ICAS Paper No. 68-19

OPTIMUM SHAPES OF BODIES IN HYPERSONIC GAS FLOW

by

A. L. Gonor
USSR Commission on Exploration and Use of Outer Space
USSR Academy of Sciences
Moscow, U.S.S.R.

The Sixth Congress of the International Council of the Aeronautical Sciences

DEUTSCHES MUSEUM, MÜNCHEN, GERMANY/SEPTEMBER 9-13, 1968

Preis: DM 2.00

OPTIMUM SHAPES OF THREEDIMENSIONAL BODIES IN HYPERSONIC GAS FLOW

A.L. Gonor

Moscow University, USSR

The total drag of a body flying in an atmosphere at high supersonic speeds can achieve large values many times exceeding it's weight. That's why the determination of the minimum drag shapes became an important problem in gas dynamics. Nevertheless previously made efforts were generally directed to finding of optimum contours of two-dimensional and axisymmetric bodies (1). The formulation and solution of the variational problem of the minimum drag configuration of a threedimensional body have been made not long ago.(2-4) It's an established fact that in supersonic flow the drag of a body of noncircular cross-section filling a definite part of a given finite volume can be less than the drag of the corresponding body of revolution. Therefore formulation of the following problem of variations calculus is of interest: let's find a conic body of minimum drag inserted into a fixed circular cone and filling a definite part of the cone's volume.

A more general class of bodies can be introduced, namely the bodies whose surfaces are homothetic. The exact solution of this problem is very difficult and has not been received yet. The formulation using Newton's pressure law is considered. This approximation reduces the problem of finding surface of a three-dimensional optimum body into two separate problems: the determination of the optimum longitudinal contour and of the determination of the optimum transversal contour. Newtonian approximation gives optimum longitudinal contour for a slender body in the form of $3/4$ power law. Variational problem for the transversal contour is formulated alongside with an isoperimetric constraint and a fixed reference width. The latter is essential for a correct formulation.

It is being proved that the problem has no continuous solution and the extremal arc consists of subarcs symmetric with respect to the ray passing through a corner point. A complete extremal contour is constructed by repeating "n" times similar cycles of two symmetric subarcs. As a result the transversal contour aquires a starlike configuration with pronounced rays (Fig. 1).

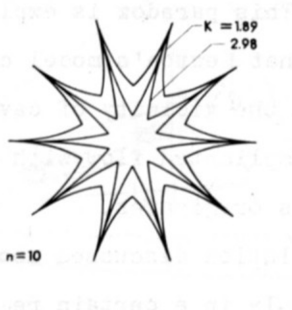


Figure 1.

It is shown that there exists a countable multitude of extremal arcs satisfying all conditions of the variational problem. The equation of extremal arcs for some limiting cases is presented in explicit form which enables to calculate the transversal contour and aerodynamic characteristics rather easily.

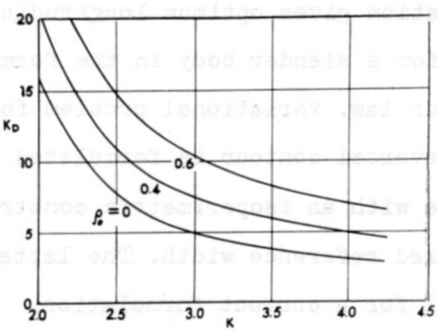


Figure 2.

The drag of an optimum body is of considerable interest (Fig.2). Calculations show that with the number of rays n = 10 the drag of the optimum starlike body is 10 times less than the drag of the equivalent optimum body of revolution. Furthermore, with the growth of number of cycles the pressure drag of the optimum body becomes smaller and vanishes while $n \to \infty$ This paradox is explained by the fact that Newton's model cannot be applied in the vicinity of cavities, where a complicated flow with internal shock waves originates.

The solution discussed above proved to exist only in a certain region of initial parameters; outside of it there exists a second class of solutions (the boundary extremum) in which the transversal contour consists of circular arcs and "star's" rays (Fig. 3).

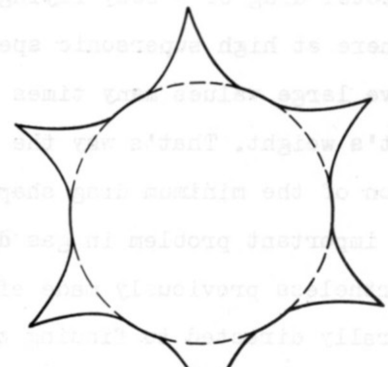


Figure 3.

