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A CRITICAL EVALUATION OF METHODS FOR COMPUTING WING-BODY INTERFERENCE AT SUPERSONIC SPEEDS

by

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Abstract

This paper summarizes the research effort, both theoretical and experimental, that has been reported since the 1954 survey by Lawrence and Flax[2] and indicates some of the promising ideas for future progress in the subject of wing-body interference at supersonic speeds. Some early methods and results are compared to computer-oriented methods for obtaining solutions to wing-body interference problems. The problem of assessing the accuracy of methods based on linear theory is introduced, and the importance of nonlinear effects is discussed. Proposals for accounting for variations from linear theory without taking the full step to exact solutions are reviewed.

Introduction

When a proposed aircraft or missile is in the preliminary design and development phase it is necessary to have fairly accurate estimates of the aerodynamic loading that will be encountered during flight. The cost of obtaining such information from extensive wind tunnel tests has always been high and seems to increase greatly with each new project.[1] The slowing or reversal of this trend lies in the development of theoretical methods for predicting the aerodynamic characteristics of proposed vehicles. There is a large body of literature on the aerodynamics of wings and bodies but a relative scarcity of methods for predicting the interaction between these components. The early work of Ferrari, Nielsen, and others was in some respects discouraging, since it indicated that great amounts of analysis and computation were required to obtain solutions for even the simplest configurations. The work of this era was very well summarized by Lawrence and Flax in their 1954 survey article.[2] At that time the subject seems to have entered a somnolescent period from which it has only recently been aroused. The new interest is due, of course, to the availability of powerful computing machinery to automate the extensive numerical work that appears inherent in the solution of a problem of this complexity.

Notation

a	radius of cylindrical body
a_∞	speed of sound
a_{ij}	aerodynamic influence coefficient
C_p	pressure coefficient
f_{2n}	functions denoting the axial variation of the 2nth Fourier component of velocity induced on the body by the wing
F_{2n}	Laplace transform of f_{2n}
K_{2n}	modified Bessel function of order 2n
K'_{2n}	derivative of the modified Bessel function of order 2n
M	Mach number
n	coordinate in direction of outward normal
$(n_i)_x,(n_i)_y,(n_i)_z$	components of the outward normal vector to panel i
P	increment in pressure coefficient due to angle of attack
$(\Delta p)_j$	pressure differential across panel j
q	dynamic pressure
r,θ,x	cylindrical coordinate system, x parallel to free stream
Re	Reynolds number
s	Laplace transform of x coordinate
u,v,w	perturbation velocity components in the x, y, and z directions
U	magnitude of local velocity $[U^2 = (U_\infty + u)^2 + v^2 + w^2]$
U_∞	free stream velocity
$\tilde{v}_i$	perturbation velocity normal to defining surface of panel i
v_{radial}	radial component of perturbation velocity $(v_{radial}^2 = v^2 + w^2)$
W_{2n}	special function defined in Ref. 5
x,y,z	cartesian coordinate system, x parallel to free stream velocity, see Fig. 1
x'	x-coordinate with $x' = 0$ taken at leading edge of wing
X_{ij}	x-component of velocity induced on panel i by unit load on panel j
α	angle of attack
β	$\sqrt{M^2 - 1}$
γ	ratio of specific heats
η	ratio of distance from body centerline to wing semispan
ξ	variable of integration
φ	perturbation velocity potential
$\varphi_B^{(k)}$	body-alone velocity potential generated by kth step in iterative method
Φ_1	Laplace transform of φ_1 (transform on the x-coordinate)

φ_i interference velocity potential (transform method)

φ_W wing-alone velocity potential (transform method)

$\varphi_W^{(k)}$ wing-alone velocity potential generated by kth step in iterative method

General Problem of Wing-Body Interference

The wing-body combination perturbs a uniform free stream of velocity U_∞ parallel to the x-axis (Fig. 1). The perturbation velocities in the x, y, and z directions are labeled u, v, and w, respectively. In most of the investigations reported here, it has been necessary to use the equations of gas-dynamics in the linearized form although some of the numerical methods to be described are free from this limitation. The general objective is the development of computing procedures that are applicable to a wide variety of configurations, rapid and straightforward in application, and sufficiently accurate to be of use in the design and analysis of practical airplane designs. In many investigations, it has been necessary to restrict the geometry to a specific configuration in order to simplify the analysis to a point where numerical results could be obtained.

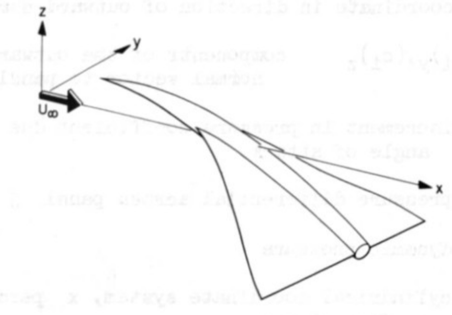

FIGURE 1. COORDINATE SYSTEM

Computing Methods Based on Linearized Equations of Flow

The Iterative Method

In solving complex problems in aerodynamics, it is the usual practice to exploit previously derived results to as great an extent as possible. In solving problems in wing-body interference, a logical first approach is to determine how a solution could be constructed using the methods developed for isolated wings in conjunction with methods developed for isolated bodies. The wing-alone solution will satisfy the condition of tangential flow on the surface of the wing but not on the body, and vice versa for the body-alone solution. In the iterative method, these wing and body solutions are used in alternate steps. In each step a flow field is created that exactly cancels the velocity components of the previous step that penetrate the surface. In mathematical terms, the velocity potential of the configuration is expressed as a sum

$$\varphi_N = \sum_{k=0}^{N} \left(\varphi_B^{(k)} + \varphi_W^{(k)} \right)$$

The terms corresponding to $k = 0$ in the summation are the wing-alone and body-alone potentials for the isolated components. Subsequent terms in the sum satisfy boundary conditions from the velocity flow field of the previous term. For example, $\varphi_W^{(k)}$ satisfied the boundary condition

$$\frac{\partial \varphi_W^{(k)}}{\partial n} = - \frac{\partial \varphi_B^{(k-1)}}{\partial n} \quad \text{on the wing}$$

