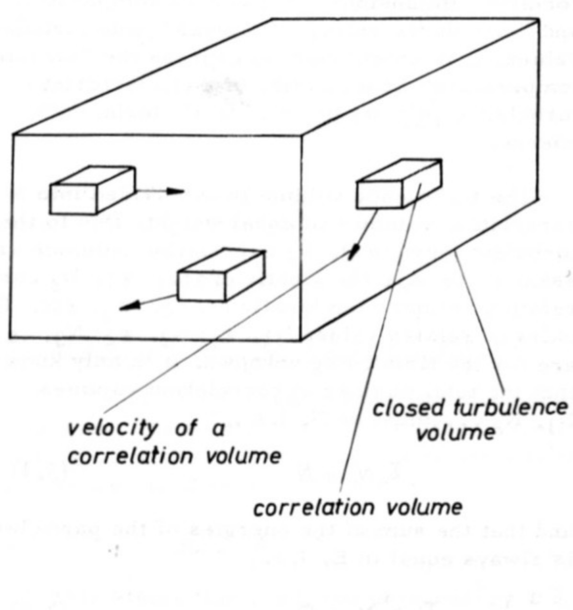
The restriction to low turbulence velocities is made in order to simplify the mathematics. However, the relationships gained in this way are correct to the first approximation. Any number of higher terms may be taken into consideration if sufficient effort can be expended on calculation.

3.2 Working Hypothesis: Canonical Distribution of Turbulence Energy

Let a closed system consist of identical and mutually independent particles. Let each of these particles have a quantitative dimension, for example a certain energy. This energy may be freely transferred from one particle to another. However, the total energy of all particles is constant. Such a system can be presented in various permutations due to the free transferability of energy. On the average, however, that distribution of the energy to the individual particles will occur which has the greatest probability. Such a system is designated as the canonical totality and the distribution as the canonical distribution.

Gibbs demonstrated that such a canonical system behaves macroscopically just like a thermodynamic system. The distribution modulus, for instance, which is a typical feature of the canonical distribution, has the same parameters as the thermodynamic, absolute temperature scale. The probable condition can be attributed to the entropy of the thermodynamic system.

These relations become evident when it is recalled for example, that gases consist of individual particles - the molecules of which move independently from one another and freely transfer their energy by impact, while the total energy of all molecules remains constant. A detailed analysis shows that temperature, entropy and the thermodynamic principles apply not only to the so-called "ideal gas" but generally to any complex systems (e.g., van der Waals forces, fluids, solids, frozen degrees of freedom). The hypothesis is proposed that a closed, homogeneous, isotropic and stationary turbulence volume represents a canonical system and that the turbulence energy is subject to a canonical distribution.



The turbulence volume under consideration is separated into partial volumes, which are identical with one another. Since it was assumed that the homogeneous and isotropic condition exists throughout the turbulence field, it is possible to realize this uniformity. Initially, the number of individual partial volumes is arbitrary. Due to the requirement that all particles in a canonical system be independent from one another, a certain volume size is specified. It may be designated "correlation volume", and is defined as that volume beyond which correlation ceases to exist between adjacent correlation volumes. The combined mass at the center of gravity of a correlation volume is designated m.

Due to the forces acting between the individual correlation volumes, the turbulence energy may be freely transferred from one particle to another.

As the turbulence field under investigation is assumed to be stationary, the condition that overall system energy be constant is satisfied. It is thus evident that all characteristics of a canonical system – identical independent particles, free transfer of energy, constant overall energy - are realized in the assumed homogeneous, isotropic and stationary turbulence field. Thus it is possible to postulate canonical distribution for such an idealized turbulence quantity, and a "turbulence temperature" a "turbulence entropy" and a "1st and 2nd law of turbulence dynamics" may be derived in analogy to statistical thermodynamics. These functions will have the same characteristics as temperature, entropy and the laws of thermodynamics. Macroscopically, a turbulency quantity is fully defined by these characteristics.

"Turbulence Temperature" θ

The first task is to relate the still unknown condition dimensions, "turbulence temperature" and "turbulence entropy" to measurable turbulence values. It is appropriate to express the "turbulence temperature" as a function of a characteristic turbulence velocity or of a kinetic turbulence energy.

The turbulence volume is subdivided into N correlation volumes of equal weight. Due to their turbulent movement, N_1 correlation volumes are assumed to have the kinetic energy ε_1, N_2 correlation volumes the kinetic energy ε_2, etc. The pairs of related values N_1, ε_1; N_2, ε_2; N_3, ε_3 ... are for the time being unknown. It is only known that the total number of correlation volumes N_1, N_2 ... must be N, i.e.,

$$\Sigma_n N_n = N \tag{3.1}$$

and that the sum of the energies of the particles is always equal to E, i.e.,

$$\sum_n \varepsilon_n N_n = E \tag{3.2}$$

As is known, distribution of the number of possible cases W and of the equal energy values N_1, N_2 ... to the N correlation volumes is

$$W = \frac{N}{N_1! N_2! \ldots} \tag{3.3}$$

As free exchange of energy between the correlation volumes is possible, in dynamic equilibrium that distribution will result, for which the probability for W is the greatest. It is appropriate to examine the maximum value of $\ln W$ in the place of the maximum W. Such a procedure is permissible as both functions are monotonic.

$$\ln W = \ln N! - \sum_n \ln N_n ! \tag{3.4}$$

In accordance with Stirling's formula we may write for value N:

$$\ln N! = N \ln N - N \tag{3.5}$$

The equation (3.4) thus changes into

$$\ln W = N \ln N - \sum_n N_n \ln N_n \tag{3.6}$$

By means of Lagrange multipliers, the maximum of W can be ascertained, taking into consideration the boundary conditions of equations (3.1) and (3.2). The calculation results in

$$N_n = N\, e^{\frac{\varphi - \varepsilon_n}{\theta}} \tag{3.7}$$

The parameter θ is the distribution modulus. Gibbs has shown that the distribution modulus has the same characteristics as the absolute thermodynamic temperature. Therefore, θ can be identified with the "turbulence temperature".