In the limiting case ray's amplitude is reduced to zero and the drag coefficient tends to become equal to that of the equivalent body of revolution. Later the thickness constraint is taken off and the optimum shape in the class of arbitrary conic bodies is found. The solution of the extremal problem gives results similar to those for a slender body. In particular the transversal contour again has a starlike shape and the drag is reduced even more than in case of a slender body.

Starlike shapes can find important applications if they are shown to have smaller total drag alongside with large pressure drag reduction. However calculations show that the found bodies have noticibly larger surface area as compared with equivalent bodies of revolution. Therefore friction forces may considerably enlarge the total drag and their influence should be taken into consideration while choosing the optimum shape.

The variational problem of the optimum body shape with friction forces has been solved in works by A.Miele[1] and the author[5].

It is assumed that the local friction drag coefficient is constant upon the body's surface and equal to the mean friction drag coefficient upon a plate at the same flow conditions. The other assumptions are the same as in the problem of configuration of the minimum pressure drag body. As a result it is found out that the transversal contour is constructed of separate smooth arcs filling a certain angle and has a starlike shape. The conditions of the problem are satisfied by a countable multitude of solutions with different number of rays n . Some initial values correspond to the boundary extremum. Then the cross-section is constructed of "star's" rays and circular arcs. In the limiting case the cross-section degenerates into a circle.

An approximate equation for a transversal contour arc is found with a help of assymptotic relationships. A calculation is made showing that the "star's" rays are slightly concave (nearly straight).

The total drag calculated at usual for applications values of the friction coefficient ($2 \cdot 10^{-3}$) allows to conclude that a body of a starlike shape has considerably lesser drag as compared with an equivalent body of revolution.

The above three-dimensional configurations have given a direct impulse to improvement of the theory. It was clear that in cavities Newtonian shock layer model cannot be applied. In this connection a scheme has been developed which took into consideration the secondary impacts and led to appearance of concentrated forces along the line of intersection of surfaces.[6]

The following hypotheses have been put in the foundation of the calculation: first, it is assumed that on each surface there exists a regular shock layer with a known distribution of flow field along the normal; secondly, the secondary impacts occur in an infinitely thin layer according to the scheme of absolutely nonelastic impact.

These assumptions allow to calculate the value of the impulse which is lost in the process of secondary impact and to determine the concentrated forces distributed along the intersection line (Fig.4).

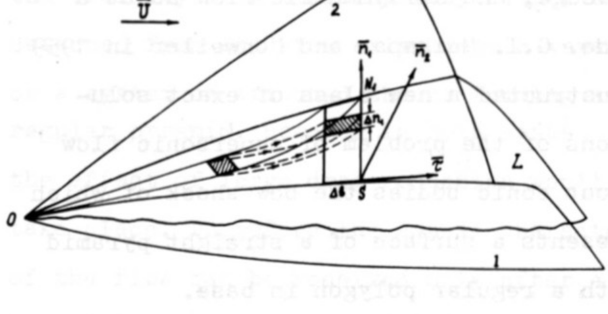


Figure 4.

The results received are applied to calculation of drag of a stepped cone which is compared with appropriate values of numerical solution. The comparison showed an excelent agreement of the data with the exact values and allowed to formulate a simple similarity rule. It is stated that disregard of the concentrated forces enlarges the divergence with respect to the exact data by 20%.

The influence of concentrated forces upon drag of starlike bodies is being studied. The drag is shown to become smaller and tending to a finite value as number of rays increases with fixed volume and diameter of the encircling ring. The paradox stated for starlike bodies in usual Newtonian theory vanishes as concentrated forces are introduced. However the drag of "the star" in this case is also some times less then the drag of the equivalent body of revolution.

Specific exact solutions of problems of supersonic flow about bodies accounting for origination of shock waves of finite strength are of great interest in gas dynamics. Not long ago only two exact solutions were known, namely, flow about a wedge, and axisymmetric flow about a cone. G.I. Maikapar and Nonweiler in 1959 constructed a new class of exact solutions of the problem of supersonic flow about conic bodies the bow shock of which presents a surface of a straight pyramid with a regular polygon in base.

