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PROPAGATION OF THE SONIC BOOM IN THE REAL ATMOSPHERE

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Abstract

By virtue of linear theory of a sonic boom, the pattern of shock and rarefaction waves occurring in the supersonic flow past an aircraft, may be treated as a wavefront. If intensity of a sonic boom is not analysed, the problem of propagation of a sonic boom is merely geometrical problem.

This paper presents the analitic method of determination of a wavefront and application of that to the following models of an atmosphere: 1/still homogeneous atmosphere, 2/still atmosphere with gradient of sound velocity, 3/still atmosphere with temperature gradient, 4/homogeneous atmosphere with vertical wind velocity distribution and 5/atmosphere with gradient of sound velocity and vertical wind velocity distribution.

I. Introduction

Sonic boom is caused by the shock waves occurring when an aircraft /rocket, projectile/ flies at the supersonic speed. As it is well known, sonic boom may be a danger to the other aircraft in the air and to the people and objects on the ground. In order to determine the range of a sonic boom it is necessary to know how do the shock waves propagate. Propagation of the shock waves depends on the history of a flight and atmospheric conditions, mainly on the temperature gradient and wind velocity distribution.

General theory of a sonic boom is presented in paper of Warren and Randall(1). In this theory there are introduced the notions of a wavefront and rays. The ray may be treated as a path of element of a wavefront. Influence of temperature gradient on propagation of a sonic boom was investigated in above mentioned paper, in another paper of Warren(2) and by author of present paper(3), (4) as well. Influence of wind velocity was considered by Dressler and Fredholm(5).

The aim of the present paper is the investigation of influence of temperature gradient and vertical wind velocity distribution on propagation of a sonic boom. It is assumed that a flight path lies in the vertical plane that is parallel to the vector of wind velocity. Propagation of a sonic boom is considered merely in the plane of a flight path. Intensity of a sonic boom is not analysed.

II. Symbols

a -sound velocity
a_A-value of a at altitude of aircraft
$\bar{a} = a/a_A$
k -non-dimensional coefficient /Table 1/
l -coefficient /Table 1/
t -time
$\bar{t}$ -non-dimensional time /Table 1/
t_A-time corresponding to the certain position of an aircraft
$w = w_G - w_A$
w_A-value of w_G corresponding to the altitude of aircraft
w_G -wind velocity relative to the ground
$\bar{w} = w/a_A$
x, y -Cartesian coordinates moving at the speed w_A relative to the ground
$\bar{x}, \bar{y}$ -non-dimensional coordinates /Table 1/
x_A -horizontal coordinate of an aircraft
x_G -horizontal axis connected with the ground

$C = 1/\sin(\alpha \mp \Gamma)$, where upper and lower signs apply to the regions, respectively, above and under a flight path

H -altitude of an aircraft

H_0-altitude of an atmosphere with temperature gradient /Table 1/

$M = U/a_A$ -Mach number of an aircraft

U -flight speed relative to the air

V -normal velocity of a wavefront that is measured in x, y-coordinates

α = arc sin 1/M -Mach angle

$\varepsilon = \pm 1$, where upper and lower signs apply, respectively, to the aft- and headwind

Γ-angle of climb

φ-angle between normal to the wavefront and x-axis

ψ-angle between tangent to the ray and x-axis

ψ_A-value of ψ corresponding to the altitude of an aircraft

III. Assumptions

In atmosphere there is a vertical temperature gradient, i.e. a=a/y/ and vertical wind velocity distribution, i.e. $w_G = w_G/y/$. The value of w_G at altitude of an aircraft is equal to w_A. The flight path lies in the vertical plane /x, y/, x-axis is horizontal and moves at the speed w_A relative to the ground. In x, y-coordinates the wind velocity is equal to $w/y/ = w_G/y/ - w_A$. Vector of wind velocity lies in /x, y/-plane.

At the instant t_A an aircraft is at the point $/x_A, 0/$, its Mach number is equal to $M \geqslant 1$, Mach angle α, altitude H and angle of climb Γ. According to the definition of a ray, at the point $/x_A, 0/$ the angle between the tangents to a ray and a flight path is equal to $/\frac{\pi}{2} - \alpha/$, the angle between the tangent to a ray and x-axis is equal to /Figure 1/

$$\psi_A = \pm\left[\frac{\pi}{2} - (\alpha \mp \Gamma)\right] \qquad (1)$$

where the upper and lower signs apply to the wavefronts propagating, respectively, above and under the tangent to a flight path.

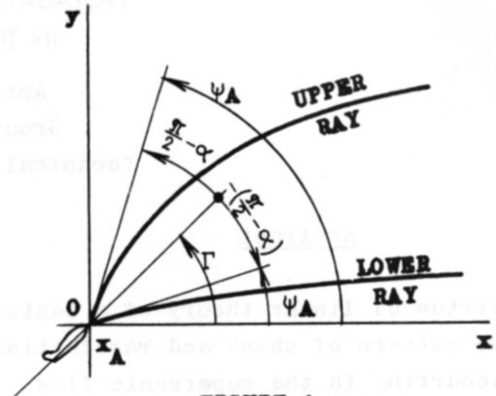

FIGURE 1.

IV. General Snell's Law

Let us consider Figure 2. The normal velocity of a wavefront measured in x, y-coordinates is equal to

$$V = a + w \cos\varphi \qquad (2)$$

One exists the relation between V and φ [5]. After suitable modifications this relation may be expressed in the form

$$V/\cos\varphi = \text{const} \qquad (3)$$

and that is fulfilled along a ray. Substituting (2) to (3) and calculating the constant from the condition that at the beginning of a ray there is w=0, a=0 and $\varphi = \psi = \psi_A$, we get the following formula that will be called the general Snell's law

$$a/\cos\varphi + w = a_A/\cos\psi_A \qquad (4)$$

where ψ_A is determined by (1). The rays are not orthogonal to the wavefront and they are the segments of space lines. Two rays for that the angle ψ_A lies in /x, y/-plane are the segments of plane lines; these rays will be called, respectively, upper and lower ray /Figure 1/.

If w=0, then formula (4) takes the form of ordinary Snell's law

$$a/\cos\Psi = a_A/\cos\Psi_A$$

In this case the rays are orthogonal to the wavefront and they are the segments of plane lines lying in the vertical planes.

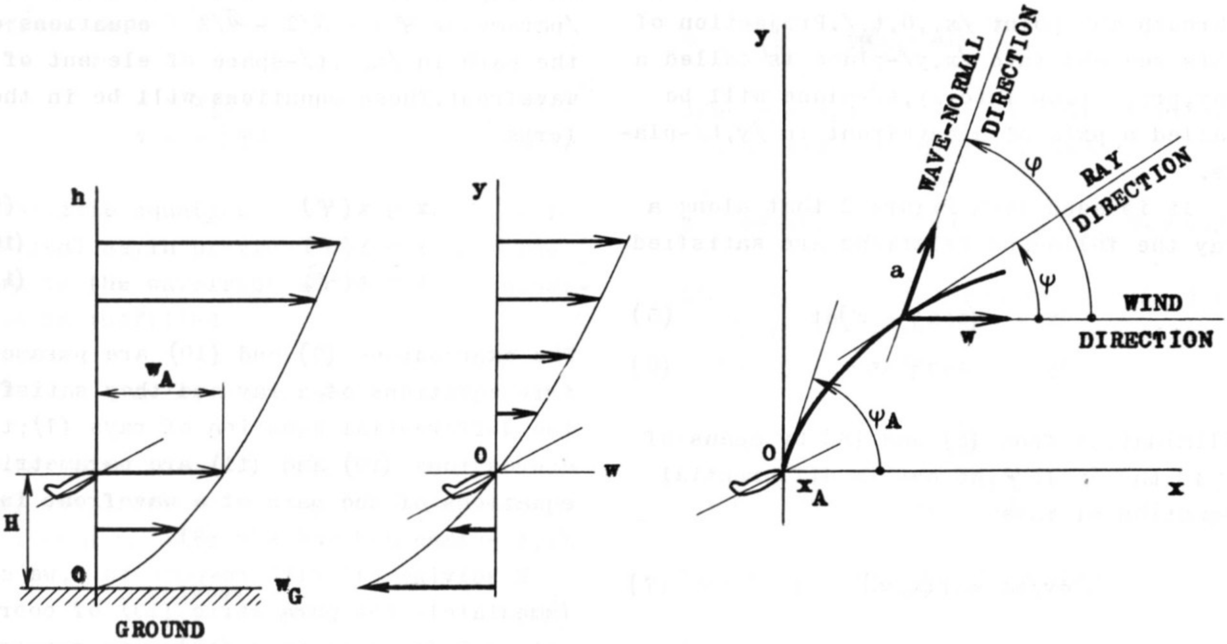

FIGURE 2.

