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STUDY OF INSPECTION INTERVALS FOR FAIL-SAFE STRUCTURES*

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Abstract

A fail-safe structure requires inspection, since otherwise the risk of complete failure due to inadequate residual strength of a member with extensive fatigue damage will soon become too large. It is a strong interest for the operator, however, to minimize the number of inspections, which are very costly. This is especially the case for the unscheduled inspections carried out on a whole fleet of aircraft when a serious crack has been found in an aircraft of the same model.

Based on a method of evaluating the reliability of fail-safe structures, developed earlier at FFA, a theoretical investigation, applicable to an aircraft wing, has been made of the influence of the length of inspection intervals which are both constant and varying. An optimum study shows that the least number of regular inspections during the service life is obtained by making especially the first interval considerably longer than the following ones. This result presupposes that all fatigue cracks occurring may be statistically anticipated. Unexpected cracks are likely to appear, however, during an initial "debugging period", and have to be considered when determining the first inspection intervals.

Unscheduled inspections, randomly distributed in time, decrease the risk of total failure but are shown to be much less effective than regular inspections. This is also demonstrated in a numerical example assuming cyclic inspection on a fleet of 100 aircraft with a sampling ratio of 0.25 for the regular inspections and further extra random inspections. It is concluded that unscheduled inspections should not be carried out, unless a completely unexpected crack has been found, which will mainly happen during the first part of the service life.

I. Introduction

Fatigue cracks cannot be entirely eliminated in primary aircraft structures of aluminium alloys, which are subjected to a large number of load cycles during a long service life, The risk of total failure due to fatigue should not be allowed however, to rise too much above the risk of static failure during the whole service life. (1), (2) It may even be maintained that the risk of fatigue failure should be lower, due to psychological reasons, since the general public will probably be more inclined to accept the inevitableness of a static failure during a violent thunderstorm than of a fatigue failure, which usually occurs as a consequence of a gust of quite ordinary intensity.(3) The latter type of accident is more difficult to explain and may initiate demands that somebody be made responsible for it, although both types are stochastic events, During the period 1946-1960 the average rate of fatigue failures for civil air carriers in U. S. A. amounted approximately to one in 10^7 hours of flight. Within a big fleet of heavy bombers the frequency of complete fatigue failures was about five times as large. (2) Considering a service life of the order of magnitude of 10, 000 hours for the civil transport aircraft and 2000 hours for the bombers, the probability of fatigue failure during the service life would be in both cases around 10^{-3}. This figure is unacceptably high, at least for the civil transports, Discussions within the ICAO has resulted in an agreement of a tentative value of the probability of failure of $P_L = 10^{-5}$. (4) In practice it might not be possible to achieve a risk lower than 10^{-4} within the near future.

The prevailing method at present of securing an aircraft a long service life with a satisfactory safety against fatigue failure is built on the fail-safe principle. All details where cracks may be anticipated, are designed to be damage tolerant and easily inspectable. The combination of a slow crack propagation and careful inspections at short intervals may, in principle, render a very low risk level for indefinite time, provided that all cracks discovered are repaired in such a way that the original ultimate strength is restored, A modern airframe contains, however, a large number of fatigue sensitive points, For service lives approaching the mean life until crack initiation the number of repairs will eventually be so large that it is no longer economical to keep the aircraft in operation. It is obvious, therefore, that the manufacturer should not aim in the design at a higher risk of crack initiation during the whole service life than at most 10 per cent in any single point. Assuming normal scatter, this implies that for a logarithmical mean value of 50, 000 hours until crack initiation the service life must not exceed some 30, 000 hours. Even with this limitation it is necessary to carry out quite a large number of inspections of the fatigue sensitive parts in order to keep the risk of complete failure below a level $P_L = 10^{-5} - 10^{-4}$. Inspections are expensive, however. The direct cost of fatigue crack inspections and resulting repairs and modifications during the service life seems to be able to amount to a sum of the same order of magnitude as the initial price of the aircraft. (5) To this cost must be added the lost

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revenue during the extra ground time for inspections and repairs which do not fit into the normal scheme of operation. It is necessary, therefore, to take the cost of the inspections into account in an analysis of the optimum length of the service life from the economical point of view. The aim of this investigation is to provide a basis for such an economical analysis by studying a statistical method for determining the length of the inspection intervals with regard to the safety against complete failure. Earlier investigations have treated periodic inspections. (6), (7) This report covers also inspection intervals of varying length, including the problem of minimizing the total number of inspections during a fixed life.

It is not feasible to inspect simultaneously all aircraft of a certain model within the fleet of an airline even if they were delivered simultaneously and thus should reach their inspection ages at the same time. Both from operational view and in order to provide a continuous employment of the overhaul capacity available, it is necessary to spread the inspections in time as evenly as possible. This means that one has to start the inspection of the first aircraft much earlier than would be required with respect to safety, in order to be able to finish the last aircraft within due time. After the first inspection, however, it is possible to apply for each separate aircraft the regular inspection intervals determined. The fact that only one or a few samples from a large fleet are selected for inspection on each occasion has given birth to the idea that one might use the statistical procedure developed for industrial quality control in order to reduce the total number of inspections for the whole fleet. (8), (9) This procedure utilizes the information obtained from a small sample size to judge the properties of the whole population. At the sample inspections an information is obtained whether there is a crack in the airframe or not. If a crack is found, all other aircraft of the same model are supposed to be inspected as soon as possible. This does not apply only to the fleet of the particular airline under consideration. The crack is usually reported to the manufacturer, who may distribute a service bulletin to all operators of the same model, prescribing inspections and possible modifications, if the crack is serious. This practice results in a large number of nonscheduled extra inspections which occur at random intervals. The extra inspections obviously contribute to reduce the risk of complete failure making it possible to increase the intervals between the regular inspections. If the number of aircraft of the same design is large, the number of regular inspections could be made very small. It might be questioned, however, if it is a rational approach to rely mainly on extra inspections. A comparison has therefore been made between the total number of inspections required for the two systems:

1. Regular inspections only
2. Regular inspections + extra random inspections

The comparison has been performed in the first place assuming the regular inspections to be evenly distributed in time on the various aircraft of the fleet, which means that there is practically a continuous inspection all the time. So called cyclic inspection has also been studied numerically in an example where the inspections were divided into four blocks. A particular detail of an aircraft is thus inspected at every fourth occasion an inspection takes place.

In the analysis of the probability of total failure for different inspection intervals it has been assumed that realistic fatigue tests in full scale have been carried out, before the aircraft was introduced in service, to such an extent that all fatigue sensitive points are known and that the mean of the service life until crack initiation as well as the crack propagation rate may be fairly well estimated. This is an ideal assumption which never holds true entirely in practice. During a "running-in" period it is probably inevitable that quite unexpected fatigue damage will occur which might rapidly lead to complete failure. Such a serious crack should absolutely be followed up by an inspection of the entire fleet, since one will need, in this case, all information acquirable to be able to satisfy the safety requirements. During this initial period the operator will also be prepared to accept the disturbance caused by extra crack inspections more easily than later on in the service life, since breaks will frequently occur, nevertheless, due to trouble with the engines, the hydraulic or electric systems etc. A systematic procedure for determining the risk of complete failure due to unexpected fatigue cracks has not been developed, but a discussion is carried out regarding the possibilities to control the safety level using cyclic inspections including extra inspections when unexpected cracks are discovered.

II. Basic Assumptions

Damage tolerant structures may be achieved in different ways. It is customary at present to design some primary aircraft, structures, especially the wing, as a box beam with two, three, or more longitudinal webs (Fig. 1). The bottom and top cover skins are provided with closely spaced longitudinal stiffeners in order to make the whole width effectively load-carrying. It is thus avoided to concentrate the cross-sectional area into heavy spar booms, where cracks, difficult to detect, might cause a very rapid reduction of the residual strength. In the distributed, "diffuse", structure the fatigue cracks normally are initiated at the rivet or bolt holes of the tension skin, where the stress is kept at a level which gives a rather slow crack propagation. When the crack reaches the next stringer it is delayed for a considerable time and the strength reduction is still quite moderate.

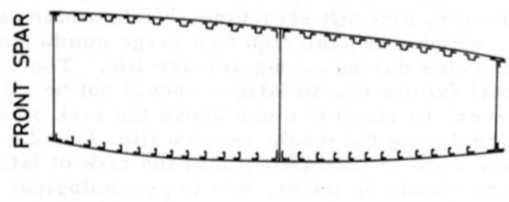

Figure 1. Schematic section of three-spar "diffuse" wing structure

The chance to discover the crack before it has reached so far is further good, if inspections are made regularly. It is also maintained that leakage of fuel from the wing tanks may often provide an automatic crack warning at an early stage. It is very important, however, that the crack is repaired as soon as possible after its initiation. The cross section has been given such a strength that the risk of ultimate failure due to the highest gust or manoeuvre loads that may be anticipated during the service life, is extremely remote as long as no reduction of the original strength has occurred. This risk is called the risk of static failure. Fatigue failure is defined as a complete failure which takes place in a structure which has been weakened by a fatigue crack. Where the crack propagation rate is relatively slow, also the fatigue failure is ultimately caused by a rather high load. The risk of fatigue failure obviously grows rapidly with the time the damaged structure is kept in service, since, with decreasing residual strength, there is a fast increase of the frequency of the loads which can cause total failure.