and $\varphi_B^{(k)}$ satisfies

$$\frac{\partial \varphi_B^{(k)}}{\partial n} = - \frac{\partial \varphi_W^{(k-1)}}{\partial n} \quad \text{on the body}$$

where $\partial/\partial n$ signifies differentiation with respect to the direction of the outward normal vector. The process will eventually converge, although it has never been carried out for terms higher than $k = 1$ because of the length and difficulty of the numerical computations. It is implicitly assumed that there is available a wing procedure and a body procedure for solving linear theory with arbitrary boundary conditions. In the original paper on this subject,[3] Ferrari proposed the use of multipoles for the solution of $\varphi_B^{(k)}$ and a Fourier analysis of the spanwise load distribution for the determination of $\varphi_W^{(k)}$. These particular choices are appropriate only to the case of high aspect ratio rectangular wings of zero thickness mounted in the midwing position. The adaptation of this method to more general configurations does not appear feasible because of the lack of satisfactory computing procedures for the isolated components.

The Transform Method

At each step in the iterative procedure, the wing flow field generated is exactly that which will cancel the flow through the wing induced by the body of the previous step. If it turned out that the flow through the wing at any step were zero, the correction required would also be zero and the series would terminate. For the special case of a wing nominally in the $z = 0$ (horizontal) plane on a body of revolution at zero angle of attack this condition of zero flow can be satisfied,[4] thereby giving a complete solution in a finite number of steps. This special case is not as restricted as it might appear because the general case of a wing-body combination at angle of attack can be decomposed into two separate solutions - one with the body alone at the desired angle of attack and another with the body at zero angle of attack and an appropriate distribution of wing incidence (see Fig. 2).

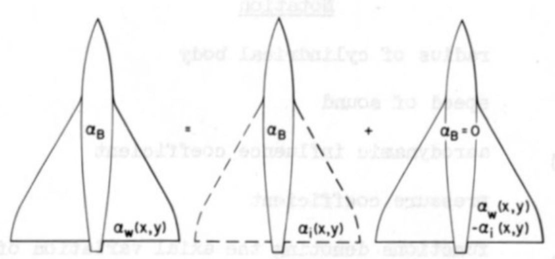

FIGURE 2. DECOMPOSITION INTO SIMPLER COMBINATIONS

The case of the cambered and twisted wing on a body at zero incidence in analyzed by considering the perturbation velocity potential the sum of two parts: φ_W due to the wing alone and φ_i due to interference. As in the iterative method, it is assumed that a procedure is available for the wing-alone solution. This flow field due to the wing alone will produce a velocity component normal to the body and therefore will violate the boundary condition on the body. The requirement that the net radial velocity of the combination must be zero on the body provides the boundary condition for the interference potential, namely,

$$\frac{\partial \varphi_i}{\partial r} = - \frac{\partial \varphi_w}{\partial r}, \qquad r = a$$

Up to this point, the procedure is identical to that of the iterative method, but now an additional requirement is added to ensure that the interference flow field must have zero downwash in the plane of the wing. Stated mathematically,

$$\frac{\partial \varphi_i}{\partial \theta}(r,0) = 0, \qquad r > a$$

This condition can be satisfied by assuming that the interference potential can be expressed as a Fourier cosine series at each longitudinal station

$$\varphi_i(r,\theta,x) = \sum_{n=0}^{\infty} \psi_n(r,x) \cos n\theta$$

Since the velocity distribution must have right-left symmetry, all odd-numbered terms in this series must be zero. One must also note, however, that the assumption of a cosine series implies that the radial velocity will have an even variation with θ. This is the velocity that is to cancel the wing velocities, and therefore the radial component of velocity induced by the wing must have an even variation with θ. This is true only for the wing thickness case and not for a thin lifting surface. However, if the leading edge of the wing is supersonic, the upper and lower surfaces of the lifting wing configuration may be analyzed separately. If the leading edge of the wing is subsonic then only the wing thickness case can be solved by this method.

To complete the solution, following Nielsen's development,[5] the problem is transformed by use of the Laplace transformation applied to the x-variable (x is parallel to free-stream velocity). The transform of the potential is also a cosine series of even multiples of θ and a general solution to the transformed partial differential equation is shown to be given in terms of modified Bessel functions. The boundary condition

$$\frac{\partial \varphi_i}{\partial r}(a,\theta) = - \frac{\partial \varphi_w}{\partial r}(a,\theta) = \sum_{n=0}^{\infty} f_{2n}\left(\frac{x}{\beta a}\right) \cos 2n\theta$$

supplies the final solution

$$\Phi_i(s,r,\theta) = \sum_{n=0}^{\infty} F_{2n}(s) \frac{K_{2n}(sr)}{sK'_{2n}(s)} \cos 2n\theta$$

where $F_{2n}(s)$ is the Laplace transform of $f_{2n}(x/\beta a)$. The inversion of this equation is accomplished by use of the so-called "W functions" defined by

$$W_{2n}(x,r) = \mathcal{L}^{-1}\left[e^{s(r-1)} \frac{K_{2n}(sr)}{K'_{2n}(s)} + \frac{1}{\sqrt{r}}\right]$$

$$n = 0,1,2,\ldots$$

The solution to the problem can be written

$$\beta \frac{\partial \varphi_i}{\partial x} = \sum_{n=0}^{\infty} \cos 2n\theta \left[\int_0^{\frac{x}{\beta a} - \frac{r}{a} + 1} f_{2n}(\xi) W_{2n}\left(\frac{x}{\beta a} - \frac{r}{a} + 1 - \xi, \frac{r}{a}\right) d\xi - \frac{f_{2n}\left(\frac{x}{\beta a} - \frac{r}{a} + 1\right)}{\sqrt{r/a}}\right]$$

The W functions are tabulated up to W_{10} in Ref. 6. From a computational point of view, one first determines the wing-alone flow field, from which one obtains $\partial\varphi_w/\partial r\ (a,\theta,x)$. At a fixed value of x, the velocity is analyzed as a Fourier cosine series and the coefficients of the various harmonics are the functions $f_0, f_2, f_4, \ldots$. These functions are used in carrying out the integration shown in the solution above.