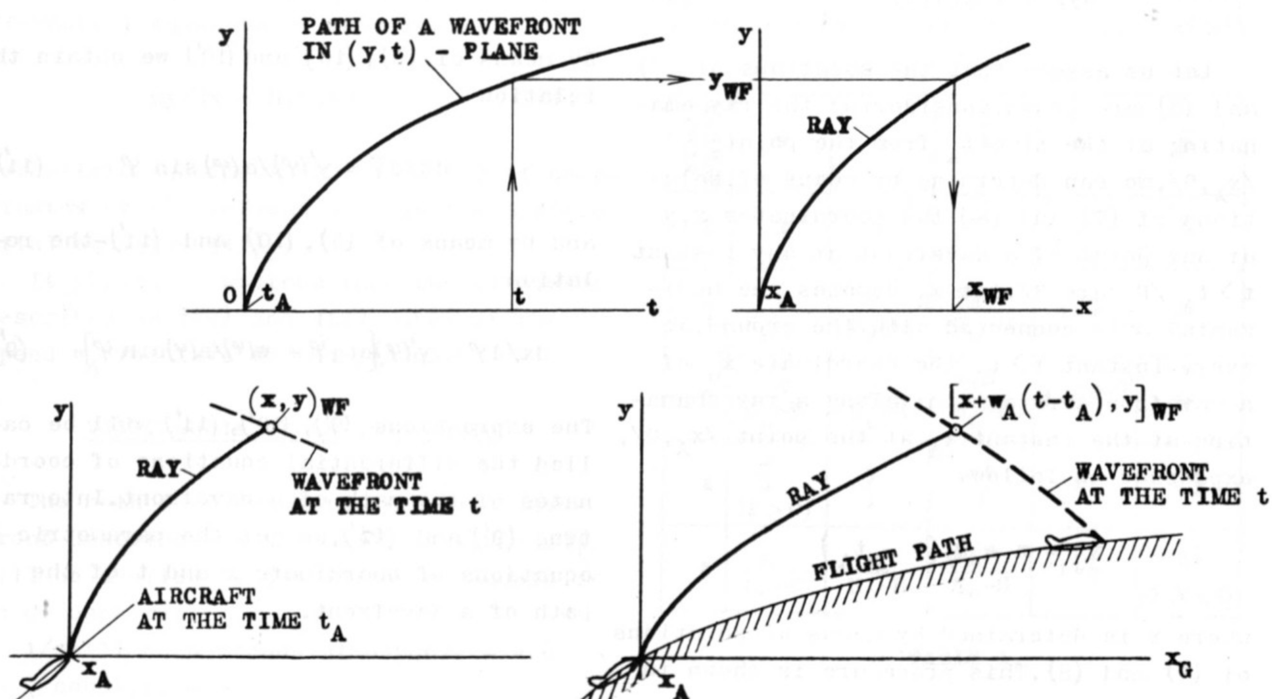

FIGURE 3

V. Method of Determination of a Wavefront

In $/x,y,t/$-space the path of element of a wavefront is the segment of line going through the point $/x_A,0,t_A/$. Projection of this segment into $/x,y/$-plane is called a ray, projection into $/y,t/$-plane will be called a path of a wavefront in $/y,t/$-plane.

It is seen from Figure 2 that along a ray the following relations are satisfied

$$dx = a(\cos\varphi + w)dt \tag{5}$$

$$dy = a \sin\varphi \, dt \tag{6}$$

Eliminating from (5) and (6) by means of (4) the angle φ, we obtain differential equation of rays

$$dy/dx = f(x,y) \tag{7}$$

and differential equation of paths of a wavefront in $/y,t/$-plane

$$dy/dt = g(y,t) \tag{8}$$

Let us assume that the solutions of (7) and (8) are known. Considering the ray emanating at the time t_A from the point $/x_A,0/$, we can determine by means of solutions of (7) and (8) the coordinates x,y of any point of a wavefront at any instant $t > t_A$ /Figure 3/. Let x_G denotes the horizontal axis connected with the ground. At every instant $t > t_A$ the coordinate x_G of a wavefront propagating along a ray emanating at the instant t_A at the point $/x_A,0/$, expresses as follows

$$x_G = x + w_A(t - t_A)$$

where x is determined by means of solutions of (7) and (8). This procedure is shown in Figure 3.

VI. Parametric Equations of the Path of Element of a Wavefront

In order to find the solutions of (7) and (8), we shall derive the parametric /parameter $\varphi = -\pi/2 \div \pi/2$ / equations of the path in $/x,y,t/$-space of element of a wavefront. These equations will be in the forms

$$x = x(\varphi) \tag{9}$$

$$y = y(\varphi) \tag{10}$$

$$t = t(\varphi) \tag{11}$$

The expressions (9) and (10) are parametric equations of a ray and they satisfy the differential equation of rays (7); the expressions (10) and (11) are parametric equations of the path of a wavefront in $/y,t/$-plane and satisfy (8).

Resolving (4) with respect to y, we get immediately the parametric equation (10) of coordinate y. Differentiating (10) with respect to φ, we receive the following expression

$$dy/d\varphi = y'(\varphi) \tag{10'}$$

By means of (6), (10) and (10') we obtain the relation

$$dt/d\varphi = y'(\varphi)/a(\varphi)\sin\varphi \tag{11'}$$

and by means of (5), (10) and (11') - the relation

$$dx/d\varphi = y'(\varphi)\left[\operatorname{ctg}\varphi + w(\varphi)/a(\varphi)\sin\varphi\right] \tag{9'}$$

The expressions (9'), (10'), (11') will be called the differential equations of coordinates of the path of a wavefront. Integrating (9') and (11'), we get the parametric equations of coordinate x and t of the path of a wavefront.

VII. Parametric Equations of a Wavefront in Straight Level Flight at Constant Mach Number

In this case of flight we shall derive directly the parametric equations of a wavefront in the forms

$$x = x_1(\varphi) \qquad (12)$$
$$y = y(\varphi) \qquad (10)$$

Parametric equation of coordinate y is identical as in previous heading. With regard to the wavefront the following condition is fulfilled

$$dy/dx = -\mathrm{ctg}\,\varphi \qquad (13)$$

Taking into account (10′), we obtain from (13) the differential equation of coordinate x of a wavefront

$$dx/d\varphi = -y'(\varphi)\,\mathrm{tg}\,\varphi \qquad (12')$$

and after integrating - the parametric equation (12) of coordinate x of a wavefront. By means of (4) and (13) we derive the differential equation of wavefronts

$$dy/dx = h(x,y) \qquad (14)$$

Parametric equations (10) and (12) of coordinates of a wavefront satisfy the differential equation (14).

It should be noticed that the wavefront described by (10) and (12) moves at the speed w_A relative to the ground.