To be able to analyse statistically the risk of fatigue failure it is necessary to possess knowledge of the distribution function of service time until crack initiation, of the reduction rate of residual strength and of the frequency of high loads on the structure. It is an urgent research object to provide more reliable information within these three areas. This investigation includes some simplified assumptions which may have to be refined when more experimental support is available, but both simplifications and parameter values introduced in the numerical calculations are believed to be quite realistic and to give a good qualitative estimation of the risk of failure. (6), (7)

Crack initiation is defined as the moment when a crack may be detected at an ordinary maintenance inspection. The service time T until crack initiation is assumed to have a log-normal distribution with a mean value μ_c and a standard deviation σ_c of $\log_{10} T$. The probability of crack initiation is thus written

$$P_c = \Phi\left(\frac{\log_{10} T - \mu_c}{\sigma_c}\right) =$$

$$= \frac{1}{\sigma_c \sqrt{2\pi}} \int_{-\infty}^{\log_{10} T} \exp\left[-(\log_{10} T - \mu_c)^2 / 2\sigma_c^2\right] dT \quad (1)$$

In the computations is used throughout $\sigma_c = 0.2$, which is a somewhat conservative value. For μ_c is assumed a typical value of 4.699, corresponding to $T_{c50} = 50,000$ h. In the discussion of unexpected cracks also smaller values are introduced, The probability of crack initiation is shown in Fig. 2 as a function of T for values of T_{c50} from 10,000 to 50,000 h.

At the crack initiation a certain reduction of the ultimate strength has already occurred. Aluminium alloys of the type 2024, which are normally used for the bottom skin in aircraft wings, show a rather insignificant decrease for a crack of detectable length, however. It has therefore been assumed that the original ultimate load S_u is constant until the moment of crack initiation, t = 0, and that it then decreases linearly with t. If the highest mean load in flight is S_m the static margin is originally

$$S = S_u - S_m \quad (2)$$

The residual static margin will decrease to zero in R hours of flight

$$S_t = S(1 - t/R) \quad (3)$$

The normalized margin, t hours after crack initiation, may be written

$$s_t = S_t/S = 1 - t/R \quad (4)$$

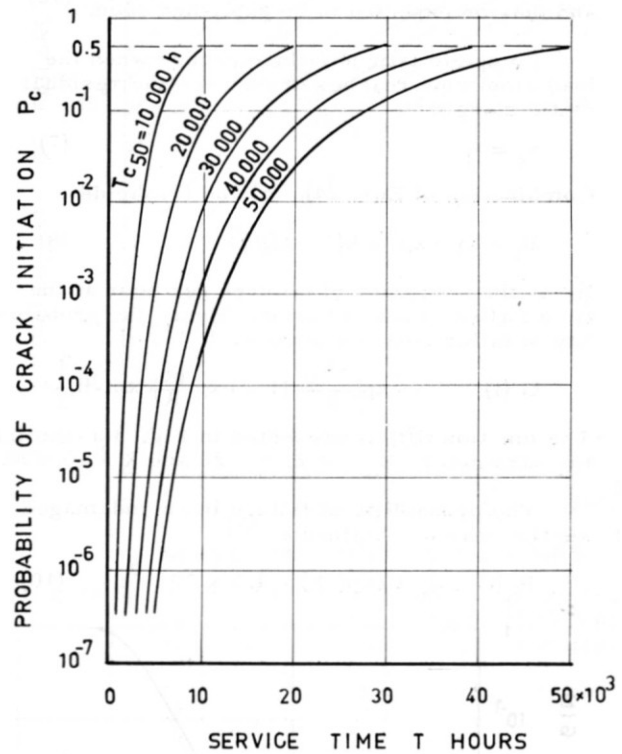

Figure 2. Probability of crack initiation before service time T

The crack propagation time in a fatigue test with constant or varying amplitudes is somewhat shorter, since the final failure is always caused by a combination of a mean load and a load amplitude, $S_m + S_a$. The parameter R is, in fact, a stochastic variable, which shows, however, considerably less scatter than the time to crack initiation. On the basis of available test results a constant value R = 10,000 h has been chosen for all numerical calculations as representative for a modern transport aircraft.

The frequency of high loads on the wing depends to a large extent on the operational use of the airplane. For civil transport aircraft gusts cause the most dangerous loads. The variation is likely to follow an extreme value distribution, but a simple exponential distribution forms an approximation which is sufficient for the present purpose. The expected number of times H that a normalized load amplitude s_a is exceeded, may then be written

$$H = H_o \exp(-h s_a) \quad (5)$$

where $s_a = S_a/S$ (6)

H_o and h are two parameters of which H_o depends on flight profile and flight route, while h varies with the design stress level, speed and weight of the aircraft. The values H = 0.2 and h = 20 have been considered applicable for civil transport aircraft on ranges of medium length, on the basis of measurements from thunderstorms. (10), (11) An ultimate design load factor $n_u = 3.75$ has been adopted. The information available concerning extreme gust loads is not satisfactory at present, but extensive measurement are being carried out and may be expected to be published soon. (12)

Complete fatigue failure occurs when the load amplitude reaches or exceeds the residual static margin

$$s_a \geq s_t \quad (7)$$

Combination of Eqs. (4), (5) and (7) yields

$$H_t = H_o \exp\left[- h(1 - t/R)\right] \quad (8)$$

H_t is the frequency of failures per hour at the time t after crack initiation. Hence the probability of failure may be deduced (6), (11)

$$G(t) = 1 - \exp\left[\frac{RH_o}{h}\left(1 - \exp\frac{ht}{R}\right)\exp(-h)\right] \quad (9)$$

The function G(t) is presented in Fig. 3 assuming the parameters $H_o = 0.2$, h = 20 and R = 10,000.

The probability of failure in an undamaged section may be obtained as (7)

$$P_u(t) = H_o\, t \exp(-h) = 4.1 \times 10^{-10} t \quad (10)$$

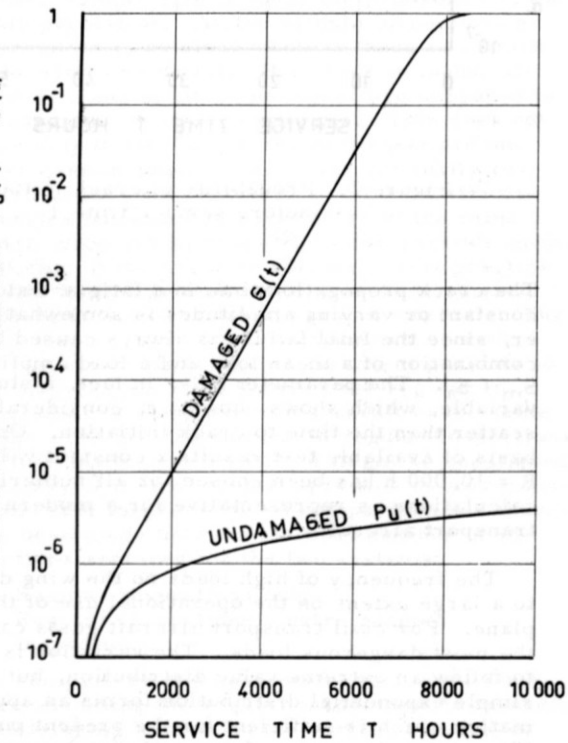

Figure 3. Probability of failure due to gust load in damaged and undamaged wing structure before service time t
$H_o = 0.2 \quad h = 20 \quad R = 10,000$

This function has also been included in Fig. 3. In the damaged section the probability of failure grows rapidly with time and has almost reached 100 per cent at t = 8000 h. For t = R/2 = 5000 h the risk is about 0.5 per cent. Already after 2000 h the risk of failure in a damaged section is ten times the risk in an undamaged section, 10^{-5} against 10^{-6}. If it is assumed, conservatively, that a fatigue crack is initiated immediately after an inspection, one should not allow the inspection interval to exceed some 20 per cent of the crack propagation time R.

III. Risk of failure during one inspection interval

The service life of the aircraft is divided into a number of inspection intervals. One arbitrary interval no. ν is considered (Fig. 4).