φ, is a trivial parameter in this connection which may be construed as "free energy".

The turbulence temperature θ and the free energy φ are established by the boundary conditions (3.1) and (3.2). By inserting (3.7) into (3.1) we obtain:

$$\sum_n e^{\frac{\varphi - \varepsilon_n}{\theta}} = 1 \tag{3.8}$$

Inserting (3.7) into (3.2) we obtain:

$$\sum_n \varepsilon_n e^{\frac{\varphi - \varepsilon_n}{\theta}} = \frac{E}{N} = \bar{\varepsilon} \tag{3.9}$$

The parameter φ, which is of no interest here can be eliminated from equations (3.8) and (3.9), resulting in the following equation for the mean particle energy

$$\bar{\varepsilon} = \frac{\sum_n \varepsilon_n e^{-\frac{\varepsilon_n}{\theta}}}{\sum_n e^{-\frac{\varepsilon_n}{\theta}}} \tag{3.10}$$

The equation (3. 10) shows in the numerator the derivation of the denominator so that

$$\bar{\varepsilon} = - \frac{d}{d(\frac{1}{\Theta})} \ln \sum_n e^{-\frac{\varepsilon_n}{\Theta}} \tag{3.11}$$

In order to display the essential point, a finite number of discrete energy values ε_1, ε_2 .... was used in the above presentation. But actually a continuous multitude of ε -values exists. In such case it is not permissible to define and count cases in the described manner.

Passing the limit from the discrete ε -values to the continuous ε -values gives:

$$\bar{\varepsilon} = - \frac{d}{d(\frac{1}{\Theta})} \ln \iiint_{-\infty}^{+\infty} e^{- \frac{q_x^2 + q_y^2 + q_z^2}{2m\Theta}} dq_x \, dq_y \, dq_z \tag{3.12}$$

q_x, q_y, q_z are the pulse coordinates defined:

$$q_x = m v_x$$
$$q_y = m v_y$$
$$q_z = m v_z \tag{3.13}$$

Equation (3. 12) formally agrees with equation (3. 11). The summation is merely replaced by an integration. The expression

$$\frac{q_x^2 + q_y^2 + q_z^2}{2m} = \frac{1}{2} m v^2 = \varepsilon \tag{3.14}$$

in the exponent of equation (3. 12) represents the kinetic energy of a correlation volume.

Solution of the integrals defined in equation (3. 12) gives

$$\bar{\varepsilon} = \frac{3}{2} \Theta \tag{3.15}$$

The mean kinetic energy of a correlation volume can be stated directly

$$\bar{\varepsilon} = \frac{1}{2} m \bar{v}^2 \tag{3.16}$$

where $\bar{v}^2$ is the mean square translation velocity of the correlation volumes.

Equating 3. 15 and 3. 16 gives the desired relation between the "turbulence temperature" and a characteristic turbulence velocity $\bar{v}$.

$$\Theta = \frac{1}{3} m \bar{v}^2 \tag{3.17}$$

Since the mass of the correlation volumes is initially unknown, and since unknown proportionality factors also occur in various integrations, equation 3. 17 will be used in the further derivations in the form:

$$\Theta \sim v^2 \tag{3.18}$$

which states that the "turbulence temperature" Θ is proportional to the square of a characteristic turbulence velocity.

"Turbulence Entropy" Σ

It has become evident that the turbulence temperature and the canonical distribution modulus have the same characteristics. The "turbulence entropy" can also be attributed to the probability of the turbulence. If two turbulence quantities with the probabilities W_1 and W_2 are combined, according to the laws of statistics, the overall system has the probability,

$$W = W_1 \cdot W_2 \tag{3.19}$$

or,

$$\ln W = \ln W_1 + \ln W_2 \tag{3.20}$$

The logarithm of probability is the sum of the respective values of the partial quantities. The entropy has just the same parameter. As is known,

$$S = S_1 + S_2 \tag{3.21}$$

where S is the entropy of the overall system and S_1 and S_2 are the entropies of the partial systems.

Analogous to the well known Boltzmann's relation,

$$S = K \ln W \tag{3.22}$$

In thermodynamics the "turbulence entropy" of a turbulence quantity is

$$\Sigma \sim \ln W \tag{3.23}$$

This "turbulence entropy" Σ satisfies the same conditions as the entropy function in thermodynamics. In the ensuing derivations, the following relation will be especially required

$$d\Sigma = \frac{dQ}{\Theta} \tag{3.24}$$

"1. and 2. Laws of Turbulence Dynamics"

According to the principle of the conservation of energy, the relation for a turbulence quantity analogous to the First Law of Thermodynamics can be written directly

$$dQ = d\phi_i + dA_i \tag{3.25}$$

This states that if a turbulence energy dQ_i is introduced, its portion $d\Phi_i$ will augment the internal turbulence energy, whereas the portion dA_i performs external work. The subscript i denotes a certain phase since in multi-phase systems, for instance, a turbulence field composed of a longitudinal (=acoustical) and a transversal (=vortex) phase, this relation is valid for all phases. As the present analysis deals only with the sonic phenomena of turbulence, it is understood that the relation

$$dQ = d\Phi + dA \qquad (3.26)$$

without subscripts, signifies the acoustic component.