VIII. Models of an Atmosphere

The method of determination of a wavefront described in the headings V, VI and VII, will be applied to the following models of an atmosphere:

1/still homogeneous atmosphere - $a = \mathrm{const}, w_G = 0$

2/still atmosphere with gradient of sound velocity

$$a = a_A\left[1 - y/(H_0 - H)\right], \qquad w_G = 0$$

3/still atmosphere with temperature gradient

$$a = a_A\left[1 - y/(H_0 - H)\right]^{1/2}, \qquad w_G = 0$$

4/homogeneous atmosphere with vertical wind velocity distribution

$$a = \mathrm{const}, \quad w_G = \varepsilon\, a(y + H)/l, \quad w = \varepsilon\, ay/l$$

5/atmosphere with gradient of sound velocity and vertical wind velocity distribution

$$a = a_A\left[1 - y/(H_0 - H)\right]$$

$$w_G = \varepsilon\, a_A k(y + H)/(H_0 - H), \quad w = \varepsilon\, a_A ky/(H_0 - H)$$

IX. Propagation of a Sonic Boom

With regard to the models 2,3,4,5 of an atmosphere it will be introduced non-dimensional quantities: coordinates $\bar{x}, \bar{y}$; time $\bar{t}$, sound velocity $\bar{a}$, wind velocity $\bar{w}$; their scales are tabulated in Table 1. We shall assume the notation

ATMOSPHERE	$\bar{x}$	$\bar{y}$	$\bar{t}$	$\bar{a}$	$\bar{w}$
1	x	y	t	a	0
2	$\frac{x}{H_0-H}$	$\frac{y}{H_0-H}$	$\frac{a_A t}{H_0-H}$	$1-\bar{y}$	0
3	$\frac{x}{H_0-H}$	$\frac{y}{H_0-H}$	$\frac{a_A t}{H_0-H}$	$(1-\bar{y})^{1/2}$	0
4	$\frac{x}{l}$ $(l>0)$	$\frac{y}{l}$	$\frac{at}{l}$	1	$\varepsilon\bar{y}$
5	$\frac{x}{H_0-H}$	$\frac{y}{H_0-H}$	$\frac{a_A t}{H_0-H}$	$1-\bar{y}$	$\varepsilon k\bar{y}$ $(0<k\leqslant 1)$

TABLE 1.

$$C = 1/\sin(\alpha \mp \Gamma) \qquad (15)$$

where upper and lower signs apply, respectively, to the upper and lower ray.

In Table 2 there are tabulated the differential equations of rays (7), paths of a wavefront in /y,t/-plane (8) and wavefronts (14). Equations (7) and (8) correspond to any flight, quantity C defined by (15) may change along a flight path.

In the case of model 1 of an atmosphere the equation (14) corresponds to the straight flight at constant Mach number and there is C = const along a flight path. For models 2,3,4,5 the equation (14) corresponds to the straight level flight at constant Mach number and C = M = const along a flight path.

In Table 3 there are represented the differential equations (9'), (10'), (11') and

ATMOSPHERE	DIFFERENTIAL EQUATIONS OF RAYS $d\bar{y}/d\bar{x} = \pm$	PATHS OF A WAVEFRONT IN $(\bar{y},\bar{t})$- PLANE $d\bar{y}/d\bar{t} = \pm$	WAVEFRONTS (C=CONST, M=CONST) $d\bar{y}/d\bar{x} = \mp$
1	$(C^2 - 1)^{1/2}$	$a\left(1 - \frac{1}{C^2}\right)^{1/2}$	$(C^2 - 1)^{-1/2}$
2	$\left[\frac{C^2}{(1-\bar{y})^2} - 1\right]^{1/2}$	$(1-\bar{y})\left[1 - \frac{(1-\bar{y})^2}{C^2}\right]^{1/2}$	$\left[\frac{M^2}{(1-\bar{y})^2} - 1\right]^{-1/2}$
3	$\left(\frac{C^2}{1-\bar{y}} - 1\right)^{1/2}$	$\left[(1-\bar{y})\left(1 - \frac{1-\bar{y}}{C^2}\right)\right]^{1/2}$	$\left[\frac{M^2}{1-\bar{y}} - 1\right]^{-1/2}$
4	$\frac{\left[(C - \varepsilon\bar{y})^2 - 1\right]^{1/2}}{\varepsilon\bar{y}(C - \varepsilon\bar{y}) + 1}$	$\left[1 - \frac{1}{(C - \varepsilon\bar{y})^2}\right]^{1/2}$	$\left[(M - \varepsilon\bar{y})^2 - 1\right]^{-1/2}$
5	$\frac{(1-\bar{y})\left[(C - \varepsilon k\bar{y})^2 - (1-\bar{y})^2\right]^{1/2}}{(1-\bar{y})^2 + \varepsilon k\bar{y}(C - \varepsilon k\bar{y})}$	$(1-\bar{y})\left[1 - \frac{(1-\bar{y})^2}{(C-\varepsilon k\bar{y})^2}\right]^{1/2}$	$\left[\frac{(M - \varepsilon k\bar{y})^2}{(1-\bar{y})^2} - 1\right]^{-1/2}$

TABLE 2.

ATMOSPHERE	DIFFERENTIAL EQUATIONS OF COORDINATES OF PATH OF A WAVEFRONT			WAVEFRONT (M = CONST)	
	$d\bar{x}/d\varphi =$	$d\bar{y}/d\varphi =$	$d\bar{t}/d\varphi =$	$d\bar{x}/d\varphi =$	$d\bar{y}/d\varphi =$
2	$C\cos\varphi$	$C\sin\varphi$	$1/\cos\varphi$	$-M\sin^2\varphi/\cos\varphi$	$M\sin\varphi$
3	$2C^2\cos^2\varphi$	$C^2\sin 2\varphi$	$2C$	$-2M^2\sin^2\varphi$	$M^2\sin 2\varphi$
4	$\varepsilon(\sin^2\varphi - C\cos\varphi)/\cos^3\varphi$	$-\varepsilon\sin\varphi/\cos^2\varphi$	$-\varepsilon/\cos^2\varphi$	$-\varepsilon\sin^2\varphi/\cos^3\varphi$	$-\varepsilon\sin\varphi/\cos^2\varphi$
5	$\frac{\varepsilon k(\sin^2\varphi - C\cos\varphi) + C\cos^2\varphi}{\cos\varphi(1-\varepsilon k\cos\varphi)^2}$	$\frac{(C-\varepsilon k)\sin\varphi}{(1-\varepsilon k\cos\varphi)^2}$	$\frac{1}{\cos\varphi(1-\varepsilon k\cos\varphi)}$	$\frac{(M-\varepsilon k)\sin^2\varphi}{\cos\varphi(1-\varepsilon k\cos\varphi)^2}$	$\frac{(M-\varepsilon k)\sin\varphi}{(1-\varepsilon k\cos\varphi)^2}$

TABLE 3.

ATMO-SPHERE	SOLUTIONS OF DIFFERENTIAL EQUATIONS OF					
	RAYS		PATHS OF A WAVEFRONT IN $(\bar{y},\bar{t})$ - PLANE		WAVEFRONTS $(C=CONST, M=CONST)$	
	PARAMETRICAL SOLUTIONS	OTHER SOLUTIONS	PARAMETRICAL SOLUTIONS	OTHER SOLUTIONS	PARAMETRICAL SOLUTIONS	OTHER SOLUTIONS
1		$y=\pm(C^2-1)^{1/2}x$ STRAIGHT LINE		$y=\pm a\left(1-\frac{1}{C^2}\right)^{1/2}t$ STRAIGHT LINE		$y=\mp(C^2-1)^{-1/2}x$ STRAIGHT LINE
2	$\bar{x} = C\sin\varphi$ $\bar{y} = 1-C\cos\varphi$ CRCLE	$\bar{x} = 0$ WHEN $C = \infty$ STRAIGHT LINE	$\bar{t} = \ln tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)$ $\bar{y} = 1 - C\cos\varphi$	$\bar{t} = \mp\ln(1-\bar{y})$ WHEN $C = \infty$	$\bar{x}=M\left[\sin\varphi-\ln tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\right]$ $\bar{y} = 1- M\cos\varphi$ TRACTRIX	$\bar{y} = 0$ WHEN $M = \infty$ STRAIGHT LINE
3	$\bar{x} = \frac{C^2}{2}(2\varphi+ \sin 2\varphi)$ $\bar{y} = 1-C^2\cos^2\varphi$ CYCLOID	$\bar{x} = 0$ WHEN $C = \infty$ STRAIGHT LINE	$\bar{t} = 2C\varphi$ $\bar{y} = 1-C^2\cos^2\varphi$	$\bar{t}=\mp 2(1-\bar{y})^{1/2}$ WHEN $C = \infty$ PARABOLA	$\bar{x}= \frac{M^2}{2}(\sin 2\varphi-2\varphi)$ $\bar{y}= 1-M^2\cos^2\varphi$ CYCLOID	$\bar{y} = 0$ WHEN $M = \infty$ STRAIGHT LINE
4	$\bar{x}= \frac{\varepsilon}{2}\left[tg\varphi\left(\frac{1}{\cos\varphi}-2C\right)-\ln tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\right]$ $\bar{y} = \varepsilon\left(C-\frac{1}{\cos\varphi}\right)$	$\bar{x} = \pm\bar{y}^2/2$ WHEN $C = \infty$ PARABOLA	$\bar{t} = -\varepsilon tg\varphi$ $\bar{y} = \varepsilon\left(C - \frac{1}{\cos\varphi}\right)$	$\bar{y} = \pm\bar{t}$ WHEN $C = \infty$ STRAIGHT LINE	$\bar{x}=\frac{\varepsilon}{2}\left[\frac{\sin\varphi}{\cos^2\varphi}-\ln tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\right]$ $\bar{y} = \varepsilon\left(M - \frac{1}{\cos\varphi}\right)$	$\bar{y} = 0$ WHEN $M = \infty$ STRAIGHT LINE
5 (k=1)	$\bar{x}= (C-2\varepsilon)tg\left[\frac{\pi}{4}(1+\varepsilon)-\frac{\varepsilon\varphi}{2}\right]+\varepsilon\ln tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)$ $\bar{y} = \frac{1 - C\cos\varphi}{1 - \varepsilon\cos\varphi}$	$\bar{y} = 0$ WHEN $C = 1$ AND $\varepsilon = 1$ STRAIGHT LINE; $\bar{x}=\mp\left[\bar{y}+\ln(1-\bar{y})\right]$ WHEN $C = \infty$	$\bar{t} = \ln tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)-tg\left[\frac{\pi}{4}(1+\varepsilon)-\frac{\varepsilon\varphi}{2}\right]$ $\bar{y} = \frac{1 - C\cos\varphi}{1 - \varepsilon\cos\varphi}$	$\bar{y} = 0$ WHEN $C = 1$ AND $\varepsilon = 1$ STRAIGHT LINE; $\bar{t} = \mp\ln(1-\bar{y})$ WHEN $C = \infty$	$\bar{x}= (M-\varepsilon)\left\{2tg\left[\frac{\pi}{4}(1+\varepsilon)-\frac{\varepsilon\varphi}{2}\right]-\ln tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\right\}$ $\bar{y} = \frac{1 - M\cos\varphi}{1 - \varepsilon\cos\varphi}$	$\bar{x} = 0$ WHEN $M = 1$ AND $\varepsilon = 1$ STRAIGHT LINE; $\bar{y} = 0$ WHEN $M=\infty$ STRAIGHT LINE