This method has been applied to the problem of a symmetrical nonlifting wedge-section wing mounted on an infinite cylindrical body at zero angle of attack. The wing of rectangular planform was analyzed by Nielsen,[5] and delta wings were analyzed by Randall[7] and by Chan and Sheppard.[8] Nielsen also obtained solutions for the rectangular wing-body combination at angle of attack. In general, four terms in the infinite series were found to give satisfactory convergence except near the leading edge of the wing. Here, the series converges so slowly that the result is of no value and the pressure in this region must be obtained by extrapolation.

The transform method is a quite satisfactory way to obtain solutions for the case of symmetrical wedge-section wings of rectangular or delta planform at zero angle of attack. A number of cases are presented in Ref. 8; by interpolation one may obtain solutions for other combinations of sweep angle and Mach number. The extension to the lifting case would require the computation of $\partial\varphi_w/\partial r$ for a twisted wing since the body-induced upwash varies across the span. This difficulty, plus the fact that wing solutions are not available for many configurations other than the rectangular and delta planforms, has discouraged the development of this method as a design tool. A further deficiency is the fact that the solution is valid only up to the trailing edge of the wing and cannot be used to compute afterbody loadings.

Panel Method (Aerodynamic Influence Coefficients)

In the discussion of the iterative method and the transform method, it was stated that one of the principal deficiencies for practical use was the lack of solutions for wing alone when the local angle of attack varies in a complex fashion over the wing. There are numerical lifting surface theories available[9] that could be used for this calculation. Instead of using such a theory in conjunction with either the iterative method or the transform method, a complete numerical wing-body procedure can be formulated that is free of the restrictions and limitations of either of these other methods. The procedure outlined here forms the basis for a computer program[10,11] sponsored by NASA-Ames and made available to the U.S. aviation industry.

Numerical lifting surface theories for supersonic application have been primarily of the "box" or "panel" type - that is, the configuration is subdivided into a large number of panels that cover the surface (Fig. 3). The assumption is made that the

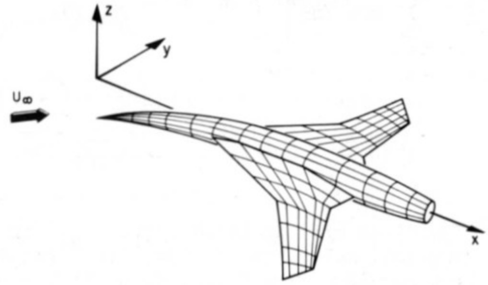

FIGURE 3. TYPICAL PANEL LAYOUT FOR WING-BODY COMBINATION

continuous functions representing the pressure, surface slope, velocity components, etc., may be approximated by functions that are constant over each individual panel. In this way, the steps in the analysis that would require the application of calculus and differential equations now require only simple arithmetic and linear algebra. These processes lend themselves very well to automatic computation.

The basic building block for the solution by panels is the solution for the induced velocity at all points in space for a given thin lifting panel of constant pressure differential. The solution must be given as one of a number of different cases depending on the position of leading and trailing edges relative to the Mach lines; these are tabulated[10] and have been programmed for automatic computation.[11] Suppose the panels are numbered from 1 to N. For each panel, say number j, define X_{ij} to be the x-component of velocity induced on panel i by a unit load on panel j (unit load means $(\Delta p)_j/q = 1$. Then, if the panels actually have pressures of $(\Delta p)_1, (\Delta p)_2, \ldots, (\Delta p)_n$ across them, the x-component of velocity on panel i is found by summing all of the individual velocities.

$$u_i = \sum_{j=1}^{N} X_{ij} \frac{(\Delta p)_j}{q}, \qquad i = 1,2, \ldots, N$$

Similar equations are developed for the y- and z-components of velocity. These 3N equations are a solution to the wing-body interference problem where the pressures are given and it is desired to find the induced velocities (and hence the shape of the configuration) at each point on the surface. However, it is the inverse of this problem that is of interest, and to solve this problem the boundary conditions must be formulated. In general, if the vector $\vec{n_i}$ that is normal to panel i is resolved into components $(n_i)_x$, $(n_i)_y$, $(n_i)_z$, then the boundary condition to be satisfied is

$$(n_i)_x(u_i + U_\infty) + (n_i)_y v_i + (n_i)_z w_i = 0$$

$$i = 1,2, \ldots, n$$

On the wing, one usually assumes that $(n_i)_y$ is negligible and that $u_i \ll U_\infty$. The boundary condition takes the form

$$(n_i)_x U_\infty + (n_i)_z w_i = 0$$

or

$$\frac{w_i}{U_\infty} = - \frac{(n_i)_x}{(n_i)_z} = \frac{dz}{dx} \text{ (local surface slope) (wing panels)}$$

On the body, the boundary condition may be written in cylindrical coordinates as

$$(n_i)_x(u_i + U_\infty) + (n_i)_r(v_{radial})_i = 0$$

since the normal vector to a body of revolution has no θ-component. Therefore,

$$\frac{(v_{radial})_i}{U_\infty} = - \frac{(n_i)_x}{(n_i)_r} \simeq \frac{dr}{dx} \text{ (local surface slope) (body panels)}$$

These two boundary conditions can be written as a single equation

$$\frac{\tilde{v}_i}{U_\infty} = (\text{local surface slope})_i$$

where $\tilde{v}_i$ represents the perturbation velocity (scalar) normal to the nominal defining surface of the configuration. This point is frequently confusing and requires further clarification. The wing-body geometry is defined at some reference condition, usually wing and body at zero incidence and no camber or twist on the wing. The introduction of incidence is done by giving values of local surface slope at each panel. The position of the panel is assumed unchanged. This is a common practice in wing theory, whereby the boundary conditions are applied at a nominal position rather than at the actual position. So, the velocity $\tilde{v}_i$ represents a velocity normal to this original defining surface, not the actual surface.