As far as the conclusions of drag reduction in case of starlike bodies are based upon the use of Newton's scheme it is necessary to find out whether the said drag reduction is actual or it is a result of growth of mistake while calculating the drag. The answer could be given by comparison with the exact solution. However the only aforementioned solution for three-dimensional bodies is limited by application to bodies whose configuration is far from optimum. Therefore construction of an exact solution for three-dimensional starlike bodies similar to those studied in Chapter 2 is of interest. Such solutions for two classes of three-dimensional flows are constructed in papers(7-9). These are of interest themselves and make it possible to study flow about some classes of starlike bodies and V-shaped wings.

Solution for the first class of flows is constructed as follows. The number of "star's" rays in the cross-section is being chosen and inclination of a rib to the flow direction is fixed (Fig.5.).

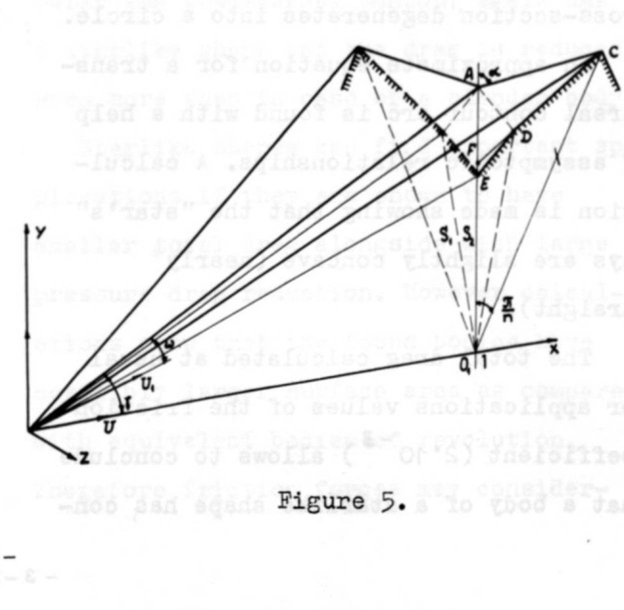


Figure 5.

The strength of the planar shock attached to the rib is selected. This strength defines angle of the body's cross-section at the end of the ray. Shocks originating on two neighbouring ribs intersect regularly and reach the beams' surface. If the transversal contour has at this station an appropriate break then further reflection does not occur. The solution of the inverse problem exists only for certain values of geometric parameters and Mach number. An appropriate region of solution existence is found for every Mach number. This region is used for calculation of body's geometry and active forces. It is shown that the constructed starlike bodies (Fig.6.) can have drag much less then drag of a circular cone of the same width and length.

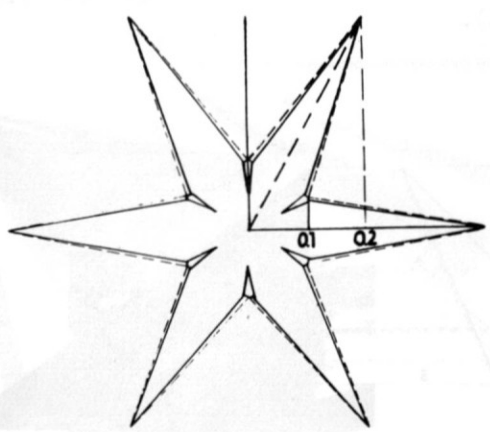


Figure 6.

For a number of bodies together with the exact pressure drag calculation the calculation with Newton's pressure law was made. The comparison showed that the mistake is about 18-20%. Thus the results received in previous chapters for starlike bodies agree with the exact solution qualitatively as well as quantitatively.

Analytical form of the exact solution allows to find rather easily the limiting flow at $M \to \infty$ and $\varkappa \to 1$. It is shown that in case of a V-shaped wing the assymptotic exact solution leads to appearance of a concentrated force coinciding as to it's value with the force received afore. This result confirms the correctness of the calculation made on the base of secondary impacts and one may expect that consideration of concentrated forces will give a real estimation of the forces produced by a flow.

Theoretical results of the study of flow about starlike bodies can't be considered final for absence of agreement between the real flow and the flow pattern given by the theory is possible. Remembering that the large drag reduction is explained by appearance of a system of weak shocks nearby a starlike body instead of a strong axisymmetric wave originated near a body of revolution one can assume that because of the boundary layer and a number of other reasons there appears in the vicinity of a starlike body either a detached from the ribs axisymmetric wave or a system of attached shocks forming a regular pyramid. However in both cases the effect of large drag reduction won't take place. Therefore the actual character of the flow can be received only after a detailed experimental study of the flow field and measurement of acting forces.