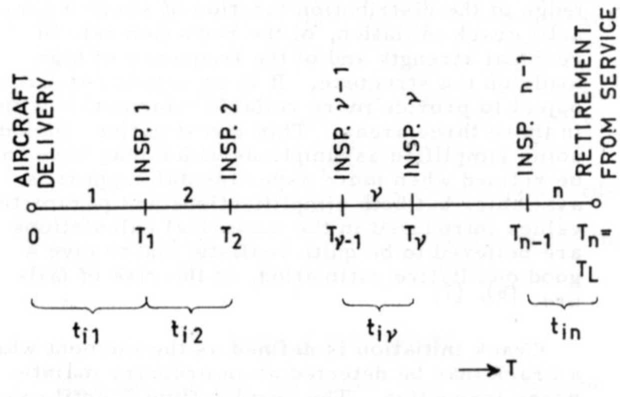

Figure 4. Time schedule for inspection of aircraft

The flight time from the first delivery of the aircraft is denoted T. The age of the aircraft at the beginning and end of interval ν is $T_{\nu-1}$ and T_ν respectively. This means that $T_o = 0$. The length of the interval is

$$t_{i\nu} = T_\nu - T_{\nu-1} \quad (11)$$

Since an inspection has been performed at $T = T_{\nu-1}$, the structure is presumed to be free from cracks at the beginning of the interval. Provided that no repairs or replacements have been made, the risk of crack initiation during a short time increment ΔT from T to T + ΔT is according to Eq. (1)

$$\Delta P_c = \frac{dP_c}{dT}\,\Delta T = p_c(T)\Delta T =$$

$$= \frac{\log_{10} e}{\sigma_c T\sqrt{2\pi}} \exp\left[-(\log_{10} T - \mu_c)^2/2\sigma_c^2\right]\Delta T \quad (12)$$

The probability ΔP that a crack which is initiated during ΔT, shall result in complete failure during the remaining part of the inspection inter- $t = T_\nu - T$, is obtained by multiplying the quantities ΔP_c and G(t) fromEqs. (12) and (9)

$$\Delta P = G(T_\nu - T)\,\Delta P_c \qquad (13)$$

Letting $\Delta T \to 0$ and integrating over the interval ν gives (6)

$$P_\nu = \int_{T_{\nu-1}}^{T_\nu} G(T_\nu - T)\, p_c(T)\, dT \qquad (14)$$

Numerical evaluations of P_ν are preferably carried out, however, by adding up values from finite steps ΔT.

$$P_\nu = \Sigma \Delta P_j = \Sigma G(T_\nu - T_j)\Delta P_{cj} \qquad (15)$$

A summation according to Eq. (15) has been made on an IBM 7090 computer with two different step lengths, $\Delta T = 500$ h and $\Delta T = 500/3 = 166.7$ h. The error was then estimated from the difference between the two results. The value of the function $G(T_\nu - T_j)$ was computed with T_j situated in the middle of the step. The calculations have been performed for ages at the beginning of the interval $T_{\nu-1}$ = 0, 1000, 2000, ..., 60,000 h, assuming inspection intervals of the lengths $t_{i\nu}$ = 500, 1000, 1500, ..., 10,000 h. Besides, for values of $T_{\nu-1} \leq 12{,}000$ h the variation of $t_{i\nu}$ has been extended as far as to 50,000 h increasing the steps between the lengths of $t_{i\nu}$ from 500 to 1000 h.

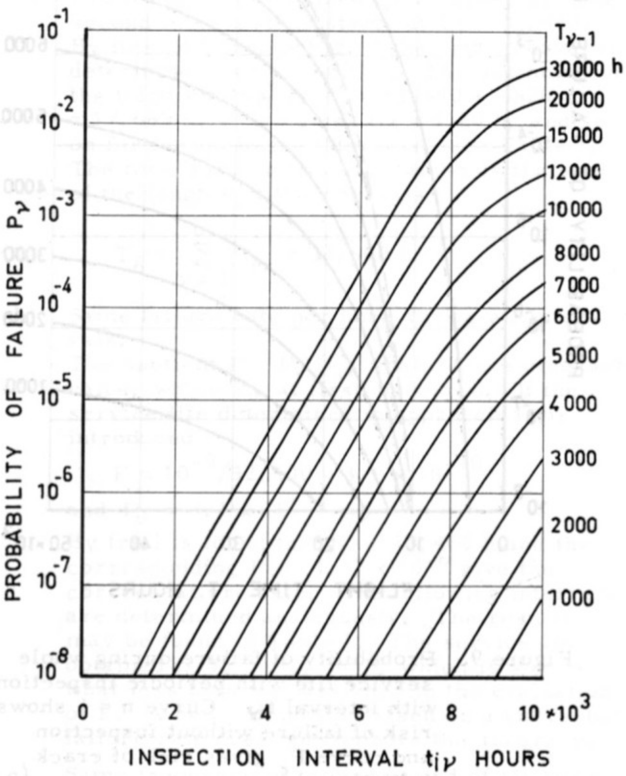

Figure 5. Probability of complete failure during an inspection interval $t_{i\nu}$ for different ages $T_{\nu-1}$ at the beginning of the interval T_{c50} = 50,000 h σ_c = 0.2 R = 10,000 H_o = 0.2 h = 20

the beginning of the inspection interval, until $T_{\nu-1}$ = 30,000 h. Fig. 5 reaches up to $t_{i\nu}$ = 10,000 h, while Fig. 6 provides P_ν -values for longer inspection intervals until 20,000 h, where only relatively low ages $T_{\nu-1}$ are of interest.

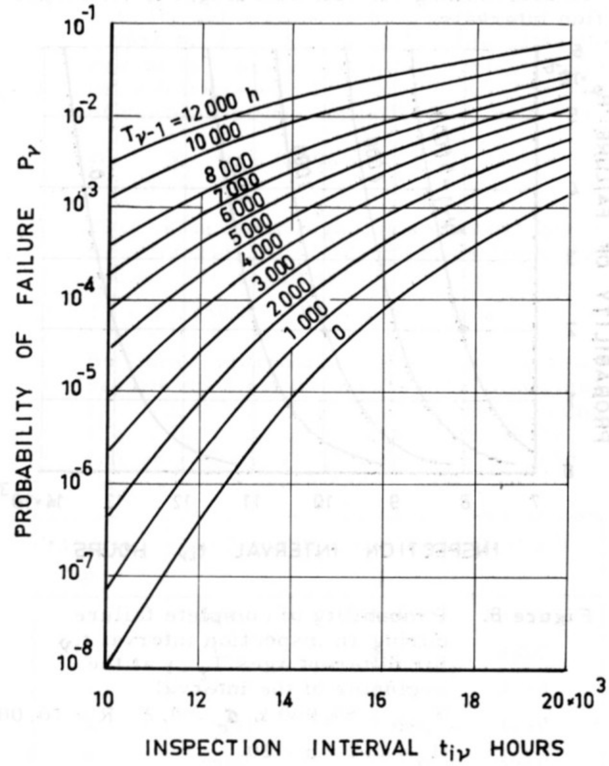

Figure 6. Probability of complete failure during an inspection interval $t_{i\nu}$ for different ages $T_{\nu-1}$ at the beginning of the interval
T_{c50} = 50,000h σ_c = 0.2 R = 10,000
H_o = 0.2 h = 20

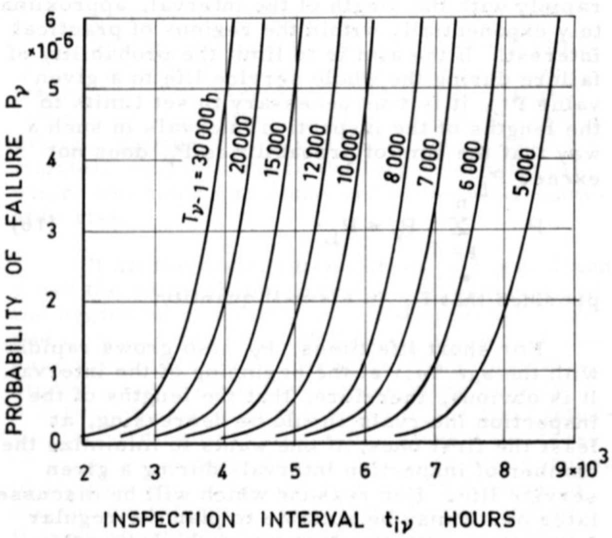

Figure 7. Probability of complete failure during an inspection interval $t_{i\nu}$ for different ages $T_{\nu-1}$ at the beginning of the interval
T_{c50} = 50,000 h σ_c = 0.2 R = 10,000
H_o = 0.2 h = 20

Corresponding diagrams are drawn in Figs. 7 and 8 with P_ν in a linear scale. These diagrams, which were obtained by interpolation in Figs. 5 and 6, have been used in a graphical approach for determining the optimum length of the inspection intervals.