For a given pressure differential on any panel, say panel j, there is a certain velocity induced normal to the nominal defining surface of every other panel. By linear theory, this velocity is proportional to $(\Delta p)_j$ and is related by a proportionality constant a_{ij}.

$$(\tilde{v}_i) \text{ induced by panel } j = a_{ij} \frac{(\Delta p)_j}{q} U_\infty$$

The total velocity at panel i induced by all the panels is

$$\tilde{v}_i = \sum_{j=1}^{N} a_{ij} \frac{(\Delta p)_j}{q} U_\infty, \qquad i = 1,2, \ldots, N$$

The nondimensional proportionality constants a_{ij} ($i = 1,2, \ldots, n$; $j = 1,2, \ldots, n$) are called <u>aerodynamic influence coefficients</u>. They depend only on the relative location of the panels and the Mach number. The equations above can be compactly arranged in matrix notation so that the technique for solution becomes apparent. For the so-called "direct case" where the local pressures are given, the square matrix A of aerodynamic influence coefficients is post-multiplied by the matrix C_p to give the matrix of local surface slopes. For the inverse case where the local slopes are given, the problem is to solve N simultaneous linear equations for the N unknowns $C_{p_1}, C_{p_2}, \ldots, C_{p_N}$ (Fig. 4).

$$\begin{bmatrix} a_{11} & \cdots & a_{1N} \\ & a_{22} & \vdots \\ & \ddots & \\ a_{N1} & \cdots & a_{NN} \end{bmatrix} \cdot \begin{bmatrix} C_{p_1} \\ C_{p_2} \\ \vdots \\ C_{p_N} \end{bmatrix} = \begin{bmatrix} \tilde{V}_1/U_\infty \\ \tilde{V}_2/U_\infty \\ \vdots \\ \tilde{V}_N/U_\infty \end{bmatrix} = \begin{bmatrix} (dz/dx)_1 \\ (dz/dx)_2 \\ \vdots \\ (dz/dx)_N \end{bmatrix}$$

FIGURE 4. MATRIX FORMULATION OF PANEL METHOD

This is a greatly simplified description of the computer program described in Refs. 10 and 11. A number of additional features are incorporated in the complete program to improve the accuracy and decrease the time and storage required for solution.

This computer program is a direct and straightforward method for the solution of wing-body interference problems by use of linear theory. There are virtually no restrictions as to the complexity of geometry that can be treated. The panel method is a very versatile procedure, but its accuracy is inherently limited by the use of the linearized differential equations of flow.

Computing Methods Based on Exact Inviscid Equations of Motion

Finite-Difference Methods

Finite-difference methods have been employed with considerable success in the analysis of one- and two-dimensional problems in fluid mechanics. The success of these methods in two dimensions has led to a number of proposals for extending them to three-dimensional problems. At this time, there are no results that apply directly to the problem of wing-body interference, but the field is very active and, in all probability, results will be forthcoming in the near future.

Finite-difference methods are based directly on the continuity, momentum, energy, and state equations and therefore do not require the assumption of a velocity potential, or the various assumptions required to linearize the equations of motion, as was necessary in the previously described methods. In the region of interest, a rectangular net is defined and the solution consists of a tabulation of the flow variables at these points (Fig. 5). The

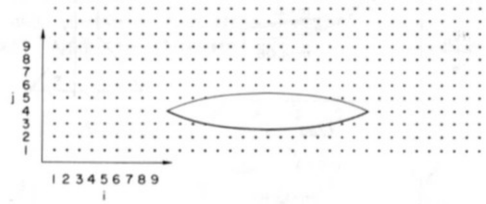

FIGURE 5. FINITE DIFFERENCE METHODS

differential equations that describe the flow are converted to difference equations, and the solution is carried out in a logical, step-by-step manner similar to methods used in the solution of initial-value problems in ordinary differential equations. These problems are nearly always formulated as unsteady problems and carried to large values of time to obtain the steady solution.

It might be supposed that solutions of this type must be based on the Eulerian form of the equations of motion because of the use of a fixed net. Actually, one of the most successful finite-difference methods employs a Lagrangian formulation for the time steps in the computation. After each time step, the flow variables at the fixed net points are obtained from the distorted Lagrangian net by interpolation. This approach, which incorporates the best features of both the Eulerian and Lagrangian equations, is the basis for the particle-in-cell method.[12]

In spite of the rapidly increasing number of applications of finite-difference methods to two-dimensional problems, the extension to three dimensions is a formidable project. An adequate description of the boundary conditions in a wing-body intersection region would require a rather fine mesh and a correspondingly large amount of computer time and storage. It appears that the successful application of finite difference methods to problems of wing-body interference will require computers of much greater speed and capacity than those available today.

Method of Characteristics

The conclusion reached regarding finite-difference methods probably applies here also. While the extension of the method of characteristics to problems of three independent variables has been proposed many times, only since the introduction of modern computers has the proposal received serious consideration. A number of investigators are preparing general purpose computer programs employing the 3D method of characteristics (3DMoC); Refs. 13, 14, and 15 are, in effect, progress reports on these ambitious projects.

It is debatable whether the finite-difference or the characteristics method first will be applied to the problems of wing-body interference. Even if the use of these methods as design tools appears to be some time in the future, the solution of even one or two special cases involving thin wings on sharp-nosed bodies would be of great value since it would indicate the magnitude of the error involved in the linearization of the flow equations.

Comparisons Between Different Methods

The method of finite differences and the method of characteristics are based on fundamental gas-dynamic equations instead of the approximate linearized equations of flow. When these methods are developed to the point where a configuration as complex as a wing-body combination can be treated, they will represent the ultimate in accuracy among methods that solve for inviscid flow.

As noted, the iterative and transform methods require such extensive analytical treatment that their application to any but the simplest configurations is virtually impossible. The transform method is particularly handicapped by the inability to handle configurations with subsonic leading edges and to predict afterbody loadings. Although the iterative method, in principle, can be used on any configuration, the problems of accounting for wings that are effectively cambered and twisted, and bodies that are highly distorted, virtually dictate the use of numerical methods for all iterations other than the first. Even granting the availability of a numerical lifting-surface theory for the warped wing, the body boundary conditions are likely to be so irregular that a large number of terms will be required if the technique of Fourier series analysis is to be used. Only the special case of the lifting wing in the $\theta = 0$ plane appears to be tractable.