If equation (3.26) is inserted into equation (3.24) then

$$d\Sigma = \frac{d\Phi + dA}{\Theta} \qquad (3.27)$$

4. Derivation of a Radiation Formula

4.1 Relationship between Sound Intensity and Density of Sound Energy

In an isentropic radiation field between the radiation intensity J and the radiant density U the following relationship exists

$$U = \frac{4\pi}{c} J \qquad (4.1)$$

The analogous equation is also valid for the spectral density

$$u(f) = \frac{4\pi}{c} i(f) \qquad (4.2)$$

4.2 Determination of Sonic Radiation Pressure

As is known, when a sound wave impinges upon a surface, it exerts a pressure, the so-called sonic radiation pressure. In general, any transfer of energy by radiation (sound, light ...) is associated with a pulse transfer, which results in a pressure effect with changes in the direction of radiation. Knowledge of the sound radiation pressure Π is required for derivation of the radiation formula. Assuming the sonic speed c to be independent of the amplitude, the basic equation

$$\frac{\partial^2 \xi(s,t)}{dt^2} = c^2 \frac{\partial^2 \xi(s,t)}{\partial s^2} \qquad (4.3)$$

provides the solution for the deflection

$$\xi = \xi_0 \sin\omega(t - s/c) \qquad (4.4)$$

The particle velocity $\dot{\xi}$ is obtained by differentiation of ξ vs. time

$$\dot{\xi} = \xi_0 \omega \cos\omega(t - s/c) \qquad (4.5)$$

The sound pressure for coplanar waves is

$$P_{\sim} = \rho c \dot{\xi}$$

$$P_{\sim} = \xi_0 \omega \rho c \cos\omega(t - s/c) \qquad (4.6)$$

In this case the local coordinate s does not apply to a fixed point in space as in the linearized theory, but to a moving volume element.

If ξ is the deflection of the volume element from the stationary zero-position x, then

$$s = x + \xi \qquad (4.7)$$

Accordingly the sound radiation pressure $\Pi_\perp$ of a coplanar wave at normal incidence is

$$\Pi_\perp = \frac{1}{2\pi} \int_0^{2\pi} P_{\sim} \, d(\omega t) \qquad (4.8)$$

$$\Pi_\perp = \frac{1}{2\pi} \omega \rho c \int \cos(\omega t - \frac{\omega}{c} \sin\omega t) \, d\omega t \qquad (4.9)$$

In this equation the unessential spatial coordinate x was assumed to be 0.

The integration of equation (4.9) gives the Bessel function of first order.

$$\Pi_\perp = \omega \rho c J_1(\frac{\omega}{c}) \qquad (4.10)$$

Definition of Bessel's function (J_1) gives:

$$J_1(\frac{\xi_0 \omega}{c}) = \sum_{K=0}^{\infty} \frac{(-1)^K}{K!(K+1)!} (\frac{\omega \xi_0}{c})^{1+2K} \qquad (4.11)$$

Limitation to low turbulence makes it unnecessary to consider higher terms of the Bessel series.

$$\Pi_\perp = \frac{1}{2} \xi_0^2 \omega^2 \rho \qquad (4.12)$$

The term $\xi_0 \omega$ is the particle velocity amplitude. It is known that the sound intensity J results from the particle velocity,

$$J = \frac{1}{2} \rho c (\xi_0 \omega)^2 \qquad (4.13)$$

The sound radiation pressure $\Pi_\perp$ at normal incidence becomes

$$\Pi_\perp = \frac{J}{c} \qquad (4.14)$$

With a sound wave impinging upon a surface at angle α, the radiation pressure is Π_α

$$\Pi_\alpha = \Pi_\perp \cos^2\alpha = \frac{J}{c}\cos^2\alpha \qquad (4.15)$$

After loss-free reflection the pressure of the reflected wave has exactly the same value as the impinging wave. The reflection pressure Π, which an isotropic sound radiation exerts upon a loss-free reflector can be ascertained by integration over the hemisphere.

$$\Pi = 2\int \Pi_\alpha \, d\Omega = 4\pi \int_0^{\pi/2} \Pi_\alpha \sin\alpha \, d\alpha \qquad (4.16)$$

$$\Pi = \frac{4\pi}{3}\,\frac{J}{c} \qquad (4.17)$$

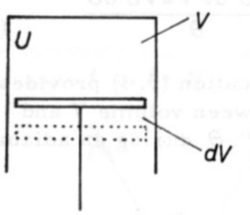
It should be remembered that in isotropic and homogeneous radiation the following relation exists between the sound intensity J and the density of sound energy U

$$U = 4\pi \frac{J}{c} \qquad (4.11)$$

Resulting therefrom is the sound radiation pressure Π in an isotropic sound radiation with the density of sound energy U.

$$\Pi = \frac{1}{3}U \qquad (4.19)$$

4.3 Derivation of the Radiation Formula



The train of thought by which Boltzmann found the law of thermal radiation will now be applied to turbulence noise.