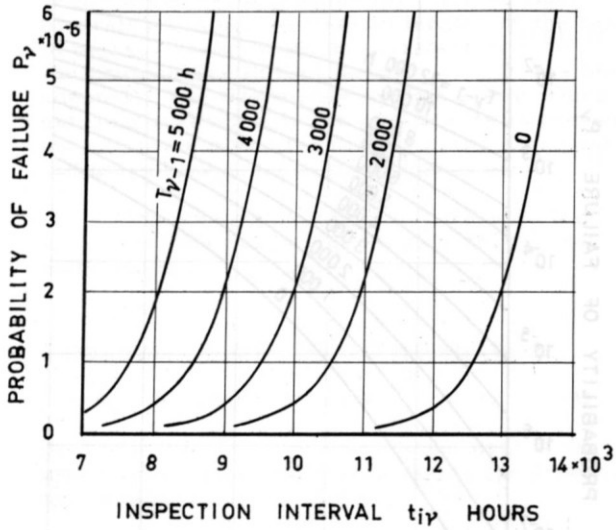

Figure 8. Probability of complete failure during an inspection interval $t_{i\nu}$ for different ages $T_{\nu-1}$ at the beginning of the interval
$T_{c50} = 50,000$ h $\sigma_c = 0.2$ R = 10,000
$H_o = 0.2$ h = 20

IV. Risk of failure during service life with regular inspections

Periodic inspections

As is evident from Figs. 5-8 the risk of complete failure during an inspection interval grows rapidly with the length of the interval, approximately exponentially within the regions of practical interest. If the aim is to limit the probability of failure during the whole service life to a given value P_L, it is thus necessary to set limits to the lengths of the inspection intervals in such a way that the sum of probabilities P_ν does not exceed P_L

$$P = \sum_{\nu=1}^{n} P_\nu \le P_L \qquad (16)$$

provided that P_L is a small quantity

For short life times, P_ν also grows rapidly with the age $T_{\nu-1}$ at the beginning of the interval. It is obvious, therefore, that the lengths of the inspection intervals should be decreasing, at least the first ones, if one wants to minimize the number of inspection intervals during a given service life. For reasons which will be discussed later on, it may be suitable to plan the regular inspections with constant intervals during the whole life. Anyhow, it is a common scheme at present to apply regular periodic inspections, supplemented by randomly occurring extra inspections. Diagrams giving the probability of failure during the whole service life assuming constant intervals have been presented in earlier reports.[(6), (7)] These diagrams were obtained from computations with larger step lengths in the integrations than those employed in Chapter III. A new, improved diagram of the probability of failure P as a function of the service time T at various constant inspection intervals t_i has therefore been designed in Fig. 9. The service life has been extended to 50,000 h, i.e. the logarithmic mean of the crack initiation time. The replacement effect has not been considered, however, which means that the accuracy is not so good when P_c is approaching 50 per cent.

If a risk of complete failure, due to fatigue, of $P_L = 10^{-5}$ is considered adequate for the wing section, which may imply a failure risk for the entire airframe of some 10^{-4}, or even more, and the service life aimed at is $T_L = 30,000$ h. Fig. 9 indicates an inspection interval required of 4100 h. The number of intervals would thus be 8, allowing a life of 33,000 h, or 7, if a life of 29,000 h is satisfactory.

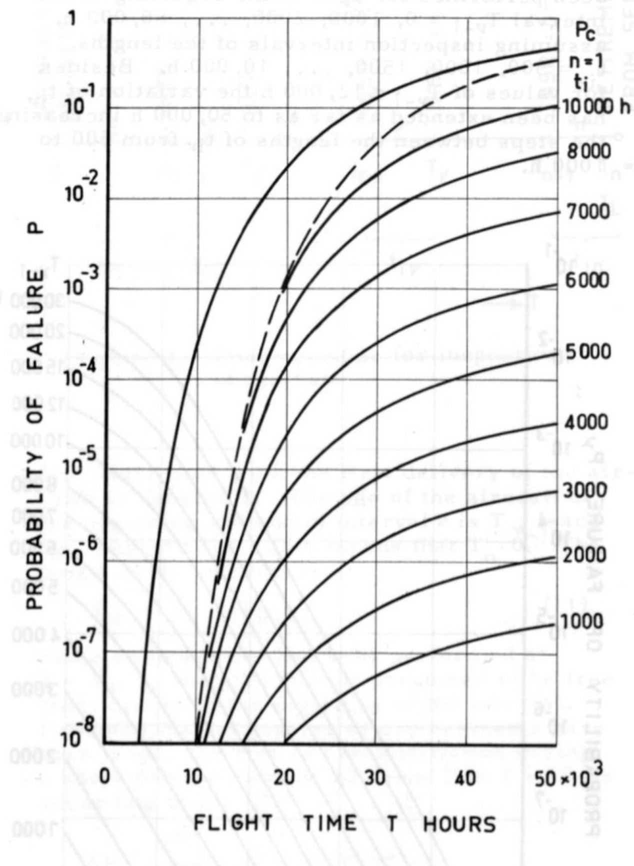

Figure 9. Probability of failure during whole service life with periodic inspection with interval $t_{i\nu}$. Curve n = 1 shows risk of failure without inspection and curve P_c gives risk of crack initiation
$T_{c50} = 50,000$ h $\sigma_c = 0.2$
R = 10,000 $H_o = 0.2$ h = 20

Inspection intervals of varying length

A strict mathematical optimization of the lengths of the inspection intervals is quite complicated since the length of earlier intervals influences the risk of failure in a following interval. It is possible, however, to arrive at approximate solutions, as accurate as the diagrams of Figs. 7 and 8 allow, by trying only a few approaches. It may be concluded from Fig. 8 that the risk of failure during the first interval, $T_{\nu-1} = T_o = 0$, is very small until $t_{i1} = 11,000 - 12,000$ h. For $t_{i1} = 12,000$ h the risk is $P_1 = 0.4 \times 10^{-6}$. Proceeding to interval no. two, one finds in Fig. 7 that $T_1 = 12,000$ h yields a risk $P_2 \approx P_1$ already at $t_{i2} = 4300$ h. For $t_{i2} = 5000$ h, $P_2 = 1.7 \times 10^{-6}$, and at $t_{i2} = 5500$ h the risk has grown to $P_2 = 4.5 \times 10^{-6}$. This latter value is obviously too high if the sum of all risks P_ν must not exceed 10^{-5}. The discussion shows that it is quite simple to find reasonable limits for the lengths of the inspection intervals.

The following numerical example has been studied:

$$P_L = 10^{-5} \quad n = 6$$

Three different approaches where introduced. The sequence of intervals giving the longest service life was considered the optimum solution, $T_L = T_{6\,max}$

a) Same probability of failure in all intervals, Risk per interval $P_\nu = 10^{-5}/6 = 1.67 \times 10^{-6}$. From Fig. 8 is obtained $t_{i1} = 12,840$ h. The second interval thus starts at $T_1 = 12,840$. By interpolation in Fig. 7 the next interval is determined as $t_{i2} = 4820$ h. The beginning of the third interval is $T_2 = 12,840 + 4820 = 17,660$ h, which gives $t_{i3} = 4120$ h, and so on for the following intervals, see Table 1. The total service life is obtained as the sum of the lengths of the intervals

$$T_6 = \sum_{\nu=1}^{6} t_{i\nu} = 32,600 \text{ h}$$

b) Same failure rate per hour during all intervals.
The quotient $F = P_\nu / t_{i\nu}$ shall have a constant value, which is, at first, unknown. If the service life determined in approach a. is introduced

$$F = 10^{-5}/32,600 = 3.1 \times 10^{-10}$$

and $P_\nu = 3.1 \times 10^{-10}\ t_{i\nu}$

By trial is found that $t_{i1} = 13,460$ h and the corresponding $P_1 = 4.17 \times 10^{-6}$ give the correct failure rate. The following intervals are determined analogously. The results may be found in Table 1. The service life obtained is $T_6 = 32,180$ h, which is somewhat less than was assumed in the evaluation of F. A new calculation based on a corrected failure rate does not improve the accuracy.

c) Same frequency of failure at end of all intervals
The slope of the distribution curves

$$\frac{dP_\nu}{dT} = p_1 = p_2 = \ldots = p_6 \text{ for } T = T_\nu .$$

In this approach one must first choose a length for the first interval, say $t_{i1} = 13,000$ h which is situated between the values obtained in a. and b. The corresponding risk is $P_1 = 2.15 \times 10^{-6}$. The slope of the curve for $T_o = 0$ is in this point about 17^o. On an interpolated curve for $T_1 = 13,000$ h a point with the same slope is then found, $t_{i2} = 4850$ h, $P_2 = 1.85 \times 10^{-6}$. The same procedure is repeated until the service life has been determined, $T_6 = 32,900$ h. The sum of the probabilities of failure is, however

$$P_L = \sum_{\nu=1}^{6} P_\nu = 1.095 \times 10^{-5}$$

This value is too high. A somewhat lower first interval, $t_{i1} = 12,850$ h, is tried secondly. The result is

$$T_6 = 32,250 \text{ h} \qquad P_L = 0.810 \times 10^{-5}$$

By linear interpolation between these two results the first interval is determined as $t_{i1} = 12,950$ h. The corresponding service life is $T_6 = 32,660$ h and the sum of the failure risks is very close to 10^{-5}. The intervals are presented in Table 1.