None of these difficulties applies to the panel method. This method is the only one of the linear theory techniques that exploits the capabilities of modern computing equipment to provide solutions for a very general range of configurations. Since the panel method is by far the easiest and most rapid method for all those presented, it is recommended as the most useful method for obtaining solutions to the linear equation of supersonic flow. The method applies with equal ease to configurations of arbitrary planform with cambered and twisted wings, cambered and boat-tailed bodies, subsonic or supersonic leading and trailing edges, etc. In addition, the panel method can be used for the analysis of certain nonplanar configurations that rely on interfering flow fields for the establishment of interesting or desirable aerodynamic characteristics (Fig. 6).

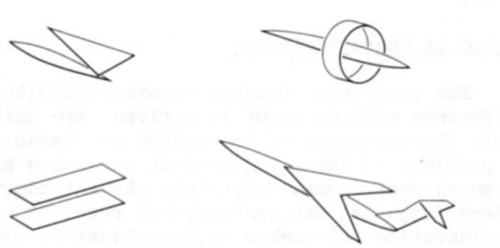

FIGURE 6. NONPLANAR INTERFERENCE CONFIGURATIONS

In recommending this method, it is appropriate to make comparisons with previously published results computed by the transform or iterative methods. Ferrari's article on interaction problems[16] includes a numerical example giving the solution for a rectangular wing on a body with an ogival nose. This configuration has also been analyzed by the panel method. The spanwise variation of pressure coefficient on the upper surface of the wing as computed by the two methods is shown in Fig. 7.

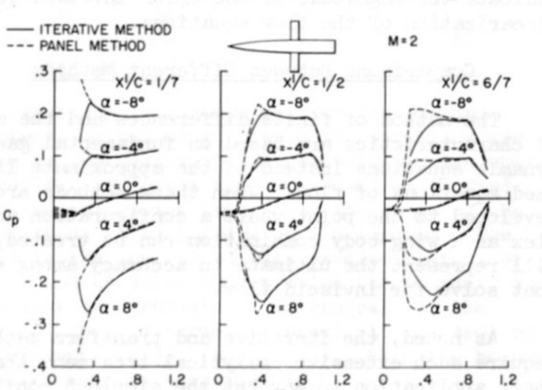

FIGURE 7. COMPARISON OF THE PANEL AND ITERATIVE METHODS

The figure shows a number of interesting features of the phenomenon of wing-body interference. For example, at zero angle of attack, there is a spanwise pressure gradient along the leading edge of the wing. This is due to the finite nose on the body. Near the tip, the flow is decelerated and hence has a higher pressure, while near the root, the flow is accelerated and has a lower pressure. As the angle of attack of the configuration is increased, the inboard portion of the wing increases its pressure loading more than the outboard portion. The upwash induced by the body decreases with the radial distance from the body, and the inboard portion of the wing effectively is at a higher angle of attack. Within the region bounded by a Mach line from the leading edge of the wing-body juncture, the wing pressures are modified by the presence of the body. The effects of the finite nose and induced upwash are predicted identically by the two methods, but the pressures in the wing-body juncture are not in such good agreement. A further comparison of this effect will be shown later.

A number of results for the transform method using W functions have been published. The case of the rectangular wing was included in Nielsen's original paper.[5] Chan and Sheppard[8] have given results for delta wings of various values of leading-edge sweep. Randall[7] also has published a result for a subsonic leading-edge delta planform. All these wings are, of course, mounted in the midwing position on infinite cylindrical bodies. The results for the wings with supersonic leading edges may be interpreted as either a symmetrical wing with wedge section or a thin lifting wing on a body at zero angle of attack. The subsonic leading-edge results can only be applied to the wing thickness case. A representative sample of these results and the corresponding results from the panel method are presented in Figs. 8 and 9.

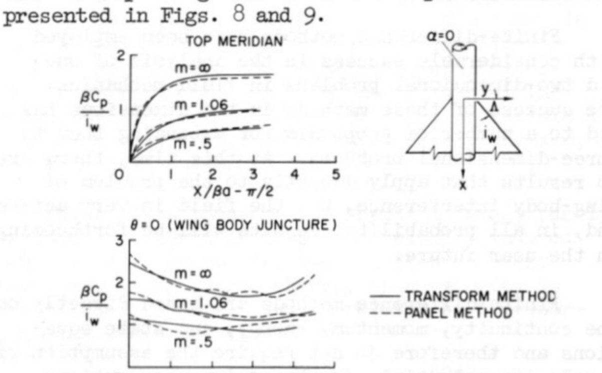

FIGURE 8. COMPARISON OF THE PANEL AND TRANSFORM METHODS - BODY PRESSURES

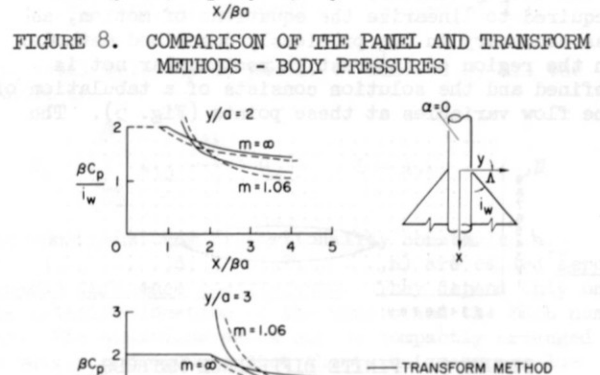

FIGURE 9. COMPARISON OF THE PANEL AND TRANSFORM METHODS - WING PRESSURES

The pressure in the wing-body juncture of the configuration with the rectangular wing at angle of attack has been computed by all three methods and the comparison is illustrated in Fig. 10. The panel method and transform method are in close agreement, while the iterative method gives substantially different results. The result shown for the iterative method is only the result after one iteration, and not the asymptotic result of a large number of iterations (a result that has never been computed). From the comparison, it may be concluded that application of the iterative method would require several iterations for proper convergence.