TABLE 4.

$(12')$ of coordinates of the path of a wavefront and coordinates of a wavefront /C=M const along a flight path/. In Table 4 there are tabulated the solutions of differential equations (7), (8) and (14). In the case of model 5 of an atmosphere one can obtain the parametric /parameter φ / solutions merely for $k \leqslant 1$. In Table 4 there are represented the parametric solutions for $k=1$.

In Table 5 that concerns the model 1 of an atmosphere there are shown the solutions of differential equations of rays, paths of a wavefront in /y,t/-plane and wavefront /C=const along a flight path/ and the examples of determining of ray, path of a wavefront in /y,t/-plane and wavefront. In Tables 6 ÷ 13 there are shown in the similar way the results concerning the models 2,3,4,5 of an atmosphere.

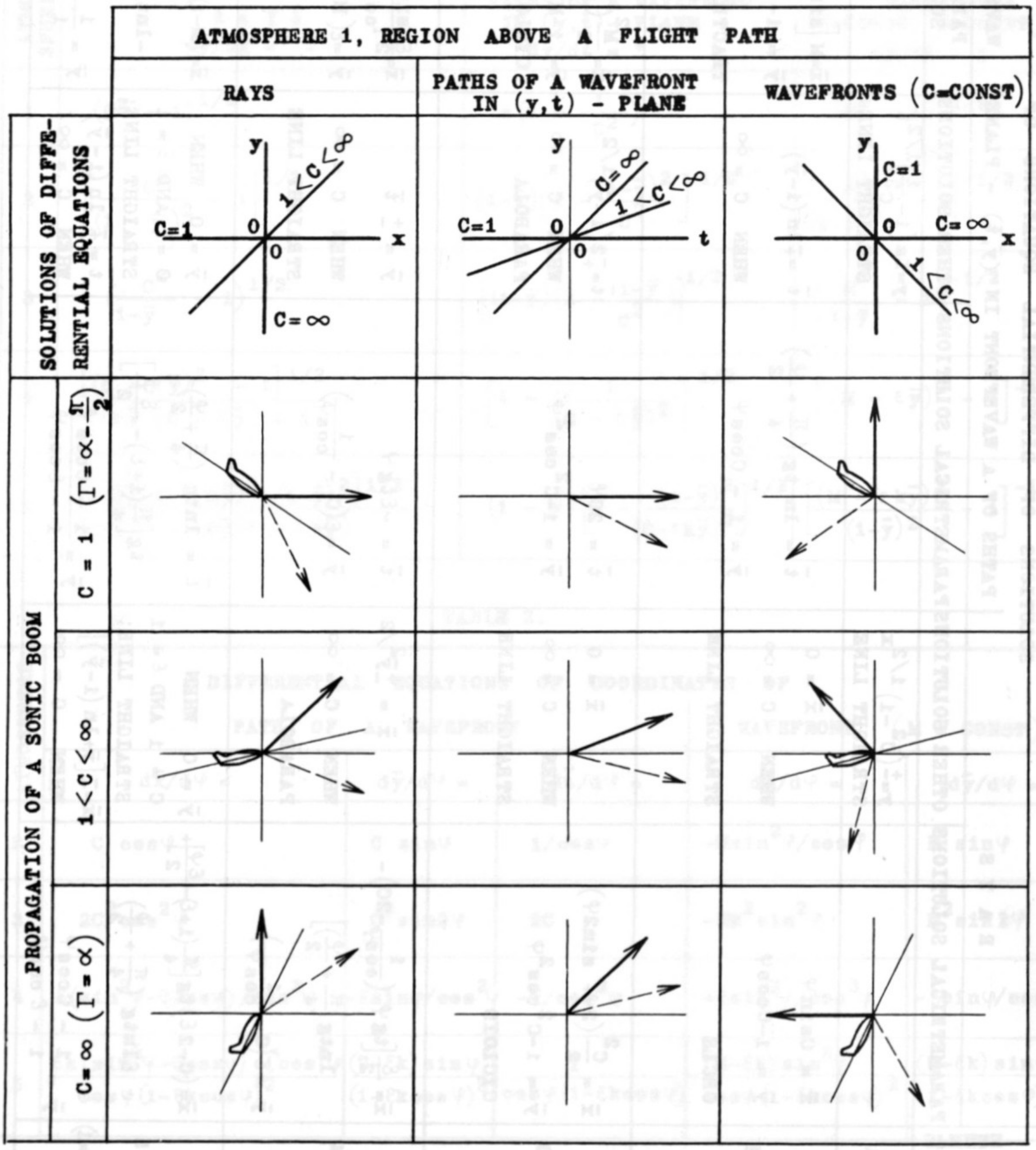

TABLE 5.