Interval	Lengths of inspection intervals		
ν	a	b	c
1	12840	13460	12950
2	4820	4660	4800
3	4120	3930	4130
4	3800	3550	3780
5	3560	3350	3550
6	3460	3230	3450
T_6 =	32600	32180	32660

Table 1. Approaches to optimum variation of inspection intervals

The approaches a. and c. give very similar variations of the lengths of the inspection intervals. The service lives differ less than 100 h from each other, i.e. within the margin of error. Approach b. gives a life about 500 h shorter, which is due to the long first interval where too much was consumed of the total allowable risk.

If all the distribution curves in Figs. 7 and 8 had the same shape, irrespective of the age at the beginning of the interval, the approaches a. and c. would have been identical, and would also have given the optimum lengths of the intervals. Studying the distribution curves more closely one finds that a small decrease of the first interval, according to approach c., gives a decrease in P_1 which is somewhat bigger than the increase of P_2 caused by a corresponding lengthening of the second interval. The first interval should thus be shortened by a small amount less than 100 h. It seems unlikely, however, that any other variation of the inspection intervals could give a total service life exceeding 32,700 h. Rounding off the values the optimum variation may be written

$\nu = 1$	$t_{i\nu}$ =	12900 h
2		4800
3		4100
4		3800
5		3600
6		3500
	T_6 =	32700 h

This variation has been checked also on the computers Bull-GE 235 and 265, using a time sharing terminal in Stockholm. The sum of the failure risks was calculated for ten other alternative solutions, where the lengths of two intervals were varied by 100 h each time, keeping the service life at 32, 700 h. The first interval was thus increased to 13, 000 h, while the second interval was shortened to 4700. It was then decreased to 12, 800 h, lengthening the second interval to 4900 h. The same procedure was repeated for the other pairs of intervals. The lowest sum of risks was obtained for the original variation which gave $P_L = 1.017 \times 10^{-5}$.

In practice it is probably not convenient to apply a continous variation of the intervals. The following approximation seems to be more realistic.

$\nu = 1$	$t_{i\nu}$ =	13000 h
2		4500
3		4500
4		3500
5		3500
6		3500
	T_6 =	32500

A still rougher optimum solution is formed by a first interval of 13, 000 h and all the following equal to 4000 h. This variation gives a sum of the risks of 1.4×10^{-5} which is slightly too high.

In comparison with constant inspection intervals the optimum variation of the lengths of the intervals saves two inspection intervals out of eight in the present example. This is mainly brought about, however, by making the first interval three times as long as in periodic inspection.

V. Effect of random extra inspections

In the previous chapter it was assumed that only regular inspections are performed, planned in advance. The inspection of an individual airplane thus occurs without regard to the information which may be obtained at the inspection of other airplanes within the same fleet. If there is only one airplane of the type under consideration in service, it is necessary to rely on regular inspections only. On the other hand, if the fatigue damage in a number of aircraft is reported to a centre, it is possible to combine a system of regular inspections with extra inspections which are undertaken as soon as possible after a fatigue crack has been detected at a regular inspection. Since the time until crack initiation is a stochastic variable, although time dependent, the extra inspections will occur randomly in time with an increasing frequency.

The individual airplane may be subjected to 0, 1, 2, ... extra inspections between two regular inspections. It is quite clear that the safety against failure is improved by the extra inspections. The higher the number of aircraft is, participating in the inspection system, the lower will the probability of failure become for a given regular inspection interval. This can consequently be extended with regard to the extra inspections.

It is also evident, however, that the system employing extra inspections cannot be maintained when the number of aircraft is approaching infinity, if the regular inspections are evenly distributed in time. A large number of regular inspections would then occur every day and cracks would be detected daily. Extra inspections would be ordered every day, which is unreasonable. A theoretical study of the frequency of the extra inspections and their influence on the risk of failure at different fleet sizes, therefore, seems to be an urgent task.

In the same way as in chapter III a regular inspection interval with a length $t_{i\nu}$ for the individual airplane has been studied. The number of aircraft is m. The inspections are assumed to be evenly distributed implying that inspections are performed in the whole fleet at constant intervals $t_{i\nu}/m$. The inspection interval $t_{i\nu}$ is divided by extra inspections into a number of ℓ part intervals of randomly varying length. The extra inspections are carried out when a crack is detected at a regular inspection on another airplane, The probability of crack initiation for the individual airplane is $P_c(T_{\nu-1})$ at the beginning of the interval and $P_c(T_\nu)$ at the end of it, The probability of crack initiation during the interval ν is thus

$$P_{c\nu} = P_c(T_\nu) - P_c(T_{\nu-1}) \qquad (17)$$

In a fleet of m aircraft the probability that no crack will occur and no extra inspections be ordered is given by

$$Q_1 = (1 - P_{c\nu})^{m-1} \qquad (18)$$

Index 1 means that the regular interval contains one single part interval. The probability of $\ell - 1$ crack initiations and ℓ part intervals is

$$Q_\ell = \binom{m-1}{\ell-1} P_{c\nu}^{\ell-1} (1 - P_{c\nu})^{m-\ell} \qquad (19)$$

In the analysis the delay between crack detection and extra inspections has been neglected. In practice some cracks which have been found during interval ν will not lead to extra inspection until next interval $\nu + 1$. This will be compensated, however, by delayed inspections originating in interval $\nu - 1$. It is also possible that the operator may exclude from extra inspection those airplanes which have comparatively recently been subjected to regular inspection. Such a reduction of extra inspections has not been considered. It would give a higher risk of failure than has been obtained. The results may thus be considered as a lower limit for an ideal system of random extra inspections.

If the conditional probability of failure during the regular interval ν, provided that the interval is divided into ℓ random parts, is $P_{\nu\ell}$, the resulting probability of failure for the interval is

$$P_\nu = \sum_{\ell=1}^{m} Q_\ell P_{\nu\ell} \qquad (20)$$

The conditional probability $P_{\nu\ell}$ is obtained from the formula

$$P_{\nu\ell} = \sum \left[\int_{T_{\nu-1}}^{T_\nu} G[N(T) - T] p_c(T) dT \right] / A \quad (21)$$

where the summation refers to all possibilities for the ℓ-1 extra inspections to occur at different times, A = number of possible occurrences, N(T) = time to first extra inspection after time T. The functions G and p_c have been defined in Eqs. (9) and (12).

The evaluation of $P_{\nu\ell}$ has been made on an IBM 7090 computer, using two different procedures, the Monte Carlo Method, and Step Integration with the trapezoidal rule. The calculations were successively refined. In the Step Integration regular interval has been divided into k equal parts with the integer ℓ assuming any value from 1 to k. The number of possibilities is

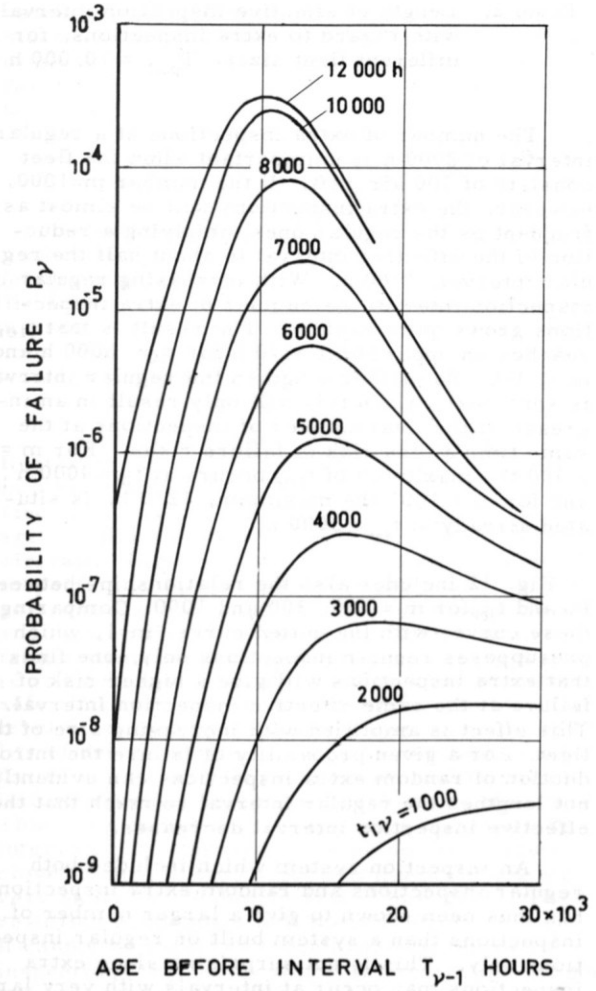

Figure 10. Probability of complete failure during a regular inspection interval $t_{i\nu}$ versus age $T_{\nu-1}$ at the beginning of the interval. Random extra inspections, number of aircraft m=100 T_{c50} = 50,000 h σ_c = 0.2 R = 10,000 H_o = 0.2 h = 20

$$A = \binom{k-1}{\ell-1}$$

The calculations have been performed with k = 10-16 part intervals.

In the Monte Carlo Method the summation has been made using A = 50 random simulations. Between the beginning and end of the interval, $T_{\nu-1}$ and T_ν, ℓ-1 random numbers have been sampled, with a rectangular distribution, independent, and rounded off to the closest number which is a multiple of 20. They have been arranged in rising sequence before they were entered into Eq. (21). In the summation values of ℓ from 1 to 18 were included.