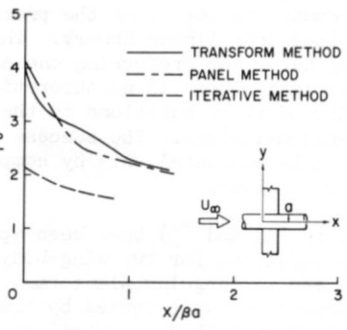

FIGURE 10. PRESSURE IN WING-BODY JUNCTURE BY THREE METHODS

Comparisons Between Theory and Experiment

Linear Theory

From the good correlation between the panel method and the transform method on simple configurations, it may be concluded that the results of the panel method are good approximations of exact solutions to the linearized equation of flow. Next, it is necessary to determine the accuracy of the differential equation itself. In the absence of any results from the method of characteristics or finite difference methods, the only data for comparison are experimentally measured pressures. Again, there is a shortage of published wind-tunnel tests in which the model has been instrumented to provide information on the phenomenon of wing-body interference. From those available, three tests have been selected to illustrate the comparison between experimentally and theoretically determined pressures.

The first test was run by the Cornell Aeronautical Laboratory. The model was of the configuration shown in Fig. 7 - that is, a rectangular wing of aspect ratio 5.7 mounted in the midwing position on a body with parabolic nose. In Fig. 11, a comparison is made between the experimental results obtained[17] and the theoretical results computed by

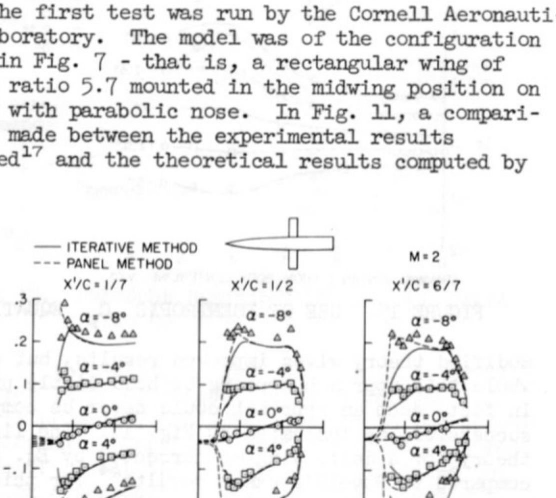

FIGURE 11. UPPER SURFACE PRESSURE DISTRIBUTION

the panel and iterative methods. The experiment and theory are in nominal agreement, but the nonlinear variation of pressure with angle of attack is quite apparent. A given flow-deflection angle produces a greater pressure variation in compression than in expansion.

Figure 12 shows this same effect. This model was run at NASA-Ames[5] and is similar to the Cornell

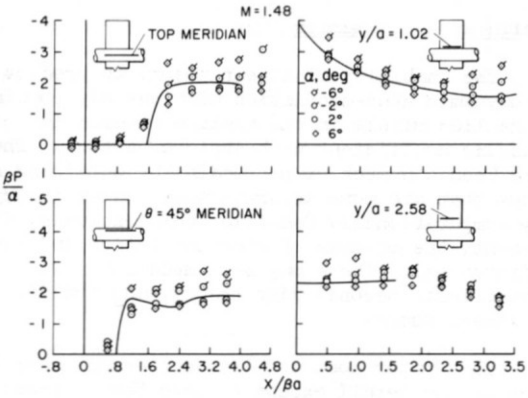

FIGURE 12. COMPARISON BETWEEN THEORY AND EXPERIMENT

model, although the aspect ratio and diameter-to-span ratio differ considerably. In these plots, the pressure is given in terms of the parameter P, which represents the difference between the pressure coefficient at a given angle of attack and at zero angle of attack. The theoretical results shown were computed by the transform method.[5] From the development of the theory of the transform method, it can be seen that the parameter $\beta P/\alpha$ should be independent of angle of attack and Mach number. The failure of the actual pressures to conform to these predictions is dramatically illustrated by plotting the data as shown in Fig. 12.

Both of the previous tests employed unswept wings. A similar test employing wings of delta planform was run at the Weapons Research Establishment (WRE) in Australia. A comparison of the results of this test with the predictions of the panel method is illustrated in Fig. 13.

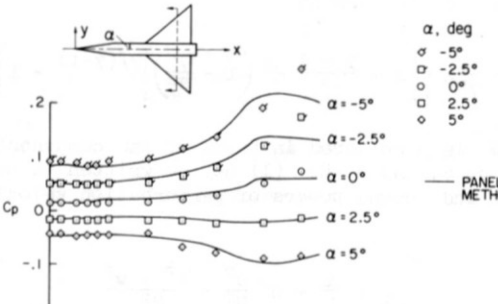

FIGURE 13. PRESSURE DISTRIBUTION ON DELTA WING-BODY

From these examples and others, one may conclude that the solutions based on linear theory give a quite satisfactory qualitative description of the flow of real gas about the actual configuration, but that the individual pressures are frequently in error by 20 to 25 percent. Of course, the usual comment regarding the use of linear theory is that the pressures on the compression side are underpredicted by about the same amount that the pressures on the expansion side are overpredicted, and that the loading (or pressure differential) is predicted quite well. While this holds for the rectangular wings, the delta-wing results do not support such a conclusion.

Nonlinear Aerodynamics

The inability of linear theory to predict the aerodynamic characteristics of wing-body combinations lies mainly in the nonlinear variation of pressure coefficient with angle of attack. There have been a number of proposals for obtaining solutions that are more accurate than linear theory and are computationally feasible (this excludes, for the present, the methods of characteristics and finite differences). These may be lumped into two categories: second-order theory, and modifications to linear theory.

This paper does not discuss second-order theories in any detail except to note that a practical method (from a computational point of view) has yet to be published. Second-order theory will always be difficult to manage for planforms of arbitrary shape since complete solutions may not be built up by superposition of elementary solutions. A direct attempt at solution of the second-order differential equation appears to be as great a computational problem as the method of characteristics. Only if the second-order solution can be reached by iteration on the first-order (or linearized) solution[18] is there any justification for this approach.