A cylinder of volume V is closed by a movable piston. The cylinder and piston walls are to reflect a sound wave without loss. The cylinder volume contains homogeneous, isotropic and stationary turbulence in equilibrium having a turbulence temperature Θ. The cylinder volume V has dimensions which are large with respect to the wavelengths. Let U be the density of sound energy. The overall sound energy Φ of the system is

$$\Phi = U \cdot V \qquad (4.20)$$

When the piston moves and the cylinder volume is reduced by d V, the density of energy U and the overall energy ϕ change by d U or dϕ respectively. The relation is described by the total differential of equation (4.20).

$$d\Phi = U\,dV + V\,dU \qquad (4.21)$$

In addition, the piston performs the work d A against the sound radiation pressure Π in the cylinder.

$$dA = \Pi \, dV \qquad (4.22)$$

The change of volume d V is, in the general case, accompanied by a change d Σ in the "turbulence entropy" Σ. According to the "first and second laws of turbulence dynamics" and the equation 3.27, the relation dΣ between the change of energy dϕ, the external energy dA and the "turbulence temperature" Θ is

$$d\Sigma = \frac{d\Phi + dA}{\Theta} \qquad (3.27)$$

After eliminating dΘ and dA according to the equations (4.21) and (4.22) we obtain

$$d\Sigma = \frac{(U+\Pi)\,dV + V\,dU}{\Theta} \qquad (4.24)$$

For the time being, the density of sound energy U is an unknown function of "turbulence temperature" Θ.

$$U = U(\Theta) \qquad (4.25)$$

The differential d U in the equation (4.24) can thus be eliminated by means of equation (4.25)

$$dU = \frac{\partial U}{\partial \Theta}\, d\Theta \qquad (4.26)$$

This results in

$$d\Sigma = \frac{(U+\Pi)\,dV + V\,\frac{\partial U}{\partial \Theta}\,d\Theta}{\Theta} \qquad (4.27)$$

From the mathematical viewpoint, "turbulence temperature" Θ has the attributes of an integral denominator. For this reason the entropy differential d Σ must be a so-called total differential. We remember that such a differential with the variables V and Θ has the format

$$d\Sigma = \frac{\partial \Sigma}{\partial V}\, dV + \frac{\partial \Sigma}{\partial \Theta}\, d\Theta \qquad (4.28)$$

The comparison of the equations (4.27) and (4.28) shows that

$$\frac{U+\Pi}{\Theta} = \frac{\partial \Sigma}{\partial V} \qquad (4.29)$$

and

$$\frac{V}{\Theta}\,\frac{\partial U}{\partial \Theta} = \frac{\partial \Sigma}{\partial \Theta} \qquad (4.30)$$

Partial differentiation of equation (4.29) by $\partial\Theta$ and of equation (4.30) by ∂V gives:

$$\frac{\Theta\left(\frac{\partial U}{\partial \Theta} + \frac{\partial \Pi}{\partial \Theta}\right) - (U+\Pi)}{\Theta^2} = \frac{\partial^2 \Sigma}{\partial V \partial \Theta} \qquad (4.31)$$

and

$$\frac{1}{\theta}\frac{\partial U}{\partial \theta} = \frac{\partial^2 \Sigma}{\partial \theta \partial V} \qquad (4.32)$$

As the sequence of the differentiations can be altered in continuous functions (U is a continuous function of the variables V and Θ) without incurring a change of the deduced values,

$$\frac{\partial^2 \Sigma}{\partial V \partial \theta} = \frac{\partial^2 \Sigma}{\partial \theta \partial V} \qquad (4.33)$$

Equating (4.31) and (4.32) gives

$$\frac{\partial \theta}{\theta} = \frac{\partial \Pi}{U+\Pi} \qquad (4.34)$$

Substituting the radiation pressure Π in accordance with equation (4.19) by the density of sound energy U

$$\Pi = \frac{1}{3} U \qquad (4.19)$$

and

$$d\Pi = \frac{1}{3} dU \qquad (4.19')$$

gives

$$\frac{d\theta}{\theta} = \frac{1}{4}\frac{dU}{U} \qquad (4.35)$$

Integration of (4.35) leads to the result that the density of sound energy U increases in proportion to the 4th power of "turbulence temperature"

$$U \sim \theta^4 \qquad (4.36)$$

Using the equation (4.12)

$$U = \frac{4\pi}{c} J \qquad (4.1)$$

we gather that the sound radiation intensity J also increases as the 4th power of "turbulence temperature" Θ .

$$J \sim \theta^4 \qquad (4.37)$$

Using the equation *(3.18)* $\theta \sim v^2$ and substituting the characteristic turbulence velocity v for the "turbulence temperature" Θ , we obtain the v^8 law which Lighthill found by quite another method in 1952: The sound intensity J emitted by a turbulence field is proportional to the 8th power of a characteristic turbulence velocity.

$$J \sim v^8 \qquad (4.38)$$

(It is impossible, as is also the case with the Stefan-Boltzmann law of radiation, to define the still unknown proportionality factor in equation (4.38) by means of previous assumptions. In section 6.1 the attempt is made to ascertain the various characteristics by dimensional analysis).

5. Deduction of Spectral Formulas

5.1 Adiabatic Process

The equation for "adiabatic" process of the turbulence field is required in the following deduction. This means that - as is the case with thermodynamic adiabats - the turbulence volume is changed without the exchange of turbulence energy from or to the outside. The "turbulence entropy" Σ along an adiabatic curve remains constant.

$$d\Sigma = 0 \qquad (5.1)$$

In section 4.3 a cylindrical volume V was changed by dV, without gain or lost of turbulence energy. The change of "turbulence entropy" is here described by equation (4.34).

$$d\Sigma = \frac{(U+\Pi)dV + VdU}{\theta} \qquad (4.34)$$

According to equation (5.1),

$$0 = \frac{(U+\Pi)dV + VdU}{\theta} \qquad (5.2)$$

and after substitution of the radiation pressure according to equation (4.19) and the density of sound energy U according to equation (4.36) we read:

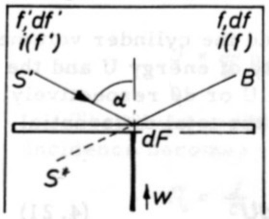
$$0 = \frac{\frac{4}{3}\theta\, dV + 4V\theta\, d\theta}{\theta} \qquad (5.3)$$

The integration of equation (5.3) provides the relation being sought between volume V and "turbulence temperature" θ during an adiabatic process

$$d\Sigma = 0 = \theta dV + 3V d\theta$$
$$\Sigma \sim V\theta^3 \qquad (5.4)$$

5.2 Deduction of a Spectral Formula

The analyses of Wien on the displacement law for thermal radiation, with reference to the theory of canonical distribution of turbulence energy on which this paper is based can also be applied to turbulence noise. The following train of thought is of interest in this connection:



A cylindrical volume V contains a homogeneous, isotropic, stationary turbulence field which is in equilibrium and has a "turbulence temperature" and the spectral sound intensity $i(f\,\Theta)$. A piston with area F moves into the cylinder at the (low) velocity w.

Let the cylinder and piston surfaces be absolutely soundproof.

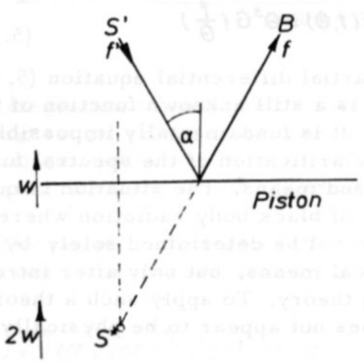
The reflection of the sound waves on the moving piston changes the spectral distribution as a result of the Doppler effect. Furthermore, the piston performs work against the sound radiation pressure. No turbulence is to be gained or lost during the whole process, i. e., the process is to run "adiabatically". In order to determine the change of spectral distribution during this process, an energy "balance" will be established, covering a fixed frequency range, i. e., from f to $f + df$, and the sound energy d^6E_- emerging from this frequency range and the energy d^6E_+ entering into this frequency range will be ascertained:

a) the sound energy d^6E_- emerging from the frequency range from f to $f + df$

The energy of all sound waves with frequencies from f to $f + df$ which strike the piston emerges by reflection on the moving piston during the time dt from the range df. This energy is raised into another spectrum range by the Doppler shift. Starting the analysis from a surface element d F of the piston surface and considering the sound waves striking d F at angle of incidence in the solid angle element $d\Omega$, the emerging energy is:

$$d\,E_- = i(f\,\Theta)\,df\,d\Omega\cos\alpha\,dF\,dt \qquad (5.5)$$

b) The incoming sound energy d^6E_+ in the frequency range f to $f + df$



A sound wave with the frequency f' starts from point S'. This wave strikes at angle of incidence α, the piston moving at velocity w, and is reflected. Seen from the fixed observation point B, the wave appears to come from S*. The virtual point S* appears to move with a velocity 2 w towards S'. The velocity component towards the observation point B is 2 $w\cos\alpha$. According to Doppler, an observer at point B perceives the frequency initially starting at S' as the frequency f. We recall that relation between f' and f is:

$$f = f'\,\frac{1}{1-2\,w/c\,\cos\alpha} \qquad (5.6)$$

As the piston moves very slowly, we can equate the angle of incidence with the reflection angle. In view of the small value of w, as a 1st approximation we may write

$$f = f'\left(1+2\,\frac{w}{c}\cos\alpha\right) \qquad (5.7)$$

for the equation (5. 6). In consequence of the Doppler effect, a frequency interval df' is transformed into another frequency range df. Differentation of equation (5. 7) results in the following relation:

$$df = df'\left(1+2\,\frac{w}{c}\cos\alpha\right) \qquad (5.8)$$

The sound radiation of the range from f' to $f' + df'$, which, due to Doppler shift caused by reflection on the moving piston, enters the range f to df, also undergoes a change of intensity, since the piston is working against the radiation pressure. A wave with the intensity $i(f'\,\Theta)$ and the spectral range df' which is reflected on the piston at the incidence angle α from a solid angle element $d\Omega$ exerts the radiation pressure $d^3\Pi$:

$$d^3\Pi = 2\,\frac{i(f'\,\Theta)\,df'}{c}\,d\Omega\cos^2\alpha \qquad (5.9)$$

In addition to the energy $i(f',\Theta)\,df'\,d\Omega\cos\alpha\,dF\,d\Omega\,dt$ which is transferred by reflection into the observed spectral range from f to $f + df$, further energy is added by the piston. Thus, the overall energy d^6E_+ is

$$d^6E_+ = i(f'\,\Theta)\,df'\,d\Omega\cos\alpha\,dF\,dt + d^3\Pi\,dF\,w\,dt \qquad (5.10)$$

If we replace df' by the equation (5. 8) and $d^3\Pi$ by the equation (5. 9), we read:

$$d^6E_+ = i(f'\,\Theta)\,df\,d\Omega\cos\alpha\,dF\,dt \qquad (5.11)$$

The overall change of energy, d^6E, in the examined spectral range is

$$d^6E = d^6E_+ - d^6E_- \qquad (5.12)$$

By use of the equations (5. 5) and (5.11)

$$d^6E = [\,i(f'\,\Theta) - i(f,\Theta)\,]\,df\,d\Omega\cos\alpha\,dF\,dt \qquad (5.12)$$

As it is assumed that the piston speed is low, there is not much difference between the frequencies f' and f before and after the reflection on the moving piston, so that we can write as a first approximation:

$$i(f',\theta)-i(f,\theta)=\frac{\partial i(f,\theta)}{\partial f}(f'-f) \qquad (5.13)$$