The step length in the integration has been in both methods ΔT = 20 h. The calculations have been carried out for regular inspection intervals with the lengths $t_{i\nu}$ = 1000, 2000, ... 12,000 h, adopting ages at the beginning of the interval $T_{\nu-1}$ = 0-30,000 h. The number of aircraft introduced was m = 10, 30, 100, 300 and 1000. In the evaluation of the probability of total failure during interval ν from Eq. (20) the binomial distribution of Q_ℓ was replaced by a Poisson distribution, which is a good approximation as long as $P_{c\nu}$ is relatively small.

In Figs. 10 and 11 P_ν is given, in a logarithmic scale, as a function of the age $T_{\nu-1}$ at the beginning of the interval for different lengths

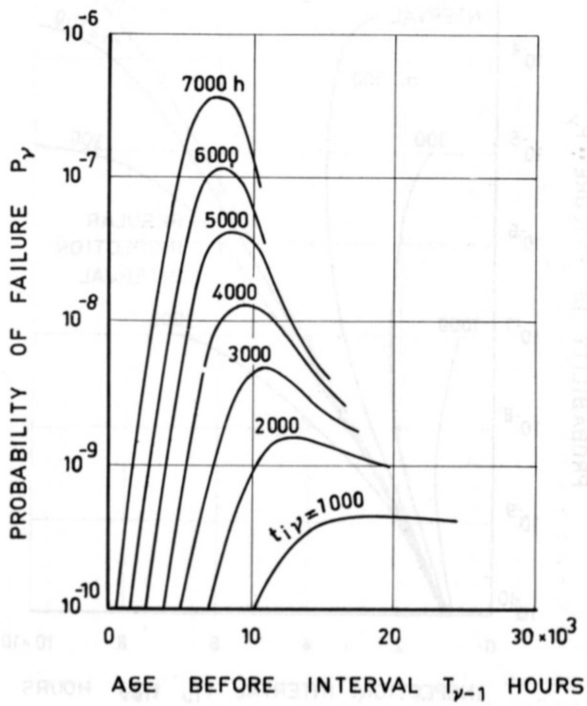

Figure 11. Probability of complete failure during a regular inspection interval $t_{i\nu}$ versus age $T_{\nu-1}$ at the beginning of the interval. Random extra inspections, number of aircraft m=1000 T_{c50} = 50,000 h σ_c = 0.2 R = 10,000 H_o = 0.2 h = 20

$t_{i\nu}$ of the inspection interval, assuming fleet sizes m = 100 and 1000 respectively. The diagrams are based on the results of the Monte Carlo Method, since this proved to give the most consistent values when varying the number of part intervals.

A comparison between the probabilities of failure P_ν for different fleet sizes has been made in Fig. 12. Curves for m = 1, 100, 300, and 1000 have been drawn considering the age $T_{\nu-1}$ = = 10,000 h at the beginning of the interval. If the probabilities are plotted versus the length of the regular inspection interval $t_{i\nu}$ a lower risk is obtained when the number of aircraft is increased, as was expected, and the reduction is more pronounced for long inspection intervals. In the comparison one should, however, take into account also the number of extra inspections. Under the assumptions on which the calculations of the failure risk have been based, the expected number of extra inspections for the individual airplane is $mP_{c\nu}$. During the interval ν there is thus together $1 + mP_{c\nu}$ inspections. The average time between two inspections, the "effective" inspection interval, is obtained by the formula

$$t_{ie\nu} = t_{i\nu}/(1 + mP_{c\nu}) \qquad (22)$$

The value of $t_{ie\nu}$ has been computed in Table 2 for the age $T_{\nu-1}$ = 10,000 h.

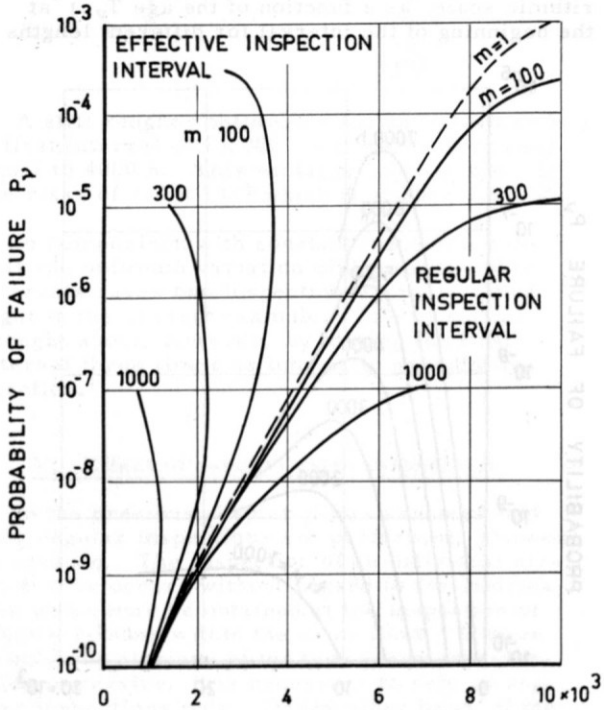

Figure 12. Probability of complete failure during regular and effective inspection interval, $t_{i\nu}$ and $t_{ie\nu}$ respectively, at age $T_{\nu-1}$ = 10,000 h before interval. Different number of aircraft m = 1, 100, 300 and 1000. Random extra inspections assumed. T_{c50} = 50,000 h σ_c = 0.2 R = 10,000 H_o = 0.2 h = 20

Regular interval $t_{i\nu}$	Effective inspection interval $t_{ie\nu}$ m = 100	300	1000
1000	975	925	790
2000	1865	1640	1160
3000	2620	2080	1210
4000	3170	2240	1100
5000	3520	2200	960
6000	3670	2060	820
7000	3650	1870	690
8000	3500	1640	580
9000	3280	1450	490
10000	3000	1260	420
11000	2810	1130	370
12000	2550	990	320

Table 2. Length of effective inspection interval, with regard to extra inspections, for different fleet sizes. $T_{\nu-1}$ = 10,000 h

The number of extra inspections at a regular interval of 2000 h is unimportant when the fleet consists of 100 aircraft. If the number m=1000, however, the extra inspections will be almost as frequent as the regular ones, implying a reduction of the effective interval to about half the regular interval, 1160 h. With increasing regular inspection interval the number of extra inspections grows quite rapidly. The result is that $t_{ie\nu}$ reaches an upper limit 3670 h for $t_{i\nu}$= 6000 h and m = 100. To further lengthen the regular interval is senseless, since this will only result in an increase of the total number of inspections at the same time as the risk of failure rises. For m = = 300 the maximum of $t_{ie\nu}$ occurs at $t_{i\nu}$= 4000 h and for m = 1000 the maximum, 1210 h, is situated already at $t_{i\nu}$ = 3000 h.

Fig. 12 includes also the relationships between P_ν and $t_{ie\nu}$ for m = 100, 300 and 1000. Comparing these curves with the dotted curve, m=1, which presupposes regular inspections only, one finds that extra inspections will give a higher risk of failure at the same effective inspection interval. This effect is amplified with increasing size of the fleet. For a given probability of failure the introduction of random extra inspections can evidently not lengthen the regular interval so much that the effective inspection interval decreases.

An inspection system which includes both regular inspections and random extra inspections, has thus been shown to give a larger number of inspections than a system built on regular inspections only. This is not surprising since extra inspections may occur at intervals with very large scatter. Single intervals which are extremely short do not contribute correspondingly to the safety. As was shown in chapter IV the probability of failure should be approximately the same in all intervals to give an optimum number of inspections. The selection of the lengths of the inspection intervals should obviously not be left to the chance. Moreover, inspections planned in advance are likely to be considerably cheaper than those generated by a random occurrence.

It may be argued that in an intelligent application of the system of extra inspections some inspections could be saved on airplanes which have recently been inspected, as was mentioned before. This would improve the efficiency of the extra inspections, especially for large fleets, compared to the results of the general study presented. It is maintained, however, that the conclusions drawn still hold true,

VI. Cyclic inspection

Cyclic inspection, also called sampling inspection, implies that the individual airplane is not subjected to a complete inspection of all fatigue sensitive points on every occasion overhaul is carried out. The big airlines apply a block overhaul plan in which a portion of the overhaul is made at each visit. Block A may include inspection of the tail area, Block B wing inspection, etc. When the whole cycle is completed, the first total inspection has been carried out. [9] If the intervals between the regular overhaul visits, the block intervals, have a length t_{ib} and the number of blocks within the cycle is b, the interval between two successive inspections of the same detail of the airframe is

$$t_i = b\, t_{ib} \qquad (23)$$

In case that only regular inspections are performed, the relevant parameter to employ in determining the risk of failure during the service life is evidently t_i and not t_{ib}. Usually the cyclic inspection seems to be combined with extra inspections, however, prescribed when cracks are detected in other aircraft.