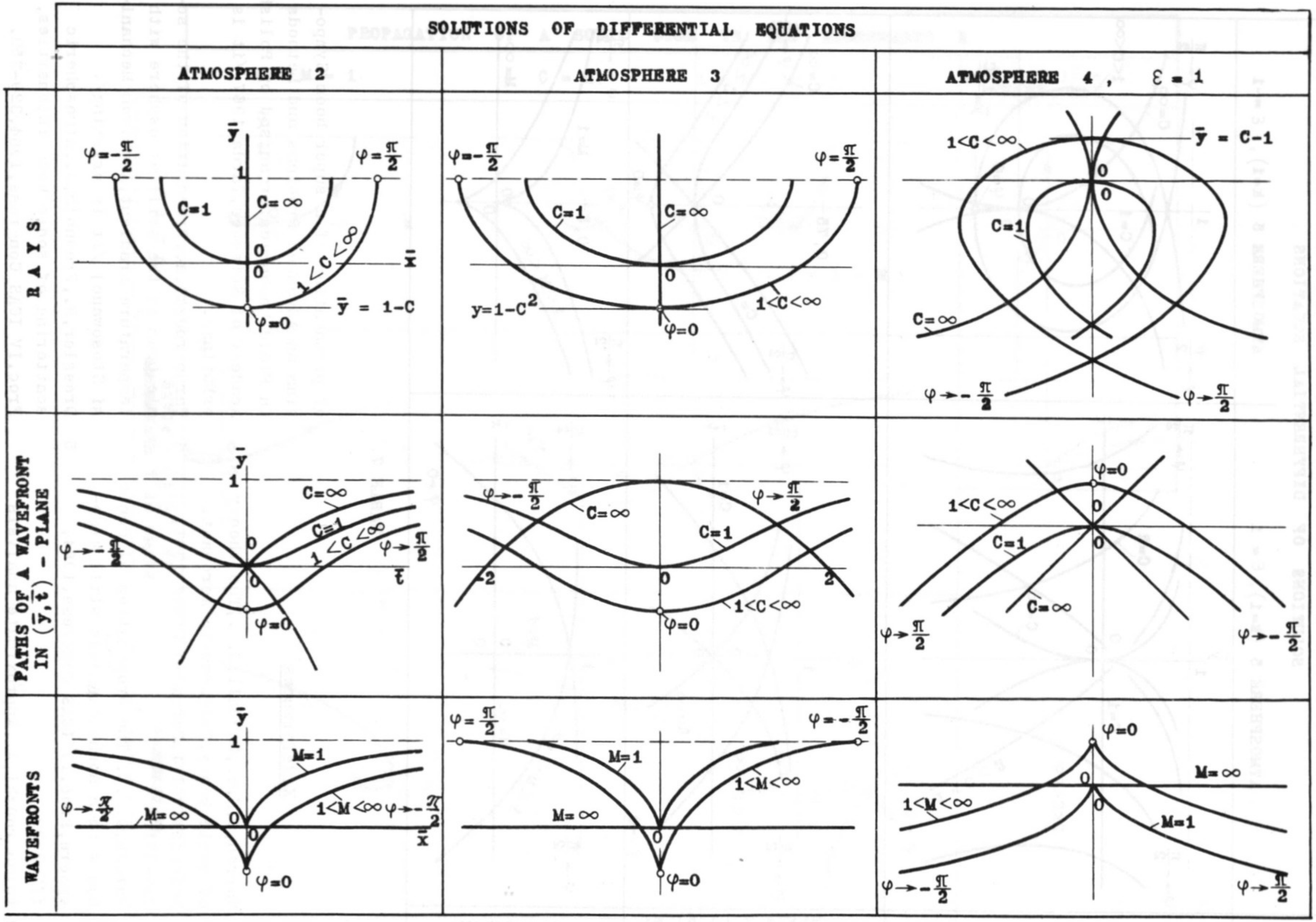
SOLUTIONS OF DIFFERENTIAL EQUATIONS
ATMOSPHERE 2
ATMOSPHERE 3
ATMOSPHERE 4 , ε = 1
R A Y S
PATHS OF A WAVEFRONT IN $(\bar{y}, \bar{t})$ - PLANE
WAVEFRONTS
$\varphi = -\frac{\pi}{2}$
$\varphi = \frac{\pi}{2}$
C=1
C=∞
1<C<∞
$\bar{y}$ = 1-C
φ=0
y=1-C²
$\bar{y}$ = C-1
$\varphi \to -\frac{\pi}{2}$
$\varphi \to \frac{\pi}{2}$
M=1
M=∞
1<M<∞

TABLE 6.

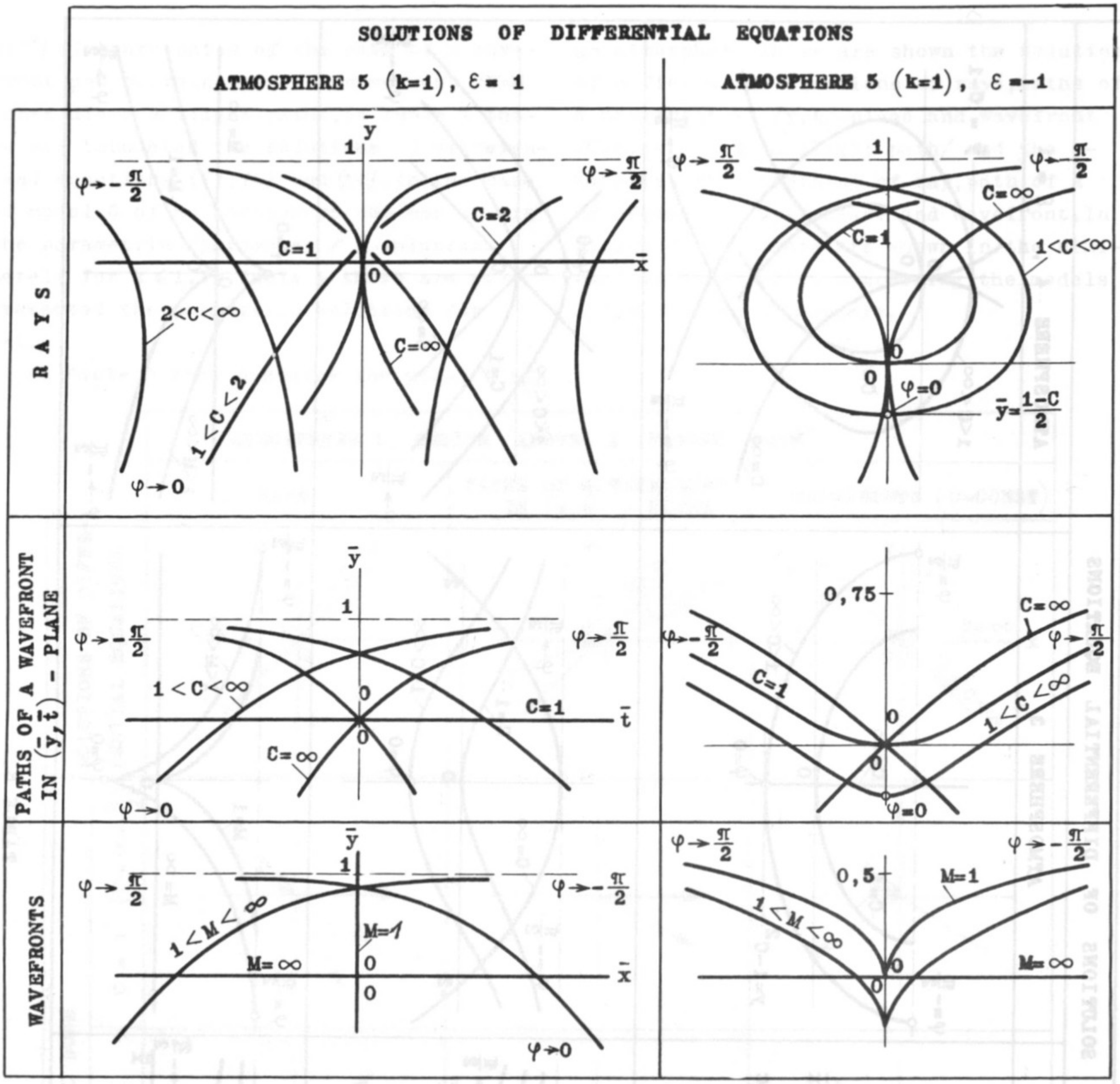

TABLE 7.