The equation (5.6) permits cancellation of the difference $f'-f$ and we obtain

$$i(f',\theta)-i(f,\theta)=2\frac{\partial i(f,\theta)}{\partial f}f\frac{w}{c}\cos\alpha \qquad (5.14)$$

Insertion of the equation (5.14) into the equation (5.12) gives:

$$d^6E=2f\frac{w}{c}\cos^2\alpha\frac{\partial i(f,\theta)}{\partial f}df\,d\Omega\,dF\,dt \qquad (5.15)$$

The energy quantity d^6E includes only that radiation in the solid angle element $d\Omega$. Integration over the hemisphere gives the energy d^4E containing the radiation from all directions. Assuming the solid angle element to be a spherical ring, we write $d\Omega=2\pi\sin\alpha d\alpha$

$$d^4E=4\pi f\frac{w}{c}\frac{\partial i(f,\theta)}{\partial f}df\,dF\,dt\int_0^{\pi/2}\cos^2\alpha\sin\alpha\,d\alpha \qquad (5.16)$$

since all directions are of equal value due to the isotropical radiation. The integration gives the relation:

$$d^4E=\frac{4\pi}{3}f\frac{w}{c}\frac{\partial i(f,\theta)}{\partial f}df\,dF\,dt \qquad (5.17)$$

Integrating furthermore the whole piston area by

$$\int dF=F \qquad (5.18)$$

and interpreting the term $F.wdt$ as a (cylindrical) volume element δV with the base F and the height wdt

$$\delta V=Fwdt \qquad (5.19)$$

gives the result:

$$d^2E=4\pi f\frac{1}{c}\frac{\partial i(f,\theta)}{\partial f}df\,\delta V \qquad (5.20)$$

d^2E is the change of energy suffered by a sound radiation in the frequency range from f to $f+df$ with an adiabatic volume change of δV. This change of energy d^2E must be equal to the change of internal acoustical energy $d^2\phi$ which the system undergoes with an adiabatic change of volume

$$d^2E=d^2\phi \qquad (5.21)$$

The internal acoustical energy $d^2\phi$ within the frequency range df is

$$d^2\phi=\delta(Vu(f,\theta))df \qquad (5.22)$$

$$d^2\phi=V\delta u(f,\theta)df+u(f,\theta)\delta Vdf \qquad (5.23)$$

Equating (5.20) and (5.23) gives

$$\frac{4\pi}{3}\frac{1}{c}\frac{\partial i(f\theta)}{\partial f}\delta V=V\delta u(f,\theta)-u(f,\theta)\delta V \qquad (5.24)$$

Using the equation (4.2)

$$u(f,\theta)=\frac{4\pi}{c}i(f,\theta) \qquad (4.2)$$

gives

$$du(f,\theta)=\frac{4\pi}{c}di(f,\theta) \qquad (5.25)$$

and

$$\frac{f}{3}\frac{\partial i(f,\theta)}{\partial f}=V\frac{\partial i(f,\theta)}{\partial f}+i(f,\theta) \qquad (5.26)$$

If, according to the "adiabatic" equation (5.3)

$$\frac{dV}{V}=-3\frac{d\theta}{\theta} \qquad (5.3)$$

the volume is replaced by the turbulence temperature, we obtain the relation

$$f\frac{\partial i(f,\theta)}{\partial f}=-\frac{\partial i(f,\theta)}{\partial\theta}+3i(f,\theta) \qquad (5.27)$$

The solution

$$i(f,\theta)=\theta^3G\left(\frac{f}{\theta}\right) \qquad (5.28)$$

satisfies the partial differential equation (5.27), where $G(f,\theta)$ is a still unknown function of the variable f/θ. It is fundamentally impossible to provide a full clarification of the spectral function with the proposed means. The situation is quite similar to that of black body radiation where the spectrum could not be determined solely by thermodynamical means, but only after introduction of the quantum theory. To apply such a theory to turbulence, does not appear to be physically sensible.

Despite the uncertainly of the spectral function $G(f/\theta)$, it is possible to obtain two valuable characteristics of the turbulence noise spectrum. The frequency f_M at which the spectrum has a peak value is obtained by differentiating the equation (5.28) on the condition that

$$\frac{\partial i(f,\theta)}{\partial f}=0 \qquad (5.29)$$

$$=\theta^2\frac{\partial G(f/\theta)}{\partial(f/\theta)}=0 \qquad (5.30)$$

The maximum is located at a certain value of $G(f_M/\Theta) = const.$ From this results the position of the maximum

$$f_M \sim \Theta \qquad (5.31)$$

The height of the intensity peak i_M is, according to equations (5.28) and (5.31),

$$i_M = \Theta^3 G(f_M/\Theta) = \Theta^3 \cdot const$$
$$i_M \sim \Theta^3 \qquad (5.32)$$

Substitution of the turbulence temperature in the resultant equations (5.28), (5.31) and (5.32) according to the equation

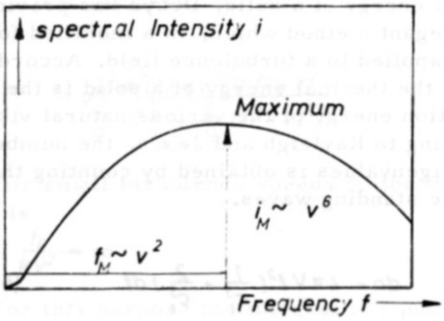
$$\Theta \sim v^2 \qquad (3.18)$$

leads to the result:

$$i(f,v) = v^6 g(f/v^2)$$

$$f_M \sim v^2$$

$$i_M \sim v^6$$



Integration of equation (5.33) with v constant throughout all frequencies from 0 to ∞ must, for validity of the results, necessarily lead to the radiation formula $J \sim v^8$ derived in Section 4.3.