Schematically it may be assumed that a fleet of m aircraft of the same age are inspected in b blocks. At the same time as block A is carried out on m/b aircraft, the other blocks B, C, ... are simultaneously applied to the same number of aircraft. If the risk of crack initiation before the inspection is P_c = 10 per cent and b = 5, i. e. 20 per cent of all aircraft are inspected each time, and further m = 100, the probability of finding a fatigue damage in some of the aircraft inspected is about 90 per cent. [8] This may sound rather safe, but does not, in fact, comply with the safety level discussed in the introduction. The probability of overlooking a crack is 10 per cent. This figure has to be multiplied by the probability of failure provided there is a crack in the structure which was initiated already during the previous interval. This probability may become rather close to unity if the inspection intervals are not very short, as is demonstrated in Fig. 3. The resulting probability will then be much too high. It cannot be expected therefore, that the extra inspections will improve the safety against fatigue failure considerably unless the number of aircraft under observation is quite large, Then the number of extra inspections will be inconveniently numerous on the other hand. This was shown in chapter V assuming that regular inspections were carried out with very short intervals in the whole fleet. In cyclic inspection the overhaul is supposed to be concentrated to a few occasions within the cycle. To study the influence of the discrete blocks a numerical example with cyclic inspection has been treated.

In a fleet of m = 100 aircraft the block interval is t_{ib} = 5000 h, constant throughout the service life, T_L = 30, 000 h. The number of blocks is b = 4, which means that the regular inspection interval for the individual aircraft is 20, 000 h according to Eq. (23). Regular inspections are performed on five occasions, T = 5000, 10, 000, 15, 000, 20, 000 and 25, 000 h. Each time there are two possible outcomes.

1. No crack detected
2. Crack detected, resulting in extra inspection on the remaining 75 aircraft

In all, the number of outcomes at five inspections is 2^5 = 32. They are recorded in Table 3. In each case the probability of occurrence as well as the conditional probability of total failure, provided the outcome occurs, have been computed. Numerical results obtained earlier in chapter III have been utilized, which implies the following combination of parameter values:

Out-come no.	Crack detected (x) at block inspection T = 5, 000 h	10, 000 h	15, 000 h	20, 000 h	25, 000 h
1					
2	x				
3		x			
4	x	x			
5			x		
6	x		x		
7		x	x		
8	x	x	x		
9				x	
10	x			x	
11		x		x	
12	x	x		x	
13			x	x	
14	x		x	x	
15		x	x	x	
16	x	x	x	x	
17					x
18	x				x
19		x			x
20	x	x			x
21			x		x
22	x		x		x
23		x	x		x
24	x	x	x		x
25				x	x
26	x			x	x
27		x		x	x
28	x	x		x	x
29			x	x	x
30	x		x	x	x
31		x	x	x	x
32	x	x	x	x	x

Table 3. Possible outcomes at cyclic inspection with 5 block inspections during service life

The most common outcome is no. 17 where no crack occurs until the fifth inspection. This combination appears in 40 per cent of all cases. Next comes no. 25 with 26 per cent and no. 9 with 13 per cent. By multiplying the two probabilities mentioned, the probability of total failure for all the different outcomes is obtained. They are added together to give the total probability of failure during the service life

$$P = 0.0367$$

To this probability is contributed 29 per cent by outcome no. 1, where no cracks occur at any inspection, while no. 17 gives 35 per cent and no. 9 23 per cent. The number of extra inspections in the whole fleet is expected to be 129, i.e. 1.29 per airplane. Since the number of regular inspections is 1.25 per plane, including the last inspection at T = 25,000 h, the effective inspection interval may be computed as

$$t_{ie} = 25,000/(1.25 + 1.29) = 9850 \approx 10,000 \text{ h}$$

Disregarding the fact that the probability of total failure is unacceptably high, a comparison with Fig. 9 shows that it is much higher than for regular periodic inspections with $t_i = 5000$ h, which gives $P = 5 \times 10^{-5}$ at $T_L = 30,000$ h. It is also higher than for $t_i = t_{ie} = 10,000$ h, where P=0.02, and even approaches the value obtained without inspections (n = 1), P = 0.042. The effect of the random extra inspections must be extremely small, and it seems preferable, therefore, to increase the number of regular inspections to twice the present number before carrying out the extra inspections.

A still better solution, theoretically, would be to vary the length of the regular intervals. If the first interval is $t_{i1} = 20,000$ h and the second one is $t_{i2} = 10,000$ h, the failure risk becomes P = 0.02, i.e. less than in the system of cyclic inspections studied, which includes extra inspections. This is remarkable, that one single inspection per airplane gives a better result than 1.25 + 1.29 = 2.54 inspections per plane. The cause may partly be the inefficiency of the random extra inspections but partly also the fact that in the cyclic inspection some inspections are made in the beginning of the service life, where they are of little importance for the safety problem considered. Further, one fourth of the fleet is left without inspection after 10,000 h, and another fourth is obtaining its last inspection at an age of 15,000 h.

More extensive investigations regarding systems of cyclic inspection are planned to be carried out presently. The single numerical example which has been presented cannot form the basis for wide conclusions. It might be stated, however, that it does not contradict the results of the general study with continuous regular inspections, which was made in chapter V.

VII. Unexpected fatigue cracks

The calculations of the probability of complete failure due to fatigue damage, which have been presented in the previous chapters, presuppose that the fatigue properties of the structure are statistically well-known under current load sequences. If this is not the case, there are no possibilities to determine in advance the economical service life of the aircraft and to plan the lengths of the inspection intervals. Big efforts are therefore made to acquire this information at an early stage. But airframes are very complicated structural systems. In spite of the refined methods of stress analysis, the accumulated knowledge of fatigue failure and the extensive fatigue testing, very often on a complete airframe, that the manufacturers employ in the development of a new aircraft model, it is quite likely that one or a few fatigue sensitive points have been overlooked. The stress level may not be correctly determined in every detail and the full-scale testing may not be able to simulate all influences of dynamic character on the airframe in service. The risk that both analysis and testing fail in the same detail, according to experience, cannot be neglected. Furthermore, errors in the production, outside normal scatter, is also a possibility not to be forgotten.

The airframe must normally be supposed to possess the fatigue life parameters that were initially evaluated and the risks of total failure computed from these parameters must be taken into account. On the top of these risks of expected fatigue failures one must also take care of the risks due to unexpected fatigue damage, especially during an initial debugging period. The regular inspection intervals determined due to expected fatigue, must be shortened and extra inspection of the whole fleet performed when these unexpected fatigue cracks appear. Modifications will usually be introduced. If this is not possible, the natural solution is to decrease the service life, originally fixed. The question is how conservative assumptions must be introduced in guarding against unexpected cracks.

If it is assumed that a crack is present in the structure already at the delivery, the probability of failure before a certain service time will grow rapidly with this time according to Fig. 3. A first inspection interval $t_{i1} = 5000$ h gives the failure risk $G = 5 \times 10^{-3}$ which is unacceptable. One has to cut down the interval to 2000 h to attain $G=10^{-5}$. Provided that there are cracks in a large per cent of all aircraft at T = 0, this circumstance ought to be discovered already at the inspection of the first few aircraft. Conservatively it might be stated that reasonable safety requirements should be satisfied by a first block interval $t_{ib} = 2000$, applying a sampling ratio of 0.20-0.25 in a comparatively large fleet.

In order to study the influence of a variation of the crack initiation time, the probability of failure P, during the first inspection interval has been plotted in Fig. 13 as a function of the length of the interval t_{i1} for different values of T_{c50} from 10,000 to 50,000 h. The other parameters are the same as were used e.g. in Fig. 5. In this diagram may be observed that with a logarithmic mean as low as 10,000 h, a first inspection interval t_{i1} = = 6000 h yields $P_1 = 1.8 \times 10^{-6}$. If the fleet contains 100 aircraft and cyclic inspection is planned to be performed on 25 aircraft at the first block inspection, then the risk that cracks which are actually iniated, will be overlooked, is the following

$$R_c = (1 - P_c)^{25} \left[1 - (1 - P_c)^{75} \right] \quad (24)$$

The probability of crack initiation is obtained from Fig. 2, $P_c = 0.134$, which gives

$$R_c \approx 0.866^{25} = 0.027$$

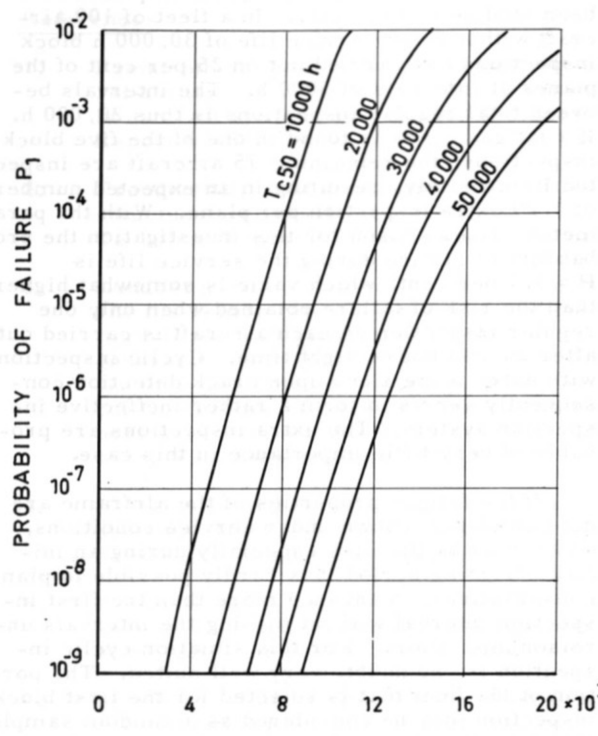

Figure 13. Probability of complete failure P_1 during first inspection interval t_{i1} for different logarithmic means to crack initiation