X. References

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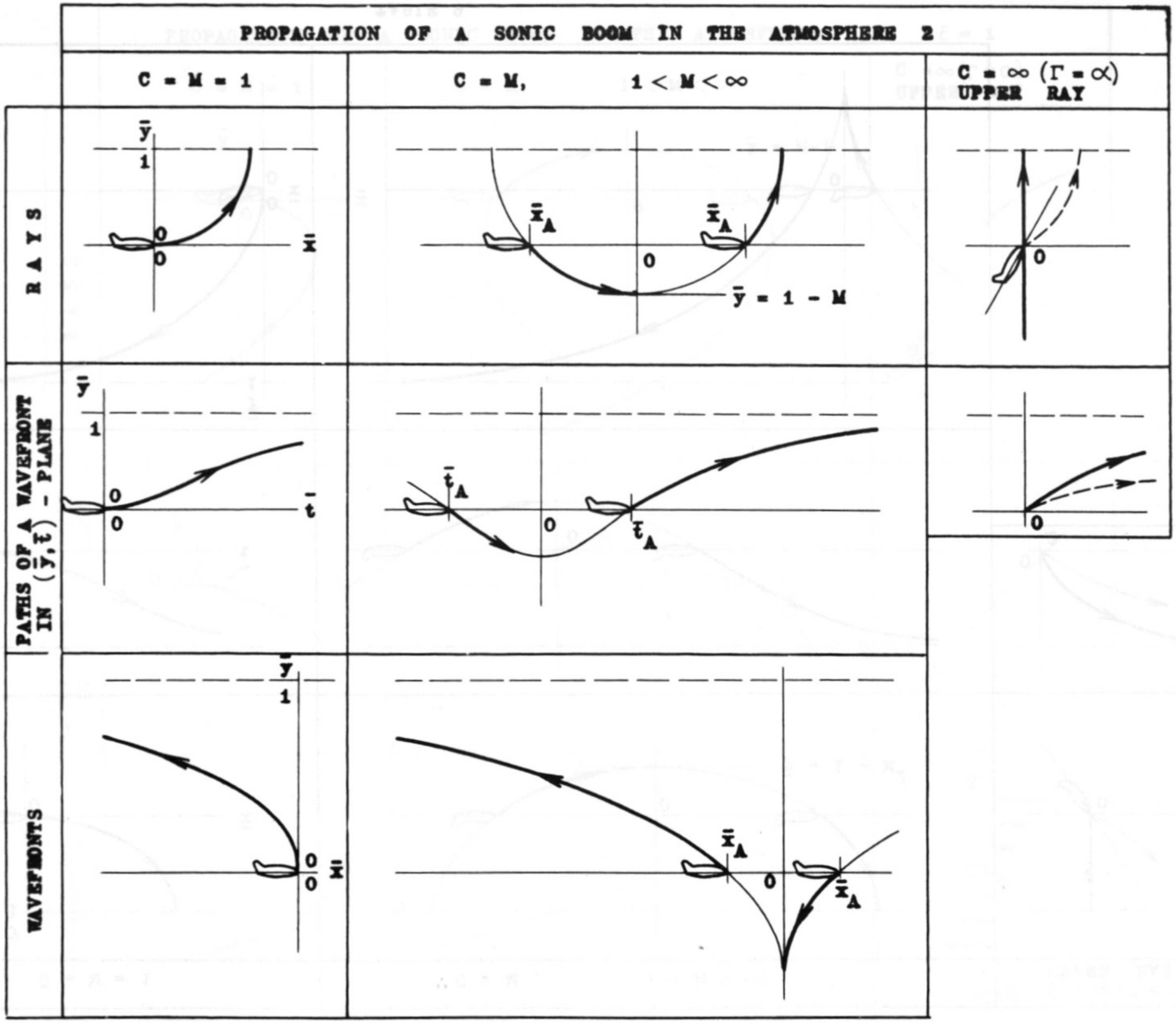
PROPAGATION OF A SONIC BOOM IN THE ATMOSPHERE 2
C = M = 1
C = M, 1 < M < ∞
C = ∞ (Γ = α)
UPPER RAY
R A Y S
PATHS OF A WAVEFRONT
IN ($\bar{y}$, $\bar{t}$) - PLANE
WAVEFRONTS
$\bar{y}$
1
0
$\bar{x}$
$\bar{x}_A$
$\bar{y} = 1 - M$
$\bar{t}$
$\bar{t}_A$

TABLE 8.

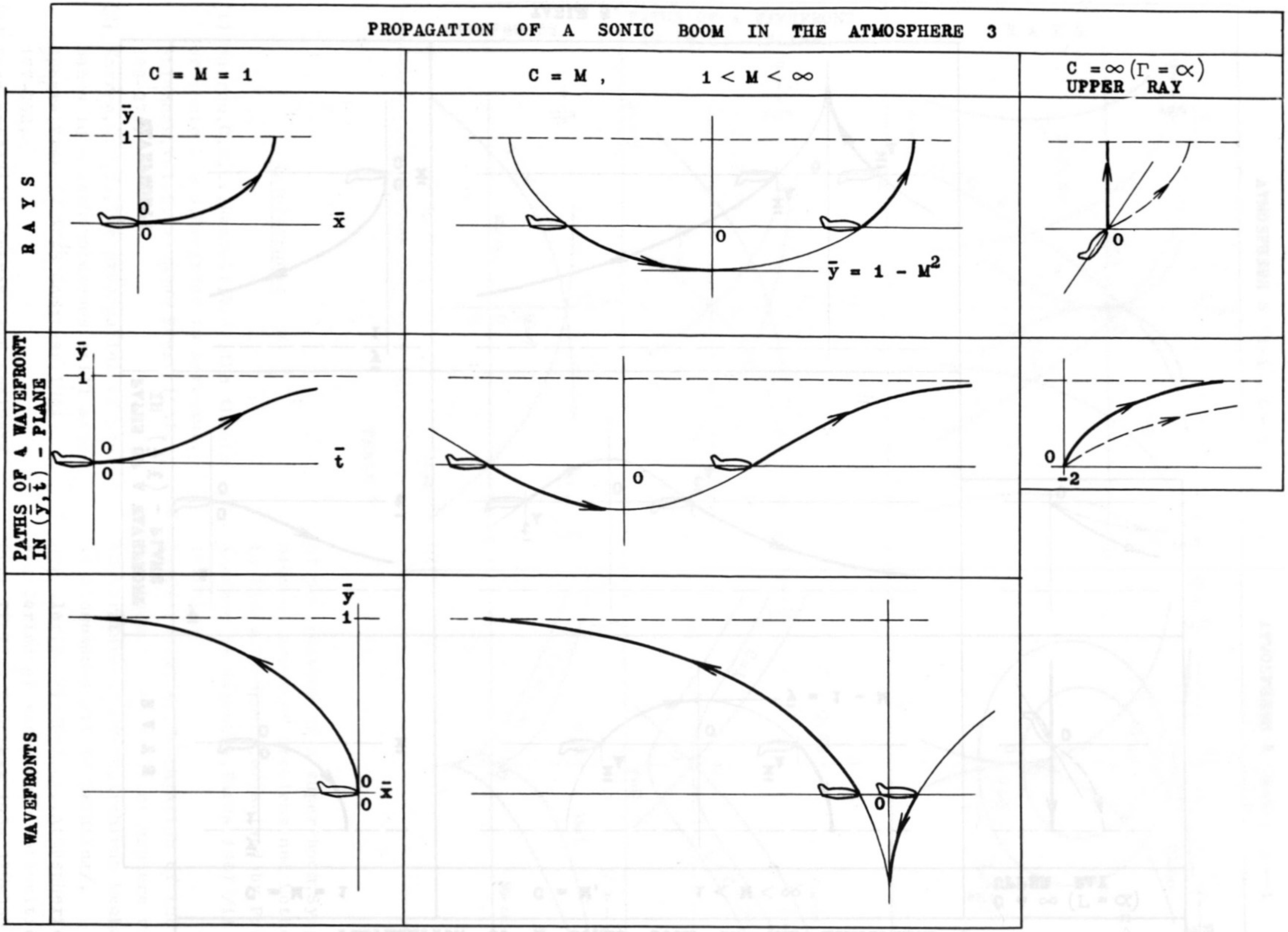
PROPAGATION OF A SONIC BOOM IN THE ATMOSPHERE 3
C = M = 1
C = M , 1 < M < ∞
C = ∞ (Γ = α)
UPPER RAY
R A Y S
PATHS OF A WAVEFRONT IN ($\bar{y}$, $\bar{t}$) - PLANE
WAVEFRONTS
$\bar{y}$
1
0
$\bar{x}$
$\bar{t}$
$\bar{y}$ = 1 - M^2
-2

TABLE 9.