In modified linear theories, the linear solution is subjected to a systematic procedure yielding a result that should be closer to the so-called "correct" solution. In the absence of exact solutions for these three-dimensional problems, the "correct" solution is obtained experimentally.

One of the first proposals for modifying the linear theory solution was to assume that the velocities computed from linear theory are correct and that the pressure coefficient should be computed from the energy equation (assuming isentropic flow) by use of the formula

$$C_p = \frac{2}{\gamma M^2}\left\{\left[1 + \frac{\gamma - 1}{2} M^2 \left(1 - \frac{U^2}{U_\infty^2}\right)\right]^{\gamma/(\gamma-1)} - 1\right\} \quad (1)$$

If U is considered in terms of its components $U_\infty + u$, v, and w, Eq. (1) may be written in terms of first and second powers of perturbation velocities as[19]

$$C_p = \frac{2u}{U_\infty} + \beta^2 \frac{u^2}{U_\infty^2} - \frac{v^2 + w^2}{U_\infty^2} \quad (2)$$

In two-dimensional airfoil theory, the well-known Busemann theory indicates that the pressure coefficient can be written[20]

$$C_p = C_{p_{linear}} + \frac{(\gamma + 1)M^4 - 4M^2 + 4}{8(M^2 - 1)} C^2_{p_{linear}} \quad (3)$$

It has also been proposed that this Busemann equation be applied to three-dimensional problems.[21] By analogy, Eq. (1) can be written as

$$C_p = C_{p_{linear}} + \frac{\beta^2}{4} C^2_{p_{linear}} - \frac{v^2 + w^2}{U_\infty^2} \quad (4)$$

Any of these equations could be used as the basis for a systematic variation of the pressure coefficient obtained from linear theory. There is no theoretical basis for preferring one equation over another; in fact, there is no theoretical basis for applying any of these equations to the results for three-dimensional wings. The success of such an approach can be evaluated only by comparisons with the "correct" values.

Equations (1) and (3) have been applied to linear theory solutions for two wing-body combinations with delta and rectangular planforms. These particular configurations were tested by the WRE;[22,23] comparisons between these corrections to linear theory and the experimental data are shown in Figs. 14 and 15. For some of the pressure taps, the

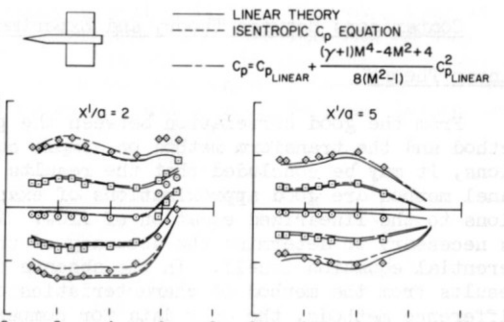

FIGURE 14. EVALUATION OF TWO MODIFIED LINEAR THEORIES

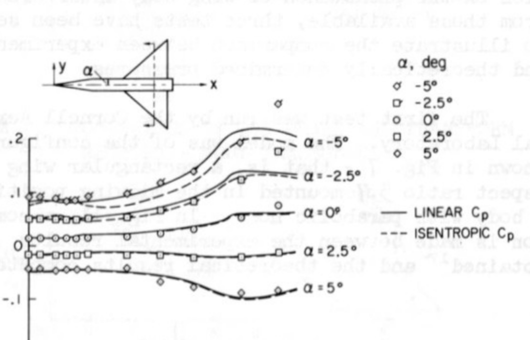

FIGURE 15. USE OF ISENTROPIC C_p EQUATION

modified theory gives improved results, but on the whole this approach appears to hold little promise. In fact, such an approach could never be completely successful, as indicated by Fig. 16. The linear theory for a delta wing as corrected by Eq. (3) is compared to Fowell's exact results[24] for this wing. The correction does improve the linear solution, but it does not modify the location of the ray that forms the boundary of the zone of influence of the apex of the wing. The upper surface of the wing (expansion surface) has local Mach numbers greater than the free-stream Mach number and therefore has a smaller region of root influence than would be predicted by drawing a Mach line from the apex.

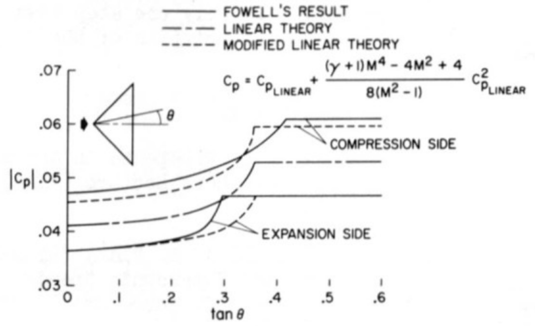

FIGURE 16. MODIFIED LINEAR THEORY FOR 45° DELTA WING AT $M = 3$, $\alpha = 4°$

This shift might be predicted by the panel method applied in an iterative manner, using the local Mach number instead of the free-stream Mach number in the calculation of the matrix of aerodynamic influence coefficients (Fig. 17). Such a calculation has been made on the wing shown in Fig. 16 and is shown in Fig. 18 as the curve labeled "modified linear theory." This approach is successful in modifying the boundary between the inner and outer

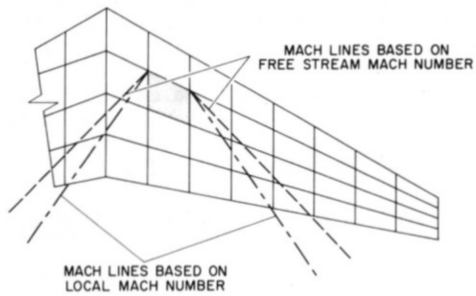

FIGURE 17. ITERATIVE PANEL METHOD USING LOCAL MACH NUMBER

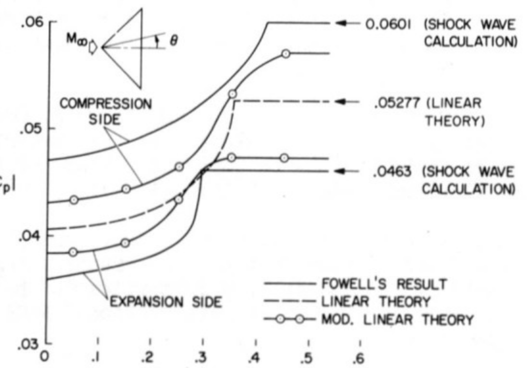

FIGURE 18. PRESSURE DISTRIBUTION ON A 45° DELTA WING AT $M = 3$, $\alpha = 4°$

domains of the solution, but still fails to predict the proper level of pressure. Nevertheless, this result is promising and lends support to the concept of developing a computing procedure based on a modification of linear theory but still utilizing the concept of superposition of elementary solutions. The accuracy of such a procedure would be intermediate between the results of linear theory, which can now be obtained easily and quickly, and the exact methods, which undoubtedly will require great amounts of computer time.