$\sigma_c = 0.2$ $R = 10,000$ $H_o = 0.2$ $h=20$

If the next inspection takes place at T = = 10,000 h the failure risk in a first interval, t_{i1} = 10,000, will amount to P_1 = 0.0042 according to Fig. 13. The resulting risk that the crack will not be discovered at the first block inspection and then cause failure before the second inspection is computed as the product

$$P_1 \times R_c = 0.0042 \times 0.027 = 1.1 \times 10^{-4}$$

Since this value is too high, the second inspectior must come earlier, say at 8000 h. The corresponding $P_1 = 0.99 \times 10^{-4}$ and thus

$$P_1 \times R_c = 2.7 \times 10^{-6}$$

If cyclic inspection is applied and the block interval is 2000 h, all 75 aircraft remaining from the first inspection will have been inspected at T = 8000 h and the probability of failure before this time will be somewhat lower than computed above.

When the first cycle has thus been completed all sorts of fatigue damage with a crack initiation time $T_{c50} \leq 10,000$ h will have been detected with a high confidence. In the analysis for the following cycle it will therefore be possible to assume a longer crack initiation time. The same procedure may be repeated until the expected value of T_{c50} has been reached.

The hypothetical discussion which has been sketched above, refers to the case where no cracks are detected at an early stage. The inspection intervals determined can be considered as conservative estimates which may be useful for the initial planning of intervals. When a crack is found in the primary structure of one, or several airplanes, this constitutes new information, which may, after a statistical treatment, form the basis for a more reliable estimation of the crack initiation than could be made before. The inspection intervals can therefore not be permanently determined in advance but must be subjected to continuous modifications due to the results of earlier inspections. According to experience, the guard against unexpected fatigue damage does usually not need to be maintained more than during a limited debugging period. The length of this period is obviously difficult to determine in advance. It is believed that a value of the order of magnitude of 10,000 h would correspond to the parameter values used in this investigation.

VIII. Conclusions

Assuming that one knows for a part of an airframe, e.g. a wing structure, the crack iniation and crack propagation times, an extensive numerical study of the risk of complete failure has been performed with special regard to the inspection intervals and procedures. In the computations certain parameter values are introduced which have been motivated in earlier reports. (6), (7) The experimental support is not satisfactory, however, and the diagrams presented should not be considered as directly applicable for practical service purposes but rather as numerical examples with a realistic background.

In Figs. 5 - 8 is shown the computed probability of failure P_ν during an inspection interval of the length $t_{i\nu}$, beginning at service time $T_{\nu-1}$. The diagrams may be used to determine the risk of failure during the whole service life of a structure which is inspected periodically or with regular intervals of varying length. Fig. 9, referred to constant intervals from 1000 to 10,000 h, has been obtained by adding the P_ν -values in Figs. 5 and 6. In the same way Figs. 7 and 8 have been utilized in an investigation of the optimum variation of regular inspection intervals during a service life slightly over 30,000 h. Periodic inspection requires eight intervals to keep the sum of the failure risks of the intervals below 10^{-5}. With an appropriate variation of the lengths of the intervals their number may be reduced to six. It can roughly be stated, however, that this reduction is achieved by making the first interval three times as long as the constant intervals. Such a variation is unsuitable, however, for other reasons.

The airlines must aim at such a distribution of the inspections of their fleets that the overhaul capacity is evenly employed and that the operations may be maintained to the extent they are demanded. Seasonal variations may occur, but it is probably reasonable to assume that for a common aircraft model inspections are carried out with short, even intervals all over the world. When a serious crack is detected at an inspection, this is usually reported to a centre from which prescriptions may be edited of immediate extra inspections on all aircraft of the particular model. This safety system has been treated numerically taking into account extra inspections, randomly occurring in time. Two different procedures have been tried, of which the Monte Carlo Method has, so far, given the most consistent results. The extra inspections result in a reduction of probability of failure during a regular inspection interval. The reduction increases with the length of the inspection interval, the age at the beginning of the interval and the number of aircraft involved. The results of the calculations are demonstrated in Figs. 10-12. As a comparison a curve denoted m = 1 has been included in Fig. 12, giving the risk of failure when information from inspections on other aircraft is neglected.

The improved safety with random extra inspections is gained at the prize of increased inspection work. A measure of the efficiency of the extra inspections is obtained by plotting the three curves for m = 100, 300 and 1000 in Fig. 12 as a function of the effective inspection interval t_{iev}, obtained by dividing the length of the regular inspection interval by the expected number of extra inspections during the interval, plus one. The three new curves to the left in the diagram provide a clear indication that the extra inspections are inferior to the regular ones. The result is not unexpected, since a random distribution of the length of the inspection intervals must obviously involve considerable irregularities. Inspections at very short intervals are almost wasted. It may be argued that such inspections will be eliminated in practice. Rational limitations are thus likely to reduce the differences between the curves for m = 1, 100, 300 and 1000, but they will not be able to change the tendency.

In cyclic inspection, which is commonly employed by the big airlines, only a certain portion of all aircraft in the fleet are inspected in the same detail simultaneously at each block inspection. When the cycle, usually consisting of 4-8 blocks, is completed, every part of the airframe which is considered fatigue sensitive, has been subjected to a thorough inspection, where all cracks of importance may be presumed to be detected. The time period of one cycle thus corresponds to what has been called the inspection interval t_i. A variation of the length of the inspection interval during the service life may be introduced but involves some complications. The normal planning seems to include only constant regular intervals, the length of which may be determined from diagrams like Fig. 9. It is maintained, however, that by receiving information from other aircraft of the same type which are inspected at the other block inspections during the interval, practically the same safety level may be attained as if the inspection interval were only equal to the time between two consecutive block inspections. This is probably correct provided the fleet under consideration is sufficiently large. The inspection system must include extra inspections on all aircraft when a crack is discovered at a block inspection. If the number of aircraft is very large, cracks will be detected at almost all inspections, which will be transformed to total inspections of the whole fleet. Thus, no inspections have been saved by the procedure.

A realistic example of cyclic inspection has been studied numerically. In a fleet of 100 aircraft with a given service life of 30,000 h block inspections are carried out on 25 per cent of the planes at intervals of 5000 h. The intervals between total regular inspections is thus 20,000 h. If a fatigue crack is found in one of the five block inspections, the remaining 75 aircraft are inspected immediately, resulting in an expected number of 1.29 extra inspection per plane. With the parameter values chosen for this investigation the probability of failure during the service life is P = 3.7 per cent, which value is somewhat higher than the risk of failure obtained when only one regular inspection on each aircraft is carried out after 20,000 hours flight time. Cyclic inspection with extra inspections upon crack detection consequently seems to form a rather ineffective inspection system. The extra inspections are probably of very little importance in this case.

If the fatigue properties of the airframe are not completely known under service conditions, which may be the case especially during an initial debugging period it is hardly possible to plan conservatively in advance more than the first inspection interval without making the intervals unreasonably short. For this situation cyclic inspection is, no doubt, very well suited. The portion of the fleet that is selected for the first block inspection may be considered as a random sample in a statistical quality control. On the basis of the results of the first inspection the next allowable inspection interval is determined etc. It is necessary to supplement the regular cyclic inspections with extra inspections in order to keep the failure risk at a satisfactorily low level. Such a procedure has, so far, only been discussed in principle with some numerical illustrations.

As a summary of the conclusions it can be stated that cyclic inspection with extra inspections, randomly caused by the detection of cracks at regular block inspections, forms a suitable means of preventing total failure due to fatigue as long as completely unexpected cracks must be taken into account during an initial debugging period. After this period when the fatigue properties of the structure, in reason, ought to be known in advance, random extra inspections implies a less effective utilization of the inspection capacity than if the same labour had been employed in regular inspections. Further, they usually involve a costly disturbance of the operations. When using cyclic inspection the interval between two consecutive inspections of a specific detail on the same airplane is the significant parameter, while the interval between inspections on different airplanes is not of direct importance. An optimum distribution of the lengths of the inspection intervals is approximately obtained by making the risk of failure the same in all intervals during the service life. An initial period of at least 10,000 hours should be excepted from this rule, however.

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