The integral $J = \int_0^\infty i(f,v)\,df$ is the overall intensity J emitted throughout all frequencies

$$J = \int_0^\infty i(f,v)\,df = \int_0^\infty v^6 g(f/v^2)\,df \qquad (5.36)$$

$$= v^8 \int_0^\infty g(f/v^2)\,d f/v^2$$

The function $g(f/v)$ has an unknown but fixed value for all quantities of f and v, where f/v^2 remains constant.

Therefore,

$$\int_0^\infty g(f/v^2)\, d f/v^2 = const.$$

Thus

$$J \sim v^8 \qquad (5.37)$$

Since the $J \sim v^8$ law established by Lighthill using a completely different method has been confirmed experimentally, it is surely proper to ascribe a higher degree of probability to the results presented here derived from a different working hypothesis.

6. Appendix

6.1 Dimensional Analysis

The proportionality factors of the Stefan-Boltzmann and Wien's laws for black body radiation could not be determined by thermodynamics means alone. It was not until formulation of Planck's quantum theory that this uncertainly could be eliminated.

Similarly, the hypothesis on which this paper is based permits various solutions. However, the number of permissible solutions can be limited by dimensional analysis.

Up until now only the canonical distribution property of the turbulence field model has been used. By introduction of a sound velocity c, a medium density ρ (medium pressure and temperature are already determined by c and therefore represent no new independent parameters) and a characteristic length d, the turbulence field can be further specified.

If dimensional analysis is performed for a model so defined, instead of the formulae

$$J \sim v^8 \qquad (4.48)$$
$$i \sim v^6 g(f/v^2) \qquad (5.33)$$
$$f_M \sim v^2 \qquad (5.34)$$
$$i_M \sim v^6 \qquad (5.35)$$

the expanded relations will be obtained

$$J \sim \rho c^{-5} v^8 \qquad (6.1)$$
$$i \sim \rho c^{-4} d v^6 g(f d c v^{-2}) \qquad (6.2)$$
$$f_M \sim v^2 d^{-1} c^{-1} \qquad (6.3)$$
$$i_M \sim \rho d c^{-4} v^6 \qquad (6.4)$$

The emission formula $J \sim \rho c^5 v^8$ agrees with the known Lighthill law.

6.2 Extension to the non-linearized case

For the derivation of the emission and spectrum formulae, of necessity only low turbulence velocities were permitted.

By means of this limitation it was possible to neglect the higher terms compared to the linear term. In the following part, this limitation shall be eliminated in order to obtain a relation for the emission formula valid for any velocity.

The calculation performed in section 4.3 remains essentially the same. For the emission-pressure Π appearing in equation (4.29).

$$\frac{d\Theta}{\Theta} = \frac{d\Pi}{U+\Pi} \qquad (4.29)$$

only the higher terms of the expansion from to U need be taken into consideration. If the radiation pressure Π is given by a general series

$$\Pi = \frac{1}{3} U + a_2 U^2 + a_3 U^3 \ldots\ldots \qquad (6.5)$$

the differential equation will be obtained with

$$d\Pi = (\frac{1}{3} + 2 a_2 U + 3 a_3 U^2 \ldots\ldots) dU \qquad (6.6)$$

$$\frac{d\Theta}{\Theta} = \frac{(\frac{1}{3} + 2a_2U + 3a_3U^2 \ldots\ldots)dU}{\frac{4}{3} U + a_2U^2 + a_3 U^3 \ldots\ldots} \qquad (6.7)$$

By means of the series

$$U = b_0 + b_1\Theta + b_2\Theta^2 + b_3\Theta^3 \ldots\ldots + b_n\Theta^n \qquad (6.8)$$

the following will be obtained by comparing the coefficients

$$b_0 = b_1 = b_2 = b_3 = 0 \qquad (6.9)$$

$$b_4 = const$$

$$b_5 = b_6 = b_7 = 0$$

$$b_8 = -\frac{21}{4} a_2 b_4^2$$

$$b_9 = b_{10} = b_{11} = 0$$

$$b_{12} = \frac{33}{8} b_4^3 (a_3 - \frac{21}{2} a_2^2)$$

$$b_{13} = b_{14} = b_{15} = 0$$

If, according to equation (3.18) the "turbulence temperature" Θ is replaced by the characteristic turbulence velocity

$$\Theta \sim v^2 \qquad (3.18)$$

and/or

$$\Theta = z v^2 \qquad (6.10)$$

then

$$U = b_4 z^4 v^8 - \frac{21}{4} a_2 b_4^2 z^8 v^{16} + \frac{33}{8} b_4^3 (a_3 - \frac{21}{2} a_2^2) z^{12} v^{24} \ldots\ldots \qquad (6.11)$$

If the density of acoustic energy U is expressed by sound intensity I in accordance with equation (4.1)

$$U = \frac{4\pi}{c} J \qquad (4.1)$$

the emission formula

$$J = \frac{c b_4}{4\pi} (z^4 v^8 - \frac{21}{4} a_2 b_4 z^8 v^{16} + \frac{33}{8} b_4^2 (a_3 - \frac{21}{2} a_2^2) z^{12} v^{24} \ldots\ldots) \qquad (6.12)$$

is obtained.