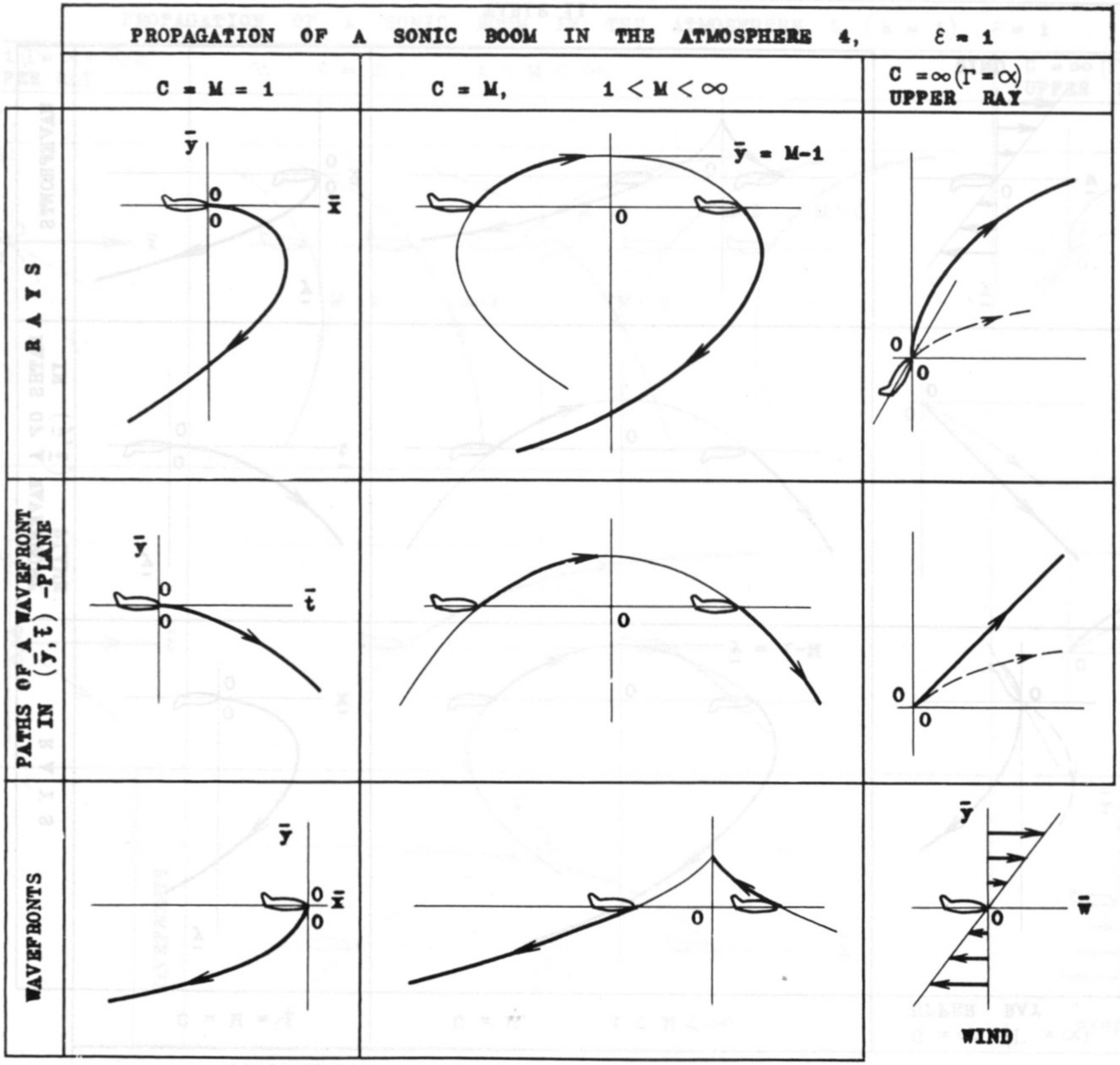
PROPAGATION OF A SONIC BOOM IN THE ATMOSPHERE 4, $\varepsilon = 1$
$C = M = 1$
$C = M, \quad 1 < M < \infty$
$C = \infty \, (\Gamma = \alpha)$
UPPER RAY
RAYS
PATHS OF A WAVEFRONT IN $(\bar{y}, \bar{t})$ -PLANE
WAVEFRONTS
$\bar{y}$
$\bar{x}$
$\bar{t}$
$\bar{y} = M-1$
0
$\bar{w}$
WIND

TABLE 10.

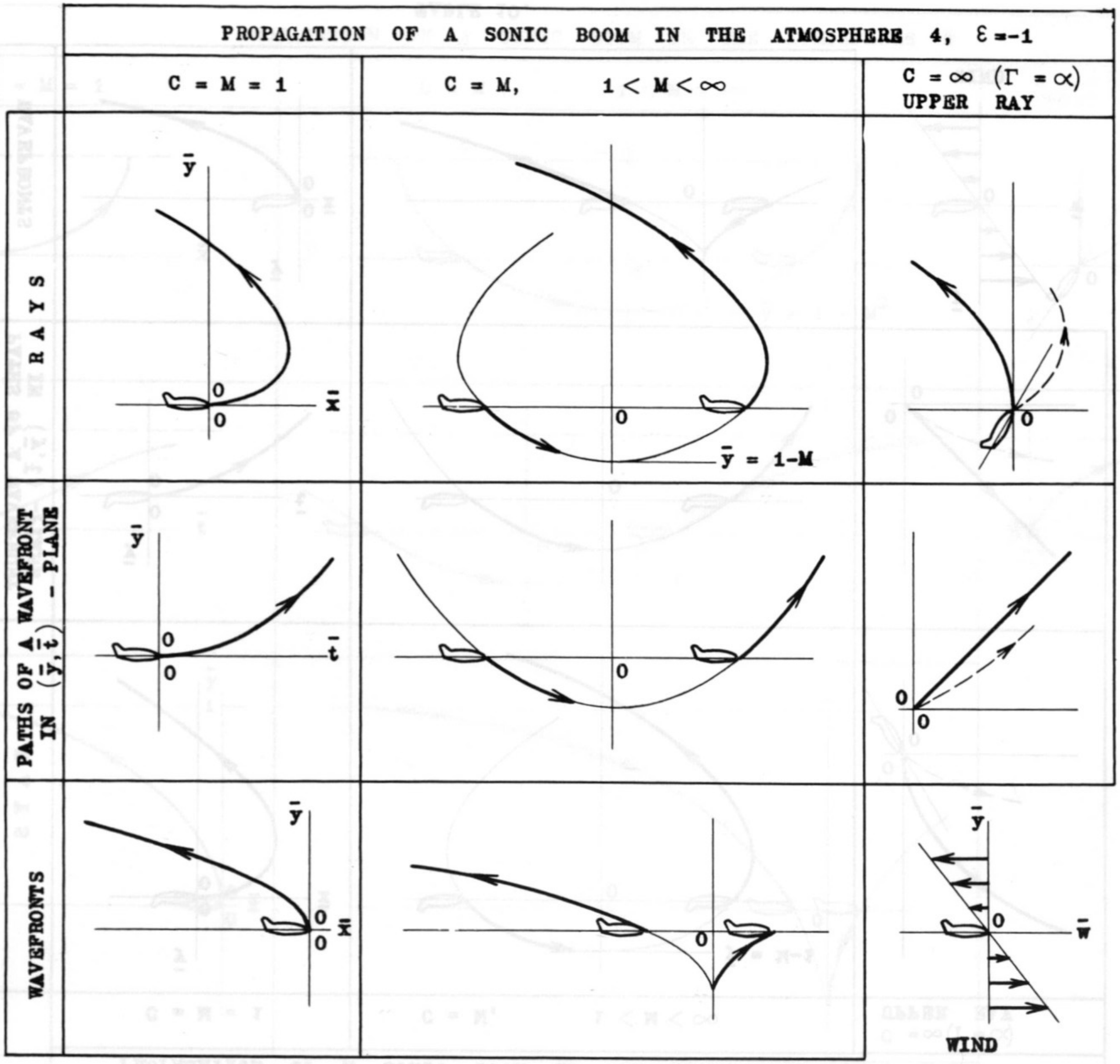
PROPAGATION OF A SONIC BOOM IN THE ATMOSPHERE 4, ε = -1
C = M = 1
C = M, 1 < M < ∞
C = ∞ (Γ = ∝)
UPPER RAY
R A Y S
PATHS OF A WAVEFRONT IN (ȳ, t̄) - PLANE
WAVEFRONTS
ȳ = 1-M
WIND

TABLE 11.