The experiments were made in wind tunnels [10-13] at Mach number $M = 4, 6, 8$. In the first group of experiments the flow about six models of starlike bodies with number of rays $n = 6, 10, 15$ at $M = 4$ has been studied (Fig. 7).

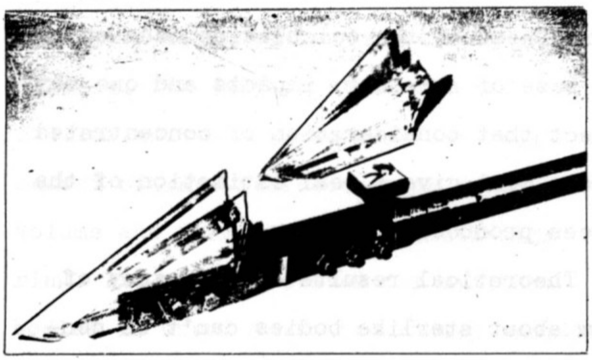


Figure 7.

It was assumed that the shock wave will be attached along body's ribs (this fact was proved experimentally) and the flow field can be studied on a model which presents one element consisting of a pair of semipetals. On the first stage two purposes were pursued: to define by optical methods the flow structure between the rays (judging by the wake behind the model) and pressure distribution on model's surface.

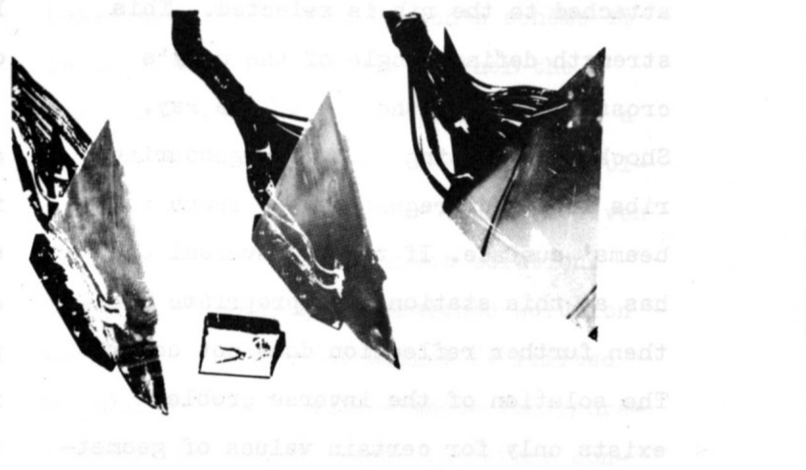


Figure 7a.

The analysis of the pictures of the flow and the schemes of the shocks made it possible to conclude that the flow similar to the calculated is realized and regular intersection takes place to which a system of weak reflected shocks generated by stochastic deflections from the calculated regime, by errors in the process of models construction etc. is added (Fig.8).

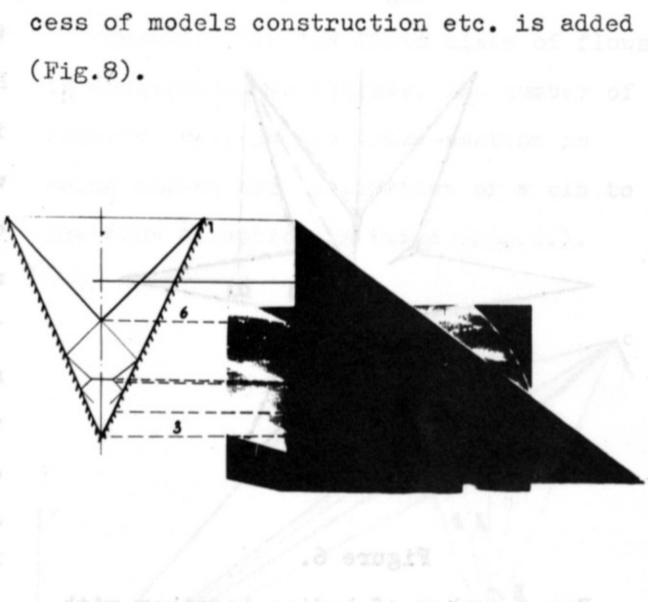


Figure 8

The study of pressure distribution made on three drained models ($n = 6, 10, 15$) showed a good agreement of experimental data with exact theory calculations (Fig.9).