In the first term of the series expansion it is again possible to see the known Lighthill v^8 law. If the distribution of the sound emission pressure Π in the range of higher intensities - i.e., if the coefficients a_2, a_3, a_4 of the series -

$$\Pi = \frac{1}{3} U + a_2 U^2 + a_3 U^3 \ldots\ldots a_n U^n \qquad (6.5)$$

are known, the emission law valid for any velocity is given.

6.3 Determination of Emission Formula by the Debye-method

To determine the specific heat and/or the thermal energy of a solid, Debye has provided a very elegant method which, in a modified form, can be applied to a turbulence field. According to Debye, the thermal energy of a solid is the oscillation energy of the various natural vibrations. According to Rayleigh and Jeans, the number of these eigenvalues is obtained by counting the possible standing waves.

$$dn = 4\pi V f^2 (\frac{1}{c_l^3} + \frac{2}{c_t^3}) df \qquad (6.13)$$

dn is the number of the natural frequencies of a body with volume V contained in the frequency interval df. c_l and c_t are the propagation velocities of the longitudinal and transverse wave. Since according to Planck every standing wave may be considered an oscillator and the oscillator energy can be calculated by means of the quantum theory, the internal energy of the solid is thus determined. Debye further assumes that the number of oscillators is limited whereby he obtains the T^4 law of internal energy at low temperatures T, named after him, and the Dulong-Petit law applicable to high temperatures.

The turbulence volume shall be handled in a similar way. The number of longitudinal, that is, the acoustic eigenvalues would be obtained by the equation (6.13): $c_l = c$ = velocity of sound. Since, however, no obvious formulation is known for eigenvalue energy, the calculation of the turbulence noise is based on the empirical spectral formula established by P.M. Morse and K. V. Ingard (40), in departure from the Debye method.

This equation established for a maximum is as follows:

$$i\left(\frac{f}{f_M}\right) = \frac{i_M (f/f_M)^2}{\left(\frac{5}{9} + \frac{4}{9}(f/f_M)^{1.5}\right)^3} \qquad (6.14)$$

With regard to the formulae derived in section 5.2

$$f_M \sim v^2 \;\; bzw. \;\; f = k\, v^2 \qquad (5.35)$$
$$i_M \sim v^6$$

the spectral distribution is as follows:

$$i(f,v) \sim \frac{v^6 (f/k v^2)^2}{\left(\frac{5}{9} + \frac{4}{9}(f/k v^2)^{1.5}\right)^3} \qquad (6.15)$$

By analogy with Debye, an upper frequency limit f_G is established and justified in that the high frequency eigenvalues are not fully excited because of the high attenuation. The sound intensity I of the turbulence field results from integration of equation (6.15) from zero in the limit frequency f_G

$$J \approx \int_0^{f_G} i(f,v)\, df$$

$$\sim v^8 \int_0^{f_G/kv^2} \frac{(f/kv^2)^2\, d\, f/kv^2}{\left(\frac{5}{9} + \frac{4}{9}(f/kv^2)^{1.5}\right)^3} \qquad (6.16)$$

For small turbulence speeds v, the integration limit is

$$\frac{f_G}{kv^2} \rightarrow \infty$$

For this purpose the integral in equation (6.16) has a constant value so that

$$J \sim v^8 \qquad for \;\; v \rightarrow 0 \qquad (6.17)$$

For the other extreme case of very high turbulence velocities, that is,

$$\frac{f}{kv^2} \rightarrow 0$$

the spectral function in equation (6.15) becomes

$$\lim_{f/kv^2 \rightarrow 0} \frac{v^6 (f/kv^2)^2}{\left(\frac{5}{9} + \frac{4}{9}(f/kv^2)^{1.5}\right)^3} = \left(\frac{9}{5}\right)^3 \frac{v^2}{k^2} f^2 \qquad (6.18)$$

The total sound intensity I for this case is

$$J \sim \int_0^{f_G} i(f,v)\, df = \left(\frac{9}{5}\right)^3 \frac{v^2}{k^2} \int_0^{f_G} f^2 df$$

$$J \sim v^2 \qquad for \;\; v \rightarrow \infty \qquad (6.19)$$

For very high turbulence velocities, the emitted sound intensity I is proportional to the second power of the turbulence speed, that is, proportional to the turbulence energy. This relation is qualitatively correct, since a proportionality in excess of the second power for high speeds results in the contradiction that the sound energy exceeds the total turbulence energy.

9. Symbols

a_1, a_2	coefficients of a power series
b_0, b_1	coefficients of a power series
c	velocity of sound
d	characteristic length
f	frequency
$g(\;\;)$	unknown function
i	spectral sound intensity
k	Boltzmann constant
k	1, 2, 3 ...
l	length
m	mass of a correlation volume
n	0 1, 2, 3 ...
dn	number of eigenfrequencies
q	impulse coordinates
s	position coordinate
t	time
u	spectral density of sound energy
v	mean, characteristic turbulence velocity
w	piston speed
x, y, z	cartesian coordinates
z	dimensionless number
A	work
E	total energy of canonical system
F	area
G	unknown function
I	total sound intensity
J	Bessel function
N_1, N_2 ...	number of particles with same energy
N	energy
O	area
Q	quantity of sound energy
S	thermodynamic entropy
U	total density of sound energy
V	volume

W	number of possible permutations
α	coefficient of absorption
α	angle of incidence
ε	energy quantity
ε	emission coefficient
φ	free energy
ζ	displacement
ρ	density of medium
ω	angular frequency
Θ	turbulence temperature
Σ	turbulence entropy
Φ	internal sound energy
Ω	solid angle
Π	sound radiation pressure
$\Pi_{\perp}$	sound radiation pressure with vertical incidence

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