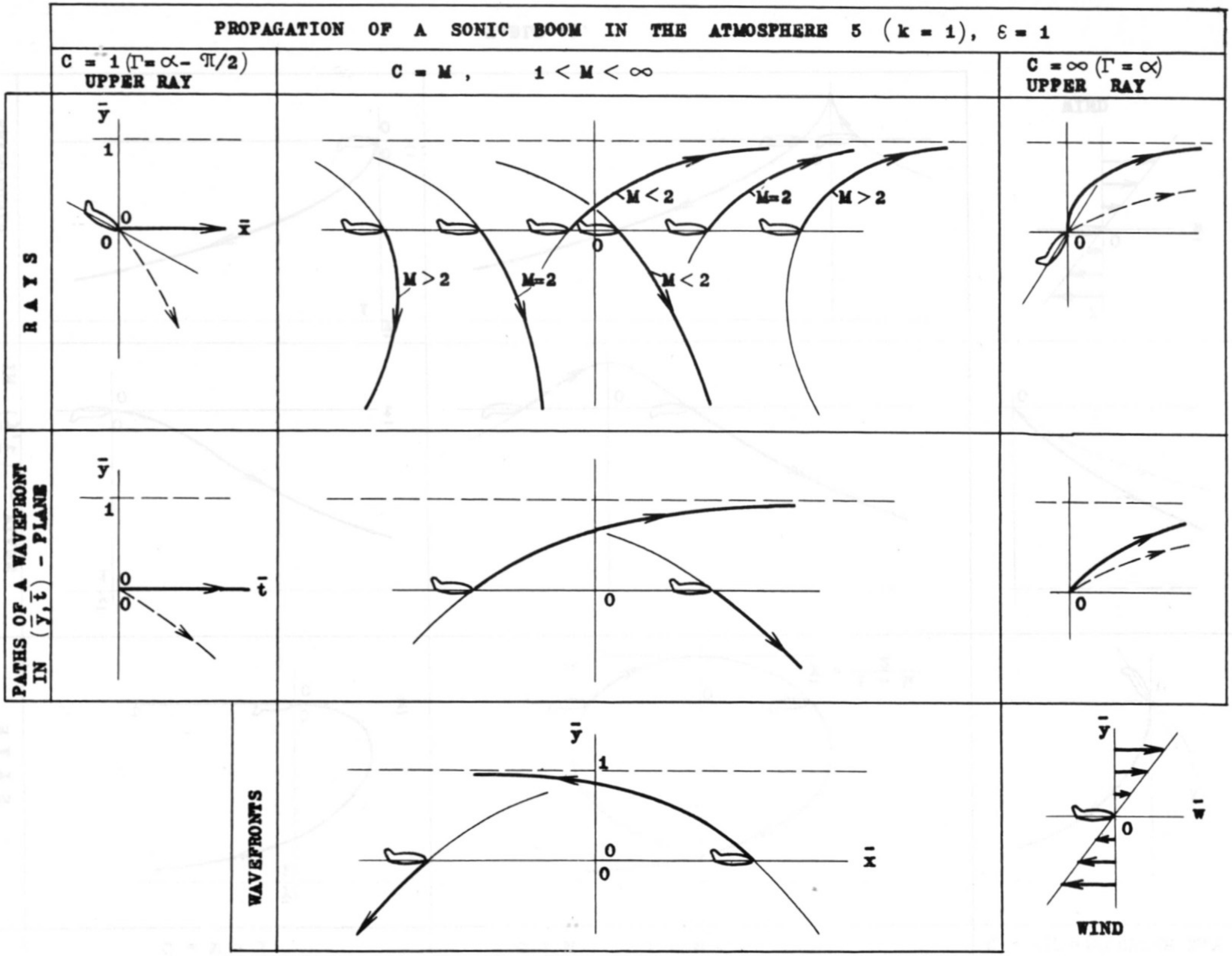
PROPAGATION OF A SONIC BOOM IN THE ATMOSPHERE 5 (k = 1), ε = 1
C = 1 (Γ = α − π/2)
UPPER RAY
C = M , 1 < M < ∞
C = ∞ (Γ = α)
UPPER RAY
RAYS
PATHS OF A WAVEFRONT IN (ȳ, t̄) - PLANE
WAVEFRONTS
WIND
M > 2
M = 2
M < 2
ȳ
x̄
t̄
w̄
1
0

TABLE 12.

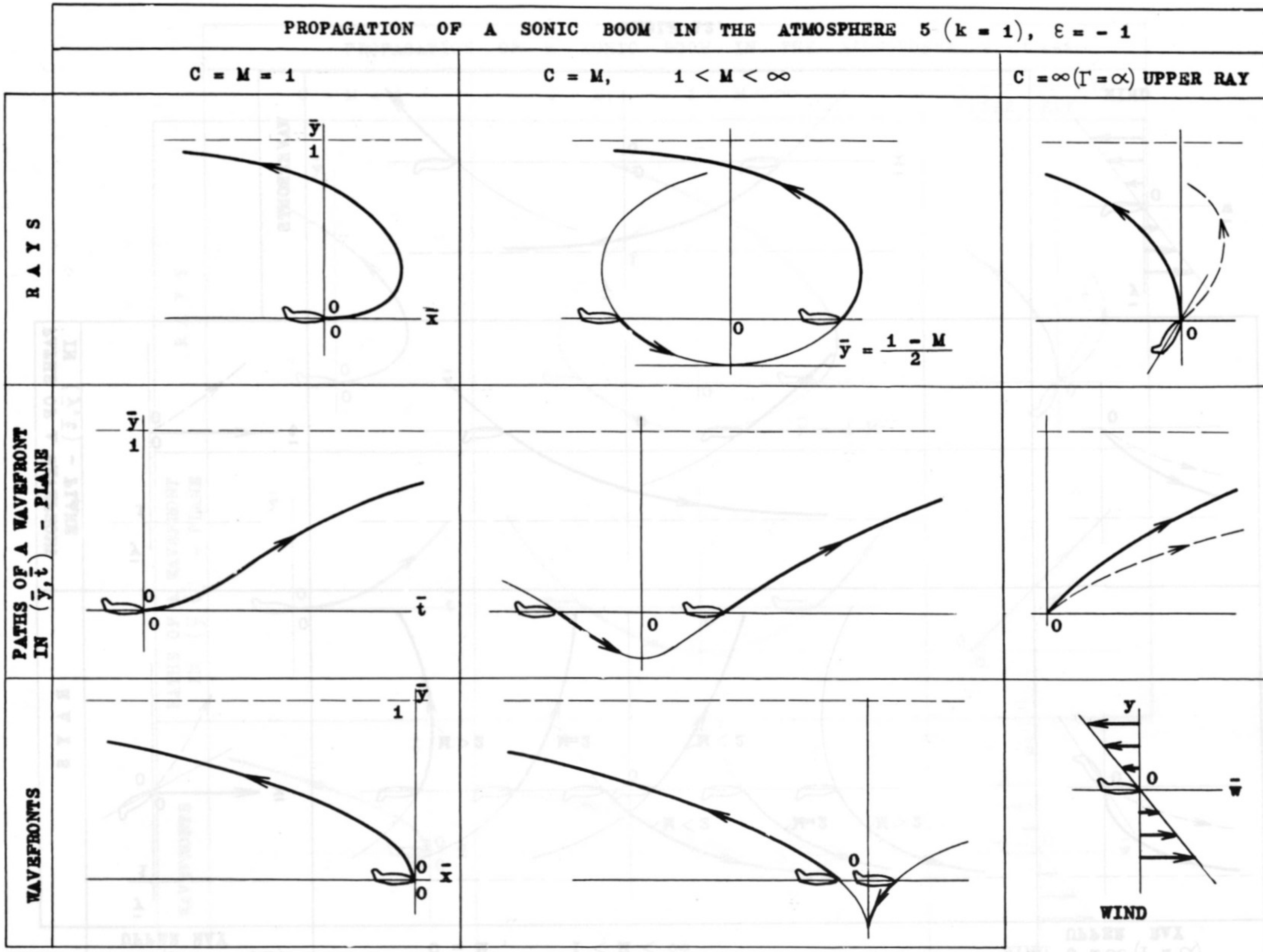
PROPAGATION OF A SONIC BOOM IN THE ATMOSPHERE 5 (k = 1), ε = - 1
C = M = 1
C = M, 1 < M < ∞
C = ∞(Γ = α) UPPER RAY
R A Y S
PATHS OF A WAVEFRONT IN (ȳ, t̄) - PLANE
WAVEFRONTS
ȳ = (1 - M)/2
WIND

TABLE 13.