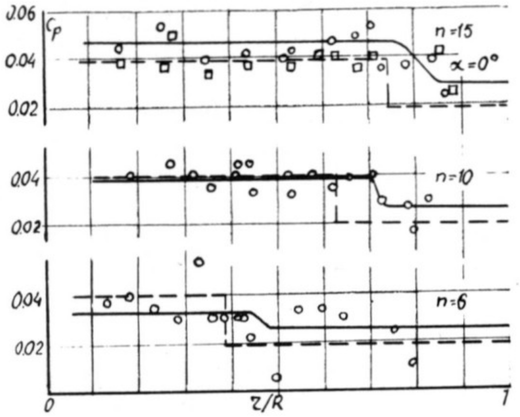


Figure 9.

The pressure distribution data were used to calculate the pressure drag coefficient of the tested bodies and comparison with the theory was made. It is confirmed experimentally that in cases predicted by the theory a manifold reduction of pressure drag is realized.

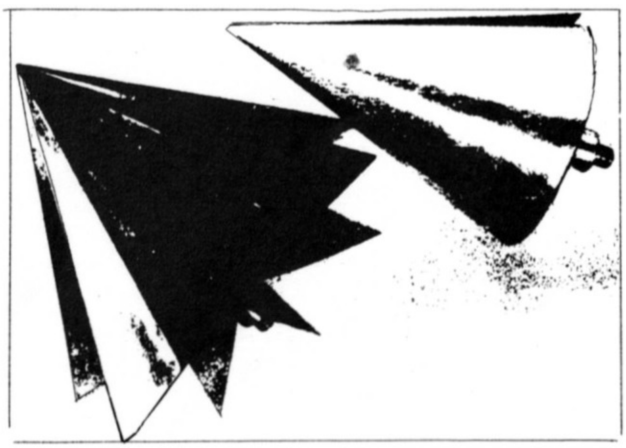
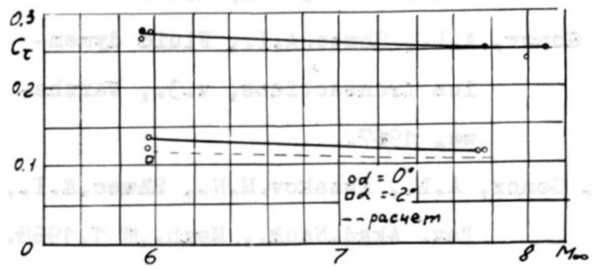


Figure 10.

The total drag measurements were made on models of a starlike body with the ray number n = 10 and of an equivalent circular cone with apex semiangle $17^\circ 30'$ (Fig. 10) at Mach number M = 6,8 and angles of attack $- 2^\circ \div 3^\circ$ (Fig. 10).



The measurements showed (Fig.11) that the total drag of a starlike body is reduced more than twice as compared with the equivalent circular cone.

Cited references

1. Theory of optimum aerodynamic Shapes (ed.A.Miele),Ac.Press,N.Y.1965.

2. Gonor, A.L., J.Appl. Math. Mech., v.27, № 1, 1963.

3. Gonor, A.L., J.Appl. Math. Mech., v.28, № 2, 1964.

4. Chernyi, G.G., and Gonor A.L., Theory of optimum aerodynamic Shapes, Chapter 19, Ac.Press,N.Y.,1965.

5. Gonor A.L., Isv. Akad. Nauk SSSR, Otd. Tekhn.Nauk, Mekh. i Mashinostr., № 4, 1965.

6. Gonor,A.L., Isv. Akad. Nauk SSSR,Mekh., № 1, 1967.

7. Gonor,A.L., J.Appl. Math. Mech.,v.28, № 5, 1964.

8. Gonor,A.L., Isv. Akad. Nauk, Mekh., № 6, 1966.

9. Gonor,A.L., Fluid dynamics transactions

v.3, Warzhawa, 1967.
10. Gonor, A.L., Shwec, A.I., Prikl. mekh. i tekhn. phys., № 6, 1965.
11. Gonor, A.L., Shwec, A.I., Isv. Akad. Nauk, Mech., № 3, 1966.
12. Gonor, A.L., Shwec, A.I., Fluid dynamics transactions, v.3., Warzhawa, 1967.
13. Gonor, A.L., Kazakov, M.N., Shwec, A.I., Isv. Akad.Nauk., Mech., № 1, 1968.