General Remarks on Comparisons Between Theory and Experiment

The use of experimental results as the standard by which theoretical methods are judged and modified requires considerable caution to avoid misleading conclusions. The process of collecting experimental data is subject to many random and systematic errors. In addition to errors, it is important to remember that the airflow through a wind tunnel is not a uniform stream of inviscid perfect gas but is a somewhat turbulent stream of viscous real gas. The assumption of inviscid flow in the theoretical development requires the parallel assumption that the effects of viscosity can be isolated and removed from the experimental data. One way to assess the effects of viscosity is to repeat the experiment at various Reynolds numbers, as in the test reported by Nielsen.[5] Figure 19 shows the results of this test,

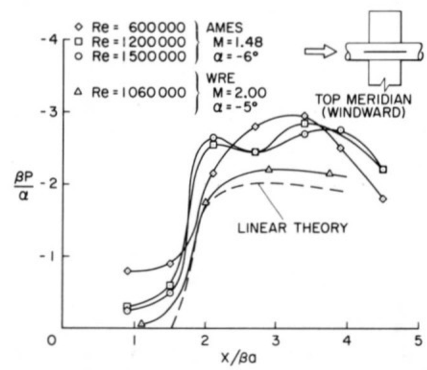

FIGURE 19. EFFECT OF REYNOLDS NUMBER ON INTERFERENCE PRESSURES

as well as WRE tests at a comparable Reynolds number. As shown, there is a significant Reynolds number effect; consequently, the failure of the linear theory to agree with the experiment may well be due more to the effect of viscosity than to a neglect of higher order terms in the differential equation. The discrepancy between the two wind-tunnel tests is not to be taken as a difference between facilities, since the tests were made with different configurations at different Mach numbers and angles of attack. The parameter $\beta P/\alpha$ simply is not invariant with angle of attack and Mach number as predicted.

Nielsen's tests were all conducted with free transition. The WRE tests were conducted with rings around the nose of the model to induce a turbulent boundary layer over the body. Remeasurement of the pressures without the transition rings gave the result shown in Fig. 20. There is a definite shift

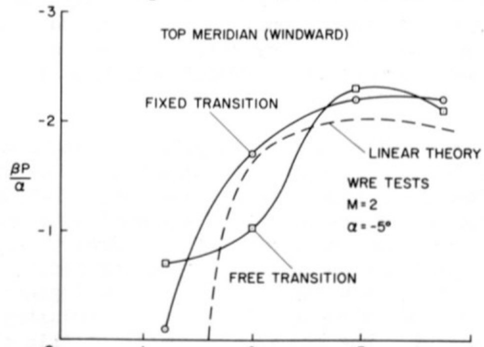

FIGURE 20. EFFECT OF TRANSITION ON INTERFERENCE PRESSURES

in the location at which the wing begins to affect the pressures on the body. A similar result is reported in Ref. 25.

It is concluded that in any experimental investigation of wing-body interference, special care must be taken to assess the importance of boundary-layer effects. The principal methods that the experimenter has at his disposal are variation of the Reynolds number and the use of various sizes of particles to induce transition from a laminar to turbulent boundary layer. It does appear that there is a significant interaction between the boundary layer and the external flow at the Reynolds numbers usually encountered in wind tunnels. This applies mainly to the fuselage; on the wing the effect seems to be minimal. It may be that these interactions are also important at flight Reynolds numbers, in which case it will be necessary to develop an integrated viscous-inviscid theory of flow about wing-body combinations.

Summary and Conclusions

The panel method, based on aerodynamic influence coefficients, is a convenient and economical way to obtain accurate solutions to virtually all problems of wing-body interference in linearized supersonic flow. Linear theory solutions are adequate for the majority of engineering studies of airplane systems although the accuracy is not sufficient to provide the information required for the detail design of a specific configuration. The principal area of difficulty is the failure of the linearized theory to predict the variation of aerodynamic loading with angle of attack. This variation is found (experimentally) to be distinctly nonlinear.

Computing techniques based on the fundamental equations of inviscid gasdynamics are under development at several institutions. Some of these programs even include the effect of the boundary layer. It does appear, however, that until computers of much greater speed and capacity are available, methods of this type (finite differences, characteristics) will not be used widely as engineering design tools. They are of great value, nevertheless, in providing benchmark solutions by which more approximate methods can be evaluated and refined.

Research effort in the problem of wing-body interference should be directed toward the development of theoretical methods that adequately describe the nonlinear effects discussed in this paper, as well as provide numerical results without extravagant amounts of analysis or computation. These techniques might be based on the direct solution of the second-order irrotational equations of motion, but is more likely that they will be based on a systematic procedure for modification of the first-order solution.

The utility of such computing procedures would repay the development costs many times over in the elimination of expensive trial-and-error wind tunnel testing in the refinement of aircraft configurations. Modern aeronautical designers must free themselves from dependence on ad hoc testing for aerodynamic data if the spiraling increase of development time and cost that threatens the aviation industry is to be arrested. The key to a reversal of this trend is the development of reliable theoretical procedures for use by those intimately involved in aircraft design. This paper on the aerodynamics of the basic wing-fuselage combination is only one step toward the prediction of the characteristics of the complete flight vehicle.

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